VIBRATION SUPPRESSION BY STIFFNESS CONTROL

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Control Interaction of Flexible Structures

The objectives of large space structure control can be divided into three general categories. They are: 1) Pointing, 2) Vibration Suppression, and 3) Shape Control.

The objective of Pointing is not new to the spacecraft control community. This objective has been part of essentially every space mission of the past 25 years. The Pointing objective includes both spacecraft orientation (in the rigid body sense) and the orbit of the spacecraft.

The other categories are new, however, with the new class of flexible structures. Both vibration and shape are related to the elastic nature of the structure and are therefore fundamentally different phenomena than those encountered in rigid body attitude control. This distinction is true even for nonlinear structures.

In short, the control objectives of Vibration Suppression and Shape Control have more in common with each other than either has in common with the objective of Pointing. We consider this point of view significant.

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CONTROL OF FLEXIBLE STRUCTURES

- Pointing [Rigid Body -- Optimal Control]

- Vibration Suppression [Flexible Body]

- Shape Control [Flexible Body]

The new control objectives have immediate implications for control actuator selection. Traditionally, "external force producing" actuators such as thrusters and momentum wheels have been used. This is in keeping with the traditional Pointing objective for which this type of actuator is required.

This type of actuator is, however, highly undesirable for Vibration Suppression or Shape Control, for two reasons. First, external forces will couple the flexible motion to the rigid motion, and second, these devices expend consumables, or otherwise cannot be operated quasi-statically.

There are "internal force producing" devices available for use as actuators which are electrically powered. Among these are piezoelectric and thermoelectric devices. Since electricity is a renewable power source in space these types of actuators are well suited for Vibration Suppression and Shape Control.

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CONTROL ACTUATOR TYPES

- **External Force Producing [Thrusters - Moment Wheel]**
  - -- Required for Rigid Body Pointing
  - -- Undesirable for Vibration Suppression [Rigid Coupling]
  - -- Unusable for Shape Control [Consumables]

- **Internal Force Producing [Piezo /Thermo - Electric]**
  - -- Unusable for Rigid Body Pointing
  - -- Desirable for Vibration Suppression [No Coupling]
  - -- Essential for Shape Control

697

The ideal actuator or sensor for Vibration Suppression would be electrically powered, would be highly efficient, would weigh nothing, and would have a very high bandwidth. The high bandwidth requirement is important so that the rolloff characteristics of the closed loop system can be designed without regard to actuator dynamics.

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IDEAL ACTUATOR/SENSOR FOR VIBRATION SUPPRESSION

- Electrically Powered

- 100% Electro-Mechanical Efficiency

- Zero Mass

- Infinite Bandwidth

Our experiment utilizes piezoelectric ceramics as both sensors and actuators. Two types of piezoelectric materials are available to choose from, piezoelectric ceramics and piezoelectric polymers, and they each have specific advantages and disadvantages.

Both types are electrically powered, are low in mass, and have high bandwidth. The principal advantages of the ceramics are that they have the highest electromechanical efficiency, and operate at low voltage. The major advantage of the polymers is the possibility of casting the material into a dual structure/control element.

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PIEZO-ELECTRIC MATERIALS AS ACTUATORS/SENSORS

- **Piezo-Electric Ceramics**
  - -- Electrically Powered
  - -- Highest Electro-Mechanical Efficiency
  - -- Low Mass
  - -- Essentially Infinite Bandwidth
  - -- Low Voltage

- **Piezo-Electric Polymers**
  - -- Electrically Powered
  - -- High Electro-Mechanical Efficiency
  - -- Lowest Mass
  - -- Essentially Infinite Bandwidth
  - -- High Voltage
  - -- Possibility of Being Cast as Structural Member

Most of the significant aspects of structural dynamics for large space structures as they pertain to Vibration Suppression can be understood from a study of the single degree-of-freedom (SDOF) damped linear oscillator. The canonical form of the forced SDOF damped linear oscillator is shown.
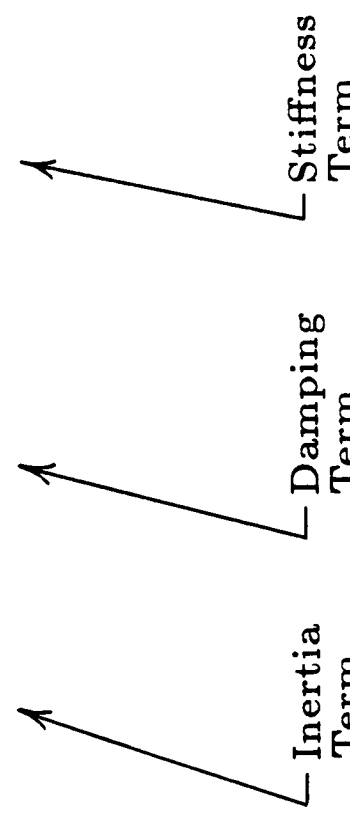
Single Degree-of-Freedom Damped Linear Oscillator

$$m\ddot{x}(t) + c\dot{x}(t) + kx(t) = F(t)$$

Substitute:

$$\frac{k}{m} \equiv \omega_n^2$$

$$\frac{c}{2\sqrt{km}} \equiv \varsigma \qquad \left(\text{Damping Ratio } \frac{c}{c_{\text{critical}}}\right)$$

$$\ddot{x}(t) + 2\varsigma\omega_n\dot{x}(t) + \omega_n^2 x(t) = \frac{1}{m}F(t) = \frac{\omega_n^2}{k}F(t)$$

└ Inertia Term

└ Damping Term

└ Stiffness Term

703

There are two types of structural dynamic response of interest. The first type is the forced response. When a structure is excited by a persistent disturbance, such as a sinusoidal forcing function, the amplitude of the response increases near structure's natural frequency. For a critical damping ratio ς less than 0.707 the maximum amplitude of response occurs at a frequency of $\omega = \omega_n \sqrt{1 - 2\varsigma^2}$. The quantity of interest in minimizing the amplitude of response is $\varsigma \omega_n^2$. For small damping (less than 0.2) the amplitude of response is inversely proportional to this quantity.

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Forced Response:

$$\ddot{x}(t) + 2\varsigma\omega_n\dot{x}(t) + \omega_n^2 x(t) = \frac{\omega_n^2}{k}P\cos\omega t$$

If $\varsigma < 0.707$

$$\left|x(t)\right|_{\max} = \frac{P/k}{2\varsigma\sqrt{1-\varsigma^2}} \qquad \text{At:} \quad \omega = \omega_n\sqrt{1-2\varsigma^2}$$

For ς small

$$\left|x(t)\right|_{\max} \approx \frac{P}{2\varsigma k} = \frac{P}{2m\varsigma\omega_n^2}$$

Quantity of Interest: $\quad \varsigma k, \quad$ or $\quad \varsigma\omega_n^2$

The other type of dynamic response of interest is the free response. If a disturbance ceases, how long does it take for the structural response to decay to zero? The quantity of interest in minimizing the settling time is $\zeta\omega_n$. The free decay of the structural response is enveloped by the term $e^{-\zeta\omega_n t}$. As $\zeta\omega_n$ is increased the settling time decreases.

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Free Response:

$$\ddot{x}(t) + 2\zeta\omega_n\dot{x}(t) + \omega_n^2 x(t) = 0$$

$$x(t) = e^{-\zeta\omega_n t}(A\cos\omega_d t + B\sin\omega_d t)$$

Where:

$$\omega_d \equiv \omega_n\sqrt{1-\zeta^2}$$

Settling Time Requirement $\Longrightarrow \zeta\omega_n =$ Desired Value

Quantity of Interest: $\qquad \zeta\sqrt{\frac{k}{m}}, \qquad or \qquad \zeta\omega_n$

The two aspects of structural response which are significant to the objective of Vibration Suppression are the steady state response amplitude characterized by the quantity $\zeta\omega_n^2$, and the free decay settling time characterized by the quantity $\zeta\omega_n$. Ideally, both of these quantities should be maximized to provide the best Vibration Suppression. In practice, however, these two quantites cannot be maximized simultaneously.

Two important observations can be made: First, the stiffness of the structure, or equivalently, its natural frequency, is much more difficult to alter than is the damping ratio ζ. This is because ζ is usually a small number whereas the stiffness is usually not. Second, the dynamic response is very sensitive to ζ.

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Dynamic Response Mode	Quantity of Interest
Response Amplitude	$\varsigma\omega_n^2$ or ςk
Settling Time	$\varsigma\omega_n$ or $\varsigma\sqrt{k/m}$

Observation: k and ω_n are much more difficult to alter than ς.

Observation: Dynamic response is very sensitive to ς.

The derivation of the SDOF damped linear oscillator's open loop poles is presented. The transfer function $T(s)$ is introduced.

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Transfer Function:

$$\ddot{x}(t) + 2\varsigma\omega_n\dot{x}(t) + \omega_n^2 x(t) = \frac{\omega_n^2}{k}F(t)$$

$$s^2\hat{x}(s) + 2\varsigma\omega_n s\hat{x}(s) + \omega_n^2\hat{x}(s) = \frac{\omega_n^2}{k}\hat{F}(s)$$

$$\frac{\hat{x}(s)}{\hat{F}(s)} = \frac{\omega_n^2/k}{s^2 + 2\varsigma\omega_n s + \omega_n^2}$$

$$T(s) = \frac{\omega_n^2/k}{(s + \varsigma\omega_n + j\omega_d)(s + \varsigma\omega_n - j\omega_d)}, \quad j \equiv \sqrt{-1}$$

Open Loop Poles:

$$s_1 = -\varsigma\omega_n + j\omega_d$$

$$s_2 = -\varsigma\omega_n - j\omega_d$$

The general objective of our Vibration Suppression system is to increase ς by active control thereby moving the system poles further into the left half-plane. The direction the pole should be moved from its open loop position in the complex plane depends on the quantity being maximized. To maximize $\varsigma\omega_n$, that is, to minimize the settling time, the pole should be moved directly left into the left half-plane. To maximize the quantity $\varsigma\omega_n^2$, that is, to minimize the response amplitude, the pole should be moved at an angle of $\tan^{-1}\varsigma\sqrt{1-\varsigma^2}$. For small damping values this angle is essentially zero and then the direction becomes straight back into the left half-plane as before.

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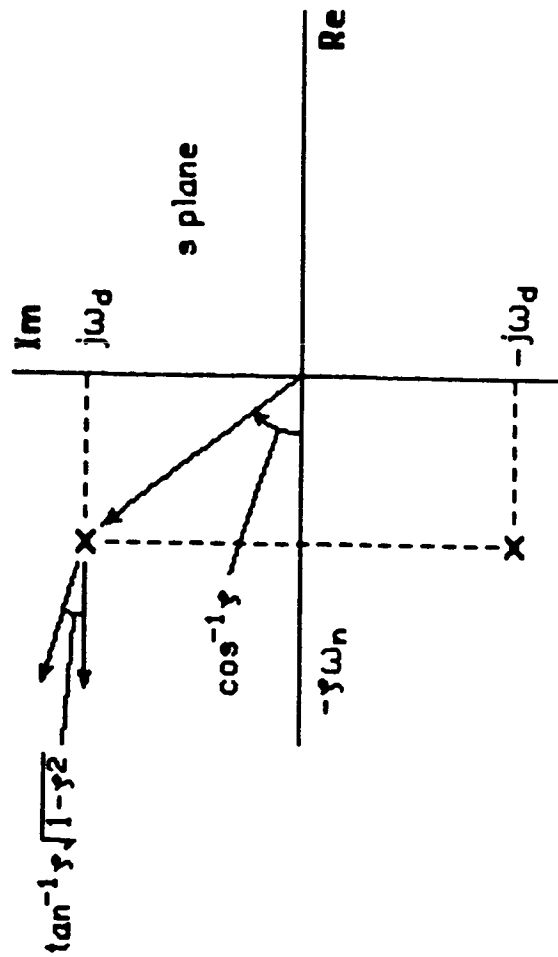
OBJECTIVE: Increase ς By Active Control Thereby Moving Poles Further Into Left Half-Plane.

Our experiment is intended to simulate some of the inherent difficulties of controlling large space structures. The specimen is a thin cantilever beam. It has low natural frequencies, low mass, and low stiffness.

We are using collocated piezoelectric ceramic sensors and actuators.

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STIFFNESS CONTROL EXPERIMENT

- Vibration Suppression of Cantilever Beam

- Test Specimen Displays LSS Pathologies
  - -- Low Stiffness
  - -- Low Mass
  - -- High Modal Density at Higher Frequency

- Collocated Piezo-Electric Ceramic Sensors and Actuators

- Various Control Strategies

The piezoelectric ceramics are 9.5 mil thick. Two ceramics are adhered to the beam (in sandwich fashion) to operate as sensors to measure bending strain. And two ceramics are adhered next to the sensors to apply a bending moment. The poling directions of the ceramics are arranged such that the actuators operate in a push-pull mode and the sensors operate in parallel.

Piezo Cantilever Beam

0.0095 TYP.

0.020

0.648

0.051

0.25

0.50

12.50

1.25

0.031

0.60

0.50

1.50

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Beam

A1

S1

R2

S2
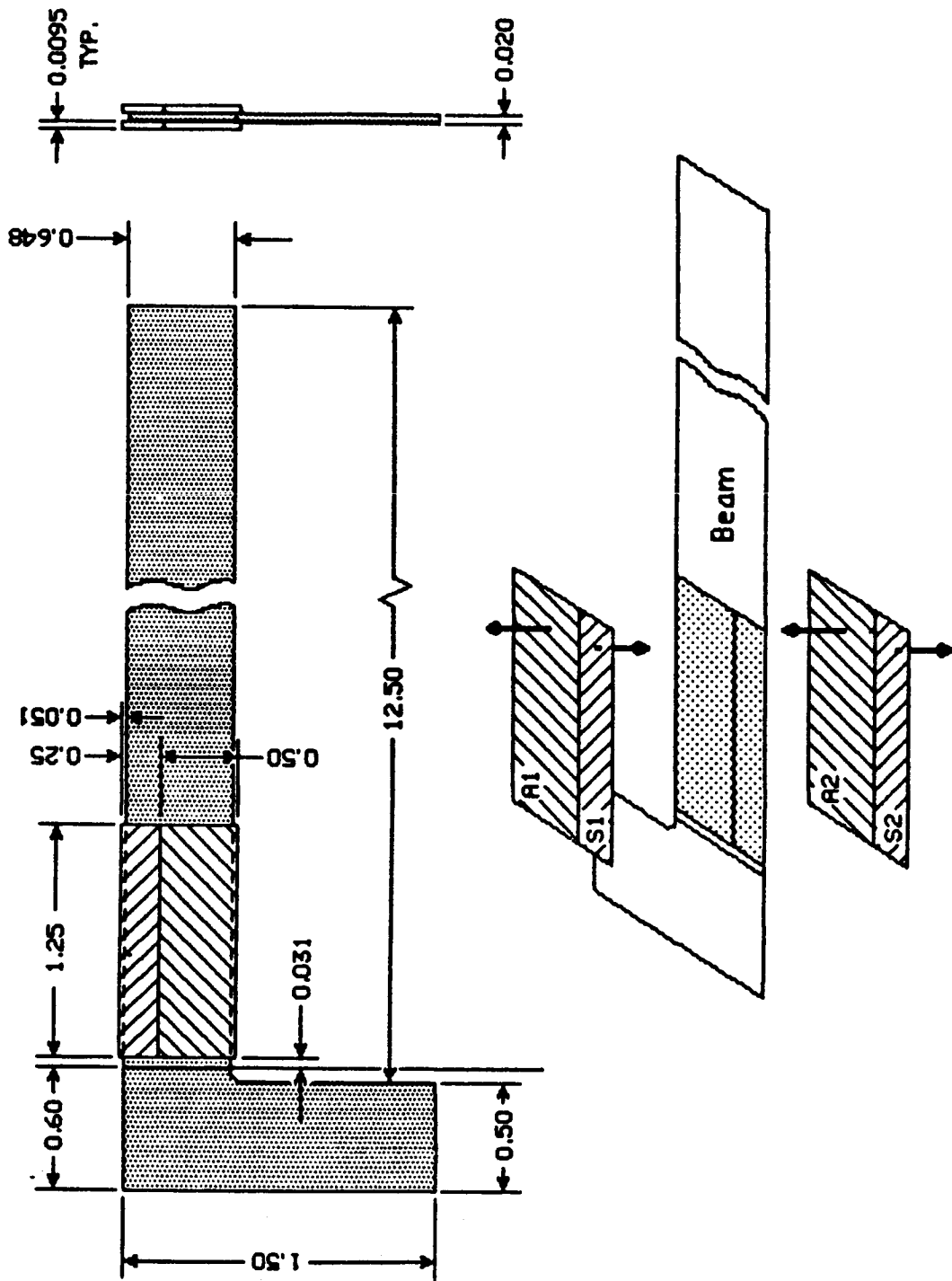
Actuator/Sensor Poling Orientations

The laboratory setup is shown. The beam is cantilevered out of a clamping fixture which is attached to a linear bearing table. The bearing table is driven by a Brüel and Kjær shaker. The ceramic actuators and sensors can be seen attached to the root of the beam. This setup allows us to excite the structure with a wide range of disturbances to measure control system performance.
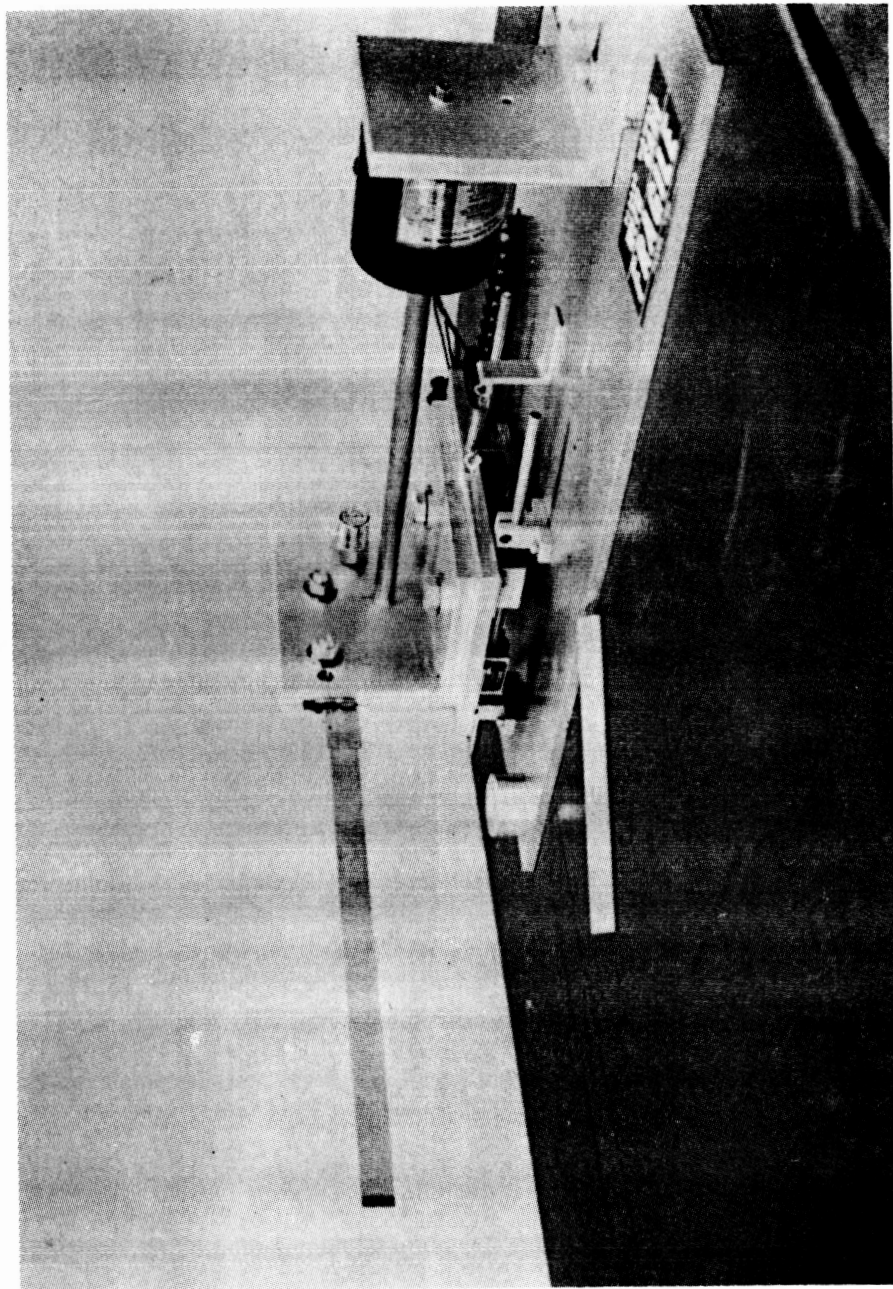
This view shows a close-up of the sensors and actuators. The larger ceramic is the actuator, the smaller is the sensor. The ceramics are attached with a standard cyanoacrylate adhesive in the same manner as strain gauges. Wires are simply soldered to the electrode surfaces.
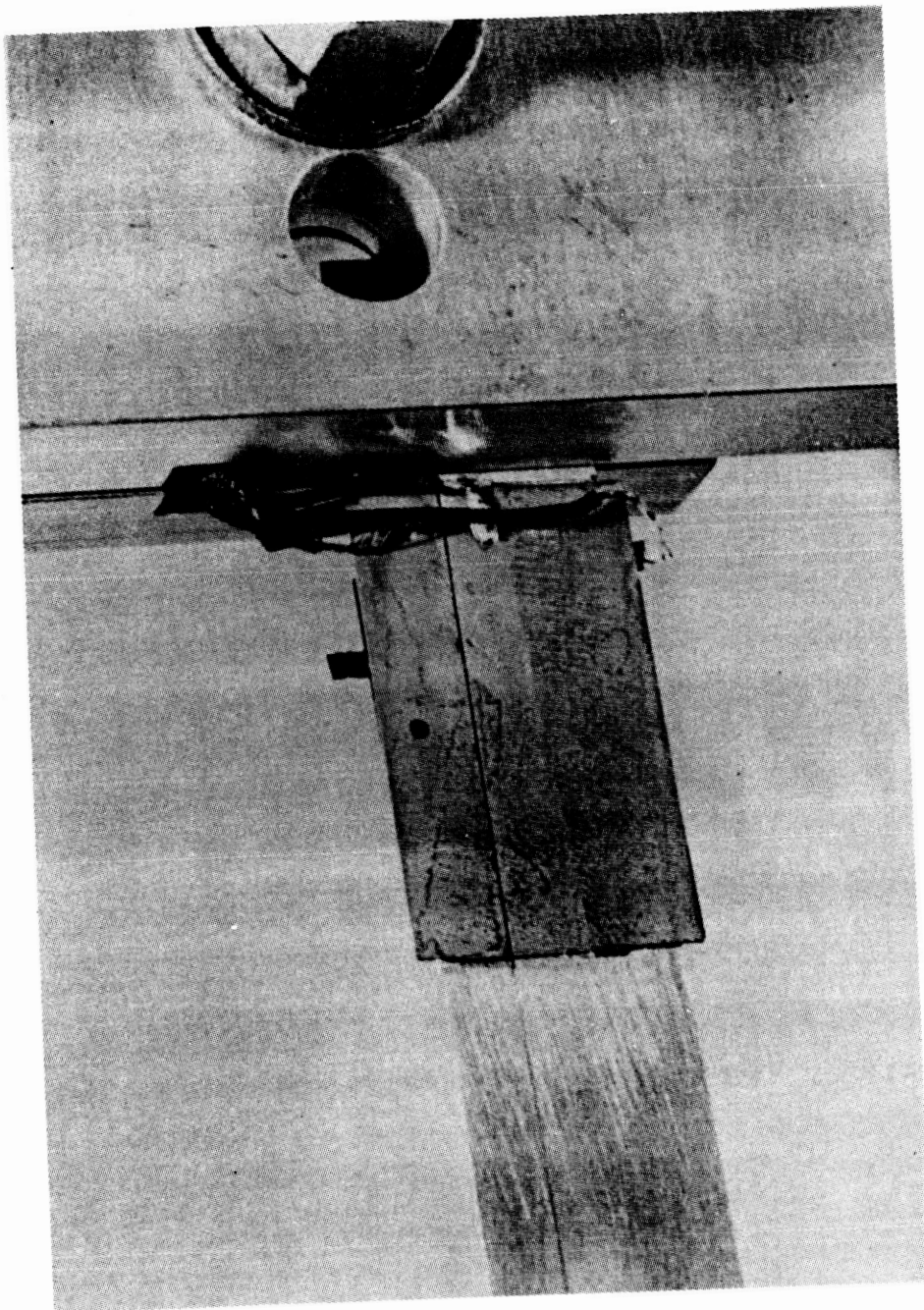
There are limitations in the state-of-the-art of structural modeling for large space structures. In particular, a detailed knowledge of the stiffness matrix of such structures seems very unlikely. Also, good knowledge of mode shapes is uncertain. This has important implications to observer-based control systems since observers are only as good as the structural models available. Furthermore, on-orbit system identification will probably be required for any complicated structure.

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REALITIES OF MODELING

- Knowledge of Stiffness Matrix Limited

- Implementation of Observers Uncertain

- On-Orbit Identification Required

In view of the preceding realities of modeling, a set of desired control system features has been developed.

Firstly, a detailed knowledge of the structure should not be required. The compensator should be designed based on the structure's natural frequencies *only* since these are the structural parameters which are most likely to be known with any accuracy. Mode shape information should be used if available.

Secondly, since the objecive is Vibration Suppression, the measurement variables should be only those quantities related to the elastic deformation of the structure. This implies that inertial sensors such as accelerometers should not be used. The only quantity intimately related to elastic deformation is strain, therefore, we measure strain only. Obtaining velocities from displacement measurements requires differentiation which amplifies noise in addition to having other problems, therefore, no velocites are used.

Thirdly, the control system should be idiot proof. That is, it should be simple to implement, should maintain stability and performance in the face of uncertainty (which defines robustness), and should not compromise the structure if an actuator fails.

Finally, the control system should not destabilize unmodelled or uncontrolled modes.

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DESIRED CONTROL SYSTEM FEATURES

- **Do Not Require Detailed Knowledge of Structure**
  - -- Use Frequencies Only
  - -- Mode Shapes if Available

- **Feedback Only Quantities Related to Deformation**
  - -- No Accelerometers (Inertial)
  - -- Measure Strain Only, No Velocities

- **Idiot-Proof**
  - -- Simple
  - -- Stability and Performance in the Face of Uncertainty

- **Do Not Destabilize Uncontrolled Modes**

Positive Position Feedback is one control strategy which satisfies the preceding set of desirable features. The strategy is outlined in the following.

The first equation is the equation of motion for the modes of the system. The right hand side is the control force vector which is defined later. For the single-input-single-output (SISO) experiment described in these viewgraphs, the measurement v_s is a scalar voltage proportional to a weighted sum of the modal coordinates of the beam excluding the rigid body modes. The structural frequencies ω are assumed known at least for the modes targeted for control.

A set of electrical filters with second order transfer functions similiar to the structural modes is constructed with designed frequencies ω_f near the structural frequencies ω. The damping values ς_f are chosed to be large, say 0.2. The tuned electrical filters are driven by the measurement voltage v_s. The filter coordinates η are multiplied by appropriate gains and added to form the control vector. This vector is then imposed on the structure.

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Positive Position Feedback:

$$\ddot{\xi} + \left[{}_{\diagdown} 2\varsigma\omega {}_{\diagup} \right] \dot{\xi} + \left[{}_{\diagdown} \omega^2 {}_{\diagup} \right] \xi = \text{control force vector}$$

$$v_s = [C]^T \xi$$

$$\ddot{\eta} + \left[{}_{\diagdown} 2\varsigma_f\omega_f {}_{\diagup} \right] \dot{\eta} + \left[{}_{\diagdown} \omega_f^2 {}_{\diagup} \right] \eta = \left[{}_{\diagdown} \omega_f^2 {}_{\diagup} \right] [S] v_s$$

$$v_a = [G]^T \eta$$

$$\text{control force vector} = [D] v_a$$

The coupled system equation is shown. It is seen that the coupled damping matrix has the standard form but the coupled stiffness matrix has off-diagonal terms. Since the measurement is a generalized displacement, the control couples into the stiffness matrix, hence the term *Stiffness Control.*

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Positive Position Feedback:

$$\begin{bmatrix} \ddot{\xi} \\ \ddot{\eta} \end{bmatrix} + \left[\begin{bmatrix} \lceil 2\varsigma\omega \lrcorner & 0 \\ 0 & \lceil 2\varsigma_f\omega_f \lrcorner \end{bmatrix} \right] \begin{bmatrix} \dot{\xi} \\ \dot{\eta} \end{bmatrix} + \left[\begin{bmatrix} \lceil \omega^2 \lrcorner & -[D][G]^T \\ -\lceil \omega_f^2 \lrcorner [S][C]^T & \lceil \omega_f^2 \lrcorner \end{bmatrix} \right] \begin{bmatrix} \xi \\ \eta \end{bmatrix} = 0$$

$$\ddot{\psi} + [C]\dot{\psi} + [K]\psi = 0$$

New Stiffness Matrix

729

A block diagram of the control loop is shown. The sensor and actuator blocks exhibit no dynamics because their bandwidth is far beyond the bandwidth of the loop. Notice that this is a *positive* feedback loop.
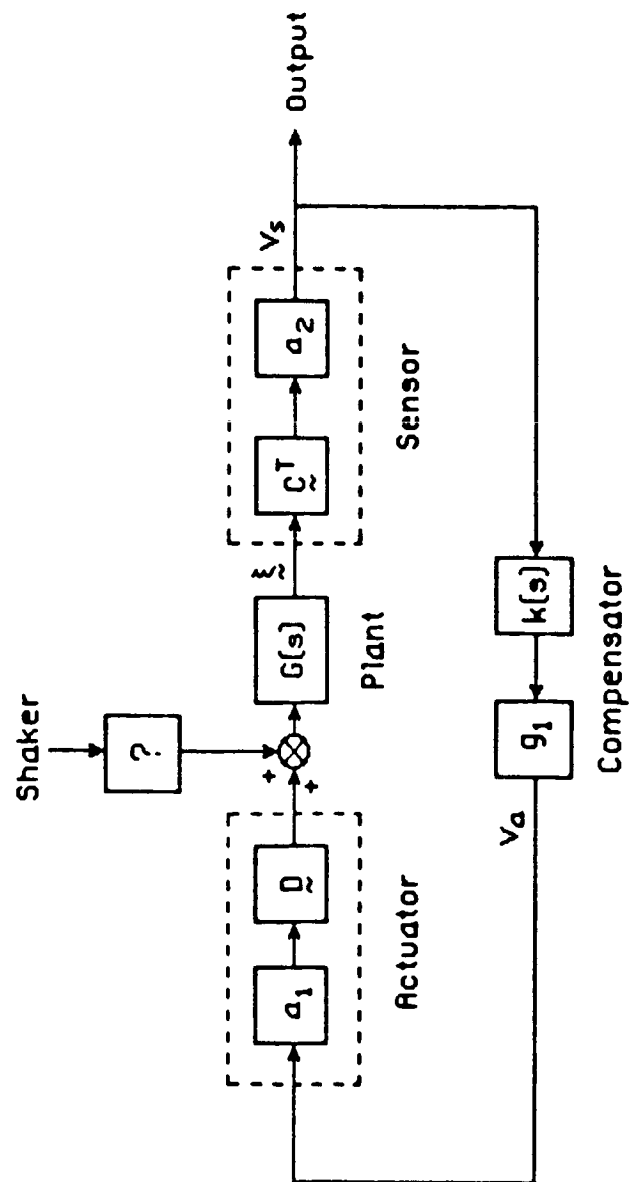
A root locus plot (which traces the location of the closed loop poles in the complex plane as a function of gain) is shown for the case of a single mode and one tuned filter. As the gain is increased the system pole moves into the left half-plane and the filter pole moves toward the imaginary axis. Approximately halfway between the open loop pole locations, the two trajectories bend, the system pole moves upward and the filter pole moves down toward the real axis. Eventually, at sufficiently high gain, the filter pole moves along the real axis into the right half-plane.

One Filter One Mode

Mode 1

Filter 1

CC

Real s

Imag s

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733
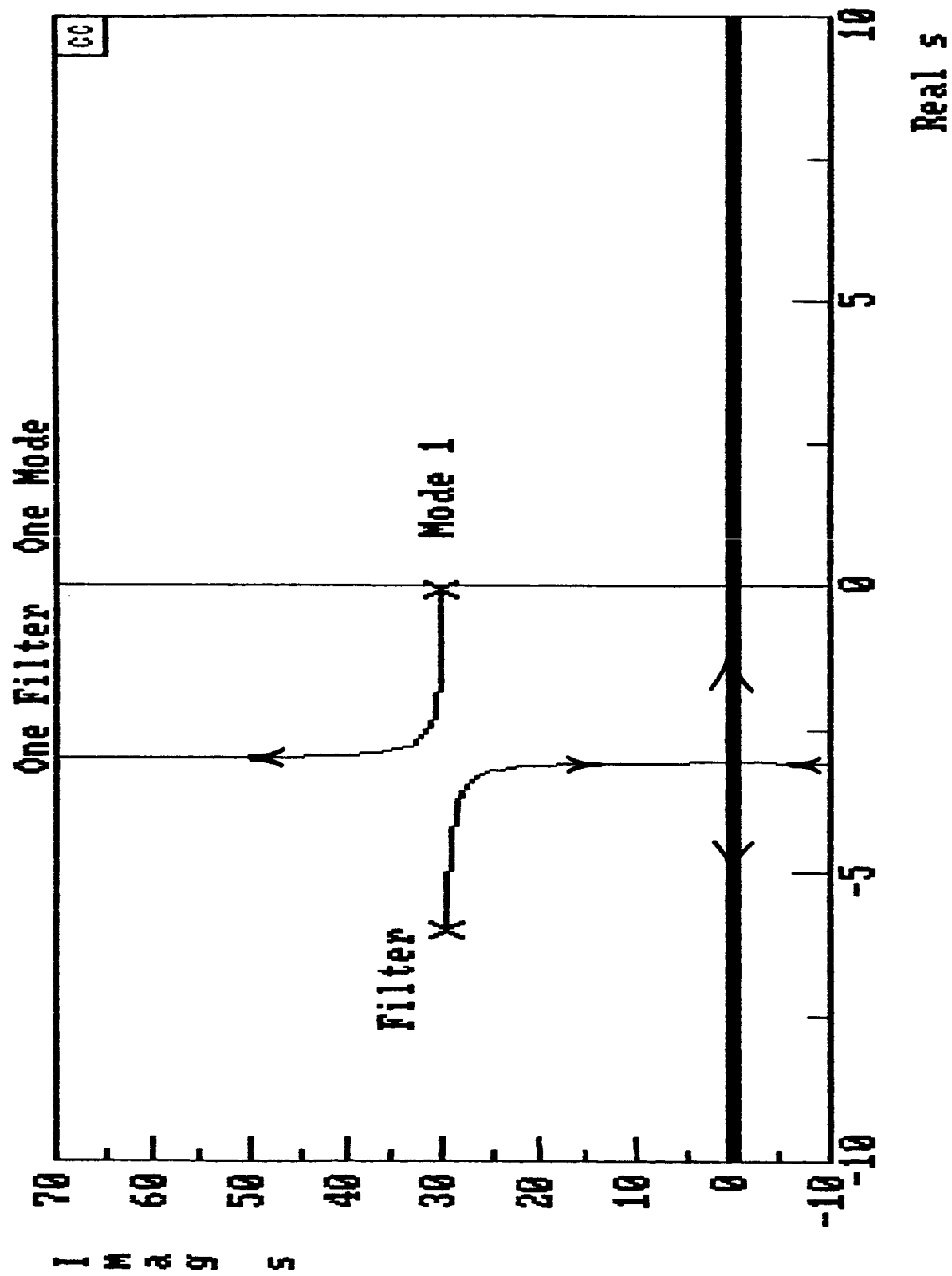
If there are densely spaced modes in the vicinity of the filter, we see that all the system poles are moved into the left half-plane with the lowest mode eventually going unstable at sufficiently high gain. The higher poles never go unstable because of the presence of transmission zeros which trap them in the left half-plane. This is characteristic of *Positive Position Feedback*. Instability will only occur either in the lowest frequency mode or the lowest frequency filter.
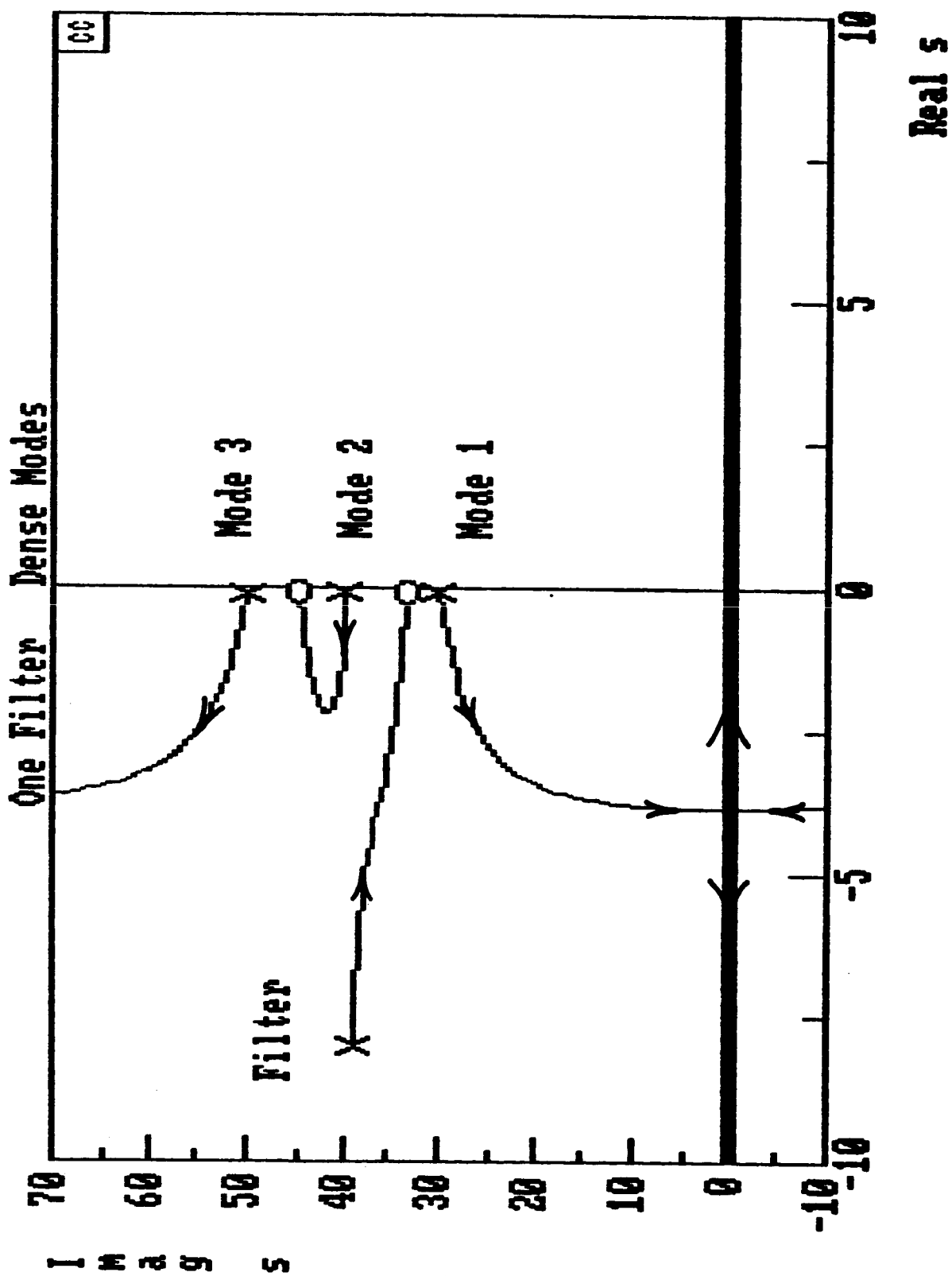
This root locus shows the effect of densely packed control filters in a region of dense structural modes. As can be seen this only improves the movement of the system poles into the left half-plane. Thus, it does no damage to place extra control filters in a region where structural modes are suspected.
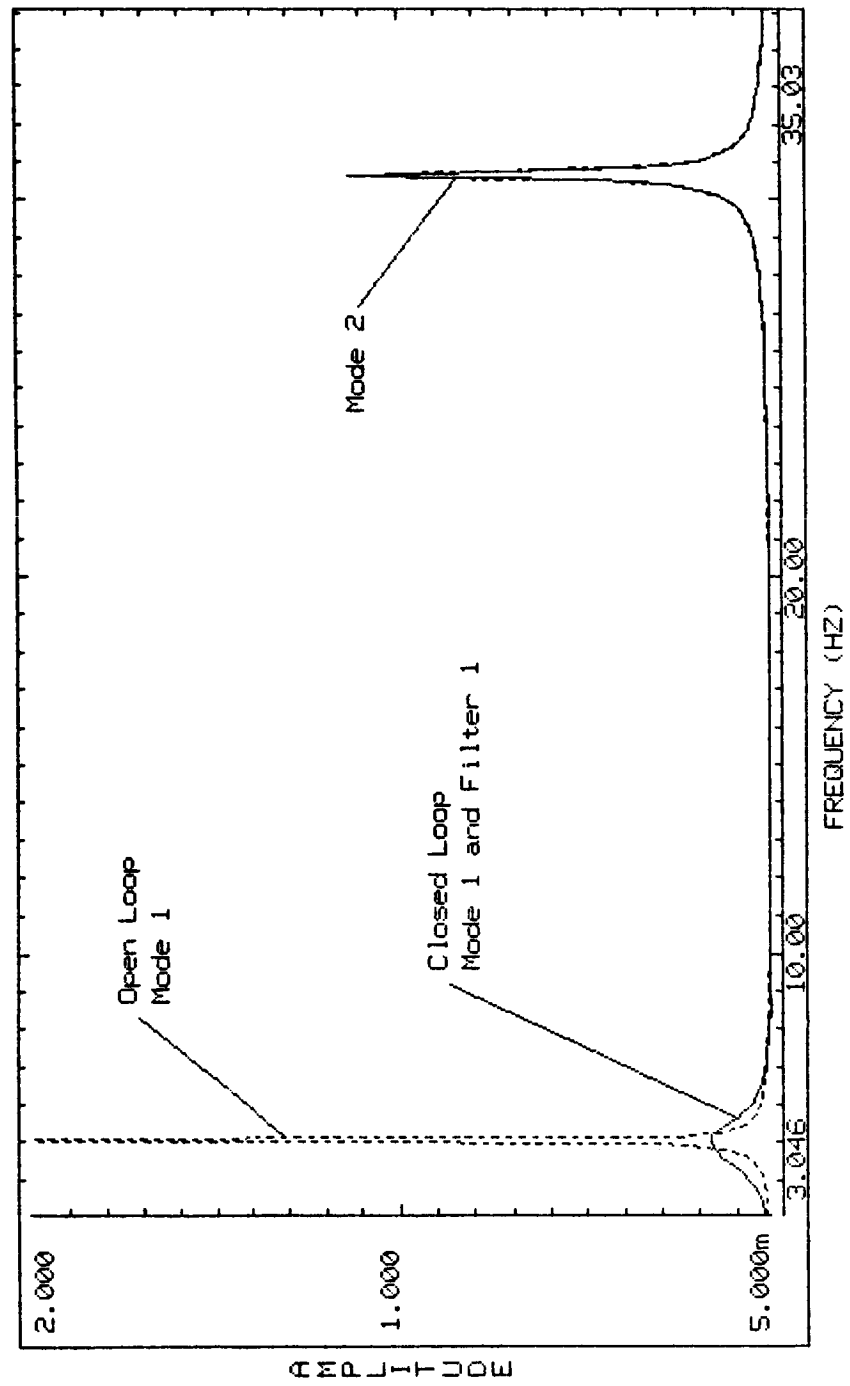
Not shown is a technique for trapping the instability by means of an "arresting pole" thus insuring that the instability can only occur in the lowest frequency filter. This means that any instability will occur at low frequency (giving more time to detect it before oscillations become large) and will occur in the electronics which can be built with saturation characteristics.
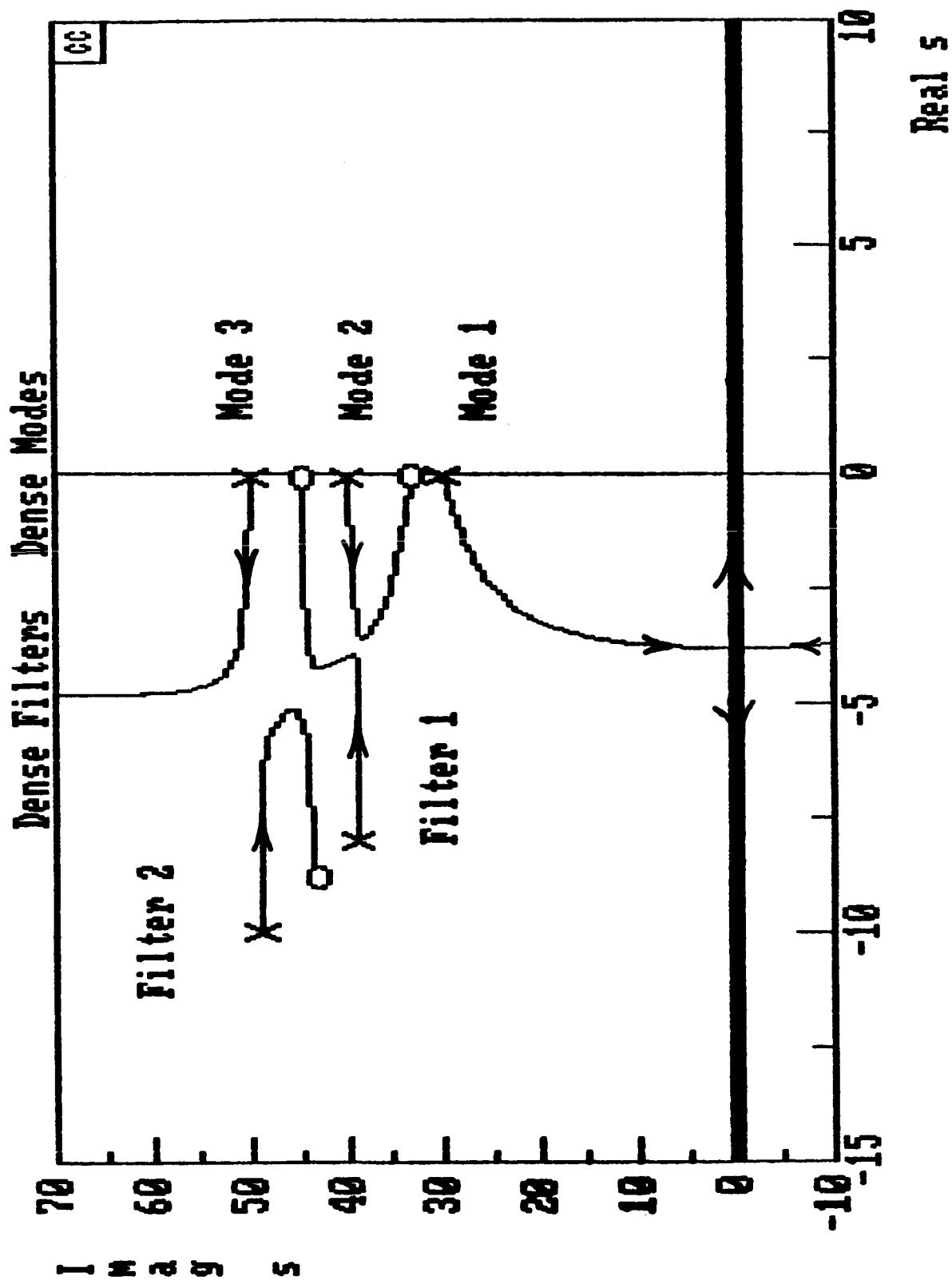
This plot shows the response amplitude for the open loop (dashed line) and closed loop (solid line) one mode control case. The first two modes are shown. Because the first two modes of a cantilever beam differ in frequency by a factor of six they are considered far apart. The mode one filter damps mode one but leaves mode two alone. Actually, the damping in mode two is increased slightly. This is another feature of *Positive Position Feedback*: all control spillover is stablizing.
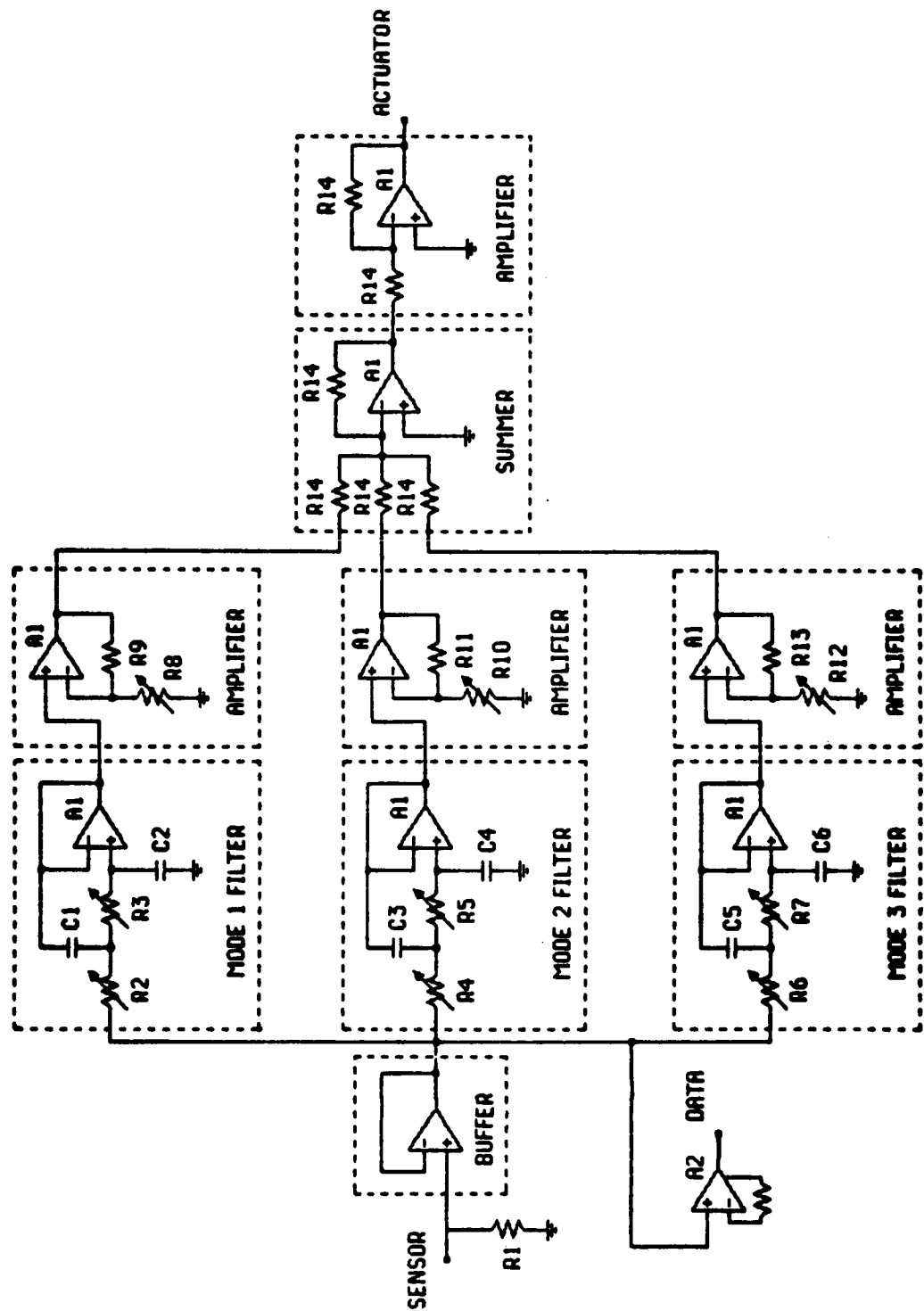
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POSITIVE POSITION CHIRP TEST
MODE 1 AND MODE 2

MSC/STI-VAMP

Open Loop
Mode 1

Closed Loop
Mode 1 and Filter 1

Mode 2

FREQUENCY (HZ)

AMPLITUDE

2.000

1.000

5.000m

3.046   10.00   20.00   35.03

The electrical circuit of the compensator is shown for three mode control. The sensor signal is buffered and then simply split into three parts and fed into three tuned filters, one for each mode since they are not densely spaced. The output of each filter is multiplied by a gain and then added to form a scalar actuator voltage.
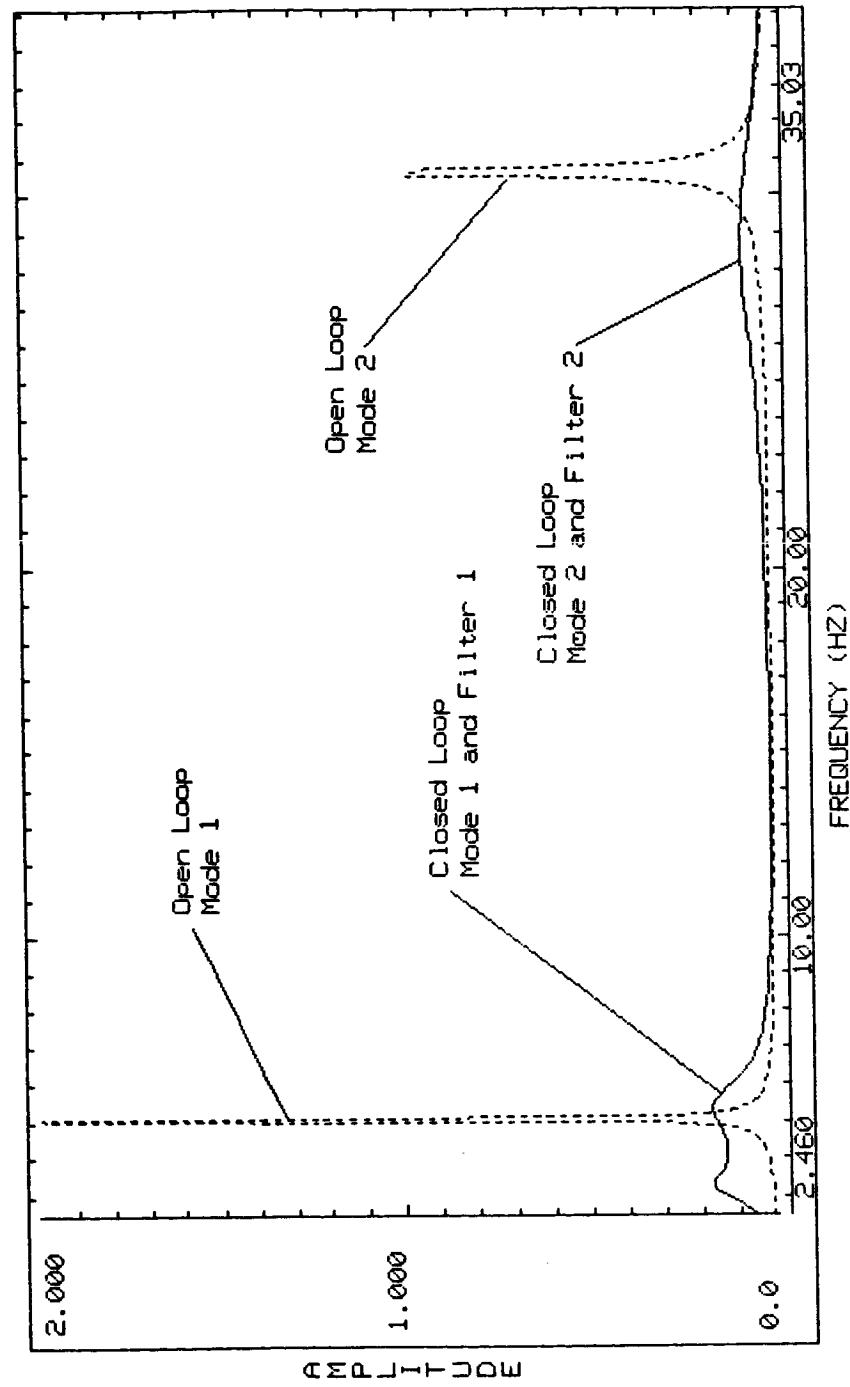
The response amplitude plot is shown for three mode control in the region of modes one and two. The dynamic response is significantly reduced.

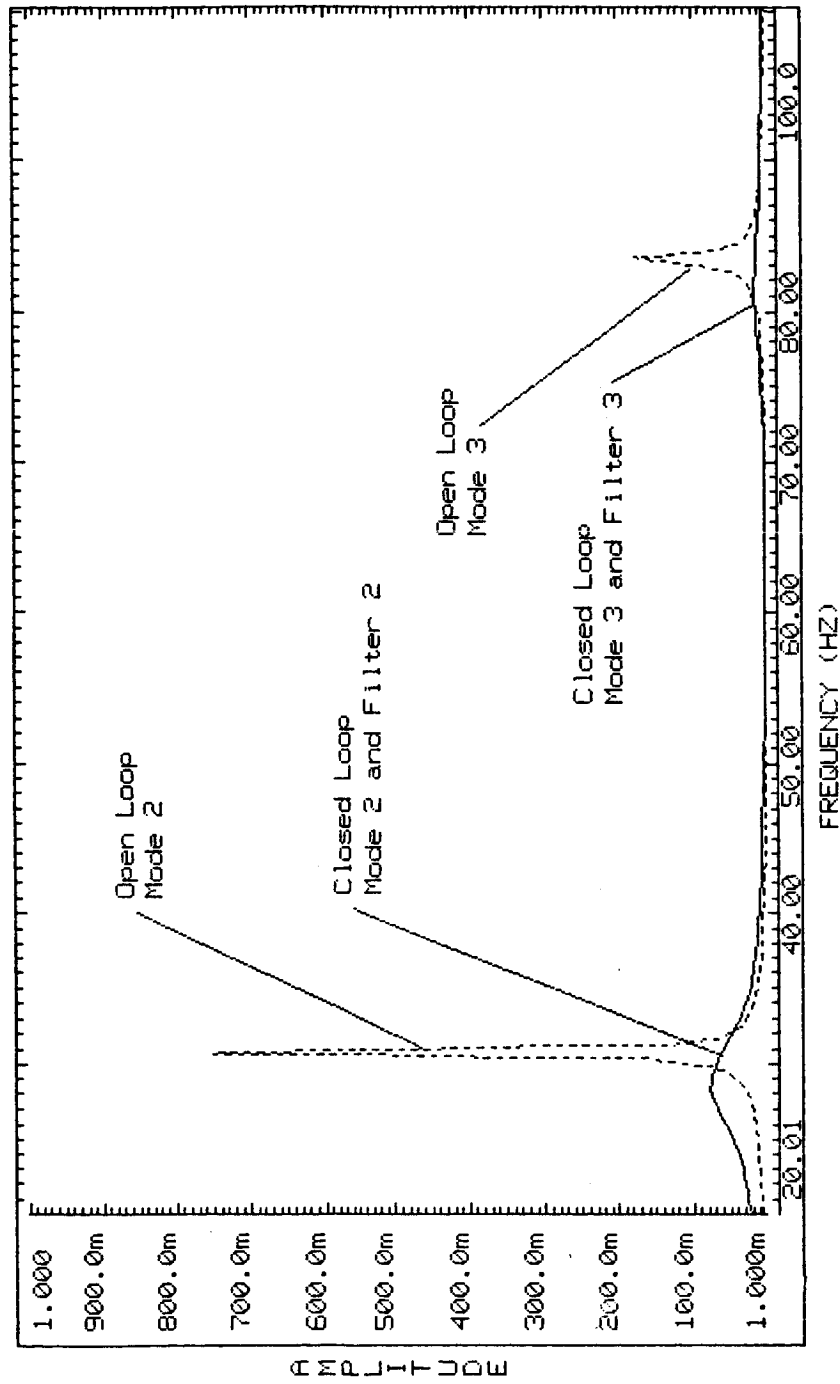JET PROPULSION LABORATORY
POSITIVE POSITION CHIRP TEST
3 MODE CONTROL

MSC/STI-VAMP

The response amplitude plot for three mode control in the region of modes two and three is shown.

JET PROPULSION LABORATORY
POSITIVE POSITION CHIRP TEST
3 MODE CONTROL

MSC/STI-VAMP

Open Loop
Mode 2

Closed Loop
Mode 2 and Filter 2

Open Loop
Mode 3

Closed Loop
Mode 3 and Filter 3

AMPLITUDE

1.000
900.0m
800.0m
700.0m
600.0m
500.0m
400.0m
300.0m
200.0m
100.0m
1.000m

20.01   40.00   50.00   60.00   70.00   80.00   100.0

FREQUENCY (HZ)

This photograph of an oscilloscope trace shows the free decay of mode one, both open loop and closed loop. The open loop settling time is approximately one minute; the closed loop settling time is approximately one second.
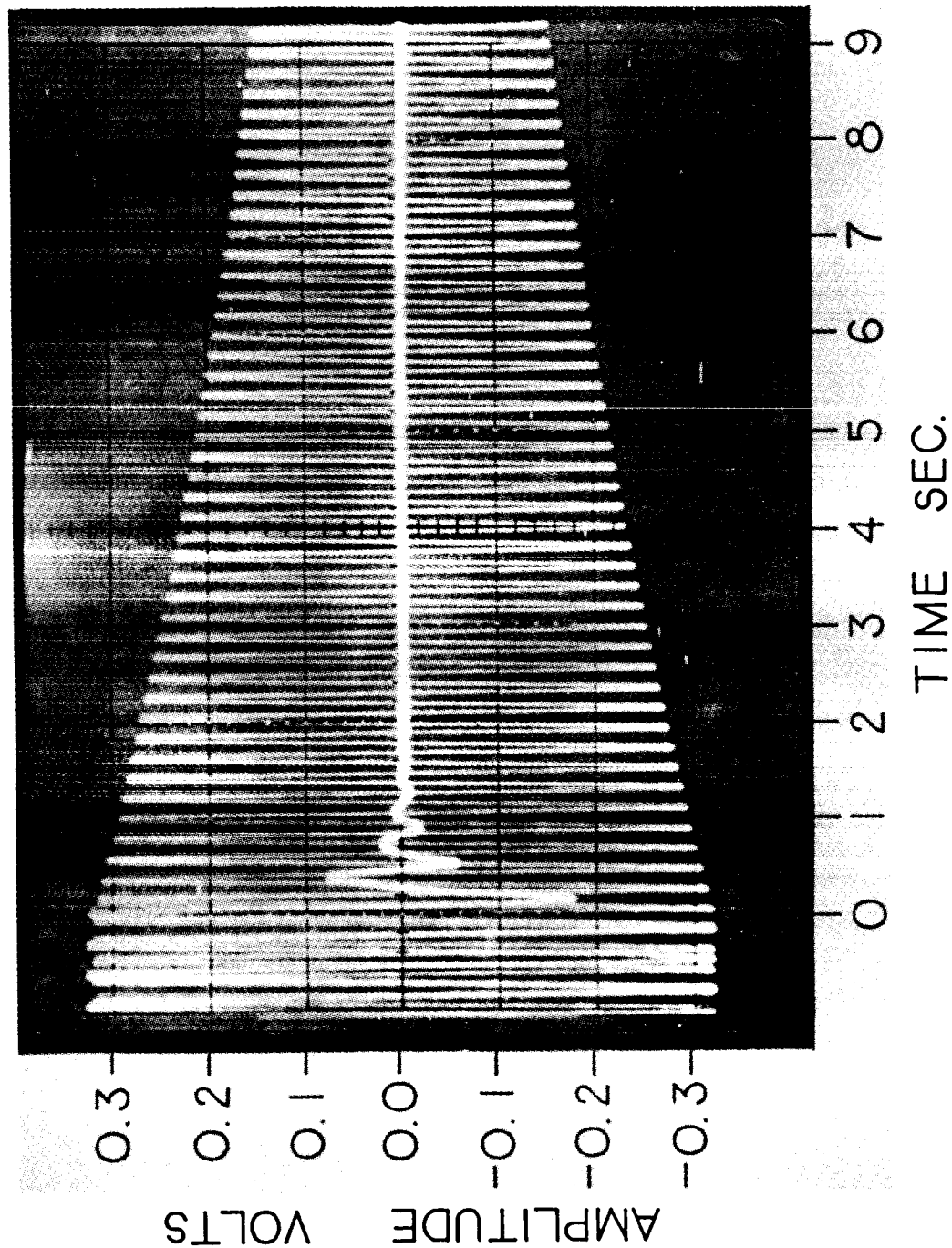
JPL SISO THREE MODE CONTROL

FREE DECAY -- MODE ONE

TIME SEC.

AMPLITUDE VOLTS
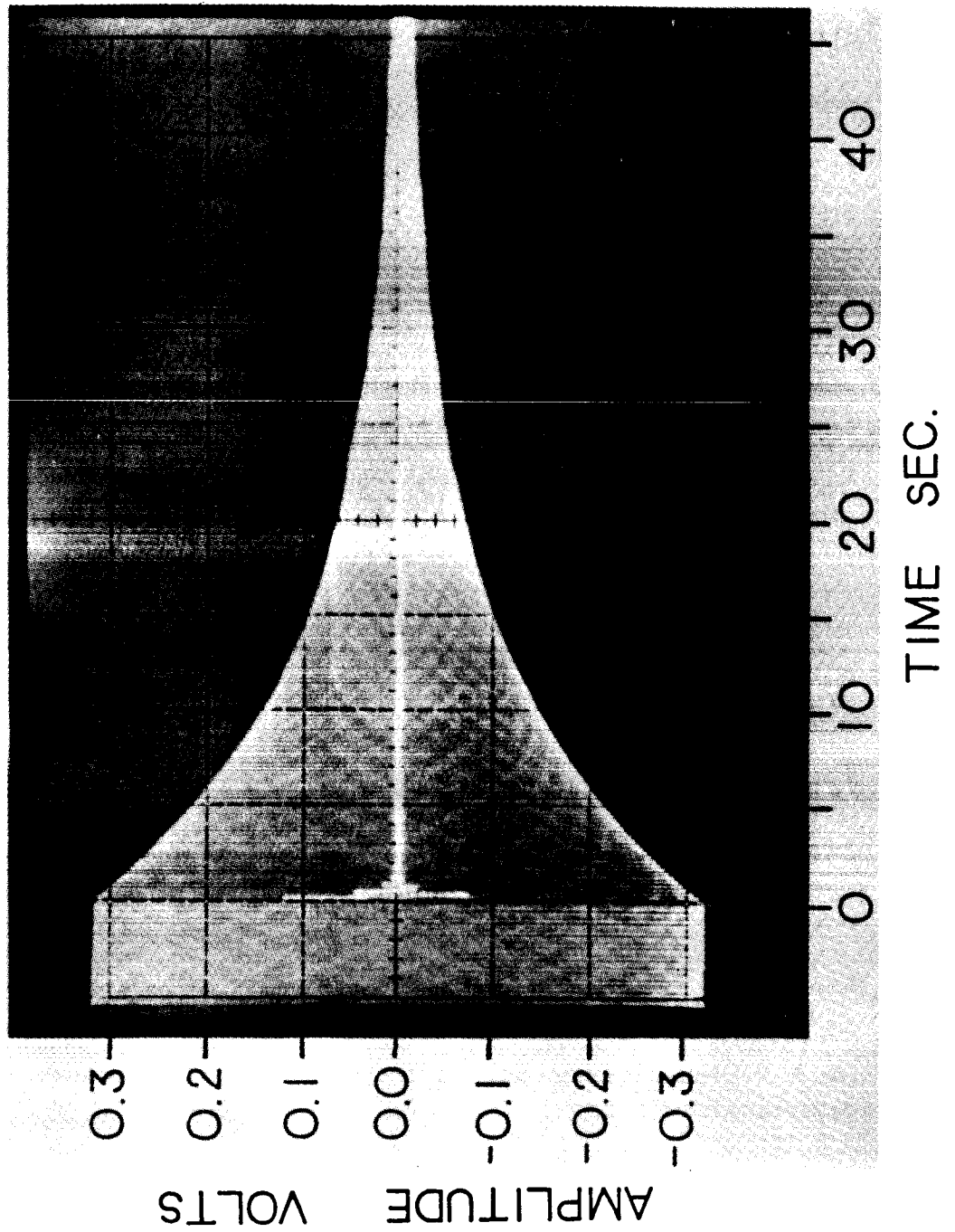
This oscilloscope photograph shows a blowup of the first part of the preceding free decay comparison. The closed loop settling time is substatially reduced.

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FREE DECAY -- MODE ONE

The three tables show a summary of the open loop vs. closed loop quantities related to dynamic response.

The damping ratio ζ is shown in the first table. The damping ratio is increased by a factor of 67 in mode one, by a factor of 45 in mode two, and by a factor of 15 in mode three.

The quantity of interest for response amplitude is shown in table two. The quantity $\zeta\omega^2$ is increased by a factor of 25 in mode one, by a factor of 44 in mode two, and by a factor of 14 in mode three.

The quantity of interest for settling time $\zeta\omega$ is shown in table three. The quantity $\zeta\omega$ is increased by a factor of 42 in mode one, by a factor of 48 in mode two, and by a factor of 15 in mode three.

Closed Loop Damping Summary

Damping Ratio	ζ_1	ζ_2	ζ_3
Open Loop	0.002	0.002	0.003
Closed Loop	0.134	0.089	0.040

Response Amplitude	$\zeta_1\omega_1^2$	$\zeta_2\omega_2^2$	$\zeta_3\omega_3^2$
Open Loop	1.993	62.87	738.0
Closed Loop	51.68	2,776	10,423

| Settling Time	$\zeta_1\omega_1$	$\zeta_2\omega_2$	$\zeta_3\omega_3$
Open Loop	0.063	0.327	1.412
Closed Loop	2.636	15.67	20.39

The Nyquist plot for the three mode control case is shown. Because the control loop is a positive feedback loop, the critical point is $(+1,0)$ instead of the more common location $(-1,0)$. The locus passes under the critical point at low frequency and continues to move farther from the critical point through mode one, mode two, and mode three. The loop begins to roll-off after mode three but the locus stays well away from the critical point.

The peculiar dogleg of the curve at low frequency is due to non-ideal behavior of the piezo-electric ceramic sensors at low frequency.

Because the locus passes below the critical point, the control system is robust with respect to time delays which tend to rotate the Nyquist plot in a clockwise direction. In our case time delays tend to move the locus away from the critical point. The system is also robust to phase lags, which are much more common in structures than phase leads.

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POSITIVE POSITION SWIFT TEST
NYQUIST PLOT   LOOP TRANSFER FUNCTION

MSC/STI-VAMP

Critical Point

1.000

1.000

0.0

-1.000

-1.000
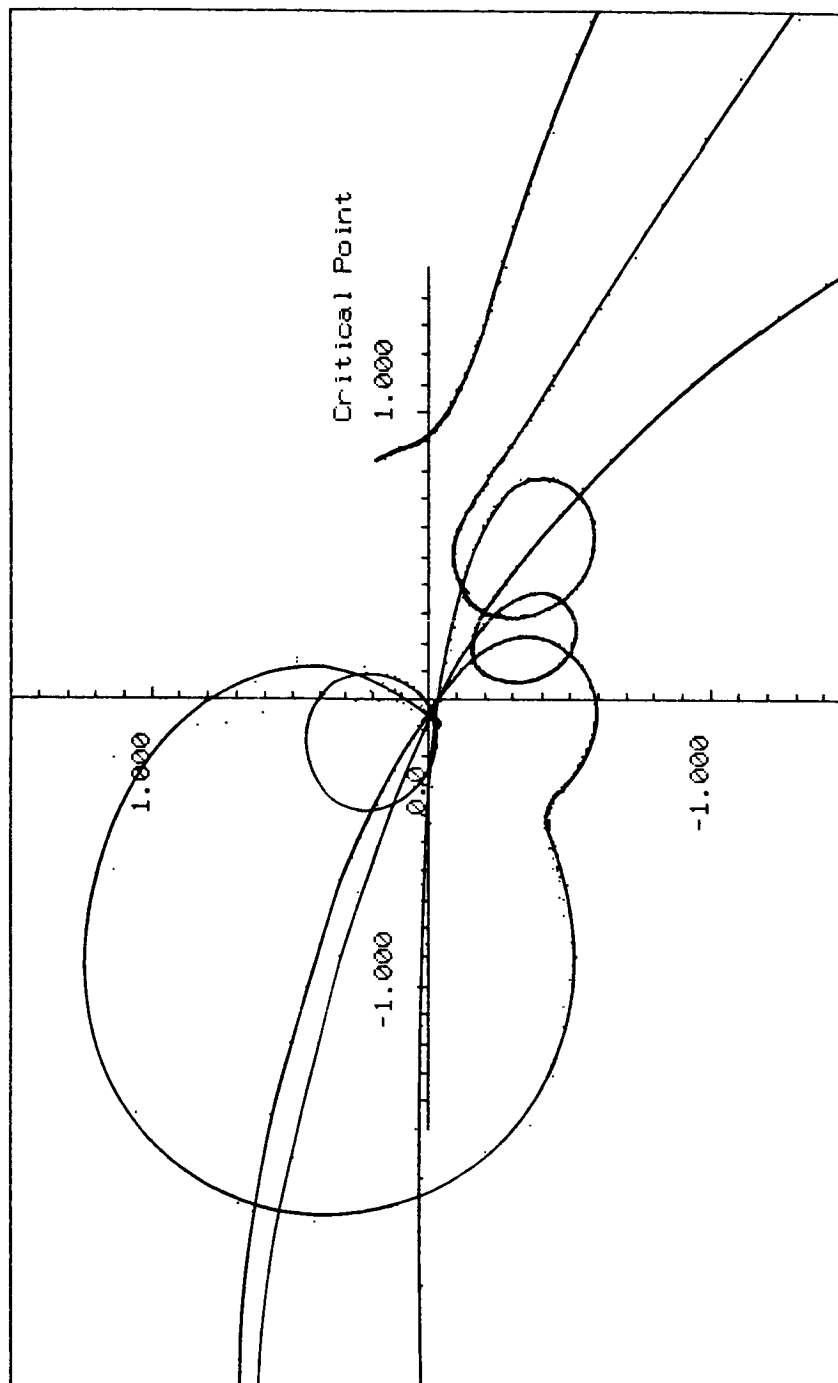
We have demonstrated the feasibility of using piezoelectric ceramics as both sensors and actuators for vibration suppression in a lightweight, flimsy structure. We have acheived multi-mode control using one sensor and actuator pair. The *Positive Position Feedback* control strategy requires only knowledge of the natural frequencies of the structure. Our implementation of *Positive Position Feedback* uses only strain measurements to acheive damping, no velocities or accelerations are needed. All spillover is stabilizing for sufficiently small gains.

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CONCLUSIONS

- Demonstrated Feasibility of Using Piezo-Electric Ceramics as Both Sensors and Actuators

- Multi-Mode Control with One Actuator/Sensor Pair

- Control Law Which Requires Only Knowledge of Natural Frequencies of Structure

- Control Law Uses Only Strain Measurements (No Velocities or Accelerations)

- All Spillover is Stabilizing

With *Positive Position Feedback* instability can only occur in the lowest frequency mode or the lowest frequency control filter. Instability can be trapped in the electronics by the use of an "arresting pole." Performance increases with increasing modal density in the sense that one control filter will increase the damping in all nearby modes no matter how densly spaced they are. Since the control loop is a *positive* feedback loop the critical point is $(+1, 0)$. The Nyquist locus passes under the critical point so phase lags and time delays tend to move the system away from instability.

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CONCLUSIONS (cont.)

- Instability Can Only Occur in the Lowest Frequency Mode or Lowest Frequency Filter

- Instabilities Can be Trapped in the Electronics

- Performance Increases with Increasing Modal Density

- Robust With Respect To Time Delays and Phase Lags