CAT-GENERATING MECHANISMS

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I'll begin with these three areas:

1. Development of instability configurations

2. The transition from unstable growth of these configurations into turbulence and a description of the nature of that turbulence

3. The question of decay of turbulence and one of the most controversial topics, the existence of what is called "fossil turbulence."

People involved in design and simulation want simple descriptions of turbulence that exists in the atmosphere and oceans, in these "clear-air" conditions. That description is not going to be forthcoming at this time. There are going to be all sorts of characteristics of these turbulent states. And, I might add, that it is amusing that the oceanographers have the advantage of us here. They are able to get in there and measure these things better than we can in the atmosphere now, when it comes to accurate measurements. And you want to keep your eye on what they are doing because a lot of the information that we gain is going to come from them.

As far as the existence of unstable configurations goes, of course, the vortex sheet has been known to be unstable for more than a century, but the first actual computation, beyond the simple fact of instability, was that of Rosenhead [1] in 1931 where he represented the vortex sheet as a sum of a lot of little vortices (Figure 1), each of which is acting on the others. And, of course, the vortex sheet is an equilibrium configuration until it is disturbed, and then the little vortices tend to move each other until it winds up in this familiar way. A lot of the literature refers to instability and/or wave breaking. These are very confusing terms really because this type of situation could conceivably be called a wave breaking. The next one (Figure 2) has totally different dynamics, namely, a wave on the surface of the ocean. Here is a laboratory wave breaking in the surface of water (Figure 2a). The dotted line is Longuet-Higgins' analytic solution to the problem [2]. Figure 2b is a picture from a surfing magazine which Longuet-Higgins picked up and fit his theoretical profile precisely to the pictured profile [3].

We are in a position to understand both of the mechanisms illustrated in Figure 2, even though they are quite different. The second one (Figure 2b) is so familiar, of course--the degeneration of that instability into a turbulent flow on the beach--that it may be surprising how little it has been studied. There are many pictures such as Figure 3 that depict the configuration of these roll-up type vortices in the atmosphere which have usually been visualized by cloud patterns [4]. This one is just off the coast of California. The atmosphere is known to have density differences like the water wave and vorticity in the basic flow like that studied by Rosenhead [1],

but in both of these cases the atmosphere has the variable continuously distributed, rather than concentrated either in a vortex sheet or an interface between the fluids of different densities. Attempts at simulation have been made in the laboratory. Figure 4 is an early example of the fact that, if one takes high resolution rather than the characteristic radiosonde resolution, one can identify layers of low Richardson number in an overall stable layer [5]. In this case, the resolution is only 400 m and the Richardson number varies over four orders of magnitude and, of course, it can go to infinity as the shear goes to zero. This has been well known. We do not know exactly how these fine layers come about, but we can expect to find them.

In the laboratory, wave breaking can be represented, for example, in the early work of Thorpe [6] a very clever device was used (Figure 5). This is the two fluid system here and that is tilted so that you can get a shearing across the interface and these little waves develop, break, mixing occurs, and they die down. Thorpe suggested that the K-H mechanism looked like this. Compared to Rosenhead's calculation, Thorpe's work is in an earlier stage because we are only looking qualitatively and not doing numerical work. In Figure 6 we have the development of a roll-up. Then the next step is pure arm-waving: the whole thing breaks down in some fashion. Quite recently, in 1983, McEwan [7], by use of a paddle, produced a breaking wave in the fluid and was able to measure the density gradient throughout. McEwan's figures are in color and so cannot be reproduced here, but Figure 7 presents an idealization of his results, which are as follows. The sequence of events is:

1. The rolling-up process produces an unstable density gradient, heavy fluid over light.

2. The breakdown of this convectively unstable region occurs on a much smaller scale, permitting irreversible diffusion of density and momentum.

3. This microstructure persists after the restoration of gross stability. The experiment shows that by this stage the motions are three-dimensional. This stage is relatively long-lasting, and is referred to by some authors (though not by McEwan) as "fossil turbulence."

4. Finally, the stratified structure is reformed, although with a slightly reduced mean density gradient in the mixed region.

This is one of the first demonstrations, even though quite recent, that the breakdown is essentially three-dimensional in character. The sequence of events is a little more clear than it was in Thorpe [6] but still not numerical. In other words, we still have not gotten in there yet and measured the character of the turbulent exchange which goes on between the breakdown of an unstable situation and the final decay of the turbulence.

We now turn to another current research approach, that of numerical simulation. This method has the great advantage of providing vast quantities of accurate data. But there are compensating disadvantages: turbulence is three-dimensional and involves a range of scales larger than non-turbulent flows; as a result, true turbulence simulation requires, at present, unconscionable amounts of time on the largest computers. Thus, it may be some

years before numerical simulation answers the questions concerning CAT that are being asked. However, progress is already evident.

In two articles, Klaassen and Peltier [8,9] have proceeded as follows. Beginning with an unstable K-H wave, they integrated numerically with a two-dimensional model. The expected roll-up occurs, bringing heavy fluid over light, but no breakdown takes place. Rather, the system oscillates, energy going back and forth between mean state and perturbation. Then, choosing a time in this development, which is of course highly nonlinear, they subject the given configuration to a three-dimensional linear stability analysis. The time development of the unstable wave is shown in Figure 8, the streamlines in the top panels and potential temperature in the bottom panels. The results of the stability analysis--which obviously requires extensive computation--are shown in Figure 9. The growth rate of the fundamental mode ω_0, at its maximum value corresponds to a wavelength in the (longitudinal) y-direction of about one-fourth the depth of the shear layer. If this maximum growth rate of this mode is converted to dimensional values, it turns out to be approximately equal to N, the Brunt frequency, showing that the breakdown is convective in its dynamics.

People who are more operationally inclined may be very impatient with these results. Of course, if you have heavy fluid over light fluid you expect a gravitational instability to result! Nevertheless, these steps are necessary in arriving at something that operationally concerned people will want to see. This is as far as the Klaassen-Peltier model can go (since it is not a simulation in itself, but the three-dimensional stability analysis of a two-dimensional configuration derived from an earlier simulation). The next step will presumably be a full-scale simulation of the turbulent breakdown, with parameterization of eddies of less than a certain scale. This would be the beginning of a quantitative characterization of the turbulence.

I will now proceed to discuss some of my own work, numerical simulations of a very different kind: the flow of a stratified fluid--e.g., the atmosphere--over an obstacle. This can be an obstacle on the ground, or an obstacle at any elevation, of course. The terrain is a natural obstacle to conceive of, but a frontal surface aloft could be the source of the disturbance, or a cloud mass. We first take a simple linear analytic solution. The wind is increasing linearly with elevation and the Brunt frequency is constant. We consider a small disturbance (Figure 10a). Nh/U_0 is the parameter which traditionally is taken to govern the linearity of the computation. If h is the height of the obstacle, the Brunt frequency is N, and the speed of the fluid at the level of which it encounters the obtacle is U_0. Here the ratio 0.1 suggests that it is a purely linear situation. And, therefore, the analytic solution is valid. The next figure will show the development of the Richardson number field from this particular streamline field (Figure 10b).

Here we have cells corresponding to the cells of the streamline field. In these cells, we have alternately Richardson number increases and decreases. You will notice there are more contour lines in the increase than in the decrease. In other words, the imposition of the gravity wave on the stable fluid increases the stability of the fluid more than it decreases the stability of the fluid. However, the fluid does have cells in which Ri decreases, and the next figure will show what happens when we increase the

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magnitude of the disturbance.   In Figure 11, $Nh/U_0 = 3$, and now we can no longer use the analytic solution; we have to use a simulation code.   Again, simulation means starting the motion from scratch and allowing the atmosphere to flow over the obstacle.   In Figure 11a the signal has only gone as far downstream as the first crest.   We see the streamlines are no longer sinusoidal but are beginning to get nearly vertical at points.   Figure 11b shows the density field.   So here we do get, not surprisingly, regions of overturning.   The point is: where the wave is trapped by the increasing velocity, by the shear itself, the situation is so stabilized by that trapping that the instability exists only in highly local regions at approximately the height of the disturbance. Nothing terribly exciting can happen. You can get a rotor cloud, but you cannot get the vast outbreaks of instability and clear-air turbulence that are characteristic of certain situations.   These two figures have represented the type of thing that can develop when increases with height in the atmosphere, therefore, providing a reflecting or trapping mechanism.   We now take a case, and this is one that has been studied more than any in which the wind is constant and the stability is constant.   The analytic solution is by Miles and Huppert [10].   Figure 12 is a flow over an ellipse where $Nh/U = 0.5$, a reasonably linear situation.   Here is our simulated solution of the same situation and this is a special simulation code.   I do not know of any other simulation in atmospheric sciences in which an orthogonal grid is generated numerically in order for the disturbing boundary to be a coordinate surface.   The computation is then done with this new grid preserving the character of the equations but with the new coordinate surface and then transferring back into the old x,z system so that the ellipse shows as an ellipse.   You simply get waves in this linear case.   However, if the disturbing obstacle is increased in elevation, we get the pattern of Figure 13, with one vertical streamline. Nh/U in this case is 0.93, the critical value for this ellipse.   Here we have simulation reproducing that situation, and we do get that vertical streamline precisely.   The second vertical streamline, or almost vertical streamline, has lost some of its energy because the energy is spreading out in two dimensions.   But that is simulated less well because the time is not long enough for the energy to fully straighten up that streamline.

The fact is, of course, that in nature the wind is not constant with height and the Brunt frequency is not constant with height.   Either increasing or decreasing wind is the rule.   We will now go to the situation in which we get a _decreasing_ wind.   If you have a wind that is linearly decreasing, it will eventually go through zero.   This gives what is called a _critical level_; it has been much studied, but less simulated; and it is a situation that is highly productive of a nonlinear type of reflection.   We have studied that first by taking a simple sinusoidal disturbance.   That is a monochromatic disturbance; but the reflection from the critical layer produces many higher frequencies.

In Figure 14, however, the disturbance generates _all_ frequencies.   The left-hand panel represents the stream function; the mean flow is seen to reverse directions at 10 km elevation, the critical level.   Well below this, at 6 to 8 km elevation, a reverse flow or rotor circulation is evident.   The density field (right-hand panel) exhibits similarly a reverse density gradient.   This would be a region of extreme turbulence.   Note that almost no disturbance penetrates above the critical level.

However, it turns out that the existence of a critical level is not necessary to produce this type of nonlinear reflection. Figure 15 represents a similar result for a flow that decreases exponentially with elevation; in the diagrammed panels, the mean flow exceeds 10 m/s at all levels. The left-hand panel shows the total horizontal velocity. At elevations near 4 km, the oncoming flow of 25 to 30 m/s has reversed itself to -25 m/s just in the lee of the obstacle! The right-hand panel shows violent vertical updrafts and downdrafts of more than 14 m/s within the horizontal distance of a few kilometers. Again, a very turbulent region would result.

We conclude that the only thing necessary for the existence of a highly reflective and potentially turbulent situation is a reasonably deep layer of decreasing wind speed. (By decreasing I mean that it is lower at higher levels than at lower levels.) This is fairly characteristic of the stratosphere. So it suggests that the structure of the lowest stratosphere is often extremely pregnant as far as clear-air turbulence is concerned. The question is: Does the disturbance, which in these cases originates at the surface of the earth, actually propagate sufficiently into the stratosphere to produce this sort of turbulence? The answer is, sometimes it does and sometimes it does not. And it is surprising how little this question has been studied. It is what we in my group are devoting ourselves to now. To what extent does the flow structure in the troposphere plus the tropopause itself act as a barrier to gravity wave energy being propagated upward?

Figure 16 will show a situation in which this is the case. The simulation used the best data we could get from Jack Ehernberger and others upwind of the famous United Airlines episode over Hannibal in 1981 [11]. In this case, there was quite a bit of damage and injury inside the plane. We tried as best we could, but there is not any source of disturbance at the surface of the earth near Hannibal. But even if we exaggerated the profile of the terrain there, we could not propagate energy into the stratosphere. Nothing much happened. However, there was an enormous cumulonimbus cloud bank, which was really a very good two-dimensional obstacle to the flow at the time, and it extended to about 9 km. We assume the cloud bank to be the obstacle; and it was sufficient to produce this very large disturbance in the stratosphere. From about 11 km up we had rapidly decreasing wind speed. The hatched areas are areas of subcritical Richardson number. I believe the plane was flying at about 13,000 feet.

The two approaches I have outlined present, I think, the present position of our understanding. We understand how very stable atmospheric flows with large Richardson numbers can be rendered unstable. We understand the process of breakdown of this instability. We can watch the turbulence develop in the laboratory and distinguish between an active stage and a "fossil" stage. But we await detailed measurement and/or simulation of the turbulence.

I feel that this workshop was well conceived and should be repeated. Perhaps by the time of the next one, the scientists will be able to answer the questions asked by the engineers at this one.

References

1. Rosenhead, L.: The Formation of Vortices from a Surface of Discontinuity, *Proc. Roy. Soc. (London)*, A134:170-192, 1931.

2. Longuet-Higgins, M. S.: On the Overturning of Gravity Waves. *Proc. Roy. Soc. A*, 371:453-478, 1981.

3. Longuet-Higgins, M. S.: Parametric Solutions for Breaking Waves. *Journal of Fluid Mechanics*, 121:403-424, 1982.

4. Gossard, E. E.; and Strauch, R. G.: *Radar Observation of Clear Air and Clouds.* New York: Elsevier Science Publishing, 1983, 280 pp.

5. Woods, J. D.: On Richardson's Number and Criterion for Laminar-Turbulent-Laminar in the Ocean and Atmosphere. *Radio Science*, 4:1289-1298, 1969.

6. Thorpe, S. A.: Experiments on the Stability of Stratified Shear Flows. *Radio Science*, 4:1327-1331, 1969.

7. McEwan, A. D.: Stratified Mixing Through Internal Wavebreaking. *Journal of Fluid Mechanics*, 128:47-57, 1983.

8. Klaassen, G. P.; and Peltier, W. R.: The Evolution of Finite Amplitude Kelvin-Helmholtz Billows in Two Spatial Dimensions. *Journal of the Atmospheric Sciences*, 42:1321-1339, 1985.

9. Klaassen, G. P.; and Peltier, W. R.: Turbulent Onset in Kelvin-Helmholtz Billows. *Journal of Fluid Mechanics*, 155:1-36, 1985.

10. Miles, J. W.; and Huppert, H. E.: Lee Waves in a Stratified Fluid, Part 3. *Journal of Fluid Mechanics*, 35:481-496, 1969.

11. Parks, E. K.; Wingrove, R. C.; Bach, R. E.; and Mehtea, R. S.: Identification of Vortex-Induced Clear Air Turbulence Using Airline Flight Records. *Journal of Aircraft*, 22:124-129, 1985.

QUESTION: David Walker (Lehigh University). Could you say something about your simulation. Is it an inviscid simulation? You don't have the no-slip condition on the surface in those calculations? Is that correct? In those obstacles, I would expect that you would get a structured kind of eddy shedding off those obstacles that I didn't see in those results.

ANSWER: This is a completely inviscid model.

WALKER: The comment I would make is we've done a number of experiments involving obstacles of that nature at Lehigh. What in fact you get is a structured kind of hairpin vortex shedding off those kinds of obstacles that penetrates after a while well up above the ground plane.

WURTELE: These are not intended to represent the flow in the immediate region of the obstacle at all. In order to do that we would have to simulate the whole atmospheric boundary layer and we haven't attempted to do that. Really these solutions are valid at distances from the obstacle. Particularly it's the vertical propagation we are concerned with here.
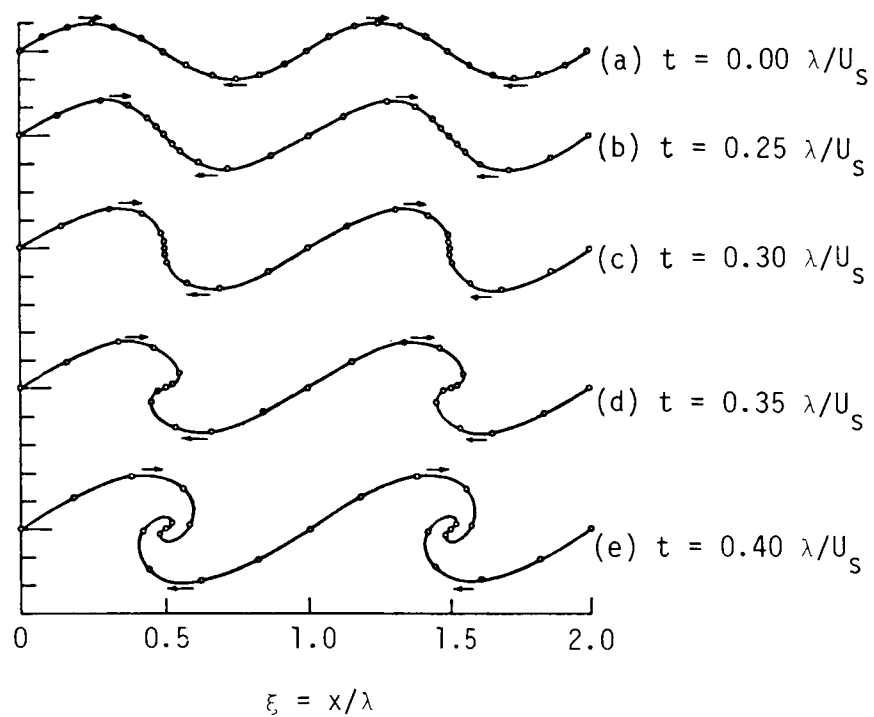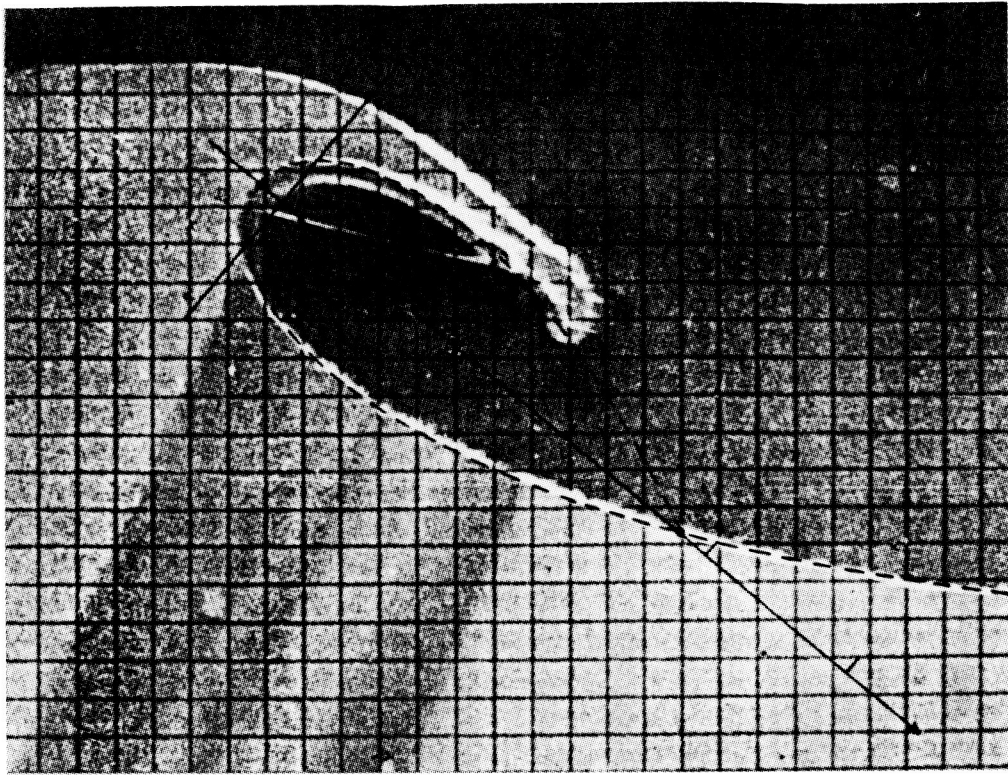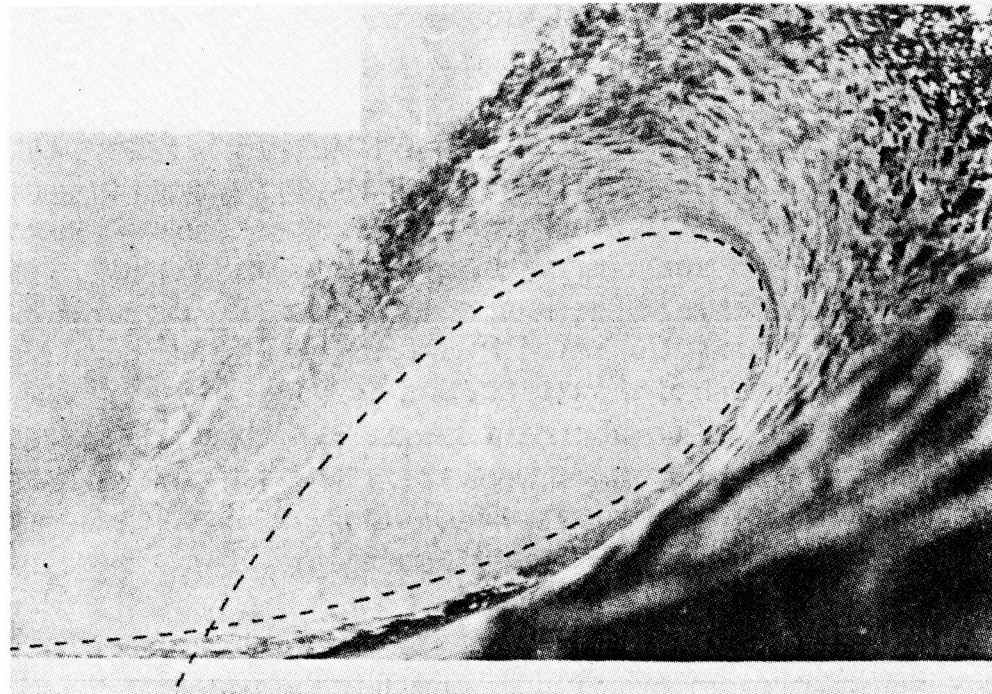
Figure 1. The rolling-up of a vortex sheet which has been given a small sinusoidal displacement [1].

(a) Laboratory wave breaking

(b) Longuet-Higgins [2] P_3 solution superimposed on a breaking wave

Figure 2. Surface waves breaking, with analytic solutions of Longuet-
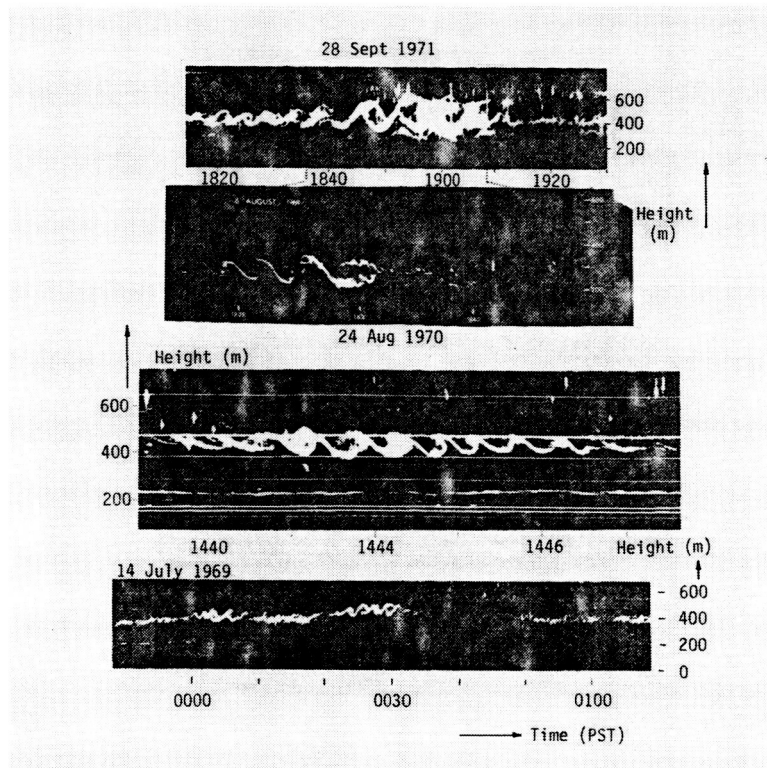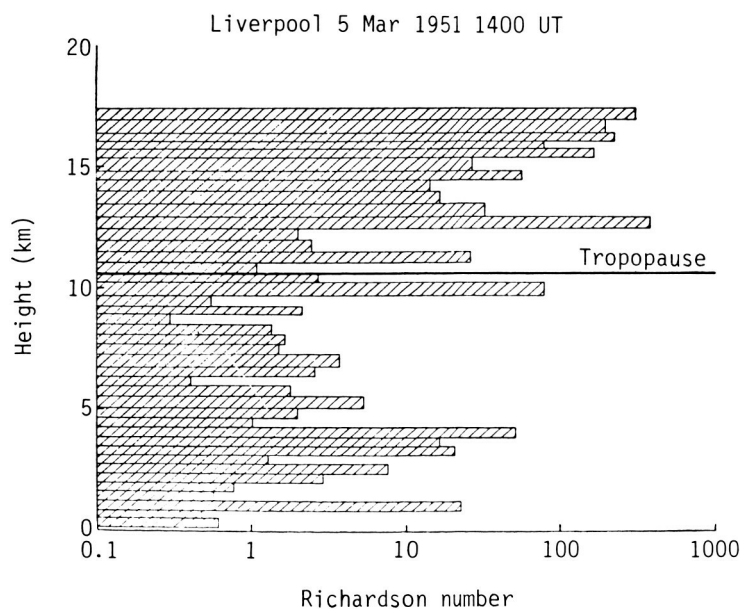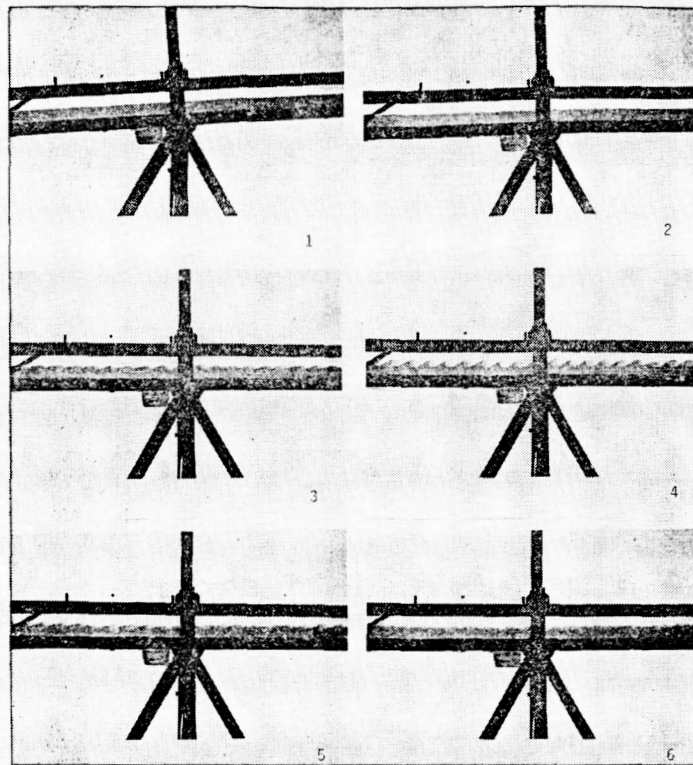Higgins [2,3] superimposed.

Figure 3. Kelvin-Helmholtz wave roll-up configurations as detected in the atmosphere by FM-CW radar [4].

A profile of gradient Richardson numbers in the atmosphere deduced from radiosonde wind and temperature data averaged over layers about 400 m thick.

Figure 4. High-resolution profile of Richardson number from Woods [5].

The growth of disturbances in a flow with J = 0.077 ± 0.01. The time between each successive photograph is about 0.5 sec and the length of the scale is 45 cm.

Figure 5.   Breaking of unstable Kelvin-Helmholtz waves in the laboratory [6].
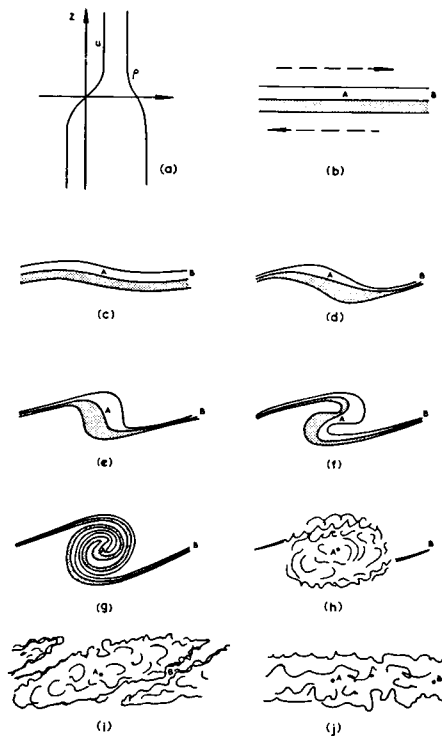
The growth of disturbances: (a) the density ρ and velocity u distribu-
tions; (b) the lines mark a fluid of constant density, points A and B
are fixed, the arrows indicate the direction of flow; drawings (c) to
(j) show the development of instability. The points A and B remain
fixed, and the lines continue to mark a fluid of constant density.

Figure 6. Schematic of generation of turbulence from breaking of
unstable Kelvin-Helmholtz waves [6].
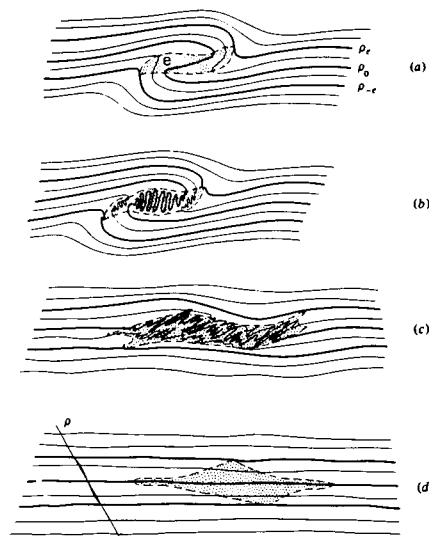
Idealization of a mixing event in a continuous stratification.
(a) Overturning. (b) Development of interleaving microstructure.
(c) Static stability is restored but microstructure is preserved.
(d) Gravitation to an equilibrium has changed the surrounding density
profile between extremum isopycnals. The distortion of the profile is
exaggerated for clarity. The intermediate isopycnals (fourth and sixth
from the top) are displaced upwards and downwards respectively from
their original positions, representing a gain in stratification
potential energy.

Figure 7. Schematic of generation and decay of turbulence from breaking
of unstable K-H waves [7]. Stage (c) is sometimes called
"fossil turbulence."

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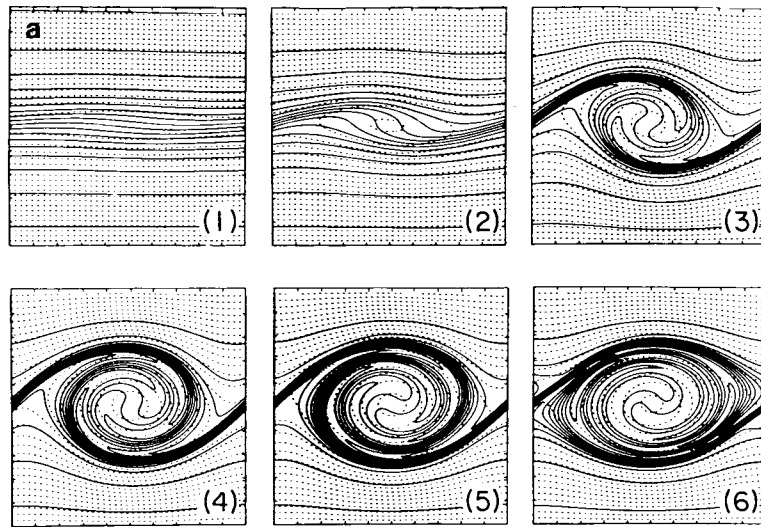
(a) Streamlines (dashed) have been overlaid on (a) the isentropes (solid), and (b) contours of the vorticity field (solid) illustrating evolution of the KH wave at Re = 500. Numerals 1-6 refer to key times. Contour intervals for the potential temperature field and streamfunction are all $\Delta\theta$ and $\Delta\psi$, respectively. The contour intervals for the vorticity field are $\Delta\zeta$ for (2) and $2\Delta\zeta$ for the remainder.
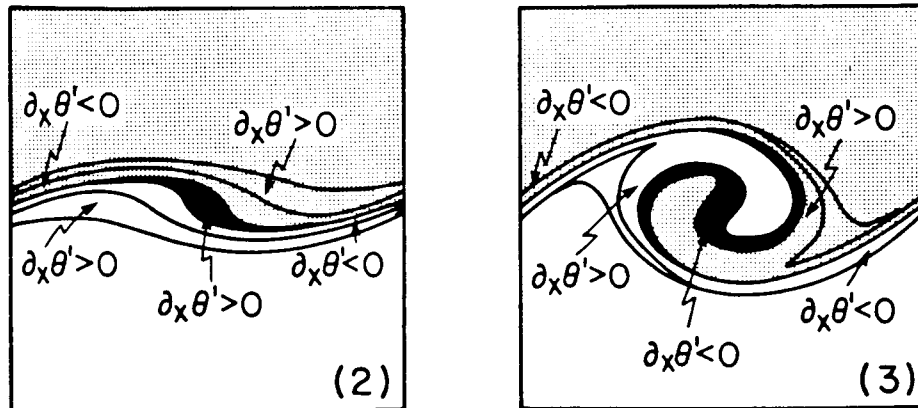

(b) Sketches of potential temperature field illustrating baroclinic sources and sinks of vorticity for a typical KH wave at key times (2) and (3) in the energy cycle. Median contour interval has been shaded darkly; regions with potential temperatures greater than the median value have been shaded lightly. Regions of baroclinic generation of vorticity $(\partial_x\theta' < 0)$ are found in the braids; regions of baroclinic destruction $(\partial_x\theta' > 0)$ are found at the right and left edges of the core.

Figure 8. Roll-up of unstable Kelvin-Helmholtz waves in simulation by Klaassen and Peltier [8]. Breakdown does not occur in two dimensions.
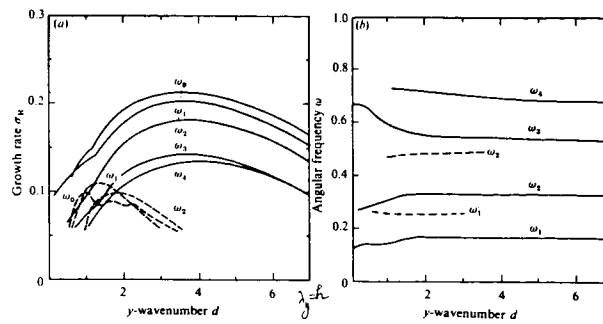
The growth rate σ_R and (b) angular frequency ω as functions of the spanwise wavenumber d for various longitudinal (b = 0) unstable modes of the Re = 500 KH wave at the key time (5) in its energy cycle. The sequency of modes labeled $\omega_0...\omega_4$ (solid lines) is associated with the primary SAR, while that for the $\omega_0'...\omega_2'$ modes (dashed lines) is associated with the secondary SAR. The truncation level used was the maximum N = 19.

Figure 9. Growth rate of three-dimensional perturbation of unstable configuration of Figure 8 [9].
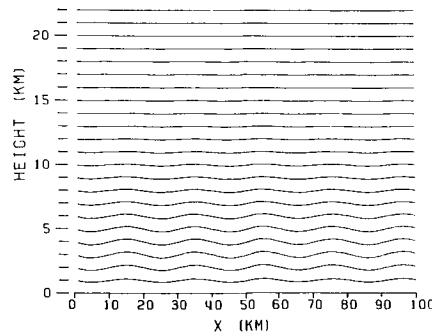
(a) Streamlines (Ri = 8, Nh/U = 0.1).
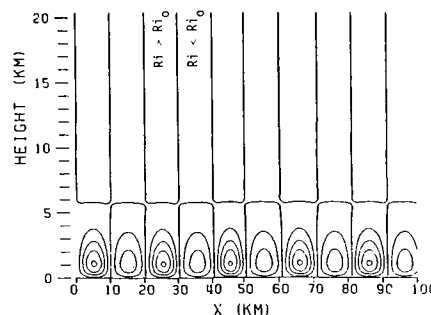
(b) Richardson number field perturbations (contours for quantity $(Ri - Ri_0)/Ri_0$ at intervals of 0.05).

Figure 10. Stratified shearing flow over an obstacle (small disturbance of height h) and corresponding perturbations of Richardson number field.

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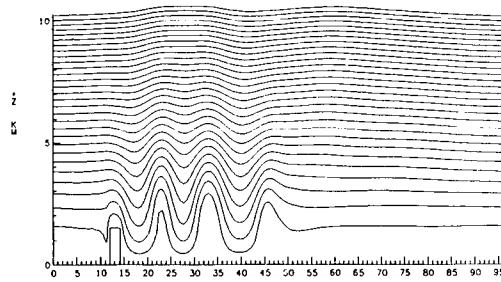
(a) Streamlines for flow of Figure 10a except that Nh/U = 3.0
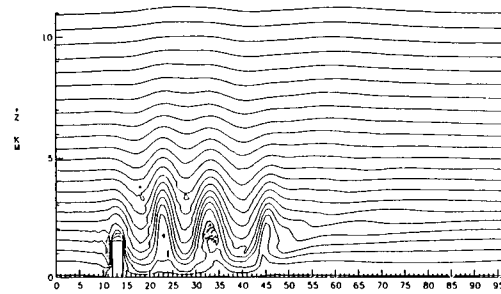
(b) Density field for flow of Figure 11a showing unstable regions
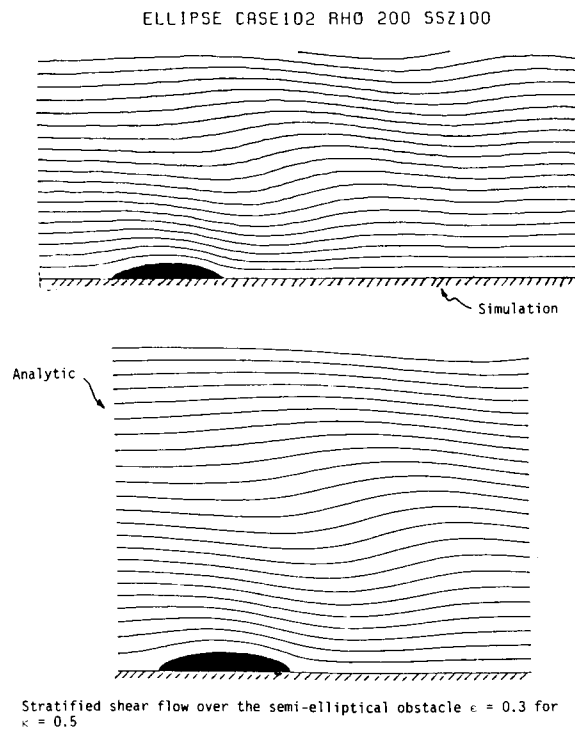
Figure 11. Streamlines and density field for flows.

ELLIPSE CASE102 RHO 200 SSZ100

Simulation

Analytic

Stratified shear flow over the semi-elliptical obstacle ϵ = 0.3 for κ = 0.5

Figure 12. Flow over an ellipse of height h with Nh/U = 0.5. Upper panel: simulation. Lower panel: analytic [10].

ELLIPSE CASE105 RHO 200 SSZ100

Simulation

Analytic

Stratified shear flow over the semi-elliptical obstacle
$\kappa = \kappa_c = 0.93$
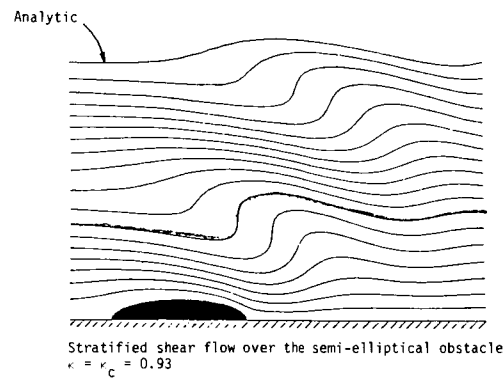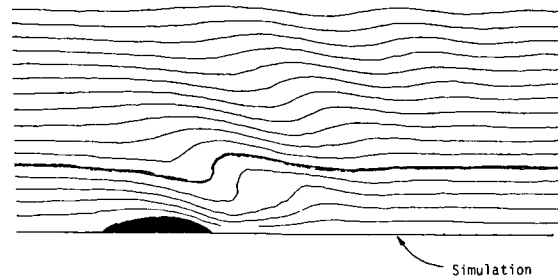
Figure 13.   Same as Figure 12 but for Nh/U = 0.93.   Unstable streamline is
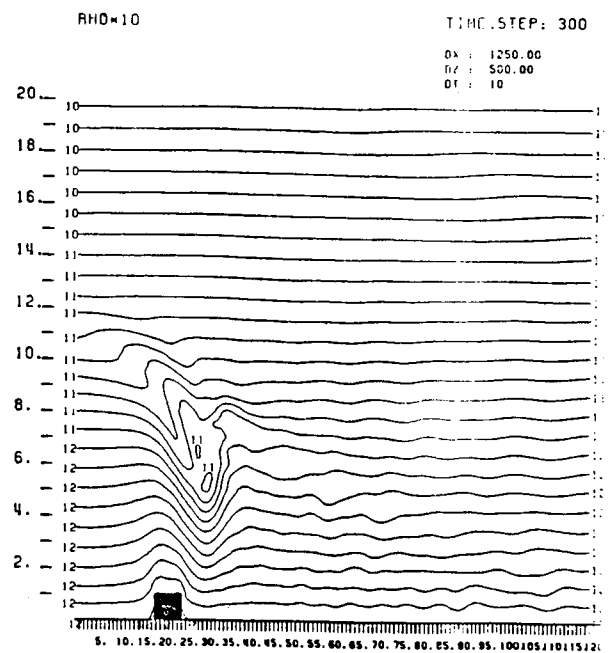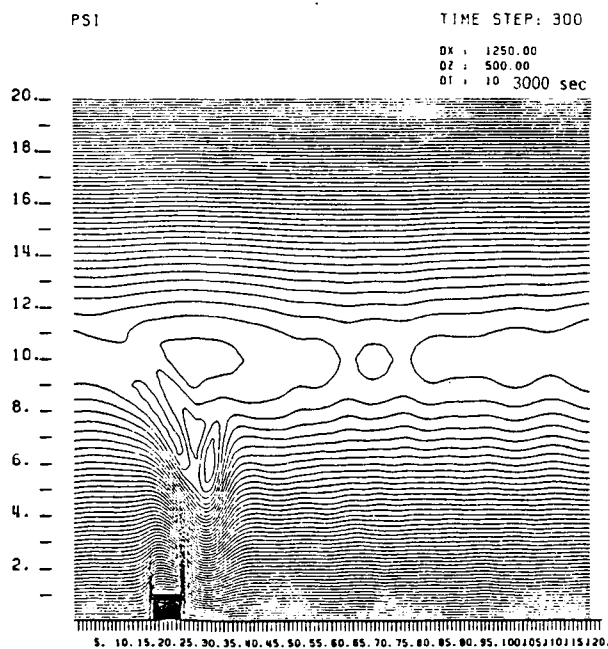darkened.   Upper panel: simulation.   Lower panel: analytic
[10].

PSI

TIME STEP: 300

DX : 1250.00
DZ : 500.00
DT : 10   3000 sec

RHO×10

TIME STEP: 300

DX : 1250.00
DZ : 500.00
DT : 10

Figure 14.   Stratified shear flow with critical level.   Left-hand panel:
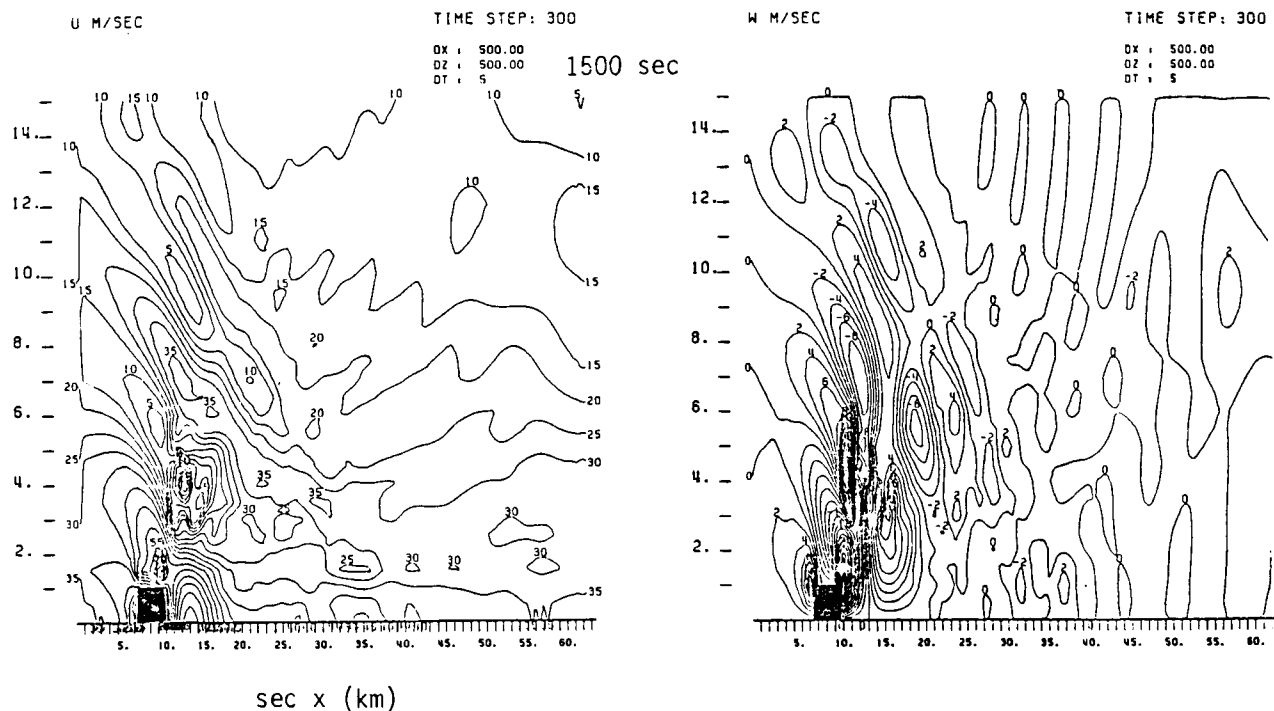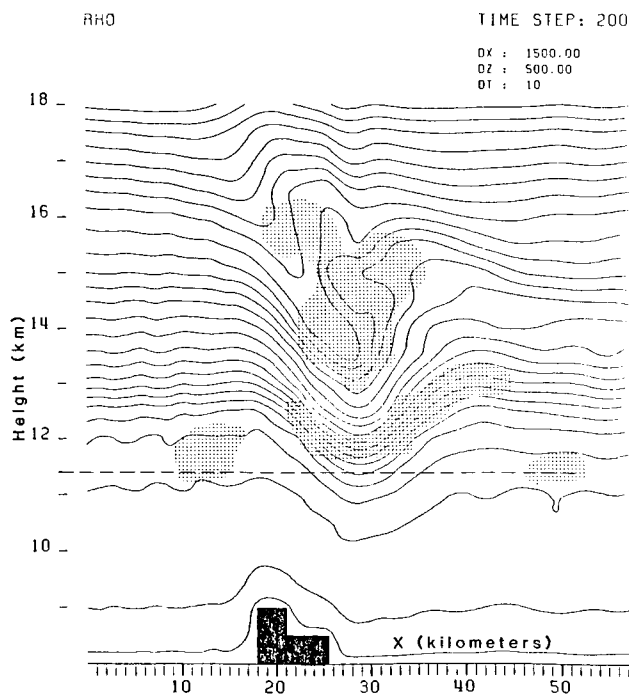streamlines.   Right-hand panel: density.

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Figure 15. Stratified shear flow with exponentially decreasing speed.
Left-hand panel: total horizontal velocity. Right-hand
panel: vertical velocity.

Figure 16. Simulation of conditions under which CAT-encounter occurred.
Regions of Ri < 1 are hatched.