VERIFICATION OF LARGE BEAM-TYPE SPACE STRUCTURES

Choon-Foo Shih, Jay C. Chen, and John A.Garba

Applied Technologies Section
Jet Propulsion Laboratory
California Institute of Technology
Pasadena, California

Abstract

This paper describes the verification
approach of large beam type space
structures. The proposed verification
approach consists of two parts. The
first part is to remove the gravity
effect on the tested substructure and
to identify the on-orbit dynamic
characteristics of the substructure by
using the measurements of the ground
test. A scaling law is also
established to define the critical
length of the structure which can be
tested in 1-g field without incurring a
buckling problem. The second part is
to develop an adequate scaling law to
extrapolate the dynamic characteristics
of the prototype structure by using
results from the substructure. The
verification approaches are
demonstrated on two typical structural
configurations, the feed support
structure of a wrap-rip antenna and a
candidate Shuttle flight experiment.
The results indicate that it is
practical to verify the on-orbit
dynamic characteristics of these
structures by using the proposed
approach.

Introduction

Structures to be used for future space
application will be very large in size,
such as space station or large
deployable antenna systems [1]. These
space structures may have dimensions on
the order of 30 to 200M. The major
technical problem which must be
overcome before large flexible
structures can be utilized for future
missions is to develop confidence in
predicting their on orbit dynamic
characteristics. Current test methods
are inadequate for such structures
because of their service configurations
and the effect of ground test
environments. Methods must be
developed to accurately predict on-
orbit dynamic characteristics of large
very flexible structures by utilizing
ground test data obtained from either
multiple supports, scale model testing,
or substructure testing. A possible
approach to this problem is addressed
in this paper.

Since many large flexible space
structures can be modeled as beams [2],
the generic structural element chosen
for this investigation is a large space
beam. The results obtained from
analyzing a large space beam are
applied to large multi-dimensional beam
type space structures, such as a
typical feed support structure for a
wrap-rib antenna [3] and the MAST, a
deployable beam shuttle flight
experiment which is being planned by
NASA as part of the Control of Flexible
Structure (COFS) program [4]. The
approach of this work is to perform a
series of analytical investigations to
examine the applicability of scale
model ground testing for the
determination of structural dynamic
characteristics and to examine the
applicability of testing a full scale
substructure in a 1-g environment.
These analyses establish dimensionless
parameters for verifying structural
characteristics of large beam type
space structures and establish the
limitations of these test methods for
structural verification.

The verification approach presented in
this paper consists of two parts. The
first part is to investigate the
gravity effect on the dynamic
characteristics of a large space beam.
A closed form solution for the dynamic
response of a large space beam
subjected to its own weight has been
derived previously [5]. The results
provide a better understanding of
structural characteristics of a large
space beam under gravity. In addition,
the relationships for the natural
frequencies in a 1-g field and a 0-g
field are formulated. This allows the
identification of the on-orbit dynamic
characteristics of large beam type
structures by utilizing the ground test
data of such structures.

The second part of the verification
approach is to develop scaling laws. A
scaling law for the critical buckling
length of large laced columns is
established. This allows the selection
of an adequate length of the structures
for ground test. Another scaling law
for the bay number of the structure
with replicable bays is also developed.
The results can be applied to
extrapolate the dynamic characteristics
of a large prototype structure by using
the testing data of a substructure. In
order to obtain more representative
results, the shear effect is accounted
for in developing this scaling law.
Alternate approaches, such as
suspending the system vertically, is
also discussed in this work. Finally,

the approaches developed in this work are demonstrated on both a typical feed support structure of a wrap-rib antenna and the MAST configuration. Numerical results from the NASTRAN code as well as the closed form solution are presented.

Gravity Effect

The free vibration of a large space beam with simply supported ends subjected to its own weight (Figure 1) has been investigated in Reference 5. The results established the relationship of the natural frequencies in the 1-g field to those in a 0-g field. They are expressed by

$$\frac{\omega_{gm}}{\omega_{om}} = \left[1 + \frac{NL^2}{m^2\pi^2 EI} + \frac{AW_i^2}{2I} \right]^{1/2} ;$$

for $i = m = 1, 3, 5, \ldots$ (1.a)

$$\frac{\omega_{gm}}{\omega_{om}} = \left[1 + \frac{NL^2}{m^2\pi^2 EI} \right]^{1/2} ;$$

for $m = 2, 4, 6, \ldots$ (1.b)

where ω_{gm} is the natural frequency of the mth mode due to gravity effect, ω_{om} is the natural frequency of the mth mode in the 0-g environment, N is the axial stretching force, L is the beam length, E is the Young's Modulus, I is the cross sectional moment of inertial, A is the cross sectional area and W_i is the series coefficient determined form the static deformation, $W(x)$, due to its own weight

$$W(x) = \sum_{i=1,3,5,\ldots} W_i \sin \frac{i\pi x}{L}$$ (2)

Equation (1) indicates that the natural frequencies of the symmetric modes (m=1,3,5...) depend on not only the axial stretching force but also the static deformation due to its own weight. However, the natural frequencies of the asymmetric modes (m=2,4,6...) are not affected by the static deformation. It should be pointed out that the results shown in Equation (1) are based on the linearized approach of the governing equation. The vibration amplitude is assumed to be relatively small compared to the static deformation due to its own weight in a 1-g field. For a large vibration amplitude, the nonlinear behavior of free vibration can be obtained from Reference 5. The present paper will consider only small amplitude vibration.

The dynamic characteristics of a vertically hanging beam (Figure 2) subjected to gravity effect can be derived by using the energy method. The normalized frequency equation can be expressed by

$$\frac{\omega_{gm}}{\omega_{om}} = \left[1 + \frac{MgL^3}{2\pi^2 m^2 EI} \right]^{1/2} \quad \text{for all } m$$ (3)

where M is the mass per unit length. It should be pointed out that for the laced columns the mass M in Equation (3) is the total mass of the structure divided by the total length of the structure.

Limitations of Ground Tests

The results discussed above allow the verification of structural characteristics of large beam type structures in space by utilizing the ground test data of such structures. However, one of the limitations of the ground test for a very large flexible structure is the buckling of the structure due to its own weight. This kind of buckling problem will restrict the length of the structure tested in a 1-g environment. In order to define the critical buckling length of the structure in 1-g field, a scaling law must be established.

Generally, the results of buckling analyses provide the eigenvalues and their corresponding buckling modes. The eigenvalue is the factor by which the pre-buckling stresses are multiplied to produce buckling. Since the loading environment is designated as 1-g, the relationship between the structure length and the critical gravity multiplier (eigenvalue) must be established in order to define the critical buckling length of the structure in the designated 1-g field.

A typical buckling mode of a 20-bay structure subjected to a field 7.2 times earth gravity is shown in Figure 3. The geometric dimensions and material properties of this structure are obtained from Reference 3 and are also shown in Figure 4. The buckling mode shown in Figure 3 is a local type buckling mode of the top longerons. This occurs because the compressive stresses in the top longerons exceed the critical buckling stresses. Numerical results based on NASTRAN results, shown in Table 1, indicate that the critical buckling stress of the longeron (N_{cr}) is not significantly affected by the structural length. Based on the assumption that the critical buckling stress of the longerons remains constant, it can be derived that the critical gravity multiplier is inversely proportional to

the square of the bay number if each bay of the structure is replicable. This can be expressed by

$$\lambda = \left(\frac{n_{cr}}{n}\right)^2 \qquad (4)$$

where λ is the critical gravity multiplier of a n-bay structure and n_{cr} is the critical buckling bay number of the structure in a 1-g field. Table 2 shows the critical gravity multiplier, based on NASTRAN results, as a function of the bay number. Applying the Equation (4), the critical bay number of this structure can be predicted. These are also listed in Table 2. Satisfactory results are observed. In addition, it is noted that the lowest buckling mode of a 2-D 20-bay structure is a global lateral buckling (Figure 5). Table 3 indicates that Equation (4) is valid for this kind of lateral buckling mode also.

Scaling Law

Since the buckling problem limits the length of the structure tested on the ground and each bay of the structure is replicable, a proper approach to successfully conduct a ground test is to test the structure with a number of bays less than the critical number of bays. Therefore, a scaling law must be established in order to extrapolate nature frequencies of the full size structure by using results from substructure testing.

It is known that the natural frequency of a uniform beam is inversely proportional to the square of the beam length. This is based on the assumption that the shear effect is negligible. However, Reference 6 indicates that the effect of shear on the deflection is much greater for a laced column than for a solid beam. Hence, this kind of shear effect must be considered in large beam type space structures, such as the typical feed support structure of a large antenna or the MAST.

It is noted that the effect of the shearing force reduces the critical buckling load of a laced column. This must be considered as the stiffness of the structure is decreased due to the action of shearing forces. In order to account for this effect in the vibration problem, the stiffness term in the frequency equation should be modified. This modified stiffness can be approximated from the buckling strength of a laced column. Following a similar approach as that used in Reference 7, the modified stiffness (EI_e) of a triangular laced column as shown in Figure 4 can be expressed by

$$EI_e = \frac{EI}{1 + \dfrac{c}{n^2}} \qquad (5.1)$$

where EI is the bending stiffness of the laced column which can be approximately expressed by $EA_\ell b^2/2$, the c/n^2 is the correction term due to the shear effect, the constant c depends upon the structural geometry and the vibration modes. For a triangular laced column, the constant c for the bending modes can be expressed by

$$c = \frac{2m^2\pi^2 EI}{3\ell^3 b^2}\left(\frac{d^3}{EA_d} + \frac{b^3}{EA_b}\right) \qquad (5.2a)$$

where ℓ is the length of the longerons, d is the length of the diagonals, b is the length of the battens, EA_d and EA_b are the axial stiffnesses of the diagonals and the battens, respectively. Equation (5.2a) can be rewritten as

$$c = \frac{\pi^2 m^2 A_\ell}{3\ell^3}\left(\frac{d^3}{A_d} + \frac{b^3}{A_b}\right) \qquad (5.2b)$$

Substituting the modified stiffness into the frequency equation of a beam subjected to lateral vibration, the scaling law can be expressed as

$$\frac{\omega_P}{\omega_{sb}} = \frac{n_{sb}^2}{n_P^2}\left(\frac{1 + \dfrac{c}{n_{sb}^2}}{1 + \dfrac{c}{n_P^2}}\right)^{1/2} \qquad (6)$$

where ω_P is the natural frequency of the full size structure, ω_{sb} is the natural frequency of the substructure, n_P is the bay number of the full size structure and n_{sb} is the bay number of the substructure. It should be noted that the first part of the righthand side of Equation (6) accounts for pure bending and the second part accounts for the shear effect.

The scaling law of Equation (6) is verified by using a 2-dimensional feed support structure. The geometric dimensions and material properties of this structure are the same as those shown in Figure 4. The constant c in Equation (6) for a 2-D laced column can be obtained directly from Reference 6. Both 20-bay and 40-bay laced columns are used to predict the natural frequencies of a 60-bay structure. Note that the structure will exhibit lateral buckling if the bay number exceeds 47, as shown in Table 3. The natural frequencies of these structures are calculated by using NASTRAN and are also listed in Table 4. The comparison between the predicted natural

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frequencies of a 60-bay structure and those from NASTRAN results are also shown in Table 4. The results indicate that the effect of shear plays a significant role on extrapolating the natural frequencies of a longer laced column. It also shows that the scaling law based on Equation (6) provides satisfactory results.

Verification Process

The results discussed above can be applied to verify the on-orbit dynamic characteristics of large beam type space structures. The verification process can be summarized in the following steps:

1. Implementation of the buckling analysis for the structure subjected to its own weight provides the critical gravity multiplier (eigenvalue) and its corresponding buckling mode.

2. Application of the scaling law for the critical buckling length, as shown in Equation (4), determines the critical buckling bay number of the structure in a 1-g field.

3. Selection of a structure with bay number less than the critical bay number for ground test to provide substructure testing measurements in 1-g environment, such as the static deformation, axial stresses and natural frequencies.

4. Application of the frequency equation, as shown in Equation (1) or (3), removes the gravity effect and determines the natural frequencies of the selected substructure in a 0-g field.

5. Application of the scaling law for bay number, as shown in Equation (6), verifies the on-orbit natural frequencies of the prototype structure.

Applications

Two large beam type space structures are examined. The first one is a typical feed support structure of a wrap-rib antenna, shown in Figure 4. The results from the buckling analysis associated with the scaling law indicate that the structure will buckle due to its own weight if the bay number of this structure exceeds 54. In order to prevent the buckling problem, a 40-bay structure is proposed for the ground test. Since no real ground testing is anticipated in the example problem, the measurements of this 40-bay structure are assumed to be those obtained from NASTRAN results as listed in Table 5. Following Steps 4 and 5 as discussed in the verification process, the on-orbit natural frequencies of a

longer structure (such as 60-bay) can be determined and these are listed in Table 6 together with the direct NASTRAN results for comparison. A good agreement is observed.

The second space structure examined in this work is based on the MAST configuration which is being considered by NASA for a future flight experiment [4]. The material properties and geometric dimensions of the MAST are listed in Table 7. The full length of the prototype MAST is approximately 60 meters (54 bays). However, a 10-bay MAST is proposed for the ground test because of the buckling limitation of the structure subjected to the gravitational environment. The ground test data of this 10-bay MAST, based on NASTRAN results, are also shown in Table 5. Following the verification process as discussed previously, the natural frequencies of this 54-bay MAST can be predicted and the results are also shown in Table 6. The higher discrepancy shown in this case is believed to be due to the smaller number of bays used in the ground test. The scaling factor due to shear effect is more accurate for a laced column with a large number of panels. For instance, if a 20-bay MAST could be tested in the 1-g field, better results could be achieved.

An alternate approach of verifying on-orbit dynamic characteristics of this MAST configuration is to test MAST substructure suspended vertically. The restriction of the MAST length, due to buckling caused by its own weight, is no longer a major concern in the vertical suspension test. A 20-bay MAST is chosen for the vertical suspension approach. Results, as shown in Table 8, indicate that the gravity effect on the natural frequencies of a 20-bay MAST hanging vertically is insignificant. The predicted natural frequencies of the prototype MAST, based on a vertical suspended approach, are shown in Table 9. Better results are observed in this case.

Conclusions

An approach for the verification of a beam type space structure has been described. The effect of gravity on the dynamic characteristics of both horizontally and vertically supported beams has been studied and the results are applied to identify the on-orbit dynamic characteristics of the structure tested on the ground. The natural frequencies of the full size structure are extrapolated from those of the substructure by using scaling laws. The results indicate that, in order to accurately predict the natural frequencies of a laced column, the shear effect should be considered in this scaling law. NASTRAN analyses are

implemented to verify the results based on the proposed verification approaches. Satisfactory results are observed in verifying the on-orbit natural frequencies of both the typical feed support structure of a wrap-rib antenna and the MAST configuration.

References

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Acknowledgement

Table 1 Critical Buckling Stress of Longeron vs. Bay Number

Bay No.	20	40	60	80	100
N_{cr}(ksi)	1.44	1.41	1.40	1.39	1.39

Table 2 Predicted Critical Bay Number of the Feed Support Structure with Different Bay Number

Bay No.	20	40	53	60	80
λ	7.20	1.75	0.992	0.772	0.432
n_{cr}	54	53	53	53	53

Table 3 Critical Buckling Bay Number of a 2-D Feed Support Structure

Bay No.	10	20	40
λ	21.70	5.20	1.28
n_{cr}	47	46	46

Table 4 Comparison of Predicted Natural Frequencies for a 60-bay Structure

m	n	ω_m (NASTRAN)(HZ)	ω^b/ω_m^*	ω^s/ω_m^*
1	20	4.10	0.785	1.051
1	40	1.24	0.950	1.007
1	60	0.58		
2	20	10.67	0.577	1.070
2	40	4.00	0.864	1.016
2	60	2.05		
3	20	17.04	0.477	1.065
3	40	7.14	0.800	1.018
3	60	3.97		

ω^b: Predicted natural frequency based on the assumption that the shear effect is negligible.

ω^s: Predicted natural frequency based on the scaling law which includes the shear effect.

ω_m^*: Natural frequency of the mth mode for the 60-bay structure (NASTRAN Results).

Table 5 Measurements Based on NASTRAN Results

Measurements	Feed Support Structure (40-bay)	MAST (10-bay)
Max. Deformation (in.)	14.64	0.0406
Max. Compressive Stress(PSI)	8054	344
Max. Tensile Stress (PSI)	4043	192
Natural Frequencies in 1-g field (Hz):		
m = 1	0.953	17.23
m = 2	3.086	47.91

Table 6 Comparison of Natural Frequency of Large Beam Type Space Structures

Structure	Mode No.	ω_p(Hz)	ω_e(Hz)
Feed Support Structure (60-bay)	1	0.418	0.415
	2	1.554	1.523
Mast (54-bay)	1	0.767	0.715
	2	3.097	2.749

ω_p: Predicted natural frequencies from verification approach.

ω_e: NASTRAN results

Table 7   Geometric Dimensions and Material
          Properties of the MAST

Overall Geometry

Total Length (L)	2380.86 in
Length of Each Bay (ℓ)	44.09 in.
Diameter enclosing the MAST (D)	55.12 in.
Bay Number	54

Cross-Section

Longerons: (3)
Length (ℓ)	44.09 in.
Inside Diameter of all Longerons	0.55 in.
Outside Diameter of Top Longerons	0.812 in.
Outside Diameter of Bottom Longerons	0.763 in.

Diagonals: (3)
Length (d)	64.91 in.
Outside Diameter (Solid)	0.287 in.

Battens: (3)
Length (b)	47.64 in.
Inside Diameter	0.25 in.
Outside Diameter	0.328 in.

Material

Graphite Epoxy
Young's Modulus (E)	9.62×10^6 psi
Poisson's Ratio (ν)	0.3

Mass

Joints	0.787 lb
Specific Weight Density	
Longeron	0.07814 lb/in^3
Diagonal	0.1604 lb/in^3
Batten	0.05954 lb/in^3

Table 8   Comparison of Natural Frequencies of a 20-bay
          MAST Structure Hanging Vertically

Mode No.	ω_o(Hz)	ω_E^1(Hz)	ω_E^2 (Hz)
1	4.883	4.888	4.887
2	16.524	16.531	16.528

ω_o: Natural frequencies in 0-g field (NASTRAN)
ω_E^1: Natural frequencies in 1-g field (NASTRAN)
ω_E^2: Natural frequencies in 1-g field (Equ. (3))

Table 9   Comparison of Natural Frequencies
          of the Prototype MAST

Mode No.	ω_p(Hz)	ω_e(Hz)	ω_p/ω_e
1	0.719	0.715	1.005
2	2.841	2.749	1.033

ω_p: Predicted natural frequencies from verification approach
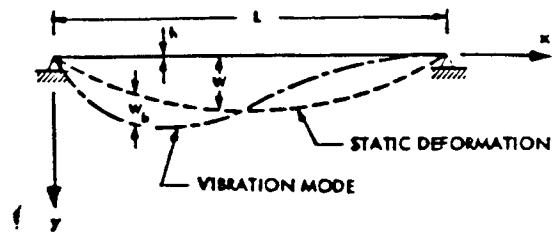ω_e: NASTRAN results
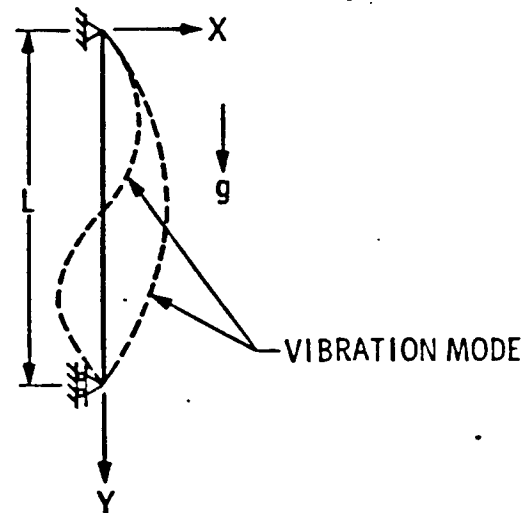

Figure 1   Simply-Supported Beam
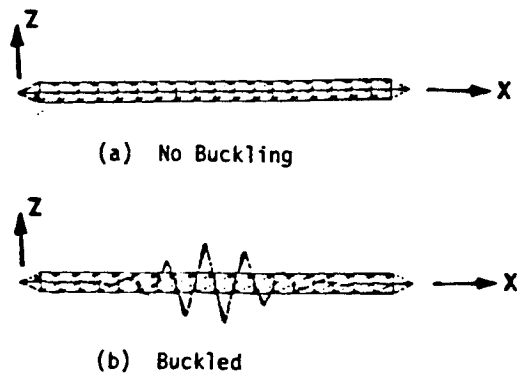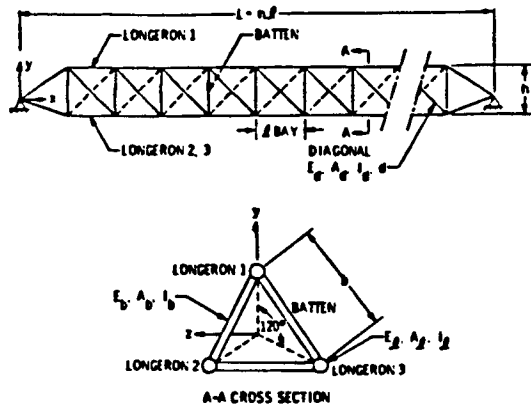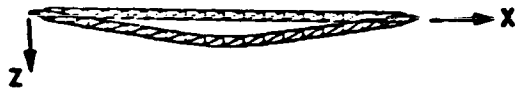

Figure 2   Vertically Hanging System

(a)   No Buckling

(b)   Buckled

Figure 3   Buckling Mode of a 20-bay
           Feed Support Structure (Top
           View)

$$ t = b = 95 \text{ in.}, \qquad d = 134 \text{ in.}, \qquad h = 82 \text{ in.} $$
$$ A_t = A_b = 0.22 \text{ in.}^2; \qquad A_d = 7.85 \times 10^{-3} \text{ in.}^2 $$
$$ I_t = I_b = 0.11 \text{ in.}^4; \qquad I_d = 4.91 \times 10^{-6} \text{ in.}^4 $$
$$ E_t = E_b = E_d = 1.66 \times 10^7 \text{ lb/in.}^2; \quad v = 0.3 $$
$$ \gamma_t = \gamma_b = \gamma_d = 0.06 \text{ lb/in.}^3 $$

Figure 4 Geometric Dimensions and Material
Properties of the Feed Support
Structure

Side View (45° w.r.t. y-axis and z-axis)

Figure 5 Lateral Buckling Mode of a
2-D Feed Support Structure

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