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COMMIT YOUR WORKS TO THE LORD, AND YOUR
THOUGHTS SHALL BE ESTABLISHED
PROV. 16:3

INTER–STABLE CONTROL SYSTEMS

BY
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APRIL 22, 1986

WORKSHOP ON STRUCTURAL DYNAMICS AND CONTROL
INTERACTION OF FLEXIBLE STRUCTURES

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INTER-STABLE CONTROL SYSTEMS

by

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Algebraic structures are discussed for control systems that maintain stability in the presence of resonance uncertainties. Dual algebraic operations serve as elementary connections that propagate the stability of inter-stable subsystems. Frequemcy responses within complex half-planes define different types of inter-stability. Dominance between incompatible types is discussed. Inter-stability produces sufficient but unnecessary stability conditions, except for conservative systems where the conditions become also necessary. Multivariable systems, colocation of actuator and sensor, and virtual colocation are treated. Instead of passivity, inter-stability relates stability to the mapping of poles and zeros by transfer functions and transfer matrices. Inter-stability determines stability on the subsystem level, is less complex even for multivariable systems, adds design flexibility, and relaxes the dynamic data problem of large systems such as space stations.

1 INTER-STABILITY IS THE PROPAGATION OF STABILITY THROUGH ALGEBRAICALLY DEFINED CONNECTIONS OF COMPATIBLE SUBSYSTEMS.

2 COMPATIBLE SUBSYSTEMS ARE STABLE AND SHARE A COMPLEX HALF-PLANE FOR ALL MAPPINGS OF THE 1st QUADRANT FROM THE COMPLEX FREQUENCY PLANE.

3 ONLY LINEAR AND CONSTANT SYSTEMS ARE CONSIDERED IN THE FORM OF TRANSFER FUNCTIONS AND TRANSFER MATRICES.

4 ADDITION, REDUCTION (I.E. INVERSE ADDITION), AND MATRIX COUPLING (E.G. INCIDENCE MATRICES) SPECIFY THE CONNECTIONS.

5 INTER-STABILITY IS DIRECTLY BASED ON THE EIGENVALUE MAPPING OF PROPERLY CONNECTED SUBSYSTEMS INSTEAD OF A PASSIVITY CONCEPT.

6 INTER-STABILITY YIELDS RESONANCE-INERT (ROBUST) CONTROL SYSTEMS WITH THE ADDED BENEFIT OF ANALYZING LOW ORDER SUBSYSTEMS.

INTER-STABLE CONNECTIONS

PHYSICAL CONNECTIONS ARE DEFINED BY THE ALGEBRAIC OPERATIONS OF ADDITION, REDUCTION, AND MATRIX COUPLING. THE OPERATIONS KEEP VECTORS IN A SEMIPLANE.

1 ADDITION + WITH THE UNIT ELEMENT Ø HAS THE DOMINANCE ELEMENT @.
A+Ø=A, @+A=@, A-A=Ø, A+B=B+A, (A+C)c=Ac+Bc, @c=@

2 REDUCTION x WITH THE UNIT ELEMENT @ HAS THE DOMINANCE ELEMENT Ø.
Ax@=A, ØxA=Ø, A\A=@, AxB=BxA, (AxB)c=AcxBc, Øc=Ø

3 ADDITION AND REDUCTION ARE PARALLEL AND SERIES CONNECTIONS WITH A DUALITY.
(AxA)+(AxA)=A, (A+A)x(A+A)=A, 1/(A+B)=(1/A)x(1/B), 1/(AxB)=(1/A)+(1/B)

4 SUBDUCTION \ IS THE DUAL OF SUBTRACTION AND CONNECTS NEGATIVE SUBSYSTEMS.
NEGATIVE SUBSYSTEMS ARE INCOMPATIBLE, BUT CAN BE DOMINATED.

5 MATRIX COUPLING IS THE TRANSFORMATION OF A DIAGONAL MATRIX D OF SUBSYSTEM ELEMENTS BY INCIDENCE, MODAL, AND ROTATION MATRICES T. M=T'·D·T

INTER-STABILITY EXPERIENCE

1965 INTER-STABILITY CRITERIA FROM HALF-PLANE IMAGES AND ALGEBRAIC CONNECTIONS WERE DEFINED AND APPLIED TO STABILIZE THE SATURN-V HYDRAULIC SUPPORT.

1970 INTER-STABILITY CRITERIA FOR SPACE VEHICLE ATTITUDE CONTROL WERE DOCUMENTED IN A SECRET REPORT (NOW DECLASSIFIED).

1972 INTER-STABILITY WAS PROPOSED FOR RESONANCE-INERT STABILITY OF SPACE STATIONS (NASA TN D-6731, JUNE 1972).

1974 FEASIBILITY OF INTER-STABLILITY FOR THE SPACE SHUTTLE WAS SHOWN AND PROPOSED TO JSC.

1976 RECEIVED A PATENT ON COLOCATED RATE GYROS FOR ATTITUDE REFERENCE WITH PLATFORM UPDATING AND ANOTHER PATENT ON VIBRATION ATTENUATION.

SYSTEMS DYNAMICS | SPACE SHUTTLE 6-DEGREES OF FREEDOM SUPPORT | NASA MSFC GLvP

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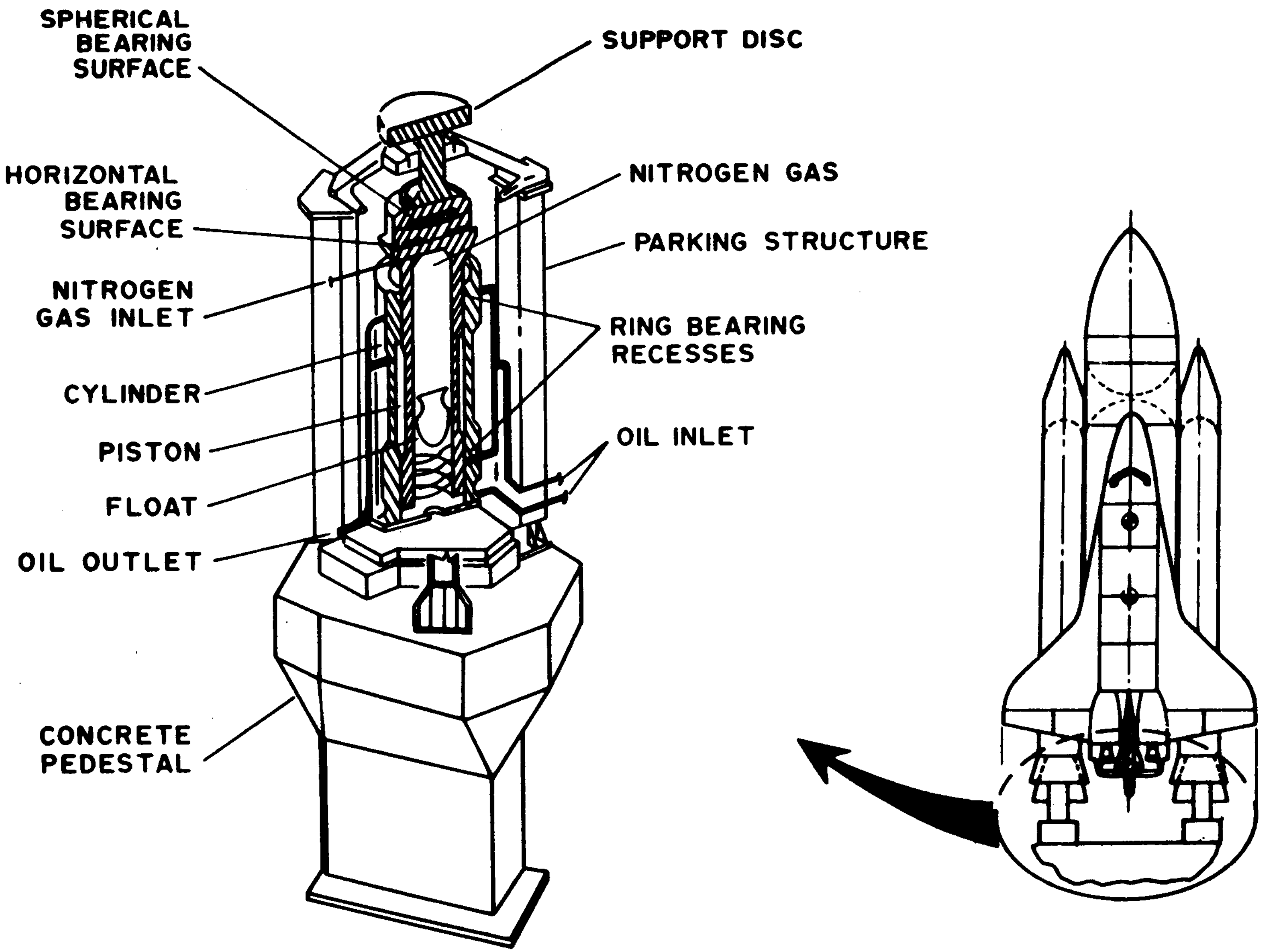
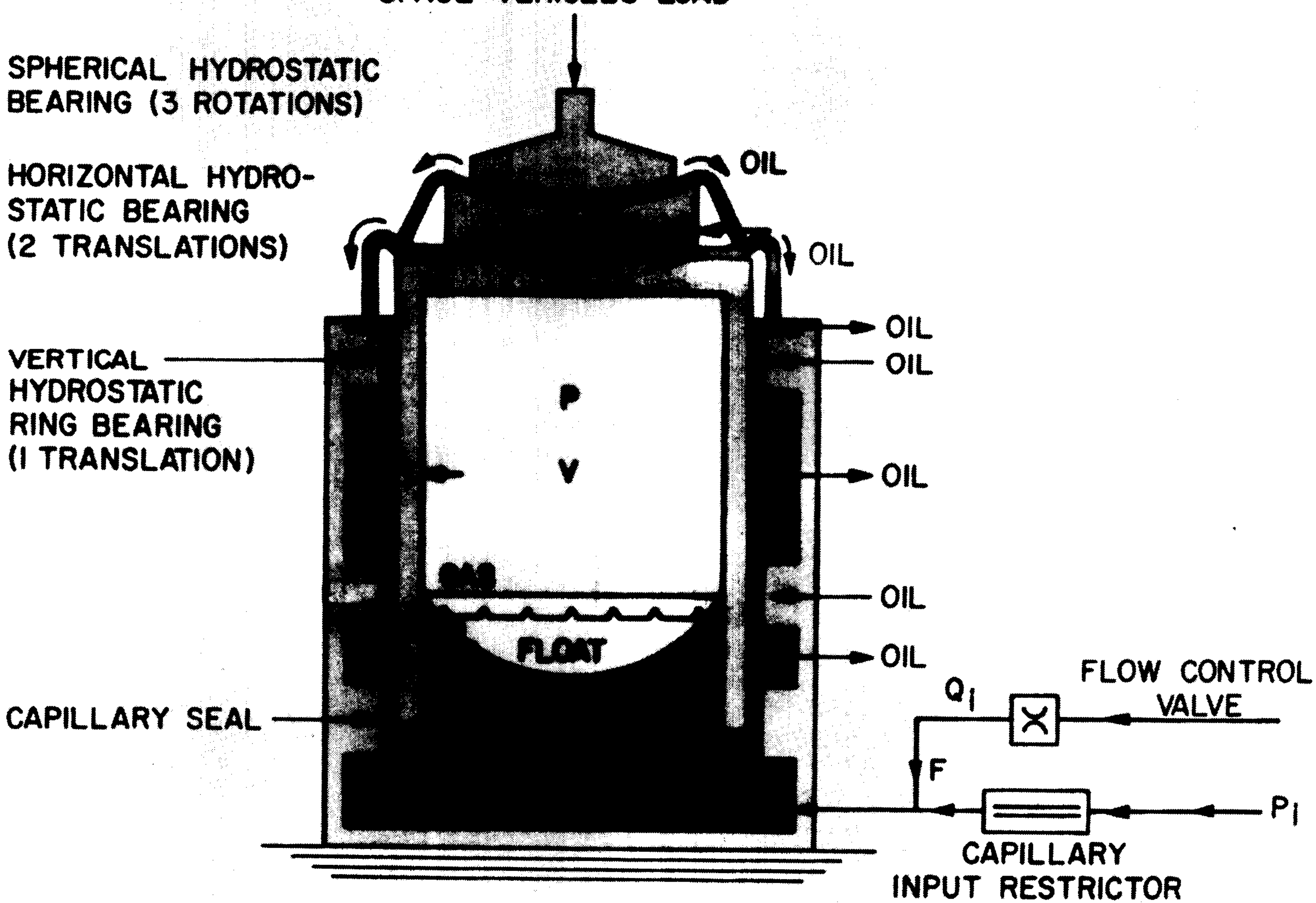
FOUR SUPPORTS HELD THE SATURN-V AND SPACE SHUTTLE DURING GROUND VIBRATION TESTS AS IN FREE FLIGHT WITHOUT FRICTION. HYDRAULIC BEARINGS AND PNEUMATIC PISTONS PROVIDED FRICTIONLESS LOW MASS SUPPORT. THE VEHICLES WERE TESTED FULL SIZE.



SYSTEMS DYNAMICS | NASA MSFC GLvP
INTER-STABLY DESIGNED HYDRAULIC SUPPORT UNIT
LABORATORY ED14 | APRIL 22, 1986

THE HYDROSTATIC BEARINGS AND THE PNEUMATIC PISTON WERE DESIGNED INTER-STABLY. THE PISTON MODEL, A 2nd/2nd ORDER QUOTIENT TRANSFER FUNCTION, WAS KEPT POSITIVE IMAGINARY LIKE THE TEST ARTICLE WHICH IS ADDITIVELY CONNECTED.



SCHEMATIC OF HYDRAULIC SUPPORT UNIT

SYSTEMS DYNAMICS — NASA MSFC GLvp

POSITIVE IMAGINARY INTER-STABLE MAPPING EXAMPLE

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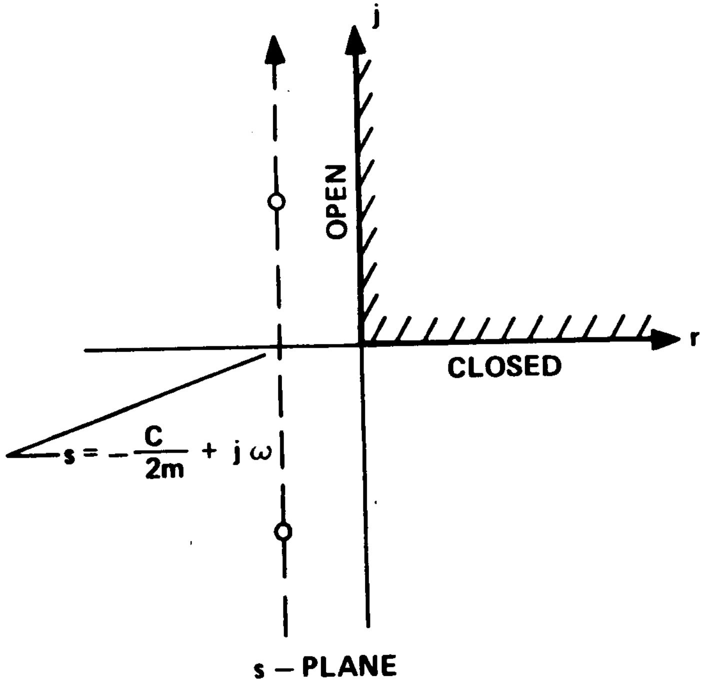
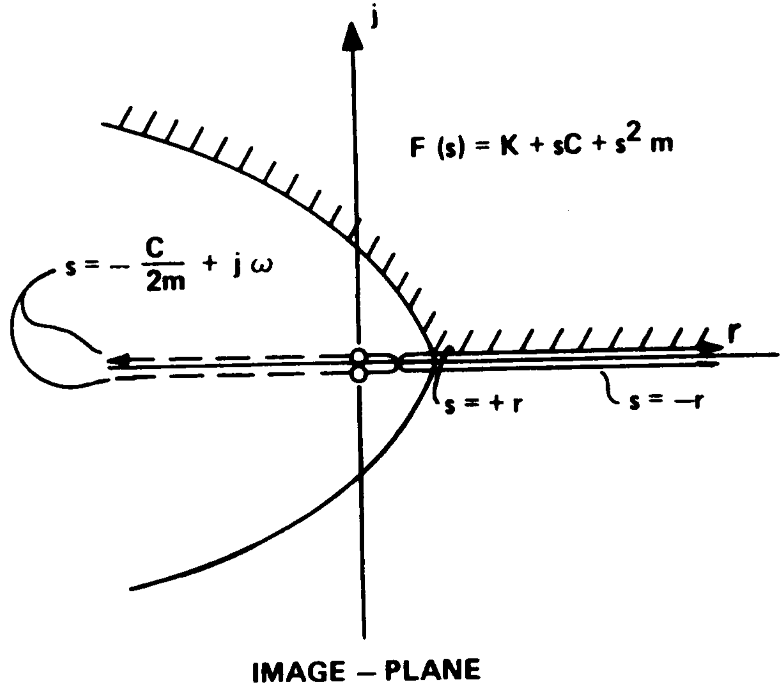
INTER-STABLE TRANSFER FUNCTIONS MAP FROM THE 1st QUADRANT OF THE COMPLEX FREQUENCY PLANE INTO A COMPLEX HALF-PLANE. THE HALF-PLANE's ORIGIN IS THE IMAGE OF THE ZEROS AND INFINITY THE IMAGE OF THE POLES. THE FIRST QUADRANT EXCLUDES THE ORIGIN AND THE IMAGINARY AXIS, BUT INCLUDES THE POSITIVE REAL AXIS. POSITIVE IMAGINARY SYSTEMS MAP INTO THE UPPER HALF-PLANE AND ONTO THE POSITIVE REAL AXIS WITHOUT THE ORIGIN. THE NEGATIVE REAL AXIS IS EXCLUDED.





SUBSYSTEM TRANSFER FUNCTIONS ARE STABLE AND MAP THE COMPLEX FREQUENCY PLANE'S 1ST QUADRANT INTO A COMPLEX HALF–PLANE. THE ORIGIN IS THUS NOT ENCIRCLED.

1 THE 1ST QUADRANT INCLUDES THE POSITIVE REAL AXIS, BUT EXCLUDES THE IMAGINARY AXIS. CONSERVATIVE SYSTEMS ARE THUS CONSIDERED AS INTER–STABLE.

2 POSITIVE REAL SYSTEMS MAP INTO THE RIGHT HALF–PLANE OUTSIDE THE IMAGINARY AXIS, e.g., $a_1 b_1 > [SQR (a_0 b_2) - SQR (a_2 b_0)]$ FOR 2ND/2ND ORDER QUOTIENTS.

3 POSITIVE IMAGINARY SYSTEMS MAP INTO THE UPPER HALF–PLANE OUTSIDE THE NEGATIVE REAL AXIS AND ORIGIN, e.g., $a_0/b_0 < a_1 / b_1 < a_2 / b_2$ FOR 2ND/2ND ORDER.

4 NEGATIVE IMAGINARY SYSTEMS MAP INTO THE LOWER HALF–PLANE OUTSIDE THE NEGATIVE REAL AXIS AND ORIGIN, e.g., $a_0 / b_0 > a_1 / b_1 > a_2 / b_2$ FOR 2ND/2ND ORDER.

5 NEGATIVE IMAGINARY SYSTEMS BECOME POSITIVE IMAGINARY SYSTEMS BY INVERSION AND VICE VERSA.

6 POSITIVE REAL SYSTEMS BECOME POSITIVE (NEGATIVE) IMAGINARY WHEN MULTIPLIED (DIVIDED) BY THE COMPLEX FREQUENCY s AND VICE VERSA.

7 NEGATIVE IMAGINARY SYSTEMS BECOME POSITIVE IMAGINARY WHEN MULTIPLED WITH THE COMPLEX FREQUENCY SQUARE s^2 AND VICE VERSA WHEN DIVIDING WITH s^2.

CONSTANT GAIN SYSTEMS ARE STABLE WHEN THE BELOW CRITERIA ARE MET. INTER-STABILITY IS MORE RESTRICTIVE, BUT UNRESTRICTED IN RESONANCES AND HIGH SYSTEM ORDERS DUE TO THE BUILDING BLOCK APPROACH.

STABLE OPERATIONS & INVERSION

Ω SET OF CONSTANT GAIN SYSTEMS

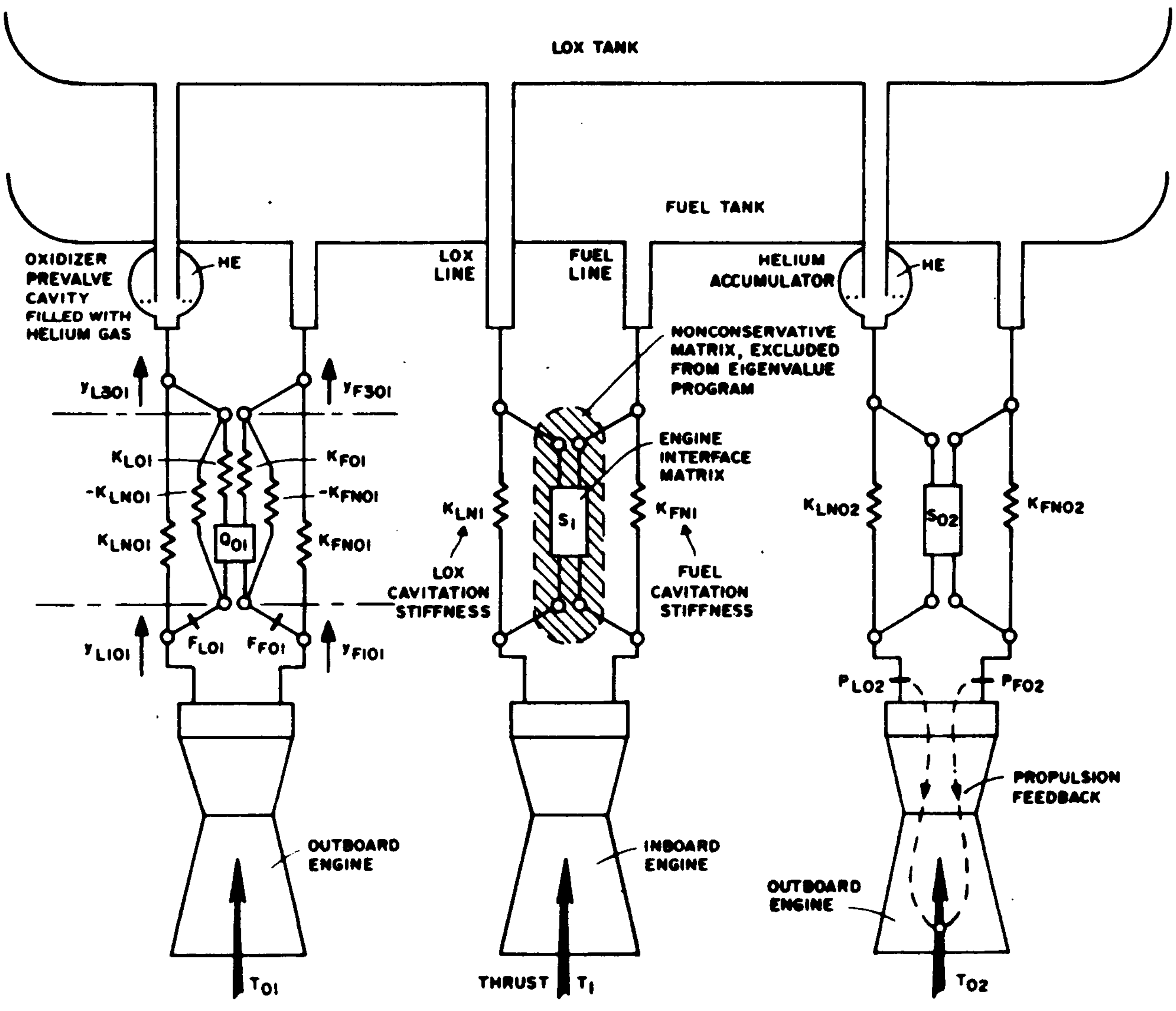
Σ SET OF STABLE SYSTEMS, $\{S, S_1, S_2\} \subset \Sigma \subset \Omega$

ADDITION	$S_1 + S_2 \in \Sigma$
MATRIX PRODUCT	$S_1 S_2 \in \Sigma$
ADJOINT	$\text{adj } S \in \Sigma$
DETERMINANT	$\lvert S \rvert \in \Sigma$
MATRIX INVERSION S^{-1}	$= \text{adj } S / \lvert S \rvert \in \Omega$

NECESSARY & SUFFICIENT FOR STABILITY: CONSIDER ZEROS OF $\lvert S \rvert$ ONLY

SYSTEMS DYNAMICS	SATURN-V POGO STABILITY CASE	NASA MSFC GLvP
LABORATORY ED14		APRIL 22, 1986

POGO IS AN AXIAL VEHICLE OSCILLATION DUE TO MECHANICAL VECTOR FEEDBACK FROM THE TANK PRESSURES OVER THE ENGINE THRUSTS TO THE AXIAL VEHICLE RESONANCES.



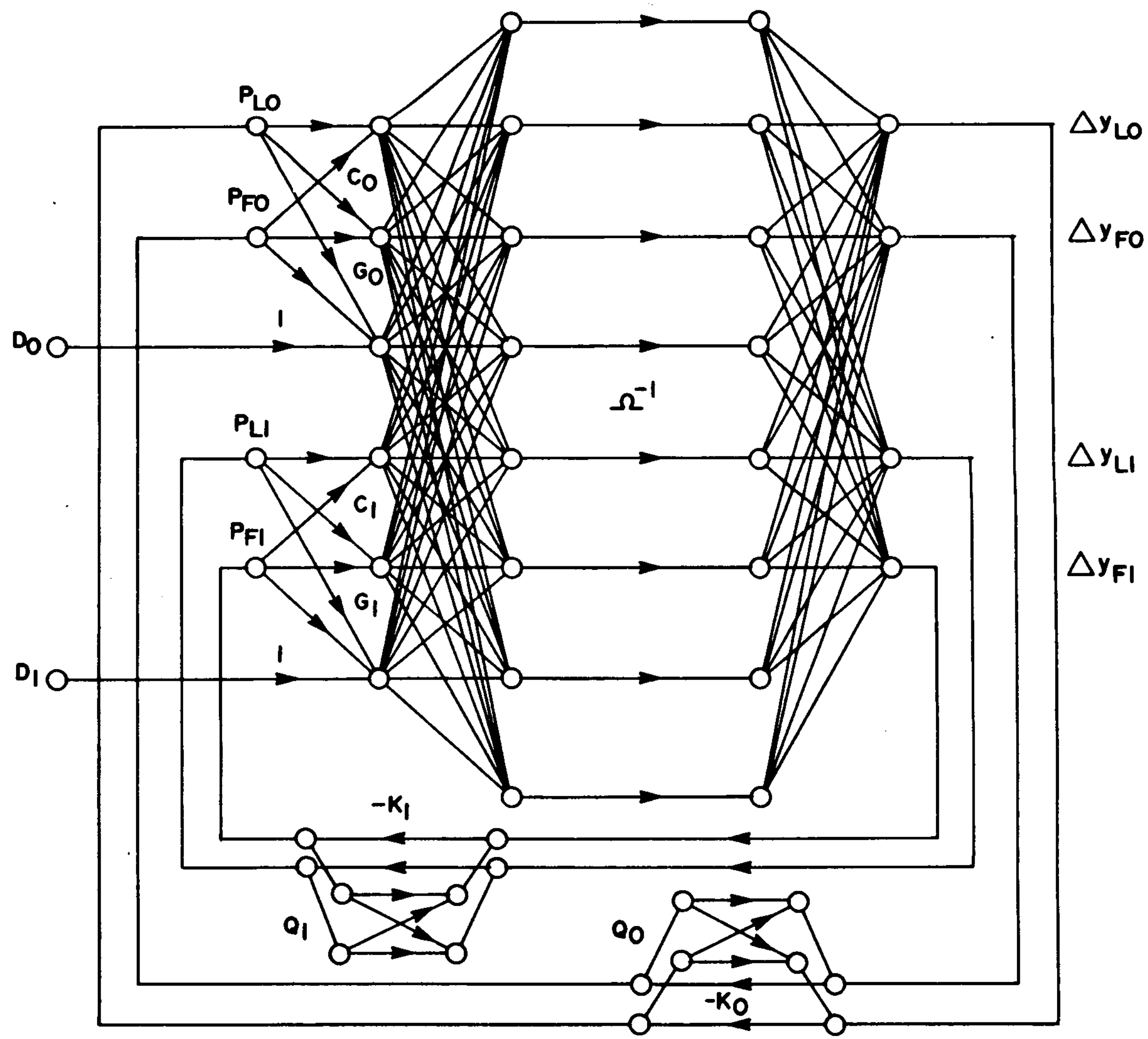
INTERFACE OF PROPULSION & VEHICLE STRUCTURE

SYSTEMS DYNAMICS NASA MSFC GLvP

LABORATORY ED14 APRIL 22, 1986

SATURN-V POGO VECTOR FEEDBACK LOOP

THE FEEDBACK VECTOR HAS FOUR P-COMPONENTS THAT REPRESENT PROPULSION GAINS FROM TANK PRESSURES TO TWO ENGINE GROUPS. THE D's ARE DISTURBANCES. OMEGA INVERSE IS A DIAGONAL MATRIX OF RIGID VEHICLE MASS AND STRUCTURAL RESONANCES. THE DIAGONAL MATRIX CLOSES THE FEEDBACK LOOP THROUGH RECTANGULAR COUPLING (MODAL) MATRICES.

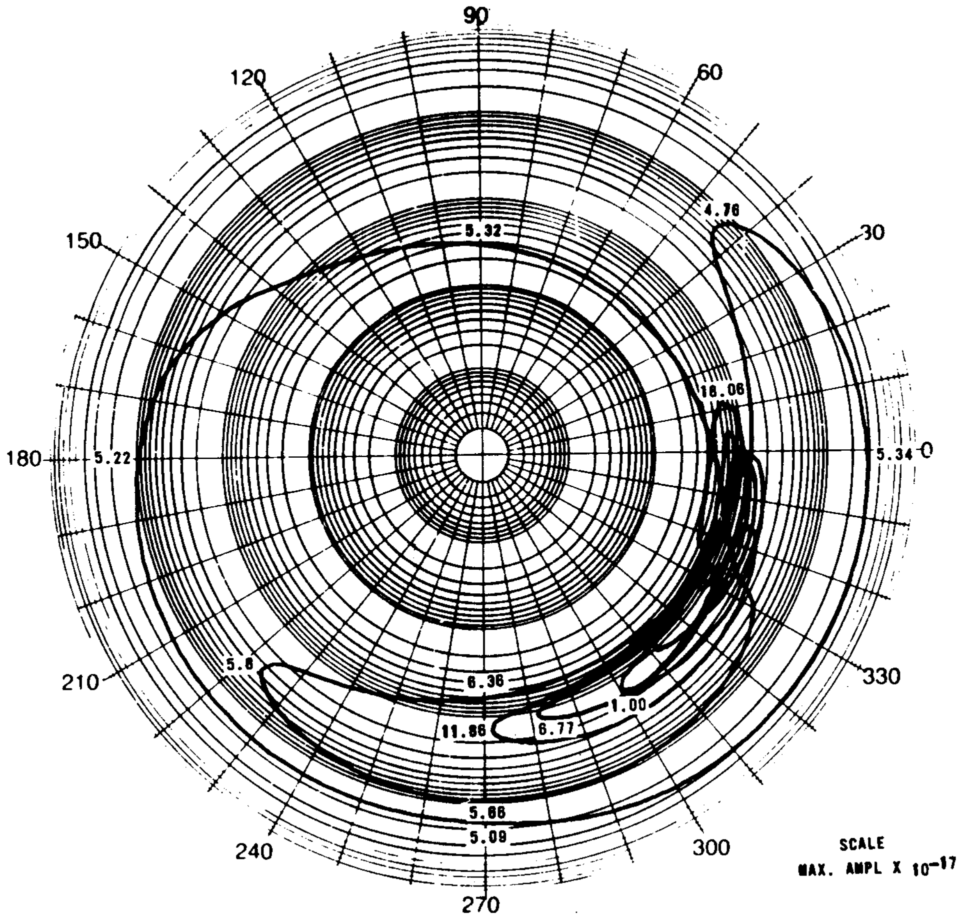


SYSTEMS DYNAMICS NASA MSFC GLvP

UNSTABLE ORIGIN ENCIRCLEMENT BY DETERMINANT PLOT

LABORATORY ED14 APRIL 22, 1986

FREQUENCY RESPONSE OF POGO SYSTEM MATRIX DETERMINANT SHOWS SYSTEM INSTABILITY BY ENCIRCLING THE ORIGIN. A STABLE TRANSFER MATRIX FORMULATION WAS USED, REQUIRING A FINAL MATRIX INVERSION. THE METHOD GENERALIZES THE NIQUIST CRITERION.



SATURN V FIRST FLIGHT STAGE OF AS–502 MISSION
AT 120 SEC FLIGHT TIME

SYSTEMS DYNAMICS | NASA MSFC GLvP

STABLE DETERMINANT PLOT OUTSIDE OF ORIGIN

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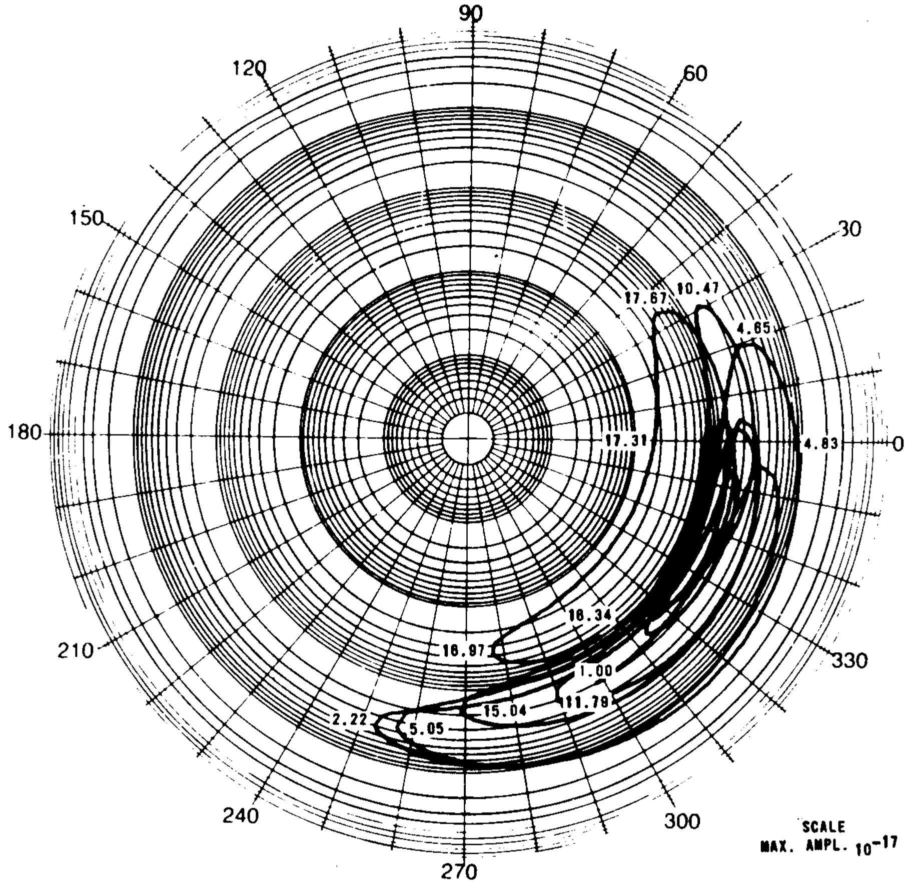
FREQUENCY RESPONSE OF POGO SYSTEM MATRIX DETERMINANT SHOWS SYSTEM STABILITY BY NOT ENCIRCLING THE ORIGIN. A STABLE TRANSFER MATRIX FORMULATION WAS USED, REQUIRING A FINAL MATRIX INVERSION. PLOT SHOWS SUCCESSFUL ELIMINATION OF POGO.



SATURN V FIRST FLIGHT STAGE OF AS–504 MISSION AT 120 SEC FLIGHT TIME

INTER–STABLE TRANSFER MATRICES ARE COMPLEX DEFINITE, HAVE INTER–STABLE TRANSFER FUNCTIONS AS ELEMENTS, AND ARE INTER–STABLE UNDER ADDITION AND REDUCTION.

1 THE TRANSFER MATRIX M IS COMPLEX DEFINITE IF THE QUADRATIC FORM $Q = u'M\,u$ MAPS INTO THE ELEMENTS' HALF–PLANE FOR ALL COMPLEX VECTORS u.

2 MATRIX INVERSION HAS A COMPLEX CONJUGATE (DENOTED BY ') QUADRATIC FORM $P = v'M^{-1}\,v = u'M'u = Q'$ FOR $v = M\,u$ AND THUS IS INTER–STABLE.

3 MATRIX ADDITION PRESERVES THE INTER–STABILITY OF THE ELEMENTS AS SEEN FROM $Q = u'\,(M + N)\,u = u'M\,u + u'N\,u$.

4 THE INVERSION OF A MATRIX SUM PRESERVES INTER–STABILITY AS SEEN FROM $Q = v'\,(M + N)^{-1}\,v = u'\,(M' + N')\,u = u'M'u + u'N'u$ FOR $v = (M + N)\,u$.

5 MATRIX REDUCTION PRESERVES INTER–STABILITY AS SEEN FROM $Q = u'\,(M \times N)\,u = v'\,(M'^{-1} + N'^{-1})\,v = v'M'^{-1}\,v + v'N'^{-1}\,v$ FOR $v = (M \times N)\,u$.

6 MATRIX REDUCTION IS COMMUTATIVE LIKE ADDITION AND IS DEFINED AS FOLLOWS $M \times N = (M^{-1} + N^{-1})^{-1} = (N^{-1} + M^{-1})^{-1} = N \times M$.

SYSTEMS DYNAMICS — POSITIVE REAL TRANSFER FUNCTION — NASA MSFC GLvP

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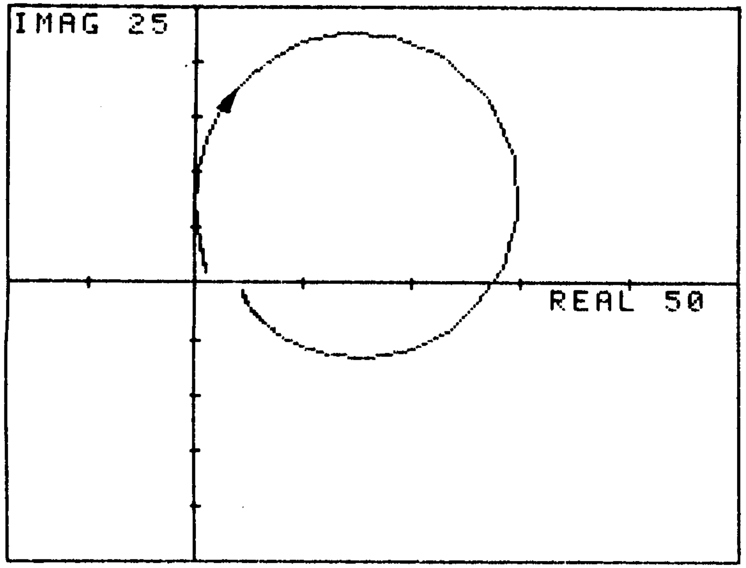
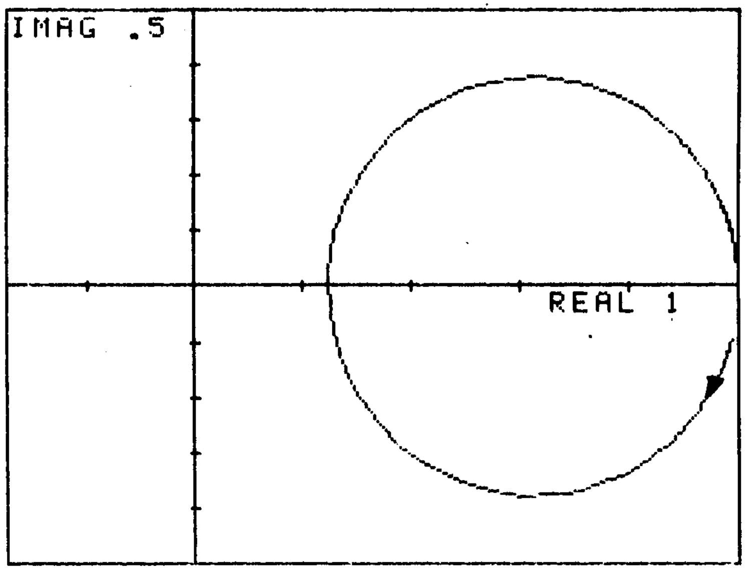
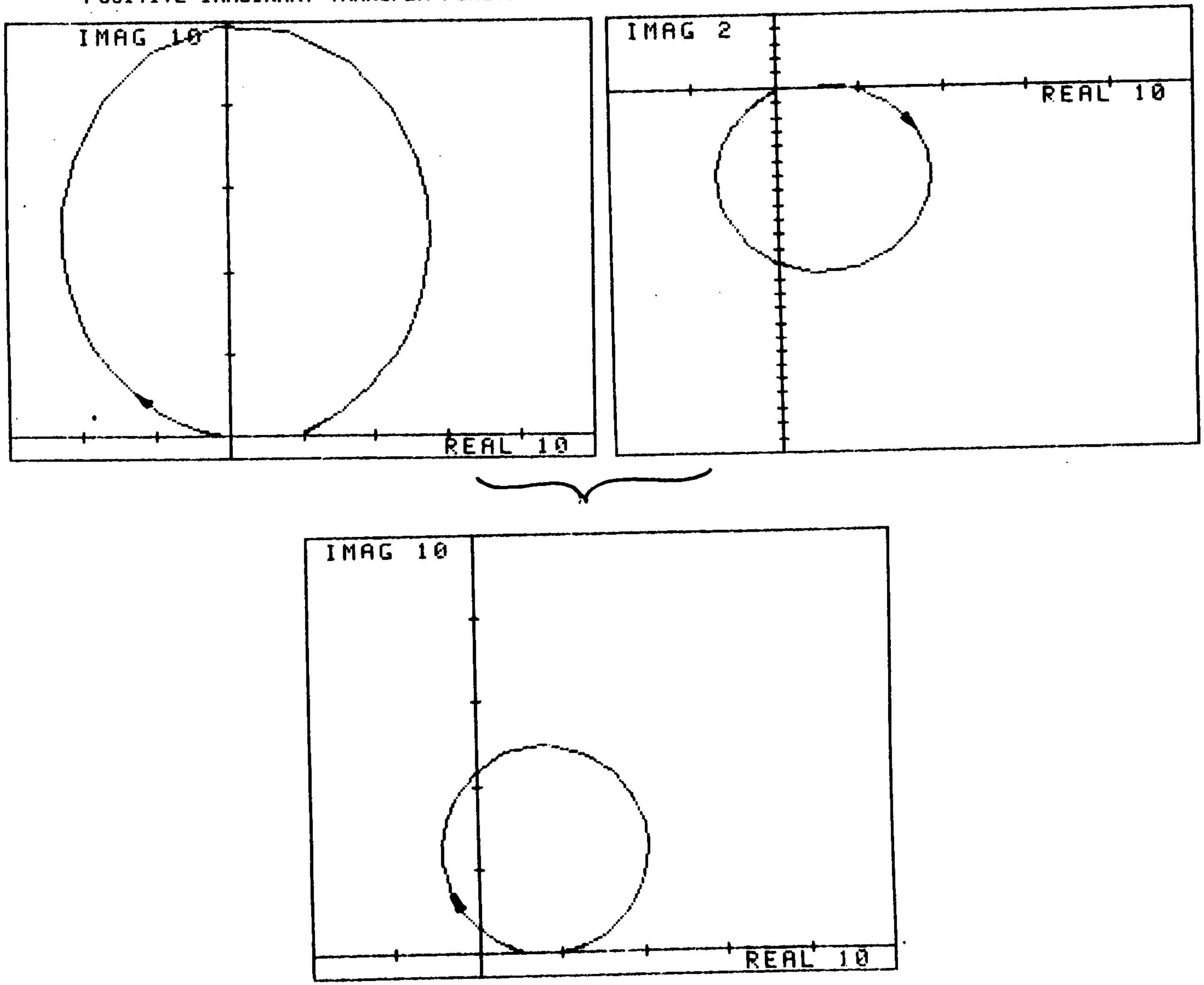
POSITIVE REAL TRANSFER FUNCTIONS FOR A LEAD NETWORK ON THE LEFT AND A NOTCH (LAG/LEAD) FILTER ON THE RIGHT.





SYSTEMS DYNAMICS | POSITIVE IMAGINARY DOMINANCE | NASA MSFC GLvP
LABORATORY ED14 | | APRIL 22, 1986

POSITIVE IMAGINARY ADDED TO A NEGATIVE IMAGINARY TRANSFER FUNCTION RESULTS IN A POSITIVE IMAGINARY TRANSFER FUNCTION. COMPONENTS ARE 2nd/2nd ORDER QUOTIENTS.



SYSTEMS DYNAMICS | INTER-STABLE CONTROL EXAMPLE | NASA MSFC GLvP

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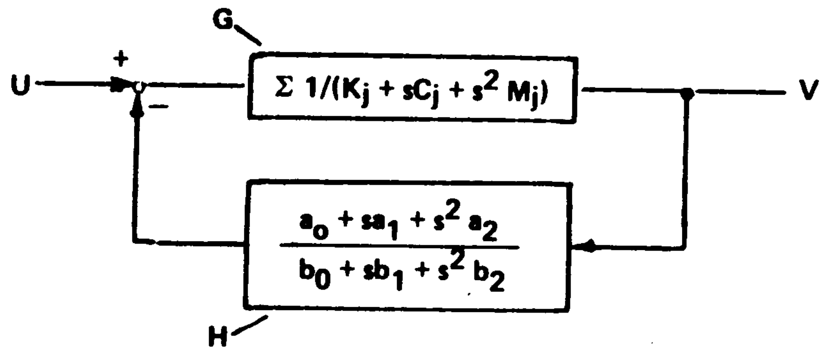


$V = G\,(U - H\,V)$

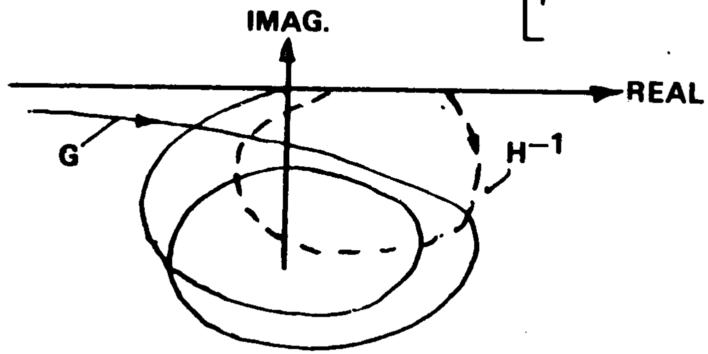
$U = (1/G + H)V$

$V = U/(1/G + H)$

$V = (G \times 1/H)U$

$$V = \left[\sum_i 1/(K_i + sC_i + s^2 M_i) \times \frac{b_0 + sb_1 + s^2 b_2}{a_0 + sa_1 + s^2 a_2} \right] U$$

COMPLEX IMAGE PLANE



INTER–STABILITY:

$b_0/a_0 > b_1/a_1 > b_2/a_2$

SYSTEMS DYNAMICS

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RATE GYRO COLOCATION RANGE FOR SPACE SHUTTLE

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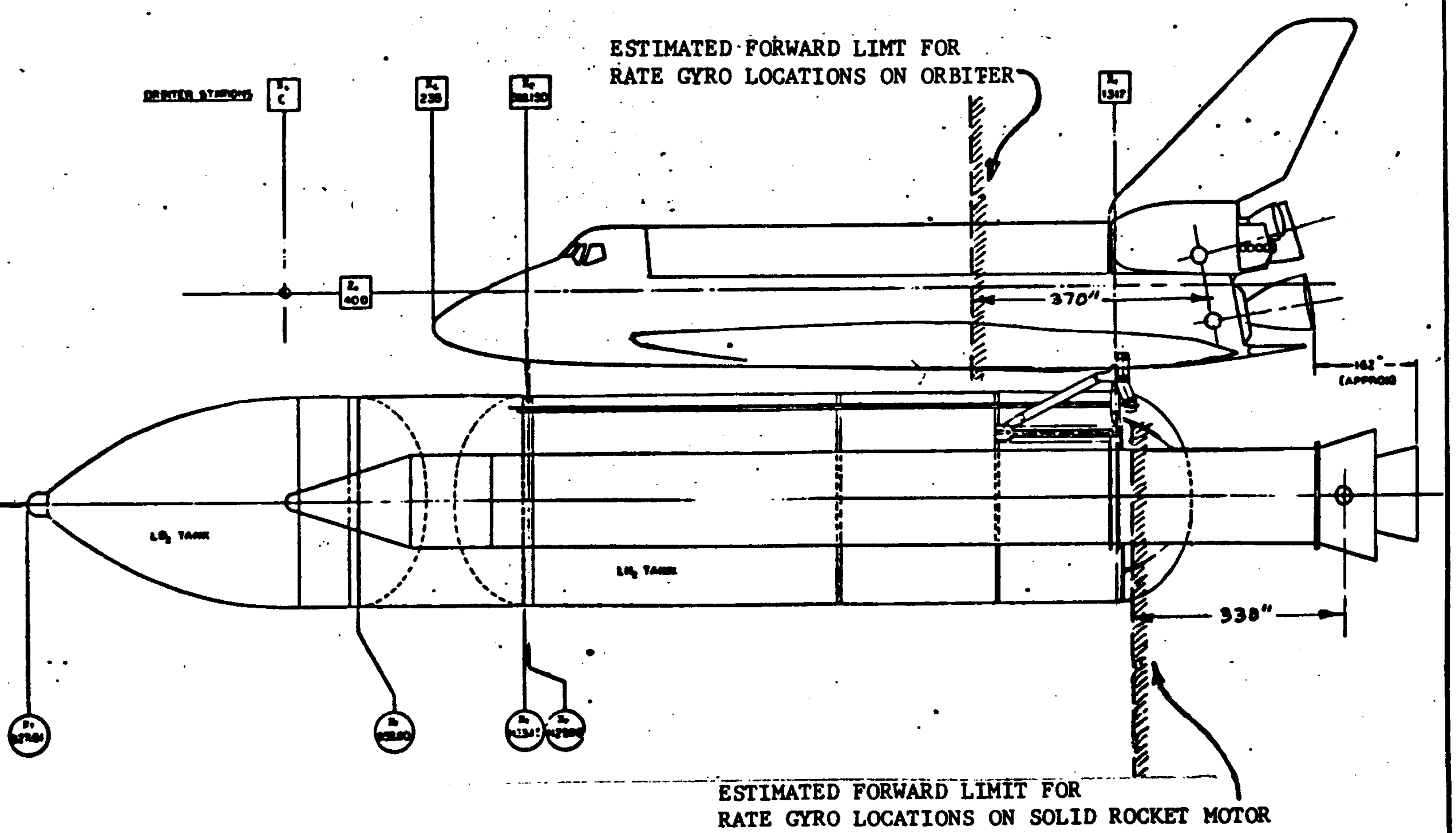
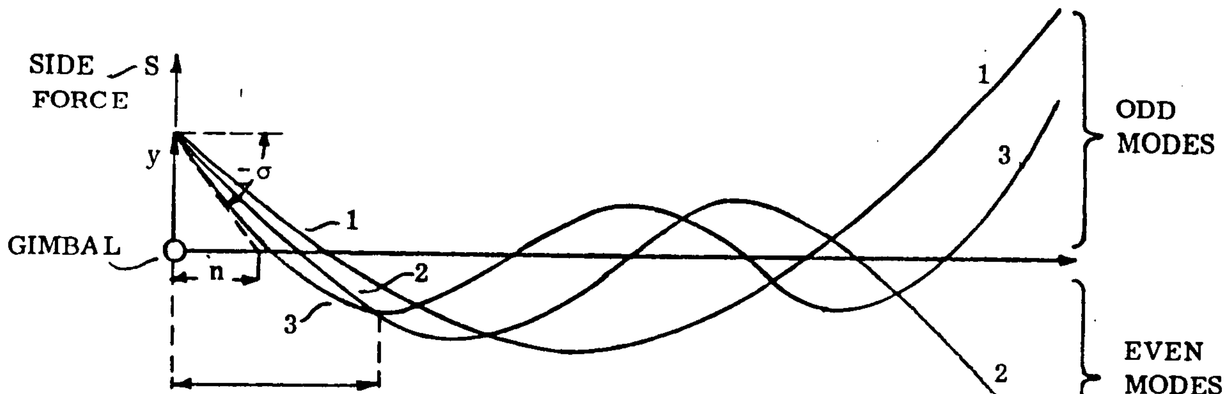
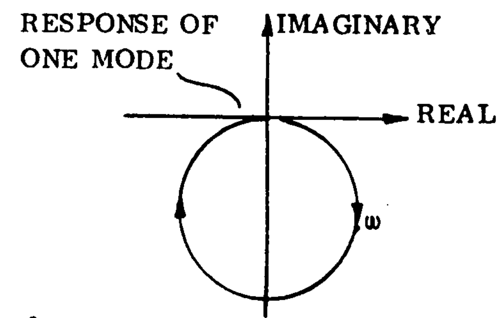


SYSTEMS DYNAMICS LABORATORY ED14 | RATE GYRO COLOCATION RANGE FEASIBILITY STUDY | NASA MSFC GLvP APRIL 22, 1986



ALLOWABLE RANGE OF RATE GYRO LOCATIONS



MODAL RESPONSE: $\frac{-\sigma}{S} = \sum_{i=1}^{\infty} \frac{-Y_i Y_i'}{m_i(\omega_i^2 + s2\xi\omega_i + s^2)}$

GIMBAL RELATION: $y = -\sigma n$, $n > 0$

RATE GYRO CONDITION: $\frac{\text{SENSOR ROTATION}}{\text{GIMBAL TRANSLATION}} < 0$ below of ~ 5 Hz

ALLOWABLE RATE GYRO LOCATION RANGE: SA-V 12m, ORB. ~ 4m, SRM ~ 8 m

SYSTEMS DYNAMICS

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INTER-STABLE SPACE SHUTTLE ATTITUDE CONTROL

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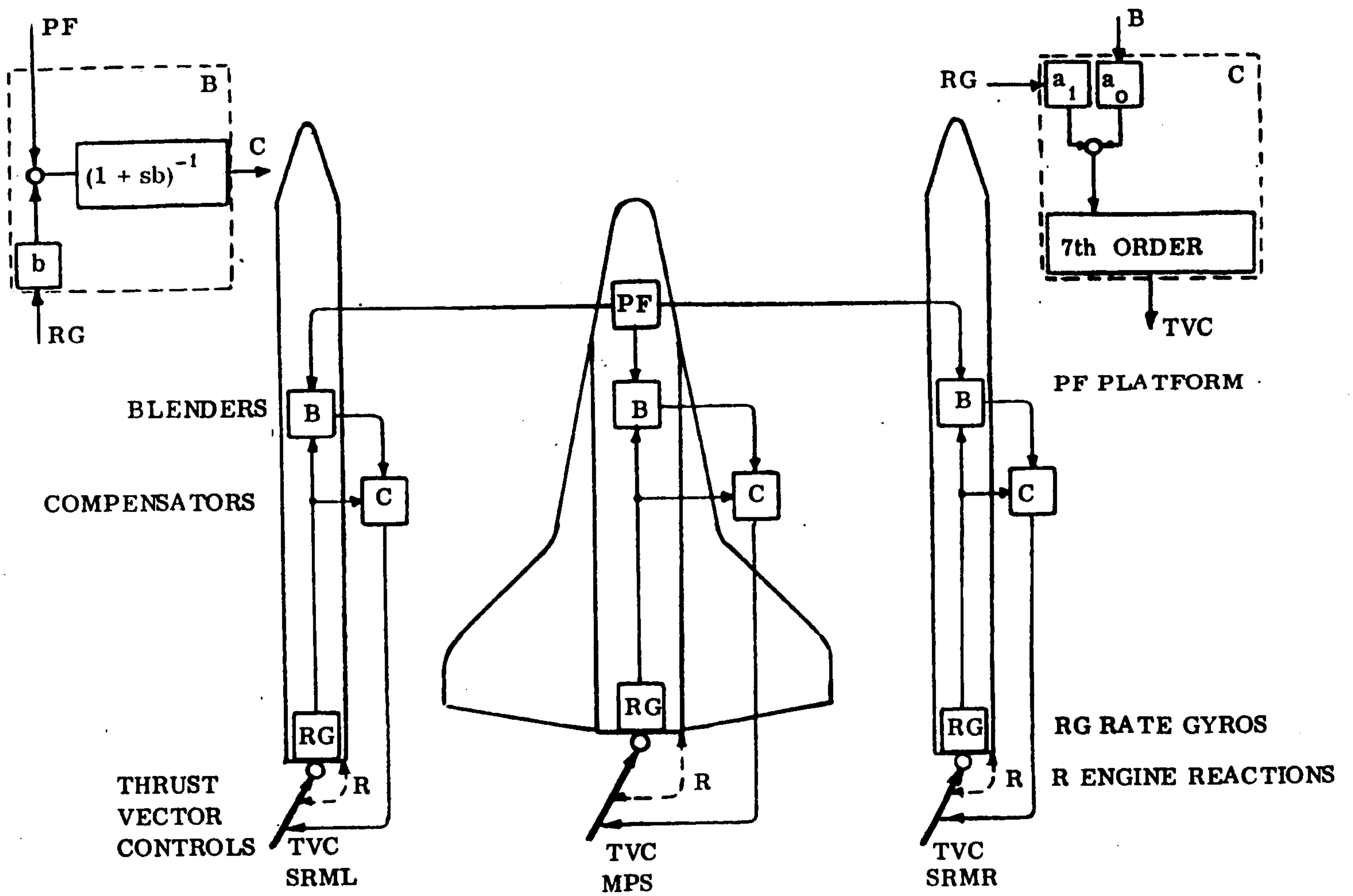

NOTE: BLENDERS AND COMPENSATORS ARE NOT PHYSICALLY SEPARATED, ONLY THEIR SIGNALS ARE SEPARATED.

SYSTEMS DYNAMICS | NASA MSFC GLvP

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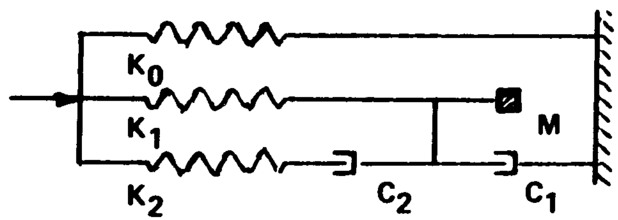
DECOMPOSITION OF A 2nd/2nd ORDER QUOTIENT

A POSITIVE IMAGINARY TRANSFER FUNCTION QUOTIENT IS DECOMPOSED INTO ADDITIVE AND REDUCTIVE BRANCHES THAT RESEMBLE MECHANICAL COMPONENTS. THE ALGORITHM DELIVERS NECESSARY AND SUFFICIENT CONDITIONS FOR INTER-STABILITY. CONTINUOUS FRACTIONS CAN BE USED, BUT YIELD ONLY SUFFICIENT CONDITIONS, EXCEPT FOR CONSERVATIVE SYSTEMS. GENERALLY, TRANSFER FUNCTION PLOTS ALWAYS YIELD NECESSARY AND SUFFICIENT INTER-STABILITY CONDITIONS.

$$F(s) = \frac{a_0 + sa_1 + s^2 a_2}{b_0 + sb_1 + s\ b_2} = K_0 + [(sC_1 + s M) \times (K_1 + [K_2 \times sC_2]))]$$

$K_0 = a_0/b_0$ $\quad C_1 = (a_1 - b_1 a_0/b_0)/b_0$ $\quad M = (a_2 - b_2 a_0/b_0)/b_0$ $\quad K_1 = a_1/b_1 - a_0/b_0$

$K_2 = a_2/b_2 - a_1/b_1$ $\quad C_2 = (a_2 - b_2 a_1/b_1)/b_1$



INTER-STABILITY PROSPECT

1 INTER-STABLE CONTROL SYSTEMS ARE INHERENTLY INSENSITIVE TO RESONANCES WHICH RELAXES THE DYNAMIC DATA PROBLEM.

2 DOCKING AND UNDOCKING CANNOT UPSET STABILITY FOR SUBSYSTEMS OF THE SAME INTER-STABILITY TYPE. THREE TYPES ARE IDENTFIED.

3 BUILDING BLOCK FEATURE REDUCES THE DIMENSION OF ANALYSES BY MAGNITUDE, HELPS THE UNDERSTANDING, AND GUIDES THE DESIGN.

4 INTER-STABILITY UNCOUPLES DIVERSE DESIGN AREAS AND DEFINES THE INTERFACES. CRITERIA ARE WELL DEFINED AND READY FOR THE SPACE STATION DEVELOPMENT.