MANEUVERING AND VIBRATION CONTROL OF FLEXIBLE SPACECRAFT*

by

L. Meirovitch & R. D. Quinn

Virginia Polytechnic Institute & State University

Department of Engineering Science & Mechanics

Blacksburg, Virginia 24061

* Supported by the NASA Research Grant NAG-1-225 sponsored by the Spacecraft Control Branch, Langley Research Center.

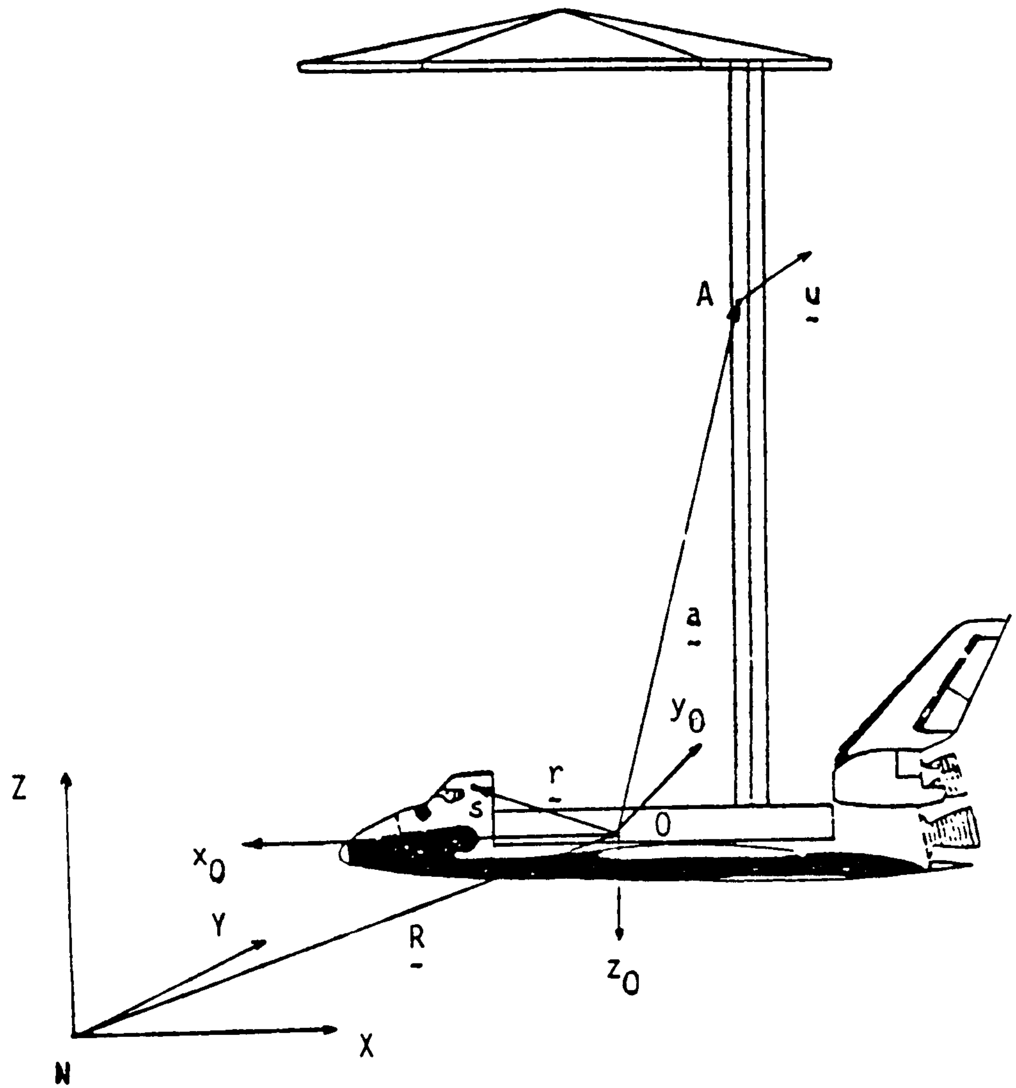


Figure 1. Spacecraft in Orbit

EQUATIONS OF MOTION

Position and Velocity of Point S: $\underset{\sim}{R}_S = \underset{\sim}{R} + \underset{\sim}{r}\ ,\ \dot{\underset{\sim}{R}}_S = \dot{\underset{\sim}{R}} + \underset{\sim}{\omega} \times \underset{\sim}{r}$

Position and Velocity of Point A: $\underset{\sim}{R}_A = \underset{\sim}{R} + \underset{\sim}{a} + \underset{\sim}{u}\ ,\ \dot{\underset{\sim}{R}}_A = \dot{\underset{\sim}{R}} + \underset{\sim}{\omega} \times (\underset{\sim}{a} + \underset{\sim}{u}) + \dot{\underset{\sim}{u}}$

$\dot{\underset{\sim}{R}},\ \underset{\sim}{\omega}$ = translational and angular velocities of frame $x_0y_0z_0$

$\underset{\sim}{u} = \Phi\underset{\sim}{q}$ = elastic displacement vector

Lagrange's Equations: $\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\underset{\sim}{R}}}\right) + \frac{\partial V}{\partial \underset{\sim}{R}} = C^T\underset{\sim}{F},\ \frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\underset{\sim}{\alpha}}}\right) - \frac{\partial T}{\partial \underset{\sim}{\alpha}} + \frac{\partial V}{\partial \underset{\sim}{\alpha}} = D^T\underset{\sim}{M}$

$$\frac{d}{dt}\left(\frac{\partial T}{\partial \dot{\underset{\sim}{q}}}\right) - \frac{\partial T}{\partial q} + \frac{\partial V}{\partial q} = \underset{\sim}{Q}$$

C = transformation matrix from XYZ to $x_0y_0z_0$

D = matrix of Euler's angles $\alpha_1, \alpha_2, \alpha_3$ ($\underset{\sim}{\omega} = D(\underset{\sim}{\alpha})\dot{\underset{\sim}{\alpha}}$)

CONTROL STRATEGY

Rigid-body motions are relatively large.

Elastic deformations are relatively small.

$\therefore$ Design a maneuver strategy as if the structure were rigid.

Then, design feedback control to suppress elastic deformations and deviations from the rigid-body maneuver.

Use a perturbation approach to separate the zero-order terms (rigid-body maneuver) from the first-order terms (elastic vibration and deviations from the rigid-body maneuver).

PERTURBATION METHOD

First-Order Perturbation: $\underset{\sim}{R} = \underset{\sim}{R}_0 + \underset{\sim}{R}_1$, $\underset{\sim}{\alpha} = \underset{\sim}{\alpha}_0 + \underset{\sim}{\alpha}_1$

Perturbed Angular Velocity Vector: $\underset{\sim}{\omega} = \underset{\sim}{\omega}_0 + \underset{\sim}{\omega}_1$, $\underset{\sim}{\omega}_1 = \tilde{\omega}_0^T \underset{\sim}{\beta} + \dot{\underset{\sim}{\beta}}$

$\underset{\sim}{\beta}$ = small angular deflection vector expressed in $x_0 y_0 z_0$ components

Zero-Order Equations (Rigid Structure):

$$m\ddot{\underset{\sim}{R}}_0 + C_0^T \tilde{S}_0 \dot{\underset{\sim}{\omega}}_0 + C_0^T \tilde{\omega}_0^T \tilde{S}_0 \underset{\sim}{\omega}_0 + \frac{Gm_e}{|\underset{\sim}{R}_0|^3} [m\underset{\sim}{R}_0 + (I - 3\hat{\underset{\sim}{R}}_0 \hat{\underset{\sim}{R}}_0^T) C_0^T \underset{\sim}{S}_0] = C_0^T \underset{\sim}{F}_0$$

$$\tilde{S}_0^T C_0 \ddot{\underset{\sim}{R}}_0 + \frac{Gm_e}{|\underset{\sim}{R}_0|^3} \tilde{S}_0^T C_0 \underset{\sim}{R}_0 + I_0 \dot{\underset{\sim}{\omega}}_0 + \tilde{\omega}_0^T I_0 \underset{\sim}{\omega}_0 = \underset{\sim}{M}_0$$

First-Order Perturbation Equation: $M\ddot{\underset{\sim}{x}} + G\dot{\underset{\sim}{x}} + (K_S + K_{NS})\underset{\sim}{x} = \underset{\sim}{F}^*$

$\underset{\sim}{x} = [\underset{\sim}{R}_1^T \ \ \underset{\sim}{\beta}^T \ \ \underset{\sim}{q}^T]^T$ = perturbation vector

$\underset{\sim}{F}^* = [\underset{\sim}{F}_1^T \ \ \underset{\sim}{M}_1^T \ \ \underset{\sim}{Q}_0^T + \underset{\sim}{Q}_1^T]^T$ = perturbing force vector

RIGID-BODY MANEUVER

Rigid-body maneuver is designed independently of vibration control.

Strategy: single-axis, minimum-time maneuver.

Maneuver Force Distribution Producing Rigid-Body Motion Only:

$$F_1(p,t) = x(p)m(p)\dot{\theta}^2(t)$$

$$F_2(p,t) = -z(p)m(p)\ddot{\theta}(t)$$

$$F_3(p,t) = y(p)m(p)\ddot{\theta}(t)$$

$\theta(t)$ = desired angular motion

$m(p)$ = mass density

$x(p)$, $y(p)$, $z(p)$ = coordinates of p relative to center of rotation

QUASI-MODAL EQUATIONS

Coordinate Transformation: $\underset{\sim}{x}(t) = X\underset{\sim}{u}(t)$

X = rectangular matrix of lower premaneuver eigenvectors

Quasi-Modal Equations: $\ddot{\underset{\sim}{u}}(t) + \bar{G}(t)\dot{\underset{\sim}{u}}(t) + [\Lambda + \bar{K}(t)]\underset{\sim}{u}(t) = \underset{\sim}{f}(t)$

$\underset{\sim}{u}(t)$ = vector of quasi-modal coordinates

$\underset{\sim}{f}(t) = X^T \underset{\sim}{F}^*(t)$ = vector of quasi-modal forces

$\bar{G}(t) = X^T G(t) X$ = reduced-order gyroscopic matrix

$\Lambda = X^T K_0 X$ = matrix of premaneuver eigenvalues

$\bar{K}(t) = X^T K_t(t) X$ = reduced-order stiffness matrix

As maneuver velocity decreases, time-varying terms decrease and equations approach an uncoupled form.

VIBRATION CONTROL

Modal Equations: $\ddot{u}_r(t) + \omega_r^2 u_r(t) = f_r(t) + f_{dr}(t)$

$f_r(t)$ = modal control force

$f_{dr}(t)$ = modal disturbance and maneuver control force (to be neglected)

Actuator Dynamics: $\dot{\underset{\sim}{F}}(t) = a\underset{\sim}{F}(t) + b\underset{\sim}{F}_c(t)$

$\underset{\sim}{F}_c(t)$ = command force vector

Modal Actuator Dynamics: $\dot{f}_r(t) = af_r(t) + bf_{cr}(t)$

Modal State Equations: $\dot{\underset{\sim}{z}}_r(t) = A_r \underset{\sim}{z}_r(t) + \underset{\sim}{b} f_{cr}(t)$

$$\underset{\sim}{z}_r = [u_r \ \dot{u}_r \ \ddot{u}_r]^T, \quad \underset{\sim}{b} = [0 \ 0 \ b]^T \quad A_r = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a\omega_r^2 & -\omega_r^2 & a \end{bmatrix}$$

VIBRATION CONTROL (CONT'D)

i. Optimal Control. Performance Index: $J = \sum_{r=1}^{\infty} \int_0^{\infty} (\underset{\sim}{z}_r^T Q_r \underset{\sim}{z}_r + R_r f_{cr}^2)dt$

$Q_r = \text{diag}\,[q_r \;\, 1 \;\, 1]$

Feedback Control Law: $f_{cr} = - \frac{1}{R_f} \underset{\sim}{b}^T K_r \underset{\sim}{z}_r = - g_{r1} u_r - g_{r2} \dot{u}_r - g_{r3} \ddot{u}_r$

$K_r = 3 \times 3$ symmetric matrix satisfying steady-state matrix Riccati equation.

Modal Gains: $g_{r1} = bk_{r13}/R_r$, $g_{r2} = bk_{r23}/R_r$, $g_{r3} = bk_{r33}/R_r$

ii. Pole Allocation: Closed-Loop Poles:

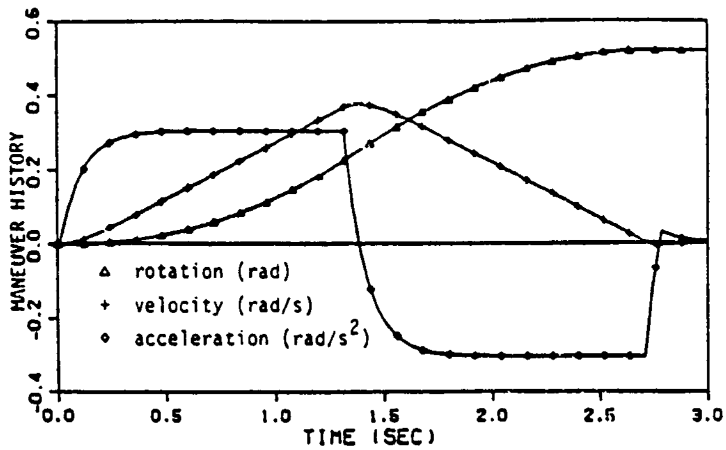
$s_{r1} = \alpha_r + i\beta_r$, $s_{r2} = \alpha_r = i\beta_r$, $s_{r3} = \gamma_r$

Modal Gains: $g_{r1} = \frac{1}{b}$ [illegible] $- \gamma_r(\alpha_r^2 + \beta_r^2)]$,

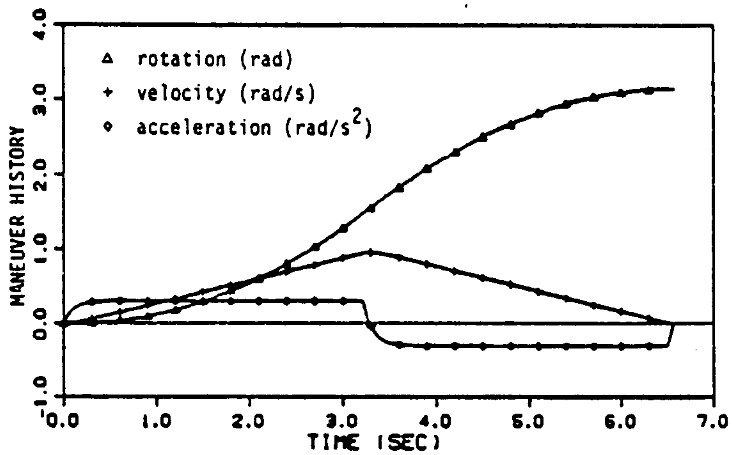
$g_{r2} = \frac{1}{b} [2\gamma_r \alpha_r + \alpha_r^2 + \beta_r^2 - \omega_r^2)$, $g_{r3} = \frac{1}{b} (a - 2\alpha_r - \gamma_r)$

iii. Direct Feedack Control: $\underset{\sim}{F}_C = - M(g_1 \underset{\sim}{x} + g_2 \dot{\underset{\sim}{x}} + g_3 \ddot{\underset{\sim}{x}})$

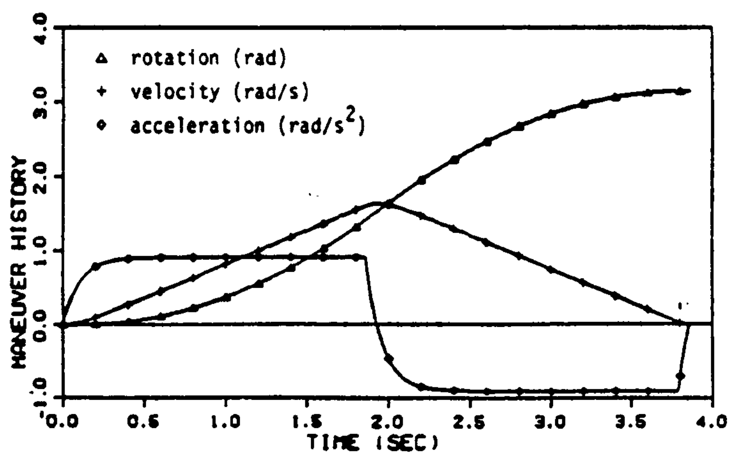
Gains for Uniform Damping: $g_1 = - a\alpha^2/b$, $g_2 = (2a\alpha + a^2)/b$, $g_3 = - 2\alpha/b$



a) 30° roll, M_{max} = 20 ft-lb.



b) 180° roll, M_{max} = 20 ft-lb.



c) 180° roll, M_{max} = 60 ft-lb.

Figure 3. Comparison of Maneuver Strategies

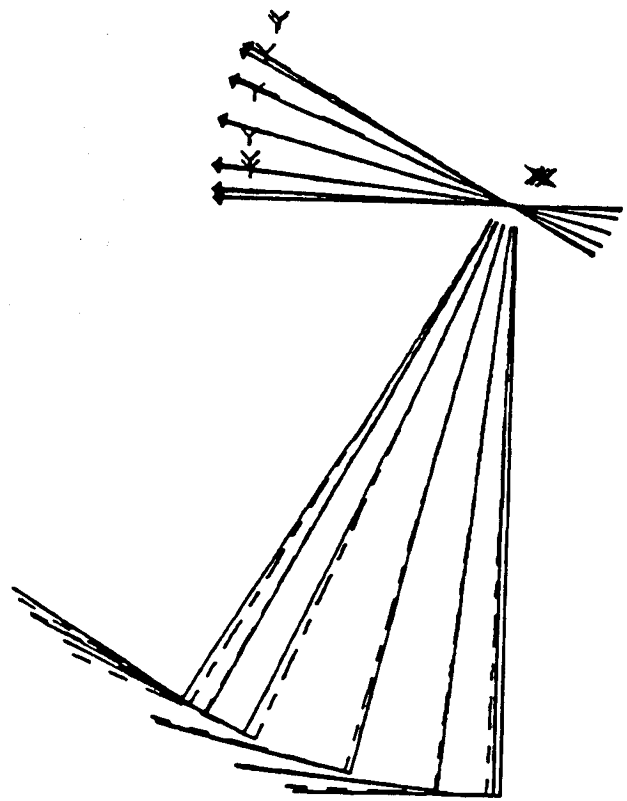


Figure 5. Time-Lapse Plot of 30° Roll Maneuver (No Vibration Control)

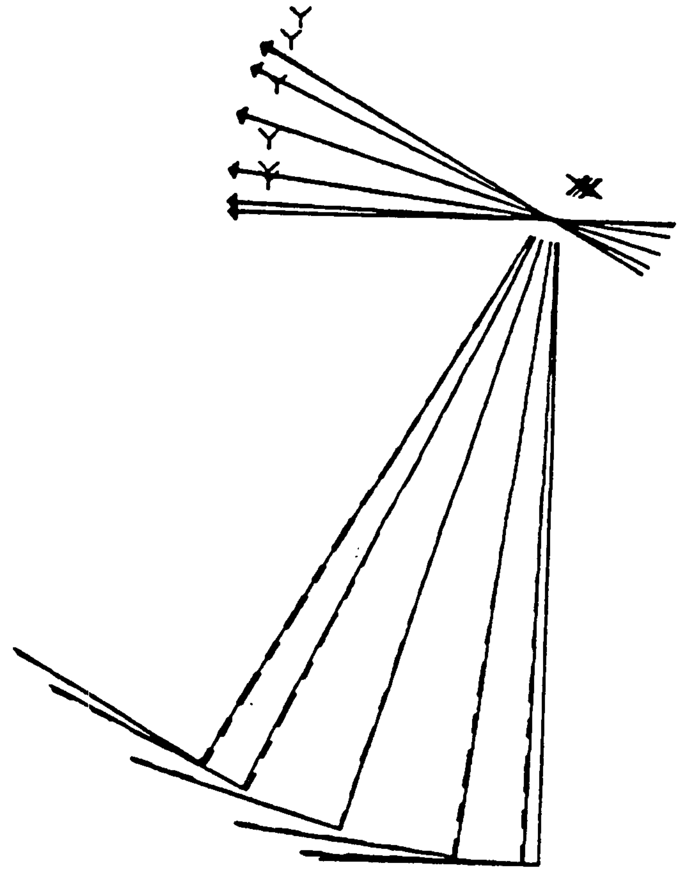


Figure 6. Time-Lapse Plot of 30° Roll Maneuver (Uniform Damping Using 10 Actuators)

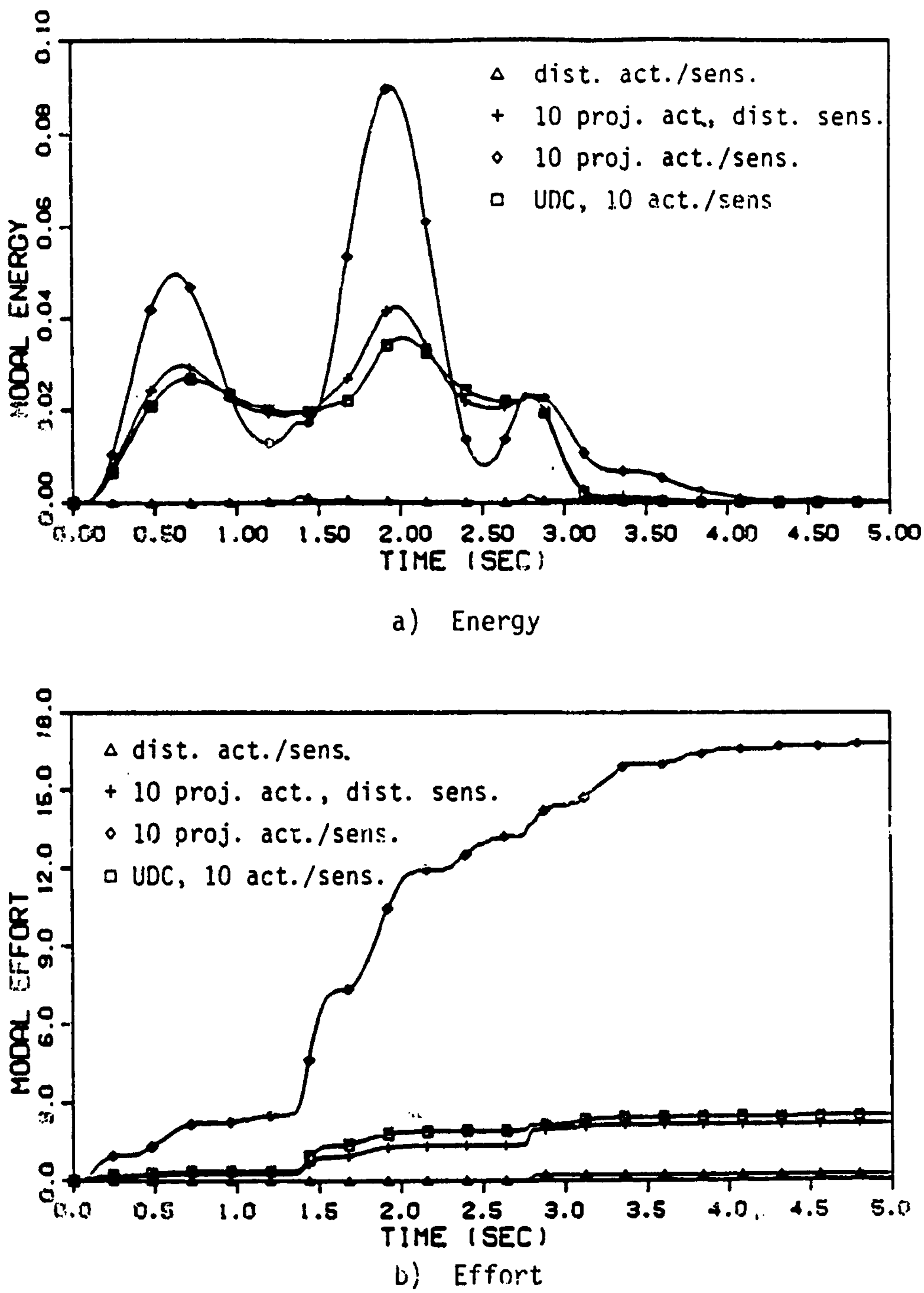
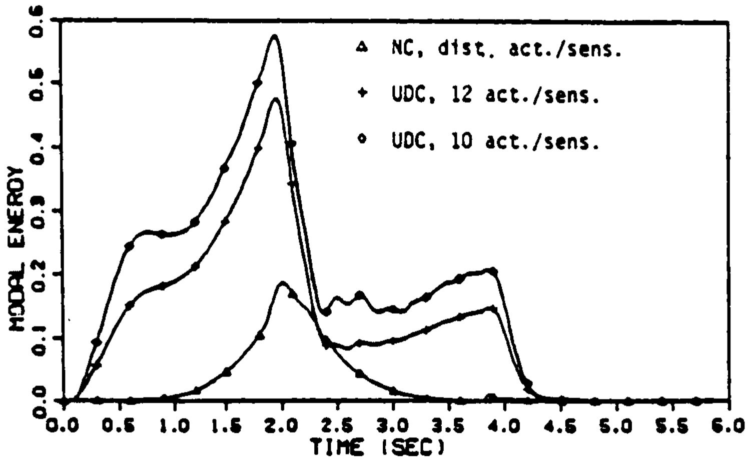
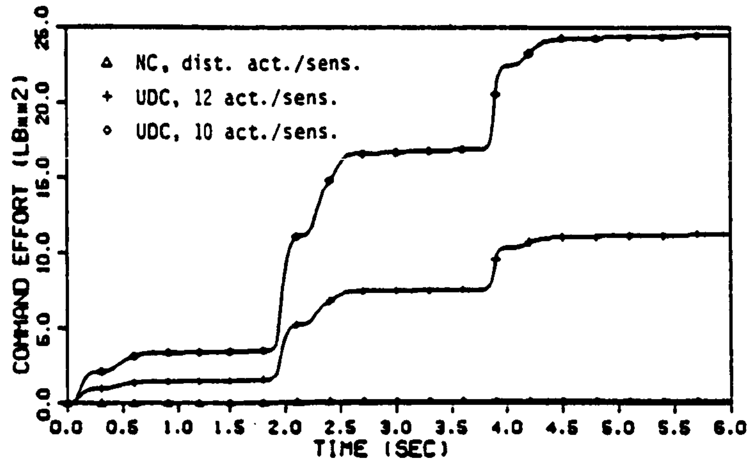


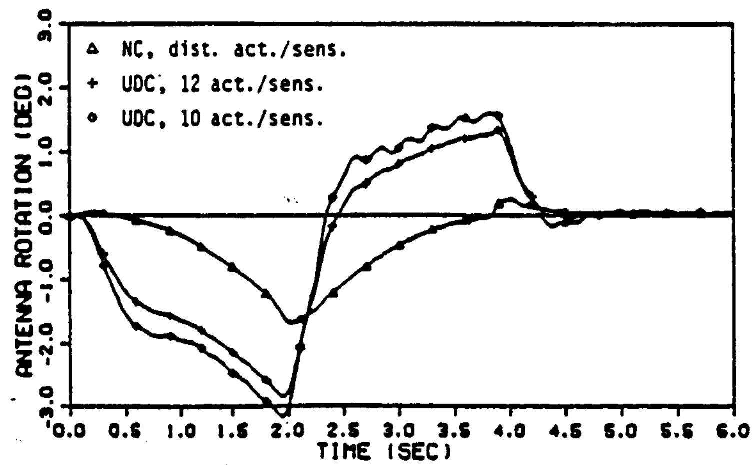
Figure 7. Comparison of Various Vibration Control Implementation Procedures for 30° Roll Maneuver



a) Energy



b) Effort



c) Antenna hub rotation in x_0 direction

Figure 12. Implementation of 180° Maneuver with Various Numbers of Actuators