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Equivalent Beam Modeling Using Numerical Reduction Techniques

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Introduction

The objective of this paper is to develop numerical procedures that can accomplish model reductions for space trusses. Three techniques will be developed that can be implemented using current capabilities within NASTRAN. The proposed techniques accomplish their model reductions numerically through use of NASTRAN structural analyses and as such are termed numerical in contrast to the analytical techniques developed in References 1-12.

The analytical techniques of Refs. 1-12 can be classified either as substitute continuum, discrete field, periodic difference, or finite difference methodologies. They are generally limited to trusses having either pinned or rigid joints and do not attempt to account for any joint flexibilities. Moreover, only specific trusses are analyzed to derive the "equivalent beam" properties. The primary reason for this limitation is the analytic complexity of treating general truss configurations with arbitrary joint characteristics. These analytic treatments did reveal, however, that equivalent truss models may require more degrees of freedom than allotted to the usual finite element beam.

To eliminate the above restrictions, numerical procedures are developed here that permit reductions of large truss models containing full modeling detail of the truss and its joints. Three techniques are presented that accomplish these model reductions with various levels of structural accuracy. These numerical techniques given in order of increasing accuracy are designated as equivalent beam, truss element reduction, and post-assembly reduction methods.

In the equivalent beam method described herein, the mass and stiffness properties of a simple finite element beam are determined so that the truss structure can be replaced with this equivalent beam element in all static and dynamic structural analyses. This approach is attractive in that once the equivalent beam properties are known, the beam length can be arbitrarily chosen by the analyst to suit the problem at hand. The approach is limited, however, to the usual six degrees of freedom describing the translational and rotational displacements for a beam node.

In the truss element reduction method, the idea of an equivalent structural element is retained but the number of truss bays to be represented must generally be chosen apriori. The advantage of this method is the capability to retain more than the six degrees of freedom alloted to the equivalent beam. Including warping and shear "degrees of freedom" in the equivalent structural element is an example of this increased capability.

The final approach does not attempt to derive an equivalent structural element for the truss. Instead, a procedure is developed that allows the analyst to identify apriori freedoms that can be reduced out of the model without loss of structural accuracy. This method thus permits a more accurate description of the truss than derived using equivalent structural elements while still allowing significant size reduction of the truss model prior to space station synthesis, modal extraction, or other static and dynamic analyses.

The numerical procedures discussed above all utilize a transformation of coordinates at some step in the reduction procedure. This coordinate transformation defines new "beamlike" degrees of freedom in terms of the original rectangular degrees of freedom describing the translational and rotational displacements of the nodes that are common between truss bays. The transformation of rectangular to beamlike degrees of freedom is described in Figures 1 and 2 for triangular trusses. The transformation for square trusses is similarly described in Figures 1 and 3.

There are two basic advantages arising from these transformations. First, the new beamlike freedoms are largely uncoupled from each other, and second, freedoms which can be reduced out through static condensation are generally more easily recognized.

The utilization of the beamlike transformation for either square or triangular trusses is discussed in Section 1.0 giving the step by step outlines for the three numerical reduction procedures. Results obtained using the three numerical reduction techniques on triangular trusses are given in Section 2.0. Square trusses are similarly discussed in Section 3.0. A preliminary analysis of a ten bay Rockwell truss using the numerical reduction techniques is then given in Section 4.0.

1.0 Step By step Descriptions of the Numerical Reduction Technique

The steps describing the three numerical reduction techniques are given in this section. The reduction procedures do not necessarily have to follow the steps as stated below since some of these steps can be combined and executed more efficiently. The steps as delineated below are given only for discussion purposes.

The first three steps in all three numerical reduction techniques are identical. The first step is to generate a detailed structural model of a single "repeating element" of the truss. The model should include as much definition of the joints as deemed necessary for accurate structural modeling. The second step reduces out all interior degrees of freedom from this single bay element using static condensation and retains freedoms only at the nodes interconnecting truss bays. The third step then connects a predetermined number of these single repeating elements and again reduces out all interior degrees of freedom. The number of bays selected in this step defines the basic mesh size to be used in all numerical reduction methods with the exception of the equivalent beam method. The finite element model resulting from the above three steps will henceforth be referred to as the basic truss cell. Further steps for each numerical procedure are described below.

1.1 Substitute Continuum Beam Method

Additional steps taken for this method are as follows:

i) Construct a truss of one or more basic cells and statically reduce out all interior freedoms resulting from this construction. The number of cells chosen requires a number of computer runs in order to demonstrate convergence of the beam properties derived below.

ii) Transform the degrees of freedom at the end of the truss to the beamlike degrees of freedom and retain only the usual six freedoms describing the translational and rotational displacements of a beam.

iii) Equate the (12 x 12) stiffness matrix resulting from this transformation and reduction to the stiffness matrix for a beam. The following equations are used to generate the E,G,I,J, and K properties of the beam:

$$AE/L = K_{11} \qquad GA/L = A/J * K_{44}$$

$$EI/L = (K_{55} - L^2/4 * K_{22})$$

$$K^{-1} = (GA/L) * \left| \frac{1}{K_{22}} - \frac{L^2}{12\,(EI/L)} \right|$$

$$1 + \upsilon = (J/A) * (AE/L)/(2 * K_{44})$$

where

A = arbitrarily chosen to be area of longerons

J/A = radius of gyration squared

K_{ii} = Diagonal terms of the (12 x 12) stiffness matrix

E = elastic modulus

G = shear modulus

v = Poisson's ratio

K = shear stiffness

$I = (EI/L) / (AE/L) * A$

L = length of segment used to generate the stiffness matrix

The resultant beam properties produce an element stiffness matrix which duplicates the stiffness matrix condensed from the explicit model. This duplication is exact for most truss structure configurations.

The mass of the equivalent beam may be calculated in two different ways. First, internally, using rigid body mass properties for either a consistent or lumped mass approach, and second, explicitly, using the (12 x 12) mass matrix describing the basic truss cell. This second approach has the disadvantage of fixing the beam length in subsequent analyses. If, however, mass per unit length is used as the beam property, then all beam properties are known independent of beam length and, the beam length can be arbitrarily chosen to suit any static or dynamic analysis at hand. This length independence property of the equivalent beam gives it a substantial advantage over the truss element reduction method in parametric studies when the effect of the length of the truss on system response is being examined. Such parametric studies are envisioned in the early design stages of the space station.

1.2 Truss Element Reduction Method

The additional steps taken in this procedure are as follows:

i) Transform the rectangular degrees of freedom of the interconnecting nodes to the beamlike coordinates.

ii) Eliminate unwanted degrees of freedom either by truncation or by static condensation. Truncation is accomplished in NASTRAN through single point constraint (SPC) and is equivalent to setting the displacement for those selected coordinates to zero. Static condensation is accomplished in NASTRAN by placing those coordinates in the OMIT set and is equivalent to setting the forces on those coordinates to zero.

iii) Form the complete truss structure using either NASTRAN image superelements or NASTRAN general elements (GENEL).

1.3 Post-assembly Reduction Method

The additional steps taken in this procedure are as follows:

i) Connect as many of the basic truss cells as required to define the complete structure and then transform coordinates. These operations may also be reversed so that a basic truss cell element can first be transformed then connected to form the complete truss.

ii) Choose freedoms to be retained for the complete structure. The freedoms retained generally have been selected by previous analytical studies of the truss or by analytical insight to the problem at hand. The reduction is then accomplished using static condensation.

2.0 Reduced Order Model For Triangular Frames and Trusses

The purpose of this section is to apply the three numerical reductions methods to triangular trusses and frames and to compare the results. The analyses are conducted only for cantilevered structures having ten and twenty bays.

Two different triangular frames and one triangular truss are examined (see Fig.4). These are identified as an unbraced Vierendeel frame, a double braced frame, and a double braced truss. A frame is distinguished from a truss by having rigid as opposed to pinned joints. Geometry and material properties are taken from Noor and Nemeth (Ref 1) in order to compare our results with theirs. The double braced frame results are also compared with the double braced truss results in order to bound the effects of joint flexibility on the modes and frequencies of a triangular structure having non-idealized joints.

The "exact" model descriptions of the cantilevered Vierendeel and double braced triangular frames are taken to be represented by finite element models having nodes only at the verticies of the battened triangles. Each node requires six degrees of freedom so that a total of 18 degrees of freedom are required to describe the deflections of one end of a frame bay segment. A total of 180 degrees of freedom are thus required to describe the cantilevered deformation of ten bays.

The primary objective of all three reduction techniques is to significantly reduce the size of the above models. Tables 1 and 2 give the total number of freedoms required by each of the three techniques to calculate the modes and frequencies of the Vierendeel and double-braced structures, respectively. These tables show that the post-assembly reduction technique allows the largest possible reduction of the three techniques considered.

Tables 1 and 2 also show the frequencies of cantilevered structures using various reduction schemes and retained freedoms. These results are also compared with the exact results of Noor and Nemeth.

No final resolution can be given at this time for the differences between our exact results and the exact results of Noor and Nemeth. It appears, however, that the differences may be attributed to the slightly different mass constructions used. MSC/NASTRAN uses a modified consistent mass approach (Ref 13) while Noor and Nemeth use the original consistent mass formulation presented by Archer (Ref 14). Alternatively, differences in modeling detail at the ends of the truss may account for the discrepancy. Detailed calculations to determine which was the more accurate were not performed.

Evaluations of the results for the various reduction schemes are also given in Tables 1 and 2. In all cases the post-assembly reduction schemes gave excellent results while the equivalent beam and truss element reduction schemes gave satisfactory results only for the double-braced structures. Detailed discussions of the various reduction schemes are given in the following subsections.

2.1 Post-Assembly Reduction

Freedoms that were retained in the post-assembly reductions were chosen simply by examining their modal participation in the frequency range of interest for the unreduced structure. In Table 1, ten, eight, and even four dof were all shown to adequately represent the Vierendeel frame when these dof were retained for every bay. A four dof representation at every other bay length was also shown to adequately represent the Vierendeel structure by showing a maximum of 5.7% error occurring for the fourth torsion mode.

Table 2 shows the results obtained for the double-braced triangular frame. One important conclusion that can be drawn from this table is that excellent results can be obtained for the frame even by considering the joints to be pinned. This conclusion is not suprising since engineers have successfully approximated frames as trusses for years. Excellent results are also expected when the four beamlike coordinates of the truss are retained at multiple bay lengths.

One important inference can be drawn from being able to use pinned instead of rigid joints for the double braced frame. The slight change in frequencies obtained by changing the joint from rigid to pinned is characteristic of a frame having a large area moment of inertia about its centroid. For in this case, the primary strain energy of the frame for low frequency modes can be accounted for by the axial extension or compression of its member elements. As a result of this energy distribution, moment capability of the individual members can be neglected and the joints can be considered pinned. In addition, the most important modeling consideration of a joint for such trusses is to accurately represent its axial stiffness. This in turn implies that free-play in the rotational directions can be ignored and that free-play in the axial direction of each member must be examined carefully to determine its effect on the the truss modes and frequencies.

In conclusion, significant model size reduction for the Vierendeel and double braced frames can be obtained by utilizing the post-assembly reduction technique. The degrees of freedom retained in the reduced models are generally easy to identify by the analyst either by previous analytical studies or by insight. Moreover, the geometrical behavior of the modes are easily recognized when expressed in terms of the beamlike coordinates and do not require mode shape plots in order to visual response.

The mass and stiffness matricies resulting from the post-assembly reduction technique are full, however, and must be repeatedly generated for trusses having different lengths. Such situations would occur in various parametric studies currently envisioned in the early stages of space station design and an "equivalent beam" approach would be preferential for such trade studies.

Model size reduction for double braced triangular frames can also be realized by considering the joints to be pinned. This approximation reduces the size of the problem by one-half when local member modes can be omitted. Further reduction can then be obtained using coordinate transformation followed by static condensation.

2.2 Equivalent Beam and Truss Element Reduction Techniques

The equivalent beam method as defined in this paper is limited to six degrees of freedom. Any extension in the number of retained degrees of freedom for an equivalent structural element necessitates use in MSC/NASTRAN of image super elements. These image super elements can be defined using the numerical truss element reduction technique as presented in this paper or they can be defined using the analytical techniques found in References 1-12. In any event, the 6-dof equivalent beam models are considered in a class of their own due to their ease of use.

The 6-dof equivalent beams are not applicable for all trusses, however, as demonstrated in Table 1 for the Vierendeel frame. In fact any 6-dof equivalent structural element may not be sufficient and additional freedoms may be required. This conclusion is supported for the Vierendeel frame by the unsatisfactory 6-dof element reduction results in Table 1 and by the satisfactory 10-dof analytical results obtained by Noor and Nemeth. It should be noted that the equivalent beam results for the Vierendeel frame are reported in Table 1 even though the beam properties did not converge to a limiting set of values when using successively longer beam segments.

The reason that the 6-dof models are unsatisfactory for the Vierendeel frame is that the frame behaves in a particularly unbeamlike manner.

Qualitatively, this difference may be attributed to the fact that the longerons bend rather than stretch for its fundamental bending modes. The cross sections of the Vierendeel beam therefore do not rotate for these fundamental modes as is normally the case for trusses. Moreover, the torsion modes are unusually coupled with cross-sectional stretching. The 10-dof analytical technique of Noor and Nemeth can account for these effects as demonstrated in Ref 1. Alternatively, the truss element reduction technique using additional retained freedoms can be effectively used as shown in Table 1.

The addition of cross-bracing to the Vierendeel frame increases the shear stiffness of the structure and, as a result, the structure behaves more like a beam. The results of Table 2 indicate that satisfactory results for the double braced frame can be obtained using either the equivalent beam method or the truss element reduction method.

3.0 Reduced Order Models for Square Cross-section Trusses

The purpose of this section is to apply the three numerical reductions methods to square cross-section trusses and to compare the results. The analyses are conducted only for cantilevered structures having ten bays.

The structures analyzed are those defined by Noor in Ref 3. The trusses are square in cross-section and vary in their bracing schemes. Repeating elements have single bracing (two bays per repeating element) and double bracing (one bay per repeating element) . Each configuration is examined with and without cross bracing. The latter configuration is kinematically stable only when rigid boundary conditions are specified. The advantage of such a configuration is that the truss may be folded flat for storage in the Shuttle cargo bay. The disadvantage is that low frequency shear and warping modes are introduced.

Tables 4 through 6 show the frequencies of the cantilevered structures using various reduction schemes and retained freedoms. These results are also compared with the exact results of Noor and Nemeth. Again unexplained differences appear between our exact results and those of Noor and Andersen but these are very small.

Evaluations of the results for the various reduction schemes are also given in Tables 4 through 6. In all cases the post-assembly and element reduction schemes gave excellent results and accounted for the shear and warping modes of the unbraced structures. Table 5 also shows that these shear and warping modes disappear when cross bracing is introduced and that the reduced order models need only account for the usual six degrees of freedom of an equivalent beam node.

The modeling assumption of using pinned instead of fixed joints was also examined for the single bay, double laced frame with cross bracing. Results are shown in Table 5. Several conclusions may be drawn from the results tabulated there. First, the primary bending and torsion modes are not affected by fixing the joint rotation freedoms. Second, many local member modes which were assumed to be high frequency modes for the pinned structure are in fact low frequency modes. The reason why the local member modes were not calculated for the pinned case is due to the fact that only translational freedoms for nodes only at the ends of each local member were retained. The local member modes would have appeared had nodes been placed midway along each member. And third, while the numerical reduction techniques presented here and Noor's equivalent beam method can all accurately predict the primary modes of a truss, they cannot account for local member modes.

In conclusion, the primary modes of the square trusses studied in this section are almost unaffected by the presence or absence of pins at the joints; warping and shear modes are of course suppressed by fixing the joints. Also, when square trusses have no cross bracing, two extra freedoms must be retained with the usual six beamlike freedoms in order to account for the warping and shear modes exhibited by such a structure.

4.0 Reduced Order Models for The Rockwell Truss

The purpose of this section is to apply the numerical reductions methods to a cantilevered Rockwell truss configuration and to examine various modeling approximations and preload effects on the modes and frequencies. These analyses were performed to get a preliminary understanding of the behavior or the truss. The Rockwell Truss is a double bay single laced square deployable truss. The batten and intermediate joints are fixed while all other joints are pinned in one direction. Several NASTRAN models of the truss were constructed using either all bar elements, all rod elements except for bars for the battens, or all rod elements. Detailed modeling of the joints were not included in these NASTRAN models of the Rockwell truss. Results of several NASTRAN analyses are summarized below: The cantilevered frequencies resulting from four different modeling schemes are presented in Table 7. The modes are plotted in Figure 5 . Differences in response between the various element configurations are due primarily to the different mass representations used. The consistent mass formulation produced a model having a higher torsional inertia and accounted for the local batten modes.

C-7

These local modes vanish from the solution when the lumped mass approach was used or when all joints were modeled as pinned. The modeling assumption of using pinned instead of fixed joints had negligible effect on the calculated stiffness of the structure. Table 8 presents the results of the preload study. The truss was subjected to a 100 pound and 200 pound axial preload and the first order nonlinear differential stiffness solution was obtained. Table 8 shows that the change in the frequency is small and varies approximately linearly with the preload. Large geometry effects under preload were not accounted for. Table 9 presents the cantilevered frequencies calculated using various numerical reduction schemes.

Translational degrees of freedom	Rotational degrees of freedom
$X = T * X_B$	$\Theta = R * \Theta_B$
$F_B = T^T * F$	$M_B = R^T * M$
$T^{-1} = D^{-1} * T^T$	$R^{-1} = S^{-1} * R^T$
$D = T^T * T$	$S = R^T * R$

Nomenclature

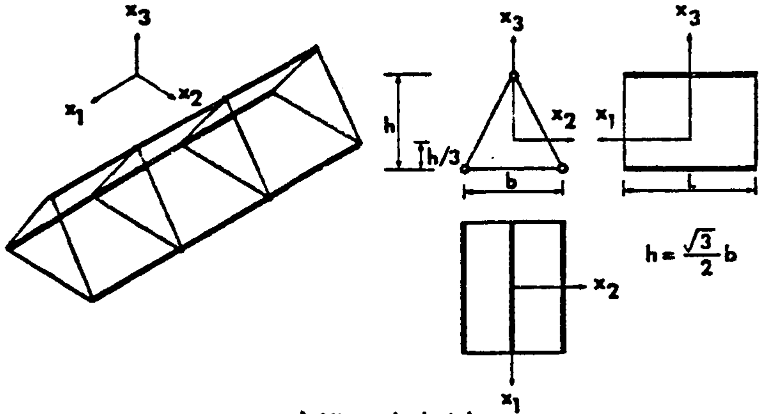
X, F = Vector of nodal translational displacements and forces, respectively,at the verticies of of the lattice cross-section.

X_B, F_B = Vector of beamlike displacements and loadings, respectively, due to translational displacements and loadings.

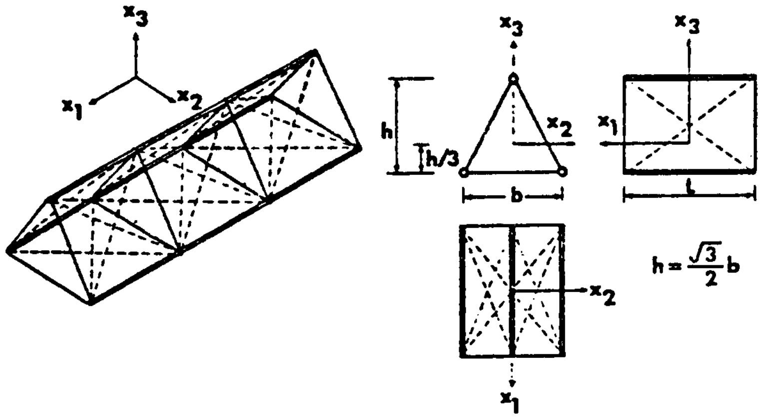
Θ, M = Vector of nodal rotational displacements and moments, respectively

Θ_B, M_B = Vector of beamlike rotational displacements and moments, respectively, due to rotatioal degrees of freedom at the nodes.

Figure 1. Beamlike Transformation Relations



a) Vierendeel girder



b) Double-laced girder

	C. Sec. Area	Length	Moments of Inertia	Torsional Constant	Material Density	Designation
Longerons	A_ℓ	L	$I_{\ell 2}, I_{\ell 3}$	J_ℓ	ρ_ℓ	═══════
Battens	A_b	b	I_{b2}, I_{b3}	J_b	ρ_b	———————
Diagonals	A_d	d	I_{d2}, I_{d3}	J_d	ρ_d	- - - - - - -

$E = 6.895 \times 10^{10}\ N/m^2$, $A_\ell = 3.0 \times 10^{-5}\ m^2$

$G = 2.652 \times 10^{10}\ N/m^2$, $A_b = A_d = 1.5 \times 10^{-5}\ m^2$

$\rho_\ell = \rho_b = \rho_d = 2768\ Kg$, $I_{\ell 2} = I_{\ell 3} = I_\ell = 6.0 \times 10^{-9}\ m^4$

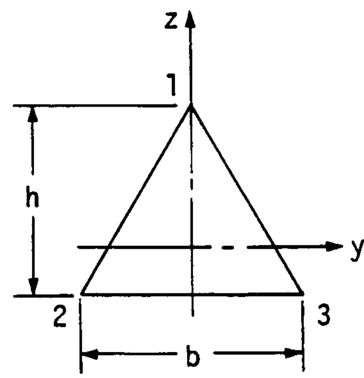
$L = 0.75\ m$, $I_{b2} = I_{b3} = I_{d2} = I_{d3} = 6.5 \times 10^{-10}\ m^4$

$b = 0.75\ m$, $J_\ell = 1.2 \times 10^{-8}\ m^4$

$J_b = J_d = 1.3 \times 10^{-9}\ m^4$

Figure 2. Beamlike Lattices used in present study.

$$
\begin{bmatrix} x^{(1)} \\ x^{(2)} \\ x^{(3)} \\ y^{(1)} \\ y^{(2)} \\ y^{(3)} \\ z^{(1)} \\ z^{(2)} \\ z^{(3)} \end{bmatrix}
=
\begin{bmatrix}
1 & & & & 2h/3 & 0 & & & \\
1 & & & & -h/3 & b/2 & & & \\
1 & & & & -h/3 & -b/2 & & & \\
 & 1 & & -\frac{2h}{3} & & & -\frac{2h}{3} & 0 & 0 \\
 & 1 & & h/3 & & & h/3 & -s/3 & s/3 \\
 & 1 & & h/3 & & & h/3 & s/3 & -s/3 \\
 & & 1 & 0 & & & 0 & 1/3 & 1/3 \\
 & & 1 & -b/2 & & & b/2 & -c/3 & -c/3 \\
 & & 1 & b/2 & & & -b/2 & -c/3 & -c/3
\end{bmatrix}
=
\begin{bmatrix} X \\ y \\ Z \\ \Theta_x \\ \Theta_y \\ \Theta_z \\ \alpha_x \\ \eta_r \\ \eta_s \end{bmatrix}
$$



where C = Cos 60 = 1/2 $\quad$ $(x^{(1)}, y^{(1)}, z^{(1)})$ = Displacements at Node 1

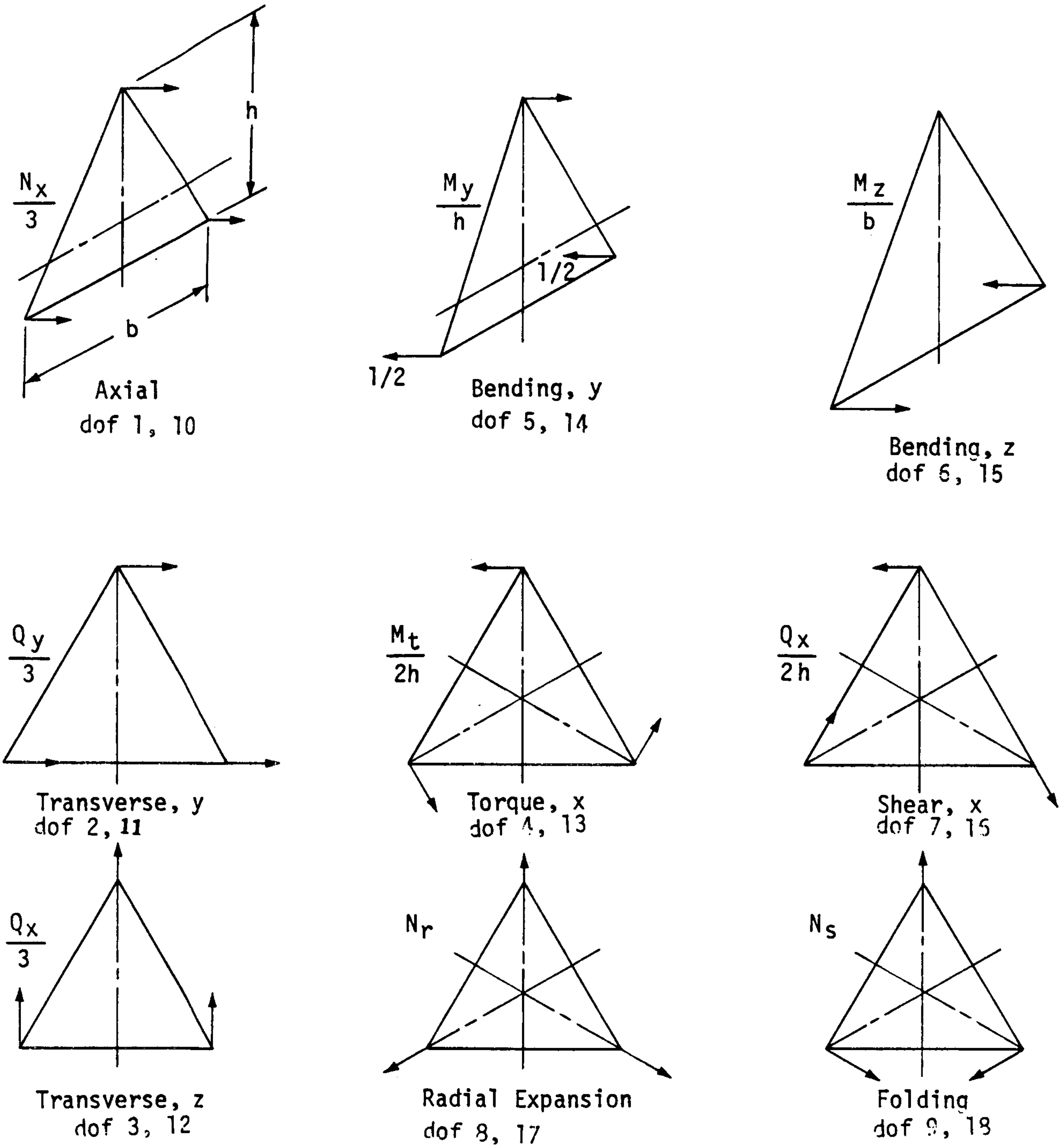
S = Sin 60 = 1/2 $\quad$ $(0_x^{(1)}, 0_y^{(1)}, 0_z^{(1)})$ = Rotations of Node 1

$h = b_s$

Figure 3 (a). Expanded Transformation For Triangular Trusses $X = X_B$

$$
\begin{bmatrix} \Theta_x^{(1)} \\ \Theta_x^{(2)} \\ \Theta_x^{(3)} \\ \Theta_y^{(1)} \\ \Theta_y^{(2)} \\ \Theta_y^{(3)} \\ \Theta_z^{(1)} \\ \Theta_z^{(2)} \\ \Theta_z^{(3)} \end{bmatrix}
\begin{bmatrix}
1 & & & & 1/2 & 0 & & & \\
1 & & & & -c/2 & 1/2 & & & \\
1 & & & & -c/2 & -1/2 & & & \\
 & 1 & & -1/2 & & & -1/2 & 0 & 0 \\
 & 1 & & c/2 & & & c/2 & -s/3 & s/3 \\
 & 1 & & c/2 & & & c/2 & s/3 & -s/3 \\
 & & 1 & 0 & & & 0 & 1/3 & 1/3 \\
 & & 1 & -s/2 & & & s/2 & -c/3 & -c/3 \\
 & & 1 & s/2 & & & -s/2 & -c/3 & -c/3
\end{bmatrix}
\begin{bmatrix} \phi_x \\ \phi_y \\ \phi_z \\ \phi_1 \\ \phi_2 \\ \phi_3 \\ \phi_4 \\ \phi_5 \\ \phi_6 \end{bmatrix}
$$

Figure 3 (b). Expanded Transformation Relation
$\Theta = R\,\Theta_B$ For Triangular Trusses



Note: Arrows on Nodes Forces for each Beamlike Loading Condition

Figure 3 (c). Beamlike Loadings For Triangular Trusses

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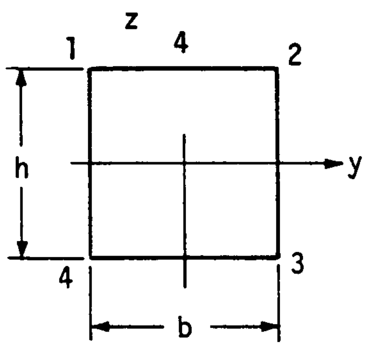
$$
\begin{bmatrix} N_x \\ Q_y \\ Q_z \\ T_x \\ M_y \\ M_z \\ B_x \\ N_y \\ N_z \\ Q_x \\ B_y \\ B_z \end{bmatrix}
=
\left[\begin{array}{cccc|cccc|cccc}
1 & 1 & 1 & 1 & & & & & & & & \\
 & & & & 1 & 1 & 1 & 1 & & & & \\
 & & & & & & & & 1 & 1 & 1 & 1 \\
 & & & & h/2 & h/2 & -h/2 & -h/2 & b/2 & -b/2 & -b/2 & b/2 \\
h/2 & h/2 & -h/2 & -h/2 & & & & & & & & \\
b/2 & -b/2 & -b/2 & b/2 & & & & & & & & \\
1 & -1 & 1 & -1 & & & & & & & & \\
 & & & & -1 & 1 & 1 & -1 & & & & \\
 & & & & & & & & 1 & 1 & -1 & -1 \\
 & & & & b/2 & b/2 & -b/2 & -b/2 & -h/2 & h/2 & h/2 & -h/2 \\
 & & & & h/2 & -h/2 & h/2 & -h/2 & & & & \\
 & & & & & & & & b/2 & -b/2 & b/2 & -b/2
\end{array}\right]
\begin{bmatrix} F_x(1) \\ F_x(2) \\ F_x(3) \\ F_x(4) \\ F_y(1) \\ F_y(2) \\ F_y(3) \\ F_y(4) \\ F_z(1) \\ F_z(2) \\ F_z(3) \\ F_z(4) \end{bmatrix}
$$



Alternate for N_y and N_z

$$
\begin{bmatrix} N_r \\ N_t \end{bmatrix}
=
\left[\begin{array}{c|cccc|cccc}
0 & -h/2 & h/2 & h/2 & -h/2 & b/2 & b/2 & -b/2 & -b/2 \\
0 & -b/2 & b/2 & b/2 & -b/2 & -h/2 & -h/2 & h/2 & h/2
\end{array}\right]
$$

Figure 4 (a). Expanded Transformation Relation $F_B = T^T F$ For Square Trusses

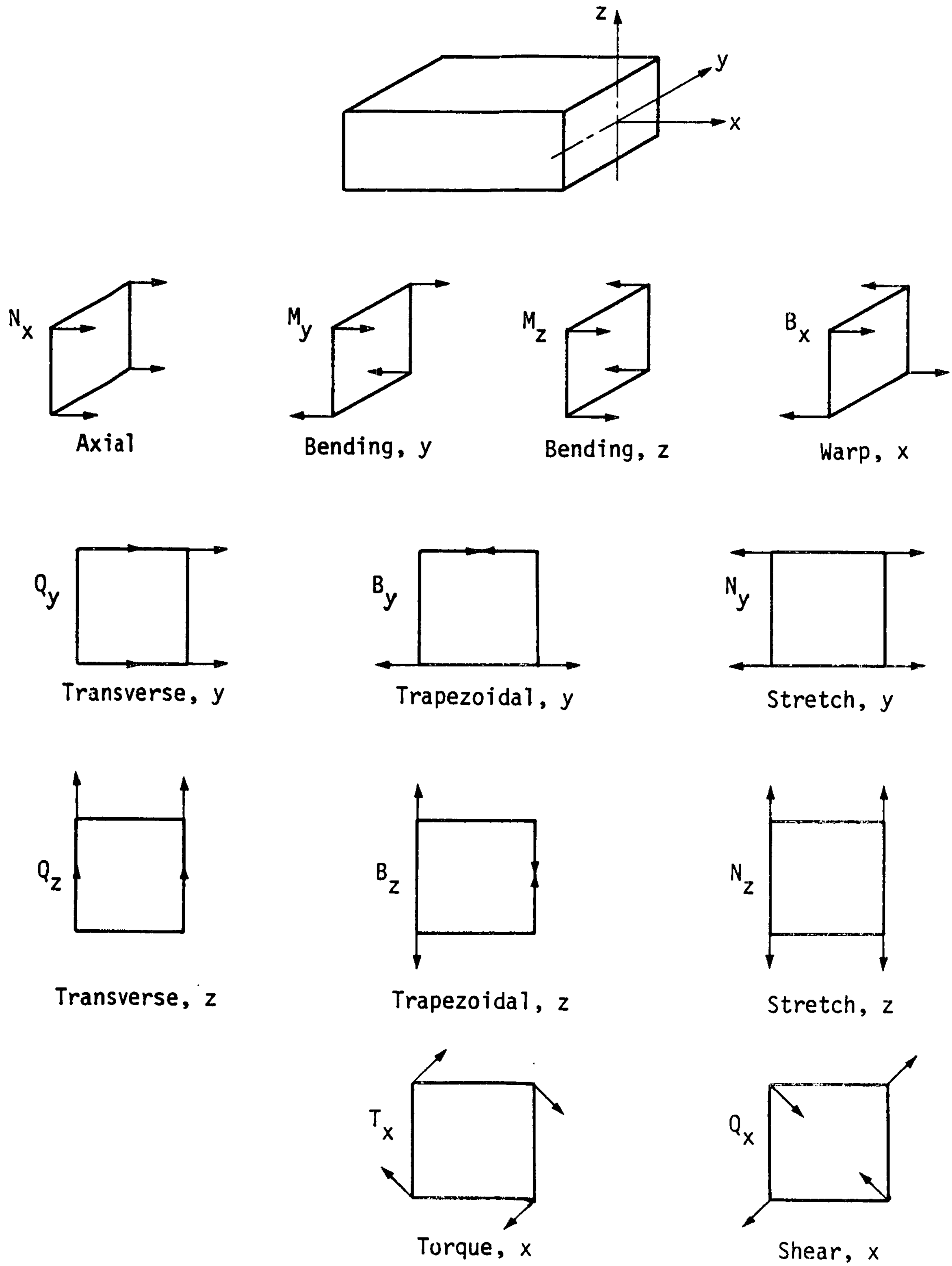


Figure 4 (b). Beamlike Loadings For Square Trusses

10 Bays with 780 lb Tip Mass

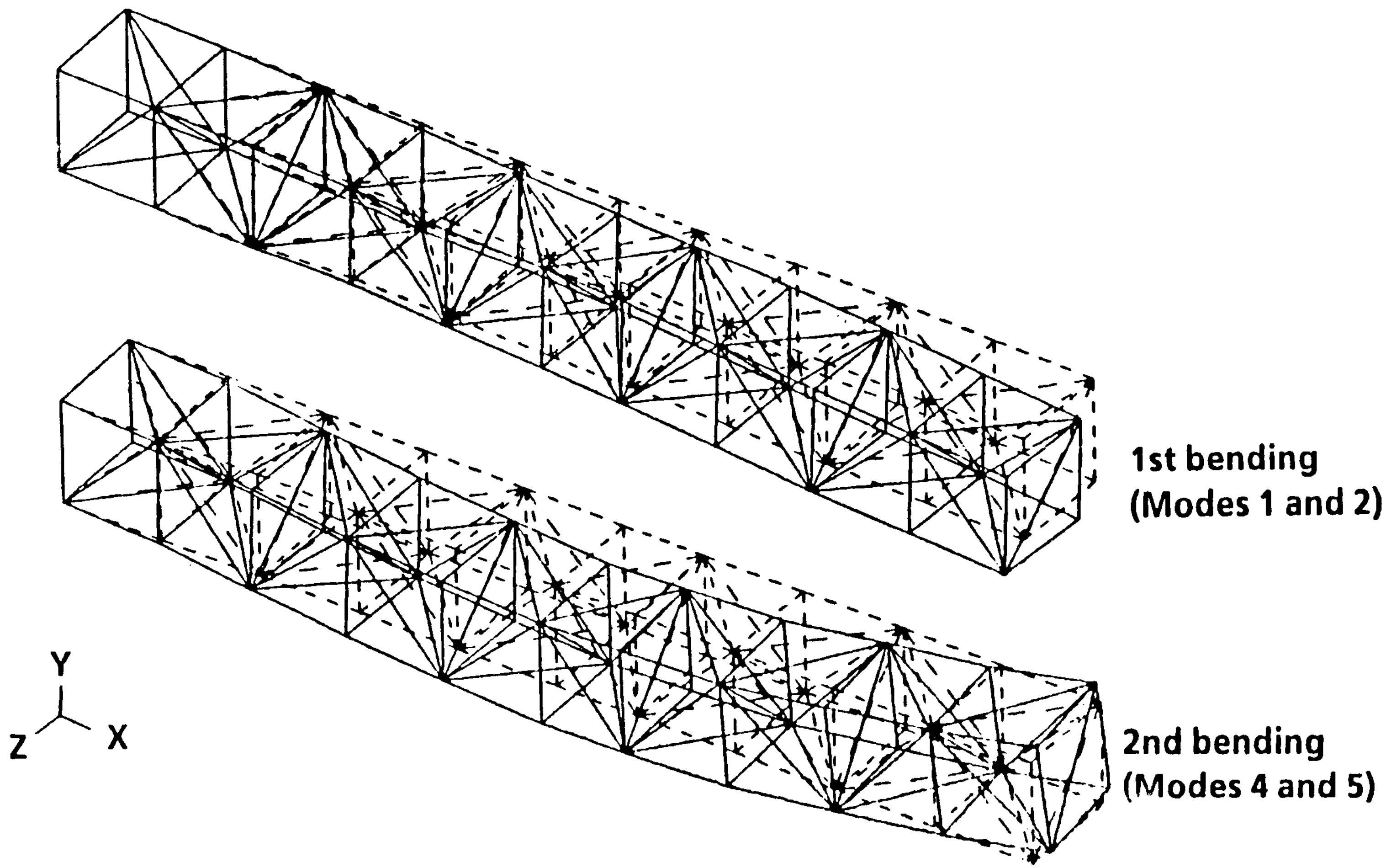


Figure 5.

TABLE 1

FREQUENCIES FOR CANTILEVERED 10-BAY VIERENDEEL FRAME

	Finite Element Model		Post-assembly Reduction				Element Reduction			Equivalent Beam	
Mode	Noor	BAC	10-DOF	8-DOF	4-DOF	4-DOF / 2 bays	Guyan 12-DOF	Guyan 6-DOF	SPC 6-DOF	Scalar Mass	(12x12 Mass
1b	2.285	2.244	2.244	2.244	2.244	2.245	2.284	1.975	4.509	2.174	2.245
1t	4.970	4.926	4.926	4.926	4.926	4.931	4.975	3.118	6.769	4.753	4.899
2b	7.554	7.464	7.465	7.465	7.465	7.490	7.550	5.945	13.851	6.471	6.742
2t	14.670	14.602	14.602	14.602	14.604	14.741	14.680	9.391	20.631	14.155	14.719
3b	14.747	14.626	14.631	14.634	14.635	14.841	14.739	10.036	24.700	10.754	11.359
4b	24.449	24.251	24.270	24.276	24.284	25.038	24.438	15.761	----	23.240	21.836
3t	24.253	24.276	24.276	24.276	24.287	25.162	24.264				
4t		34.921	34.921	34.921	34.961	36.932					
5b		36.660	36.719	36.729	36.762	37.854					
DOF used see Fig 4		1-18	1-6 10-12 17	1-4 10-12 17	1-4	1-4	1-6 10-12 14,15,17	1-6	1-6	1-6	1-6
Total DOF	100	180	100	80	40	20	120	60	60	60	60
Evaluation			E	E	E	G	E	U	U	U	U

Note: (b)=bending (t)=torsion (e)=extension ; (E)=Excellent (G)=Good (U)=Unsatisfactory
Each bending listed above represent two bending modes with identical frequecies.

TABLE 2

FREQUENCIES FOR CANTILEVERED 10-BAY DOUBLE BRACED FRAME/TRUSS

	Finite Element Model		Post-assembly Reduction					Element Reduction		Equivalent Beam	
Mode	Noor Exact Frame	BAC Exact Frame	10-DOF Frame	10-DOF Frame	9-DOF Truss	6-DOF Truss	4-DOF Truss	Guyan 6-DOF	SPC 6-DOF	Scalar Mass	(12x12) Mass
1b	8.942	8.796	8.805	8.796	8.877	8.877	8.877	8.802	12.610	8.900	8.792
1t	35.548	35.345	35.345	35.344	35.321	35.321	35.321	35.306	55.372	33.472	34.771
2b	47.930	47.323	48.661	47.327	49.490	49.492	49.492	48.716	54.632	50.276	49.814
3b	96.315(1)	95.990	----	96.039	----	----	----	106.062	110.417	99.594	90.321
1e	104.089	103.083	103.461	103.461	105.271	105.272	105.272				
2t	104.068	103.461	106.512	-----	106.946	106.846	106.846				
DOF used See Fig 4		1-18	1-6 10-12 17	2-4 10-15 17	1-9	1-4 7,9	1-4	1-6	1-6	1-6	1-6
Total DOF		180	100	100	90	60	40	60	60	60	60
Evaluation			E	E	E	E	E	E	U	G	G

Note: (b)=bending (t)=torsion (e)=extension (1)=local ; (E)=Excellent (G)=Good (U)=Unsatisfactory
Each bending listed above represent two bending modes with identical frequecies.

TABLE 3

FREQUENCIES FOR CANTILEVERED 20-BAY DOUBLE BRACED FRAME/TRUSS

Finite Element Model		Post-assembly Reduction			Element Reduction		Equivalent Beam	
Mode	BAC Exact Frame	4-DOF Frame	9-DOF Truss	4-DOF Truss	9-DOF Truss	4-DOF Truss	6-DOF Frame	6-DOF Frame
1b	2.256	2.256	2.250	2.251	2.223	2.223	2.258	2.259
2b	13.622	13.691	13.565	13.600	13.397	13.405	13.748	13.751
1t	17.756	17.830	14.421	14.476	14.232	14.232	15.551	15.539
3b	35.711	36.986	35.923	36.564	35.764	35.936	36.873	36.926
2t	52.980	53.617	43.180	44.645	43.799	43.800	46.557	46.235
1e	53.021	54.974	53.410	53.360	52.912	52.913	53.310	53.269
4b	62.732	70.280	65.528	69.101	66.483	67.518	68.305	68.625
DOF used See Fig 4	1-18	1-4 every 2 bays	1-9 every bay	1-4 every 4 bays	1-9 every 4 bays	1-4 every 4 bays	1-6 every bay	1-6 every 2 bays
Total DOF	360	20	180	20	45	20	120	60
Evaluation		E	G	G	G	G	G	G

Note: (b)=bending (t)=torsion (e)=extension (l)=local ; (E)=Excellent (G)=Good
(U)=Unsatisfactory
Each bending listed above represent two bending modes with identical frequecies.

TABLE 4

SINGLE BAY DOUBLE-LACED SQUARE TRUSSES

Pinned Joints and No Batten Cross Bracing
10 Bays Cantilevered

Mode	Finite Element Model			Post-assembly reduction	element reduction
	Noor Beam theory	Noor EXACT	BAC EXACT	8-DOF	8-DOF
1w	0.6060	0.6055	0.6035	0.6035	.6035
1b	0.8335	0.8368	0.8300	0.8286	.8440
2w	3.5742	3.6051	3.5936	3.5936	3.5940
1t	4.1545	4.1542	4.1439	4.1439	4.1439
2b	4.5723	4.6539	4.6192	4.6155	4.6805
3w	9.2143	9.4458	9.4131	9.4131	9.4131
3b	10.9937	11.4168	11.3301	11.3271	11.4420
2t	12.4635	12.4549	12.4144	12.4143	12.4145
1e	12.5104	12.5559	12.4478	12.4276	12.8483
4w	16.3566	17.0596	16.9856	16.9856	16.9857
DOF used	1-8	1-12	1-12	1-8	1-8
Total DOF		120	120	80	80
Evaluation				E	E

Note: (b)=bending (t)=torsion (e)=extension (l)=local ;
(E)=Excellent (G)=Good (U)=Unsatisfactory
Each bending listed above represent two bending modes with identical frequecies.

TABLE 5

SINGLE BAY DOUBLE-LACED SQUARE TRUSSES

With Batten Cross Bracing
10 Bays Cantilevered

Mode	Finite Element Model		Post-assembly reduction	element reduction
	Exact Pinned Joints	Exact Fixed Joints	6-DOF Pinned	6-DOF Pinned
1b	0.790	0.784	0.790	0.799
1t	4.062	4.041	4.062	4.062
2b	4.399	4.285	4.399	4.438
3b	10.779	local	10.789	10.849
2t	12.007	"	12.006	12.166
1e	12.166	"	12.166	12.197
4b	18.376	"	18.376	18.457
3t	20.200	"	20.200	20.200
5b	26.643	"	26.664	26.716
DOF used	1-12	1-24	1-6	1-6
Total DOF	120	240	60	60
Evaluation			E	E

Note: (b)=bending (t)=torsion (e)=extension (l)=local ;
(E)=Excellent (G)=Good (U)=Unsatisfactory
Each bending listed above represent two bending modes with identical frequecies.

TABLE 6

DOUBLE BAY SINGLE-LACED SQUARE TRUSSES

Pinned Joints and No Batten Cross Bracing
10 Bays Cantilevered

Mode	Finite Element Model			Post-assembly reduction		element reduction
	Noor Beam theory	Noor EXACT	BAC EXACT	8-DOF per bay	8-DOF every 2 bays	8-DOF
1w	.6658	.6655	.6603	.6602	.6603	.6603
1b	.8339	.8341	.8229	.8229	.8230	.8229
1t	2.8720	2.8585	2.8585	2.8656	2.8591	2.8586
2w	3.7418	3.7787	3.7514	3.7514	3.7659	3.7527
2b	4.2314	4.2799	4.2314	4.2314	4.2567	4.2341
2t	8.1659	8.5960	8.5542	8.5542	8.7391	8.5697
3w	9.1783	9.3900	9.3251	9.3253	9.5299	9.3446
4b	9.6439	9.8714	9.7668	9.7668	10.0551	9.7990
1e	11.7044	11.6173	11.4990	11.4990	11.5355	11.5005
3t	14.3598	14.3663	14.2857	14.2862	15.0779	14.3579
DOF used	1-8	1-12	1-12	1-8	1-8 every 2 bays	1-8
Total DOF	80	120	120	80	40	80
Evaluation				E	E	E

Note: (b)=bending (t)=torsion (e)=extension (l)=local ;
(E)=Excellent (G)=Good (U)=Unsatisfactory
Each bending listed above represent two bending modes with identi
frequecies.

TABLE 7(a)

EFFECT OF MODEL VARIATIONS ON CANTILEVERED FREQUENCIES OF THE ROCKWELL TRUSS

10 Cantilevered Bays

Mode	Model 1 (hz)	Model 2 (hz)	Model 3 (hz)	Model 4 (hz)
1b(z)	5.361	5.390	5.359	5.360
1b(y)	5.529	5.556	5.527	5.527
1t	22.248	24.248	21.020	21.014
2b(z)	26.734	26.854	26.839	26.843
2b(y)	28.489	28.502	28.597	28.574
local	38.654	----	----	----
local	42.345	----	----	----
2t	53.648	55.396	51.113	55.414

Note: (b)=bending (t)=torsion (e)=extension (l)=local;
(E)=Excellent (G)=Good (U)=Unsatisfactory

TABLE 7(b)

DESCRIPTION OF THE SELECTED NASTRAN MODELS

Description	Model 1	Model 2	Model 3	Model 4
battens	bars	bars	bars	rods
longerons	rods	bars	rods	rods
batten joints	fixed	fixed	fixed	pinned
other joints	pinned	fixed	pinned	pinned
mass dist.	coupled	consistent	lumped	consistent

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