SAFE MICROBURST PENETRATION TECHNIQUES: A DETERMINISTIC, NONLINEAR, OPTIMAL CONTROL APPROACH

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Efforts in the first area involve the design of compensators for microburst disturbance rejection. Preliminary results indicate improvements when the microburst is modeled as a colored noise process. The second task is to model the unsteady aerodynamic effects upon an aircraft's longitudinal dynamics as related to the wind shear encounter problem. The third effort concentrates on the determination of optimal trajectories through microbursts. Standard techniques of deterministic nonlinear optimal control have been used. This is the main subject of this presentation.

- STOCHASTIC LINEAR OPTIMAL CONTROL WITH COLORED NOISE ASSUMPTIONS
- UNSTEADY AERODYNAMIC EFFECTS UPON LONGITUDINAL DYNAMICS
- DETERMINISTIC, NONLINEAR, OPTIMAL CONTROL THROUGH GIVEN MICROBURSTS

DETERMINISTIC, NONLINEAR, OPTIMAL CONTROL PROBLEM

To calculate an optimal trajectory, one must have some criterion by which a determination can be made of what is optimal. For the unconstrained, fixed-time, free-end-point problem, the criterion is that the scalar, positive definite cost function, J, be minimized. In this formulation x is the state vector, u is the control vector, and $\dot{x} = f(x,u,t)$ defines the plant dynamics including disturbance inputs. J is minimized by an appropriate choice of the control vector time history. Because the aircraft plant dynamics are represented by differential equations, the continuous-time formulation of this problem seems natural. Numerical solution techniques, on the other hand, are more appropriate to the discrete-time problem. Therefore, a zero-order-hold assumption is made for the control time history, and the continuous-time problem is transformed into a discrete-time problem. Now the optimal control problem is in the general form of a static, finite-dimensional, constrained optimization problem. Standard techniques may be applied. Two such techniques are the Steepest-Descent and Newton's Second Gradient methods. The Steepest-Descent method uses an initial guess for the control time history, then differentiates the cost with respect to it to determine an optimization step that will yield the largest decrease in the cost. Newton's method is merely a generalization of the Newton-Raphson method for determination of a root of a scalar equation. In this case it is applied to the set of simultaneous equations which comprise the necessary condition for optimality: $dJ/du_k = 0.0$ for $k = 1...N-1$. A FORTRAN package was developed for the implementation of these solution techniques. It uses 4th order Runga-Kutta integration to transform the continuous time problem into the discrete time problem. The Steepest-Descent method is used for the initial improvements to the control time history because it is cheaper per optimization step and yields large changes in cost, J, per step when not in the neighborhood of the optimum. Newton's method is used to get to the final solution because it converges rapidly in the neighborhood of the solution.

CONTINUOUS TIME

GIVEN $\underline{x}(t_0) = \underline{x}_0$, $\dot{\underline{x}} = f(\underline{x},\underline{u},t)$

FIND $\underline{u}(t)$ FOR $t_0 < t < t_F$

TO MINIMIZE $J = \int_{t_0}^{t_F} L(\underline{x},\underline{u},t)dt + V(x(t_F))$

DISCRETE TIME

GIVEN $\underline{x}_1$, $\underline{x}_{k+1} = \underline{f}(\underline{x}_k,\underline{u}_k,k)$

FIND $\underline{u}_k$ FOR $k = 1 \ldots N-1$

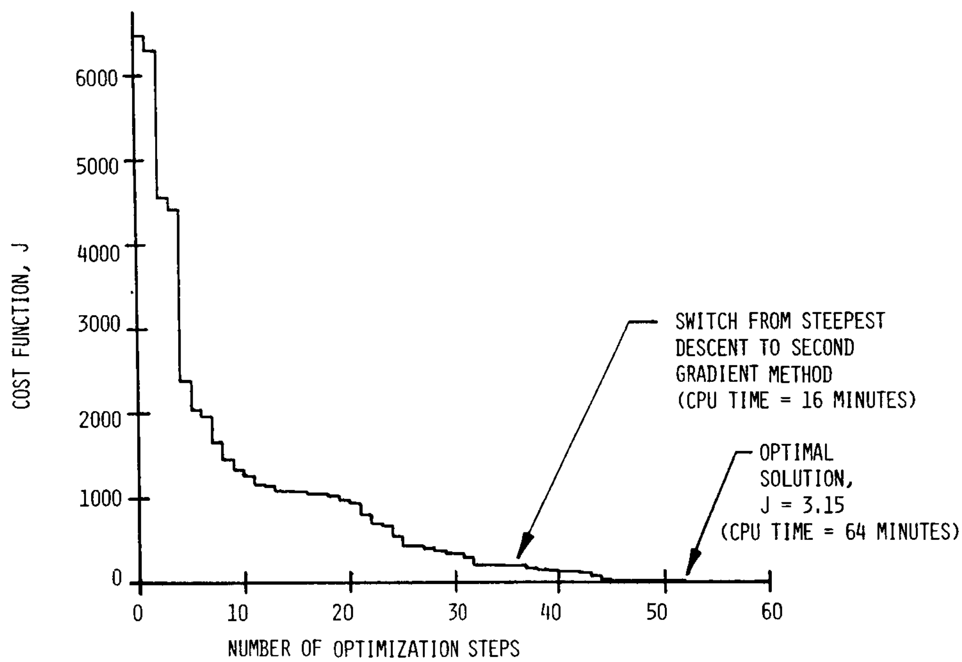
TO MINIMIZE $J = \sum_{k=1}^{N-1} L(\underline{x}_k,\underline{u}_k,k) + V(\underline{x}_N)$

SOLUTION TECHNIQUES

- STEEPEST DESCENT
- SECOND GRADIENT (NEWTON'S METHOD)

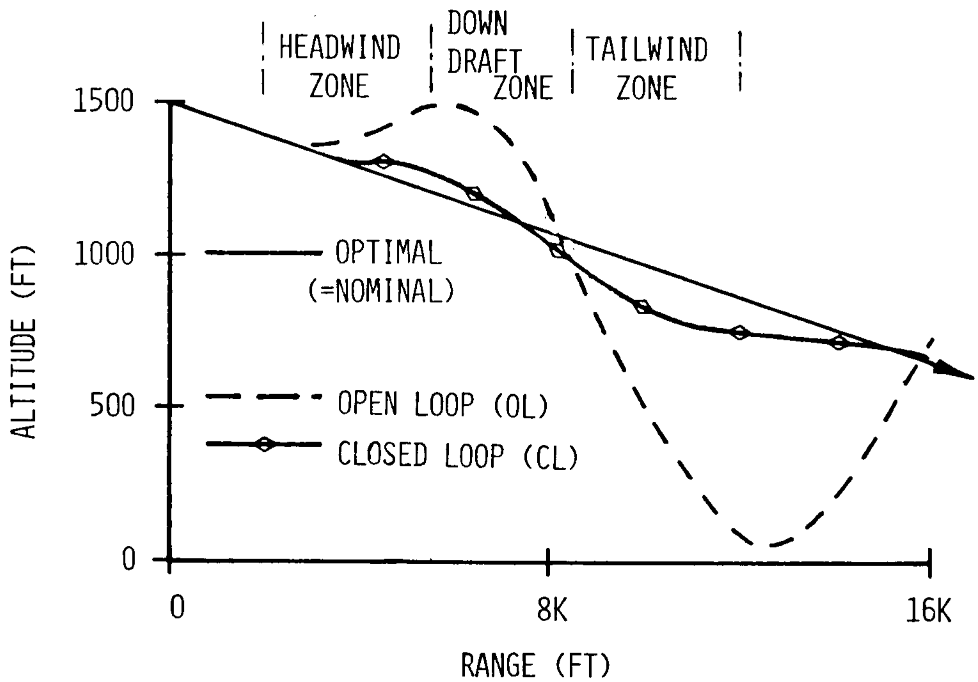
COST FUNCTION HISTORY FOR A STEEPEST-DESCENT/SECOND GRADIENT OPTIMIZATION

An optimal trajectory was calculated for the Boeing-727 model and the range dependent, sinusoidal microburst used in previous studies. After each optimization step the cost function was evaluated. This figure contains a plot of the cost as a function of the number of optimization steps. It took 51 steps to reach the optimum to within reasonable accuracy. Thirty-four steps were Steepest-Descent steps, and seventeen were taken using Newton's method. Note that the Steepest-Descent portion of the optimization required only about a third as much CPU time as the Newton's method portion, despite the fact that there were twice as many Steepest-Descent steps. This optimization probably could have been done more efficiently by waiting longer to make the switch from Steepest Descent to Newton's method. The erratic pattern of the cost function decreases, though typical of numerical optimization techniques, makes automation of the switching process difficult.



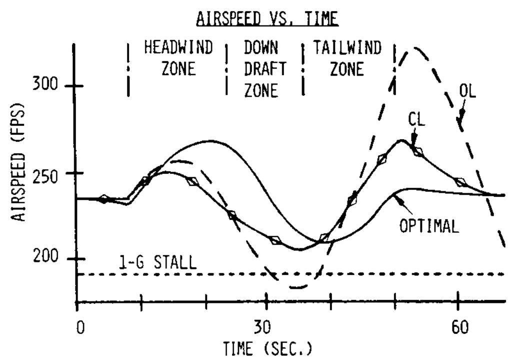
THREE TRAJECTORIES THROUGH A MICROBURST

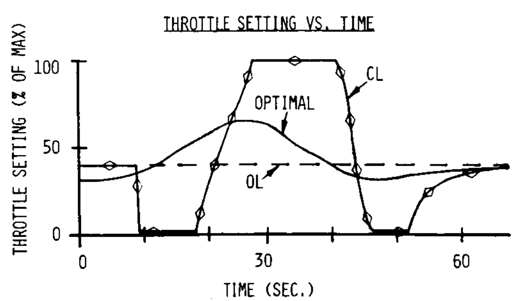
The results of this trajectory optimization are compared with the results of two previous flights by the same model through the same microburst: an open-loop flight and a closed-loop flight. The control law used in the closed-loop flight was the best so far designed by the author using classical design techniques. On this plot the optimal trajectory is indistinguishable from the nominal trajectory, a -3 deg. glide slope. In fact, it deviated no more than 1.5 ft. from the nominal. The previously best trajectory (the closed-loop run) yielded a 180 ft. perturbation, while the open-loop perturbation was about 1000 ft.



THREE AIRSPEED AND THREE THROTTLE SETTING TIME HISTORIES

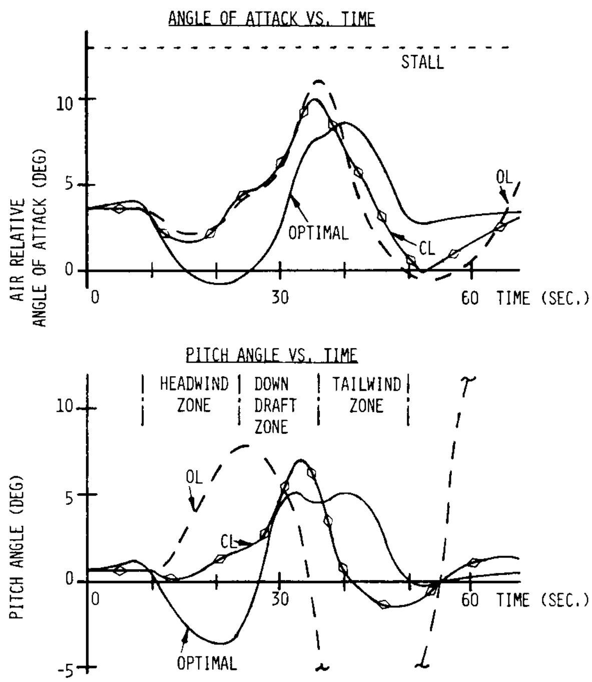
For the optimal case, it is interesting that the airspeed variation is approximatly that of the microburst itself. The optimum solution expends little effort in trying to maintain airspeed. The airspeed for this case never comes near the 1-g stall speed, so no problem is encountered. The throttle activity is much lower for the optimum case than for the poorer performing closed-loop case. Its phase lead may partially explain this. Both the airspeed and the throttle time histories for the optimum case are perturbed from the nominal prior to the initial encounter of the microburst. This is due to the nature of the deterministic optimization; the algorithm "knows" ahead of time what is about to happen and acts accordingly. This behavior is also visible on the next figure. This fact precludes the implementation of this algorithm as a control law unless sensors can be developed which sense the wind ahead of the aircraft (a possibility which will be pursued at a later date). Also note that the headwind, downdraft and tailwind zones marked are approximate. The times of their encounters vary slightly from case to case.





THREE ANGLE-OF-ATTACK AND THREE PITCH-ANGLE TIME HISTORIES

The open-loop and closed-loop angle-of-attack time histories are very similar to each other. The optimal angle-of-attack time history is out of phase with the corresponding airspeed variation. This indicates that the optimal glide slope control is primarily by angle-of-attack variation to maintain lift in the presence of airspeed variations. The pitch-angle time history bears out this interpretation, taking into account the changes in the relationship between the two angles due to the wind variations. In the open-loop and closed-loop cases, the pitch-angle is not held low enough during the headwind zone, and it is not held high long enough during the tailwind zone.



CONCLUSIONS

A relatively large amount of computer time (64 minutes of CPU activity on an IBM 3081 computer) was used for the calculation of this optimal trajectory, but it is subject to reduction with moderate effort. The Deterministic, Nonlinear, Optimal Control algorithm yielded excellent aircraft performance in trajectory tracking for the given microburst. It did so by varying the angle of attack to counteract the lift effects of microburst-induced airspeed variations. Throttle saturation and aerodynamic stall limits were not a problem for the case considered, proving that the aircraft's performance capabilities were not violated by the given wind field. All closed-loop control laws previously considered performed very poorly in comparison, and therefore do not come near to taking full advantage of aircraft performance.

- DETERMINISTIC, NONLINEAR, OPTIMAL CONTROL, AN EFFECTIVE THOUGH EXPENSIVE NOMINAL SOLUTION

- SUFFICIENT AIRCRAFT PERFORMANCE FOR SAFE ENCOUNTER OF GIVEN MICROBURST

- INSUFFICIENCY OF PRACTICAL CONTROL LAWS STUDIED TO DATE

PLANNED FUTURE WORK

Effort will be made to reduce the CPU time per trajectory optimization by improving the efficiency of the algorithm, but the basic approach will remain the same. The microbursts used thus far have been idealized. Microburst data from the JAWS project will be used to get realistic wind fields. These will be checked to see if and how any of these exceed aircraft performance limits by doing trajectory optimizations through them. Optimal trajectory solutions are also greatly affected by variations of the cost functions, $L(x,u,t)$ and $V[x(tf)]$. These effects will be considered as will the optimum for a general aviation aircraft. Once the performance capabilities are well understood, the goal will be to design practical control laws which come as close to these limits as possible. The use of lead information about the wind shear will be considered during this phase to determine what information would be useful to a closed-loop control law. Unsteady aerodynamics effects remain to be studied to determine their impact on the validity of the aircraft models used here.

- DETERMINISTIC, NONLINEAR, OPTIMAL CONTROL (DNLOC) ALGORITHM IMPROVEMENTS
- FUTURE OPTIMIZATION RUNS
  - JAWS MICROBURSTS
  - VARYING COST FUNCTIONS, $L(\underline{x},\underline{u},\tau)$, $V(\underline{x}(\tau_F))$
  - GENERAL AVIATION AIRCRAFT
- PRACTICAL CONTROL STRATEGIES APPROACHING OPTIMUM PERFORMANCE
- UNSTEADY AERODYNAMIC EFFECTS