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HIGH SPEED SIMULATION OF FLEXIBLE MULTIBODY DYNAMICS

A.D.Jacot, R.E.Jones, and C.D.Juengst
The Boeing Aerospace Company, Seattle, Wash.

ABSTRACT:

A multiflexible body dynamics code intended for fast turnaround control design trades is described. Nonlinear rigid body dynamics and linearized flexible dynamics combine to provide efficient solution of the equations of motion. Comparison with results from the DISCOS code provides verification of accuracy.

INTRODUCTION

Control design for complex multiflexible body dynamical systems requires many computer runs of simulations with high CPU usage. The high fidelity computer codes which currently address this nonlinear dynamics problem (for example, DISCOS, ref. 1, and TREETOPS, ref. 2) cannot provide sufficiently fast computer turnaround. To adequately address the control structure interaction problem, structural analyzers for this work should be embedded within a control design code having available an entire repertoire of control simulation and analysis tools. The high fidelity dynamics codes are not designed for convenient use in this way. High fidelity analysis methods are needed, however. The nonlinearities of large motion multibody dynamics suggest that control design based on linear analysis will fail to assess performance accurately, and may also fail to identify stability problems during prolonged slewing motions. Thus there appears to be a deficiency in control structure interaction design methodology for nonlinear multiflexible body systems. The SADACS (Spacecraft Appendage Dynamics And Control Simulation) code attempts to address a range of problems in this category.

Most multiflexible body dynamics problems are essentially linear in their flexible behavior even though their rigid body motions may be strongly nonlinear. SADACS was specifically developed for this type of problem. The present work is a follow-on to the approach described by Hassul and Heffernan in reference 3. SADACS is embedded within the Boeing EASY5 control design and simulation system which provides a wide range of simulation tools, linear control design methods, and nonlinear time domain integration options. The approach achieves high computational speed by solving the flexible dynamics equations in diagonalized system mode form. It solves the fully nonlinear rigid motion problem in parallel with the flexible solution, providing an accurate total motion prediction for most nonlinear dynamic response problems.

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Large angular motions cause gradual changes in the flexible system modes, and these changes are handled by updating the system modes. Very little error in the flexible motion is caused by this updating, no error is induced in the rigid solution, and little increase in computational time occurs.

SADACS is operational in a number of Boeing applications involving complex control structure interaction problems. It has been verified by comparisons with predictions of the DISCOS code. Computational speeds have varied from several times faster than DISCOS for small problems to 100 or more times faster for larger problems. It is routinely used for problems that are computationally infeasible for the high fidelity codes which solve the coupled, nonlinear structural equations of motion in terms of the flexible modes of individual appendages.

Because SADACS is an approximate approach, it must be verified when it is used for problems which have a stronger degree or different type of nonlinearity than that previously studied. To date, however, it has been found highly accurate for complex multiflexible body dynamics problems.

TECHNICAL APPROACH

The handling of structural flexibility in SADACS parallels the approach of conventional structural analysis. The several bodies of the system are represented by their component modes, retaining corresponding freedoms at the attachment points. They are then coupled to form the equations of the total system by performing a conventional structural merge. This greatly simplifies setting up the flexible equations of motion in comparison with the fully nonlinear formulation used by codes such as DISCOS.

Since this approach omits all nonlinearities, a separate nonlinear analysis is performed in parallel with the flexible solution. This solution addresses only the rigid motions, thereby retaining the nonlinearities of greatest importance in most problems. The separation of rigid and flexible solutions can only be done with the flexible formulation in system mode form. Therefore, SADACS performs an eigensolution to obtain a system normal mode representation of structural flexibility. The use of normal modes provides the improvement in computational time which is the aim of the SADACS development.

Though this approach appears both simple and logical, its implementation involves approximations in mathematical derivations which are difficult to justify and to understand as regards physical meaning and probable consequences in problem solutions. The concerns center on the handling of the rotations and their rates. Each of the technical sections which follow attempts to identify the mathematical approximations as they are introduced and to describe the physical nature and possible magnitude of errors which may occur in simulations.

The technical details of SADACS center on three main subjects:

1. Definition of the flexible structural model in terms of

the modes of its component bodies.
2. Use of the structural definition to set up the multiflexible body equations of motion, accounting for the gimbal freedoms and torques.
3. Eigensolution, truncation of system modes, and handling of truncated modes in dynamic analysis.
4. Simultaneous solution and combination of separate rigid and flexible dynamic response problems.

These subjects and the important matters of verification of accuracy and computational speed are discussed in the sections which follow.

Definition of the Structural Model

Figure 1 illustrates the type of multiflexible body system to be studied. A four body chain topology is shown, although SADACS also handles a tree topology. Each body has a coordinate frame which is used for its structural analysis. Bodies 1 and 2 in this figure are each attached to two other bodies. At their attachment points these bodies each have 6 degrees of freedom. Using the Craig-Bampton component body modal formulation (refs. 3, 4), these freedoms are defined by stiffness and mass data. They are treated as coordinates, called constraint modes, and have modeshapes which involve deformations of the interiors of the bodies. The modeshapes are computed by imposing, one at a time, the displacements and rotations of the attachment points and performing static structural analyses of the resulting deformations. Taken as a group, the attachment point freedoms combine to provide rigid body motion. For this reason, the constraint mode set cannot be easily truncated. They also provide the flexibility of the bodies in response to forces and torques applied externally to the attachment points.

To supplement the constraint modes, dynamic flexible modes of the bodies are computed with the attachment points completely fixed. These are called the fixed interface modes. Taken together, the constraint and fixed interface modes provide a complete description of the motions of the bodies in modal coordinate form. The fixed interface modal set is usually truncated.

The structural modelling described above contains implied approximations due the handling of the rotations. The use of the constraint modal coordinates to define rotations is a superposition procedure. It ignores the fact that rotations are only superimposable in a specific sequence defined by the physical construction of a gimbal device and properly accounted for in mathematical procedure. If the rotations are sufficiently small, however, they can be treated by superposition. Therefore, the modal approach taken here is only valid for the flexible portion of the motion, in which the rotations are very small. In addition, the use of the body structural analysis frame as a basis for the definition of the constraint mode coordinates ignores the fact that the actual orientations of the axes of the interbody attachment constraints are influenced by flexibilty. There are situations in which this is important, as in the case of a very flexible body attached to another body which is massive or which has large angular momentum. SADACS does not attempt to address these types of problems. Finally, the structural analysis frame is treated effectively as an inertial frame because the constraint modes referred to this basis provide the only means of rigid motion of a body. If a body has large motions in either rotation or translation, they must be represented by large values of the constraint mode coordinate values. Numerical difficulty could then be encountered in the use of the constraint mode stiffnesses. These are typically very large and are not well suited to analyses in which they must create reactions to large, nearly rigid motions. Therefore, if SADACS were to simulate a transient response using the structural model rather than the separate rigid motion solver to compute the rigid motion, errors typical of inertial grounding would likely occur. However, the code is never used in this mode, and the restriction of the structural model to simulate only the flexible motion eliminates this concern.

Equation 1 gives the relationship of the Craig-Bampton modal coordinates to the discrete physical freedoms of a single body.

$$\begin{Bmatrix} d_B \\ d_I \end{Bmatrix} = \begin{bmatrix} 0 & I \\ P_{IF} & P_{IC} \end{bmatrix} \begin{Bmatrix} q_F \\ q_C \end{Bmatrix} \tag{1}$$

where $\langle d_I \rangle$ and $\langle d_B \rangle$ are the discrete motions of the interior and boundary gridpoints, $[P_{IC}]$ and $[P_{IF}]$ are the modeshapes defining the interior motions due to the constraint and fixed interface modes, and $\langle q_C \rangle$ and $\langle q_F \rangle$ are the constraint and fixed interface modal coordinates, respectively. The vectors $\langle d_B \rangle$ and $\langle q_C \rangle$ are identical. The discrete freedoms include both displacements and rotations.

The Craig-Bampton modal coordinates are not uncoupled as structural normal modes are. The equations of motion of the body in this form have inertial coupling between the constraint and fixed interface modes. Equations 2 and 3 show the forms of the symmetric modal coordinate mass and stiffness matrices for a single body

$$[M] = \begin{bmatrix} M_{FF} & M_{FC} \\ M_{CF} & M_{CC} \end{bmatrix} \tag{2}$$

$$[K] = \begin{bmatrix} K_{FF} & 0 \\ 0 & K_{CC} \end{bmatrix} \tag{3}$$

where the subscripts FF, FC/CF, and CC indicate the fixed interface modes, the coupling between the fixed interface and constraint modes, and the constraint modes, respectively. The matrices $[K_{FF}]$ and $[M_{FF}]$ are diagonal. $[K_{CC}]$ is generally full, but there is no stiffness coupling between the constraint and fixed interface modes. The mass coupling matrices $[M_{FC}]$ and $[M_{CF}]$ and the constraint mode mass matrix $[M_{CC}]$ are generally full.

Bodies 3 and 4 in figure 1 are each attached to only one other body. These bodies also have both constraint and fixed interface modes, but they have a simpler and more familiar form in these cases. The constraint modes are simply the rigid body displacements and rotations of the bodies imposed by their attachment point motions. The fixed interface modes are simply the cantilever modes of conventional structural analysis.

SADACS handles tree topology structural systems in which there are bodies with more than two attachment interfaces. In figure 1, for example, body 1 could have several additional appendages. In such cases, a larger number of constraint modes is defined. As the number of constraint modes is thus increased, the body becomes effectively stiffer in the numerical descriptions of both its constraint and fixed interface modes. The modes become less effective descriptors of the system dynamics, modal convergence deteriorates, and retention of large modal sets becomes necessary. In the SADACS approach, this does not cause any difficulties because the system is subjected to eigensolution and truncation. However, for approaches which solve the equations of motion in component mode form, the Craig-Bampton formulation may lead to large problem size and difficulty in integrating the equations of motion.

The coordinate systems of the structural modal analysis dictate procedures for the use of the modal data in subsequent multibody analysis. Figure 1 shows that each body has a coordinate triad which is used for its structural analysis. No generality is lost if all of these triads are parallel. Thus, each body's displacements and rotations are referred to the same basis vectors, called herein the structural analyzer global basis. The structural analysis is performed with the bodies in specific relative orientations to this global basis, called herein the nominal orientations. The modal data are therefore readily used to study the system in its nominal condition. If the bodies are to be studied in off nominal orientations, the structural data are still valid and the structural analyses need not be repeated. Transformations of the appendage constraint mode vectors can return these vectors to the global basis. Figure 2 illustrates this situation. Body 2 in the figure has been rotated to an off nominal position. The structural analysis has not been repeated for this new orientation. The body 2 modal data are still valid, but are referred to the rotated body 2 basis rather than to the global basis. This requires a compensating transformation of the constraint mode data when the multibody attachment equations are formulated.

Multibody Structural Merge

Figure 1 shows all bodies of the system oriented nominally, so that their body frames are parallel to the global frame. The modal data of each body are referred to the global basis system. The fixed interface modeshapes contain displacements along, and rotations about, the axes of the global system. The constraint mode coordinate vectors contain displacements along and rotations about these axes. If any bodies are rotated to off nominal orientations, their modal data become referred to the correspondingly rotated material frames as indicated by figure 2. The derivations of this section consider both nominal and off nominal orientations in imposing the interbody connectivity conditions.

The first step of the derivation is the transformation of the constraint modal coordinates to the basis systems of the gimbals. This allows the individual body mass, damping, and stiffness matrices to be combined into total system coupled matrices by enforcing compatibility of adjacent body displacements and rotations. This procedure is called a structural merge. No transformation is required for the fixed interface modal data.

It is in the structural merge that the approximations of the flexible body linearization become difficult to visualize and understand. The present approach differs in part from the fully nonlinear formulations in that it omits second and higher order terms in the acceleration equations. This causes approximations in inertia load distributions on the structural components and therefore in the equations of motion. The higher order terms are present in high fidelity codes because at the outset they assemble the equations of motion for bodies residing in a rotating assemblage. This defines accelerations due to products of the angular rates with themselves (centrifugal and gyroscopic effects), and with the modal rates (Coriolis effects). These are omitted at the outset when the modal equations are developed in conventional structural dynamics form. The omitted terms are very small except in cases where large angular rates and large flexibility combine. Hence SADACS is seen to be limited by the combination of angular rate and structural flexibility.

Other approximations are made in the procedure for imposing connectivity between the bodies, due to the manner of handling the orientations of the gimbal axes. In the high fidelity codes, the motions of the gimbal axes are represented exactly, including the effects of large rotations and structural flexibility. The influences of the gimbal motions on the motions of the bodies are therefore computed exactly. In the approximate derivations of the present approach, however, modal data are used to define flexible rotations and modal coordinate rotation quantities are transformed as though they were the components of a vector. The influences of the gimbal orientations are therefore approximated. This treatment is accurate if the flexible rotations are very small, so that superposition of angular motions can be done without regard for the order in which they occur. The justification of this treatment is that the structural merge is used only to model the small flexible contributions to the rotations.

In addition, transformations are applied to all of the matrices of the modal coordinate formulation without including the influences of the rates of change of the transformations with time. Since differentiation of the transformation matrices generates quantities that are proportional to the angular rates, this is equivalent to omitting nonlinearities due rotational rates. The justification of this approximation is that the inertial loadings applied to the flexible modes due to the rotational nonlinearities cause very small deformations for most problems. If extremely flexible structures were to be simulated, or if the angular rates were very large, then this would not be an acceptable approximation. Therefore it is again seen that SADACS is limited in the degree of both flexibility and angular rate which it can accurately address.

Finally, the structural merge is performed at a particular set of body orientations, and the structural model is only valid for this particular condition of the system. To address the changes in the flexible character of the system as the relative orientations of the bodies change with time, SADACS incorporates an updating feature which re-merges the system and computes new flexible modes. The details of this procedure are not covered in this paper.

Figure 3 shows the coordinate systems which are required to set up the multibody structural merge. To simplify the figure only two bodies are shown, but the discussion is easily extended to the case of many bodies. Each body has the same global coordinate system, designated by the letter "N". The hinge between the bodies uses two coordinate triads to describe the gimbal rotations. Following the nomenclature of DISCOS, reference 1, a "p" triad is defined on the "inboard" body of the pair, and a "q" triad is defined on the "outboard" body of the pair. These triads are bound to the material of the bodies, and it is convenient to refer to a "p" body and a "q" body. A sequence of three Euler rotation angles, TH1, TH2, and TH3, rotate the p triad into the q triad. Since the triads are material bound, this rotates the q body, positioning it relative to the p body.

The p triad must contain the axis about which TH1 occurs. This can be any one of its axes. The p triad must also contain the axis which, after the TH1 rotation, will be the physical gimbal axis for the TH2 rotation. This can be any axis of p other than that of the TH1 rotation. The q triad must contain the axis of the final rotation of the Euler sequence, TH3, and, when TH3 is zero, also the axis of the TH2 rotation. In general, these requirements prevent the N system from being identical to either the p or the q system.

There is a degree of arbitrariness in the definition of TH1 and the p system. The p frame may contain the TH2 axis when TH1 has a zero value. In this case, the value of TH1 positions the q body with respect to the p body subsequent to the nominal positioning and the initial value of TH1 in dynamic analysis is zero. The rotation from N to p participates in the body 2 nominal positioning in this case. Figure 3a illustrates this definition. Alternatively, the p frame may not contain the axis of TH2 when TH1 has a zero value. In this case, the value of TH1 orients the q body with respect to the p body for both nominal and subsequent positioning, and the initial value of TH1 in dynamic analysis is nonzero. There need not be a rotation from N to p in the nominal orientation in this case. Figure 3b illustrates this definition.

Consider a general gimbal device which allows three Euler angular motions TH1, TH2, and TH3. The direction cosine transformations of each angular motion are easily computed and are defined here by the matrices [T1], [T2], and [T3]. Defining the basis vectors of the p and q frames by $\langle \underline{U}_p \rangle$ and $\langle \underline{U}_q \rangle$, the total direction cosine transformation from the p basis to the q basis is

$$\langle \underline{U}_q \rangle = [T3][T2][T1]\langle \underline{U}_p \rangle \quad (4)$$

The product of the direction cosine matrices above will be denoted by [T321]. Defining the p frame cartesian components of the angular rate of the q body with respect to the p body by $\langle w_p \rangle$ and the Euler angle rates as $\langle \dot{TH} \rangle$, these components are related by

$$\langle w_p \rangle = [Pi]\langle \dot{TH} \rangle \quad (5)$$

The matrix [Pi] is a function of the Euler angles TH1 and TH2 and is not orthogonal. There are 12 possible forms of [Pi] depending on the particular physical axes of the gimbal which correspond to the sequenced angles TH1, TH2, and TH3 (ref. 6). Equation 5 allows definition of the non-orthogonal basis system of the gimbal axes, defined by $\langle \underline{G} \rangle$, as

$$\langle \underline{G} \rangle = [Pi]^T\langle \underline{U}_p \rangle \quad (6)$$

The angular rate of the q body with respect to the p body can be expressed in either the p basis, as in equation 5, or in the q basis. Denoting the latter by $\langle w_q \rangle$, the [T321] transformation of equation 4 gives

$$\langle w_q \rangle = [T321]\langle w_p \rangle \quad (7)$$

Combining equations 5 and 7 gives

$$\langle w_q \rangle = [T321][Pi]\langle \dot{TH} \rangle \quad (8)$$

The matrix product above is defined as [Qi]. Thus,

$$[Qi] = [T321][Pi] \quad (9)$$

and

$$\langle w_q \rangle = [Qi]\langle \dot{TH} \rangle \quad (10)$$

The gimbal basis can now be determined in terms of the q basis. Following the forms of equations 5 and 6,

$$\langle \underline{G} \rangle = [Qi]^T\langle \underline{U}_q \rangle \quad (11)$$

The transformation from the p basis to the N basis is defined as [pTN], and that from the q basis to the N basis is defined as [qTN]. In SADACS these matrices are approximated by their rigid body definitions and are therefore constant in time. The transformations are

$$\langle \underline{U}_N \rangle = [pTN]\langle \underline{U}p \rangle \quad (12)$$

and

$$\langle \underline{U}_N \rangle = [qTN]\langle \underline{U}_q \rangle \tag{13}$$

and define the structural analysis basis, $\langle \underline{U}_N \rangle$, in terms of the two hinge cartesian bases, neglecting the effects of flexibility within the individual bodies.

The p body constraint mode transformation to the gimbal basis can now be defined. Denoting the transformed $\langle q_c \rangle$ vector for the p body by $\langle r_{cp} \rangle$,

$$\langle q_c \rangle = [[pTN][Pi]]^{*}\langle r_{cp} \rangle \tag{14}$$

where the notation [ ]* indicates that this transformation "stacks" the [pTN][Pi] matrix as 3x3 partitions along the diagonal in order to transform both the displacement and rotation freedoms.

For the q body the transformation must consider the possibility that the body may be in an off nominal orientation. This makes it most convenient to transform from the q basis to the gimbal basis. Denoting the transformed $\langle q_c \rangle$ vector for the q body by $\langle r_{cq} \rangle$,

$$\langle q_c \rangle = [[qTN][Qi]]^{*}\langle r_{cq} \rangle \tag{15}$$

The matrix [Qi] contains the positioning information which accounts for the off nominal orientation of body q.

The transformation matrix products in equations 14 and 15 will both be denoted by [rTN]* and it will be recognized in their use below that they are numerically different for the p and q bodies.

The mass matrix of a single body, equation 2, is transformed to the gimbal rotation components by

$$\begin{bmatrix} I & 0 \\ 0 & [[rTN]^{*}]^{T} \end{bmatrix} \begin{bmatrix} M_{FF} & M_{FC} \\ M_{CF} & M_{CC} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & [rTN]^{*} \end{bmatrix} \tag{16}$$

The stiffness matrix, equation 3, is also transformed by equation 16. In this case there are no off diagonal partitions in the calculation or the result.

The generalized loads applied to interior points of bodies are transformed by the matrix on the left in equation 16. The gimbal torques are defined in the gimbal bases and do not require transformation.

The compatibility condition for the attachment, or structural merge, of the bodies is

$$\begin{Bmatrix} r_{cp} \\ r_{cq} \end{Bmatrix} = \begin{bmatrix} I1 & I2 & 0 \\ I3 & 0 & I4 \end{bmatrix} \begin{Bmatrix} r_{c1} \\ r_{cp*} \\ r_{cq*} \end{Bmatrix} \tag{17}$$

where $\langle r_{c1} \rangle$, $\langle r_{cp*} \rangle$, and $\langle r_{cq*} \rangle$ are the common p body and q body freedoms which are locked and the p and q body freedoms which are free to have relative motion, respectively. I1, I2, I3, and I4 are selector matrices which define the interbody compatibility conditions.

For simplicity of notation the vector rc will be defined,

$$\langle rc \rangle = \begin{Bmatrix} r_{c1} \\ r_{cpf} \\ r_{cqf} \end{Bmatrix} \tag{18}$$

To derive the equations of motion of coupled bodies, the mass, damping, and stiffness matrices and the generalized loads of the individual bodies are first assembled without imposing the constraint conditions. The result is illustrated for two bodies by the mass matrix below.

$$\begin{bmatrix} M_{FF} & Mr_{FCp} & Mr_{FCq} \\ Mr_{CpF} & Mr_{CpCp} & 0 \\ Mr_{FCq} & 0 & Mr_{CqCq} \end{bmatrix} \tag{19}$$

The matrix partitions denoted by [Mr] have been transformed to the gimbal bases as described by equations 14-16. The subscripts F, Cp, and Cq denote the fixed interface modes, the constraint modes on the p body, and the constraint modes on the q body, respectively. The stiffness matrix corresponding to expression 18 has null off diagonal partitions. This form of the equations expects the freedoms to be ordered $\langle q_F \rangle$, $\langle r_{Cp} \rangle$, and $\langle r_{Cq} \rangle$. The extension to the entire system is accomplished by stacking additional partitions in expression 19.

The equations of motion are to be assembled by subjecting the freedoms to the constraint of equation 17. The constraint is written

$$\begin{Bmatrix} q_F \\ r_{Cp} \\ r_{Cq} \end{Bmatrix} = \begin{bmatrix} I & 0 & 0 & 0 \\ 0 & I1 & I2 & 0 \\ 0 & I3 & 0 & I4 \end{bmatrix} \begin{Bmatrix} q_F \\ r_{cp1} \\ r_{cpf} \\ r_{cqf} \end{Bmatrix} \tag{20}$$

Defining the selector matrix in equation 20 by [II], the transformations are accomplished by pre-multiplying the system mass, damping, and stiffness matrices by $[II]^T$ and post-multiplying the result by [II]. The generalized loads are transformed by pre-multiplying by $[II]^T$. This procedure involves only simple row and column operations and is most easily performed by additions rather than by the matrix multiplication process. The result of this final step is the component mode equations of motion of the SADACS flexible body solver.

System Modal Analysis

The coupled component mode equations of motion are diagonalized by eigensolution of the merged, second order structural equations of motion. No linearization of the rigid motion equations is required for this task. Damping is usually omitted in the component mode equations, leading to a generalized real symmetric eigenvalue problem with pure imaginary frequencies. Damping is then added to the system modes after the eigensolution. SADACS optionally solves the complex eigenvalue problem using assigned component modal damping. This option has not proved advantageous. The system modal damping produced usually varies greatly among the system modes, a situation felt to be unrealistic, and the complex eigenvalue problem is felt to be less reliably solved than the real form.

The flexible modeshapes of the system eigensolution are denoted by $[S_F]$. The system flexible modal coordinates are denoted by $\langle x_F \rangle$. The recovery of the component modal coordinate data for the purely flexible motion is given by

$$\begin{Bmatrix} q_F \\ r_c \end{Bmatrix} = [S_F]\langle x_F \rangle \tag{21}$$

The system flexible mode mass and stiffness matrices are computed by pre-multiplying the corresponding component mode matrices by $[S_F]^T$ and post-multiplying the result by $[S_F]$. The resulting matrices are diagonal. The generalized load vector is computed by pre-multiplying the component mode load vector by $[S_F]^T$.

The flexible system modal set is truncated to reduce computational effort in the time stepping integration procedure. Denoting the retained modes by $\langle x_{Fr} \rangle$ and the truncated modes by $\langle x_{Ft} \rangle$, the equations which are solved are

$$\langle \ddot{x}_{Fr} \rangle = \langle X_{Fr} \rangle - [C_{Fr}]\langle \dot{x}_{Fr} \rangle - [K_{Fr}]\langle x_{Fr} \rangle \tag{22}$$

and

$$\langle x_{Ft} \rangle = [\ K_{Ft}]^{-1}\langle X_{Ft} \rangle \tag{23}$$

where the symbol $\langle X \rangle$ denotes the generalized load, $[C]$ and $[K]$ denote the modal damping and stiffness matrices, and the $[K]^{-1}$ matrix in equation 23 is the diagonal of inverse modal stiffnesses of the truncated flexible system modes. The subscripts in all terms follow the definitions given above for the vector $\langle x \rangle$. Equation 22 is given for the case of unit flexible mode generalized mass, which is the normalization provided by the eigensolver.

The eigensolution, truncation, and time stepping solution procedure outlined above is extremely fast and has encountered no numerical difficulties. Several of the advantages of the approach are discussed briefly in the paragraphs which follow.

The truncation of the higher flexible modes allows the use of a much larger integration time step than would be required if all modes were retained. This benefit is not available to approaches which integrate component mode equations of motion because truncation of the component modal set can cause serious loss of accuracy. This is especially true when cantilever appendage modes are used to simulate systems with free or controlled gimbal freedoms. The loss of accuracy is due to the fact that the higher cantilever modes are important participants in the low frequency behavior of the dynamical system and therefore need to be retained in the component mode simulation. SADACS is typically used with very large numbers of component modes and severe truncation of the system modal set. This provides a substantial reduction of computational time and does not cause noticeable loss of accuracy.

The use of equation 23 is critical to the accuracy of problem solutions. This equation provides the quasi-static responses of the higher frequency system modes. If the contribution of equation 23 is omitted, the gimbal angle responses to control torques may be either under- or over-predicted by substantial amounts. In addition, the locations of transfer function zeros are made highly inaccurate by the omission of the quasi-static responses. It is the use of equation 23 which permits truncation of the modal set for the integration of equation 22, thereby speeding the computational process.

Equations 22 and 23 are easily solved because of the absence of coupling between the modes. The entire flexible solution has been reduced to a very simple form, leaving the difficult, coupled, nonlinear analysis problem to the rigid motion dynamics, where it is known to be most important in the majority of applications. The computational price which is paid for the modal analysis simplification is the effort of the eigensolution. This has proved to be very small in problems solved to date, in comparison with the computational effort of coupled modal analysis and integration with small time steps.

Separation of Rigid and Flexible Motions

The SADACS formulation uses separate rigid body (RB) and flexible body (FB) computer codes. The rigid body code currently used is the MBDY subroutine due to Likins and Fleischer (ref. 4). This code was chosen because it is a proven standard and because it could easily be integrated into the Boeing EASY5 system. It has proven reasonably fast in applications. Other rigid body codes could equally well be used. The flexible body code is the linear, small motion formulation of conventional structural dynamics, derived in the form outlined herein. It is used in system mode form and omits rigid modes. The flexible body solver is called from EASY5 as a subroutine.

Figure 4 shows a block diagram of this procedure. The figure identifies the rigid body solver, RB, the flexible body solver, FB, and the control simulation. The rigid motion prediction of FB is seen to be unused, and the flexible prediction is combined with the RB solution to create the total motion. The total motion provides the performance of the simulated system and the feedback data for the controller. It also is used to determine if the body orientation angles have changed sufficiently to require the calculation of new system modal data.

The figure indicates the geometric and modal transformations which have been discussed in the above sections. The gimbal torques are generated in the gimbal axis bases and are directly applicable to the model gimbal freedoms. They must, however, be transformed to correspond to the system modal coordinates. Torques applied to the interiors of the bodies are generated in the body bases, and these must be transformed to account for the use of constraint modal coordinates referred to the gimbal bases. They must also be transformed to correspond to the system modal coordinates.

After the time integration, the FB responses are back transformed from the system modal form to the constraint mode form referred to the gimbal bases. After this transformation, they can be combined directly with the gimbal rotation data from RB. Because the flexible contributions to the gimbal rotations are small, they can be added directly to the Euler positioning angles computed by RB. The internal body rotations similarly require a modal back transformation. Since they are needed in the body bases, they require an additional back transformation to account for the use of constraint modal coordinates referred to the gimbal basis systems.

MBDY computes the main body angular rates in the body basis. If main body angular position relative to the inertial frame is needed, an integration of these rates is done, taking proper account of the rotations of the body axes. This is not shown in the figure. The FB module computes the small flexible angular rates and positions of identified sensor points in the body basis. If it is required to obtain these quantities in the Euler angle basis which defines the inertial attitude of the body, they are transformed in the manner of equation 10. The equation is applicable to both rates and positions in this case because the flexible angles are very small.

The figure shows the modal coordinates and their rates returned to the equations of motion block. This is required to form the right hand sides of equation 22.

The decision whether to update the system modes is based on the magnitudes of the gimbal angles. If updating is not required, the solution continues with the commanded data and the feedback signals returning to the controller. If updating is required, the computational process exits to a set of updating routines. After computing new system modes and modal coordinate values and rates, the process returns to the integration routine as indicated by the figure. At the return, the new modal data have replaced the old modal data.

DISCOS-SADACS Comparisons

The approximations which have been used to reduce the computational time of SADACS cannot be quantified as to their accuracy on the basis of judgement alone. The magnitudes of errors which might occur depend on the magnitudes of the rotations and the rotational rates and on the sensitivity of the particular dynamical system to nonlinear influences. An effective way to verify that an approximate approach is accurate is to perform comparisons with high fidelity predictions of proven codes for problems of the type under study. For this purpose, an extensive verification of SADACS in comparison with DISCOS was done. The comparison used identical component mode models in SADACS and DISCOS. A number of problems were solved for the verification, all involving large angular motions at rates which caused the rigid motions to be strongly nonlinear. The results showed that the SADACS approach is extremely accurate and fast for complex multiflexible body systems with large motions at moderate rates.

All of the calculations showed that the simplifications of the SADACS approach result in a large reduction of computer time. The reduction was about 100-times for most of the problems studied. Run time reduction estimates are strongly problem sensitive. Larger problem sizes would significantly increase the speed advantage of the SADACS type approach over fully nonlinear approaches. Very small problems have shown only a two to three times speed advantage. The DISCOS open loop simulations in some cases required 40 or more CPU hours on a VAX 11-780. These problems were not large in comparison with other simulations of complex control structure interaction problems. For many problems it would not be feasible to run DISCOS simulations, or probably any other similar simulations using component modes and retaining fully nonlinear equations. An approach such as SADACS is probably the only way to attack such large control design problems.

The elimination of certain nonlinearities within a high fidelity code can speed its calculations appreciably. The TREETOPS code has such valuable options and Boeing has developed similar options within DISCOS. However, it appears that achieving a major speed increase necessitates transforming the equations to diagonalized form and truncating the system in order to increase the integration time step. Thus, the elimination of nonlinearities may not achieve the level of computational speed increase which is needed for really large control design problems. In the authors' view, the real need for high computational speed in high fidelity codes is to allow verification of approximate codes for simulations of realistic complexity. This is barely possible at the present time.

Figures 5-8 show comparisons of DISCOS-SADACS time history predictions. The figures give the responses of the main body and one appendage of a complex system. The command for this problem was a large rotational excursion of one appendage and the problem was run without control. Torques were applied to all system bodies through an inverse inertia matrix such that the rotations of the main body and the uncommanded appendages were very small for the initial stages of the motion. At the later stages, however, nonlinear effects cause all of the bodies to have large rotations, since no feedback was used to control the motions.

Figure 5 shows the main body x-rotation. Large motions occur in the later stages of the response due to the effects of nonlinearities. These motions are not predicted by linearized simulations. This figure shows that the SADACS use of simultaneous nonlinear rigid body and linearized flexible body solutions provides excellent large motion accuracy.

The triangular excursion at the start of the motion in figure 5 is due to the effects of flexibility in response to the applied torque pulse. Figure 6 shows an enlargement of this portion of the response. In this figure, the SADACS and DISCOS predictions are separated by two plot divisions in order to permit detailed examination of the flexible oscillations. It is seen that the flexible motion predictions are virtually identical. This figure verifies the FB system mode formulation and shows that the angular rates have little effect on the flexible responses.

The responses shown on figure 6 and the early portion of figure 5 are changed greatly if the correction given by equation 23 is omitted. The magnitude can change several-fold and the sign of the early response may also change due to such omission. This was observed in comparisons of DISCOS with a linearized code which omitted the quasi static deformation correction. In this case, the modal truncation was done by the structural analysis procedure before creating the data for the control design simulation.

Figure 7 shows the main body z-rotation, again in an enlarged plot to allow close examination of the flexible response. The SADACS and DISCOS predictions are indistinguishable on the plot.

Figure 8 shows the rotation about one gimbal axis of an uncommanded appendage. The late motions have a strongly nonlinear response which is predicted accurately by the SADACS simulation. The comparison verifies the accuracy of the approach of separate RB and FB solution procedures. This calculation is a more critical test of the prediction method than that of figure 5 because it emphasizes the coupling effects of the main body translational motions and the sensitivity of the rigid body inertia matrix to the angular motions of the main body and the torqued appendage.

Conclusions

A computer code for multiflexible body dynamic analysis has been developed based on linearizing flexible motions while retaining fully nonlinear rigid motions. The code is conceptually simple because its flexible formulation is that of conventional structural dynamics. It is embedded within a control design system, so that it is well adapted to both frequency and time domain control design applications.

The accuracy and computational speed of the code have been evaluated by comparisons with the predictions of DISCOS for problems with strong rigid motion nonlinearities and moderately flexible structural components. The approach has been found to be exceptionally accurate and fast.

The accuracy evaluations have shown that multibody flexible nonlinearities are usually extremely small while rigid nonlinearities are almost always sufficiently large to require simulation in control design work.

The computational speed evaluations have shown that the key factor in the slow speeds of the fully nonlinear, high fidelity multibody simulations is their integration of the coupled equations of motion in terms of component modal coordinates. This type of formulation is required to permit full retention of flexible nonlinearities. It results in excessive computation time because of the processing of the flexible rotations as large quantities, the handling of coupling terms in the equations, and especially because of the small integration time step required by the component mode representation. The actual computation of the nonlinear numerical terms in the equations of motion is not a dominating factor in the slow computational speeds of these types of formulations.

The omission of the flexible nonlinearities allows all of these time consuming computations to be eliminated. The result is a very fast approximate approach which can attack the computationally demanding problem of control design trades while maintaining sufficient accuracy for performance predictions.

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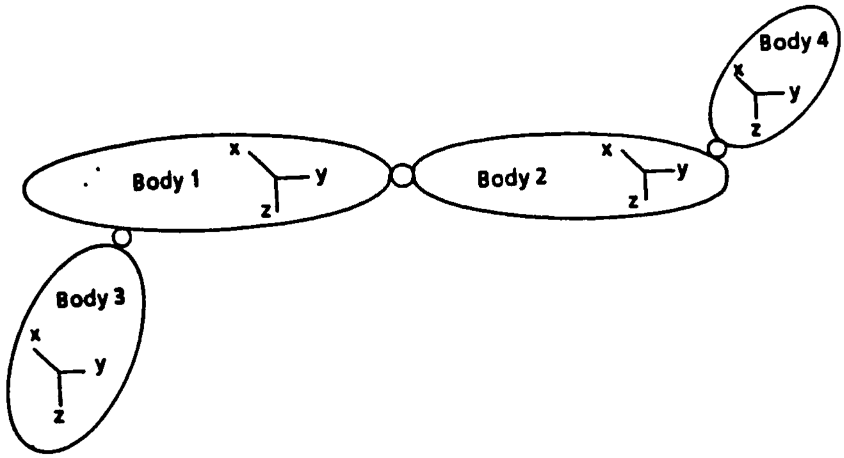

Figure 1 **Multibody System Example**

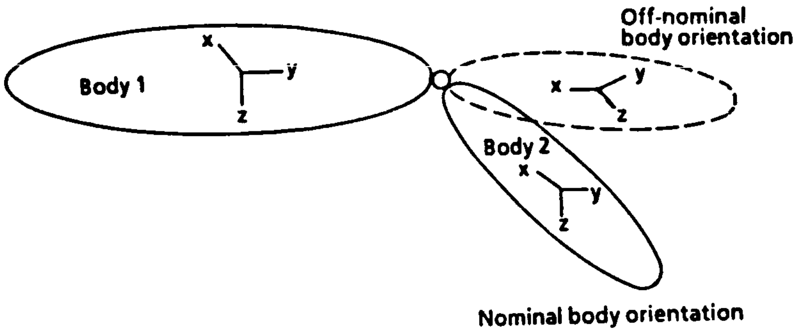

Figure 2 **Nominal and Off-Nominal Body Frames**

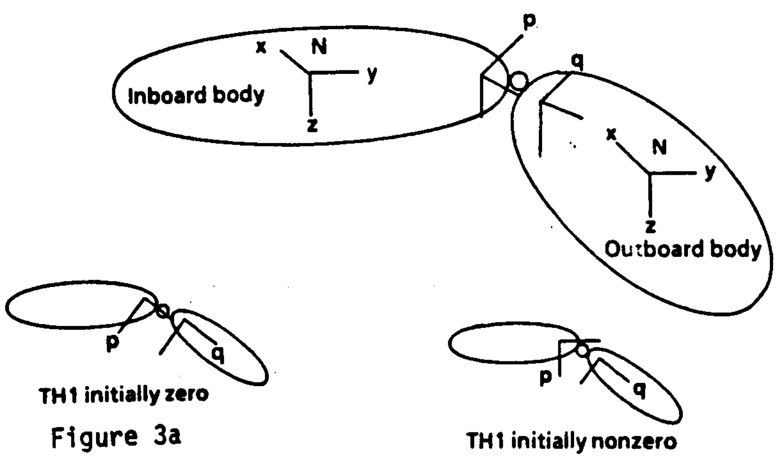

Figure 3a

Figure 3b

Figure 3 **Definitions of Gimbal Coordinate Frames**

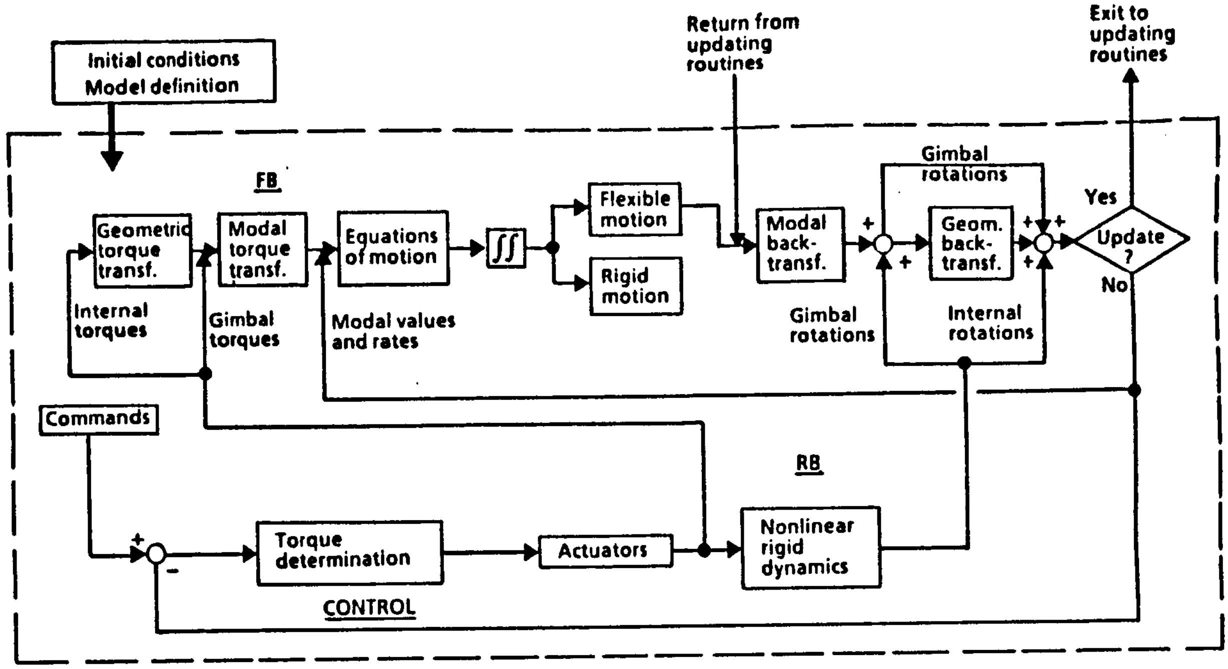

Figure 4 **SADACS Computational Flow**

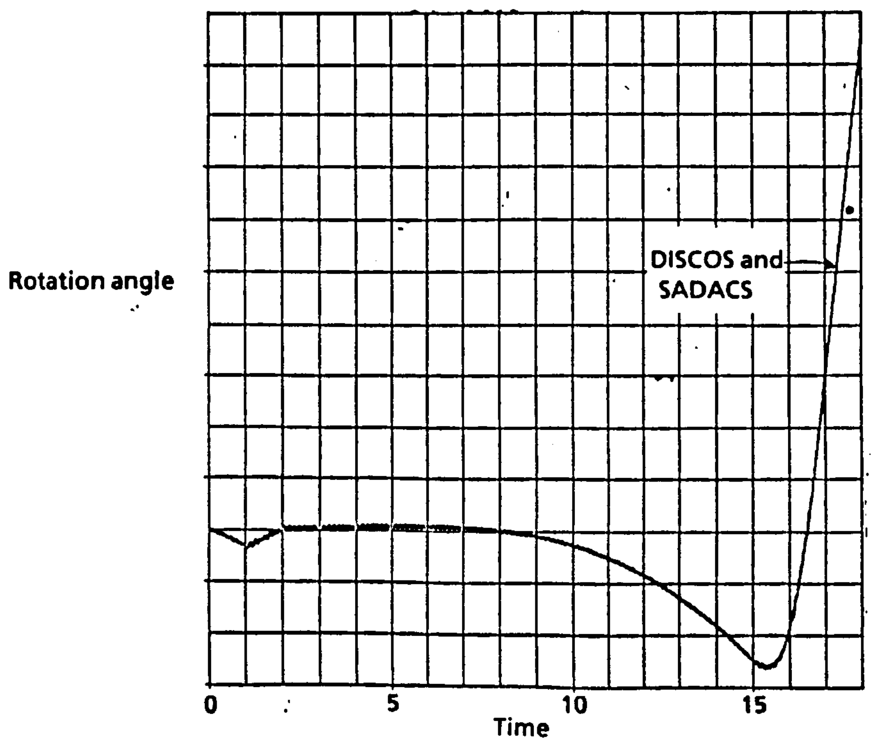

Figure 5 **DISCOS-SADACS Comparison:**
Main Body Sensor X Rotation Due To Appendage Command

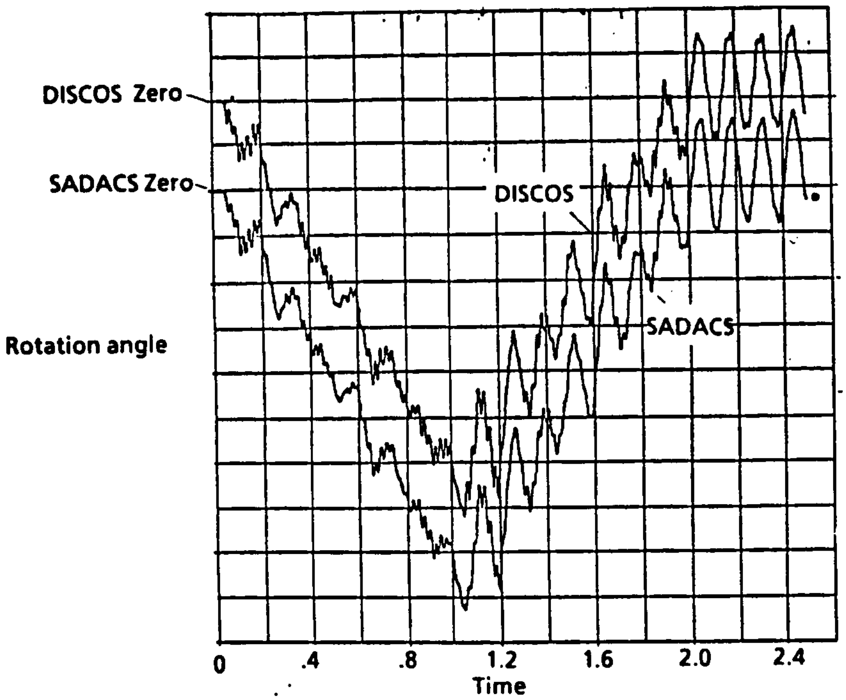

Figure 6 **DISCOS-SADACS Comparison:**
Main Body Sensor X Rotation Due To Appendage Command

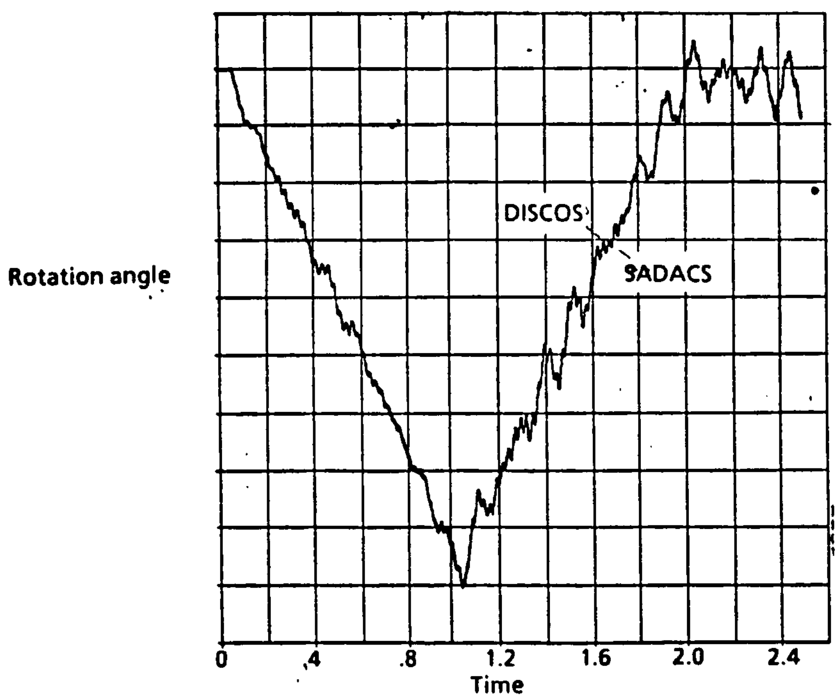

Figure 7 **DISCOS-SADACS Comparison:**
Main Body Sensor Z Rotation Due To Appendage Command

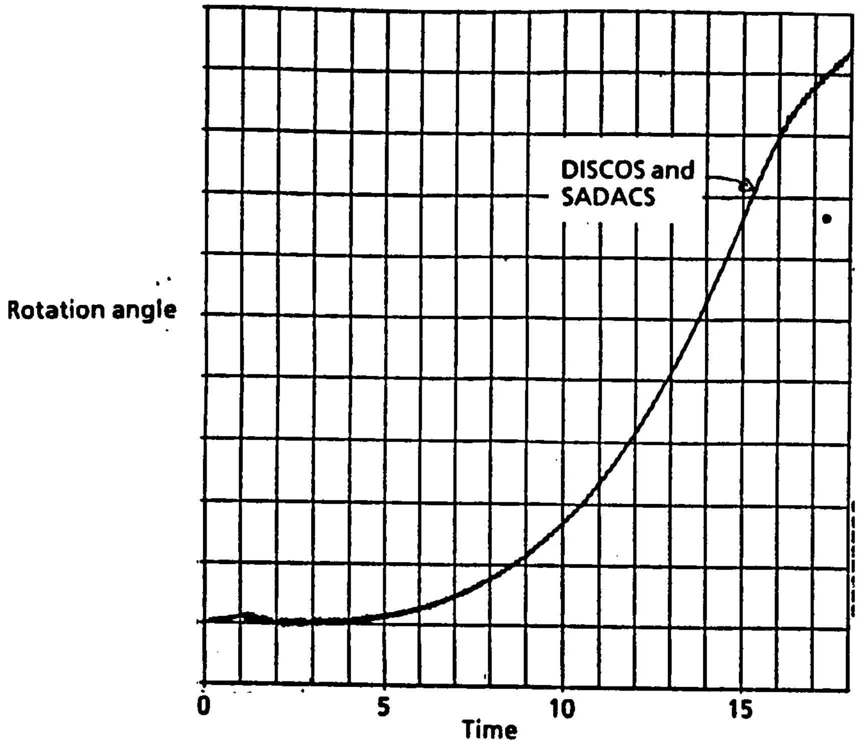

Figure 8 **DISCOS-SADACS Comparison:**
Hinge Rotation of Un-Torqued Appendage Due To Command on Another Appendage