N87-22745

MODELING OF CONTROLLED FLEXIBLE STRUCTURES WITH IMPULSIVE LOADS

M. Zak

Applied Technologies Section, Jet Propulsion Laboratory
California Institute of Technology, Pasadena, CA, USA

Abstract. The characteristic wave approach is developed as an alternative to modal methods which may lead to significant errors in the presence of impulsive or concentrated loads. The method is applied to periodic structures. Some special phenomena like cumulation effects and transitions to ergodicity are analyzed.

INTRODUCTION

Controlled large space structures, which will likely be composed of networks of long slender members, are subjected to disturbances (coming from the actuators) with a relatively small contact zone and short time interval. From the mathematical viewpoint such disturbances are characterized by discontinuities which can be considered as a very high frequency. In truncation techniques which are used in modal analysis the contribution of high frequencies is lost, and therefore, the impulsive concentrated loads are supposed to be treated by some other methods.

Since any discontinuity propagates with the characteristic speed, it is reasonable to turn to the characteristic wave approach in treating the impulsive loads. The advantage of this approach is in the fact that characteristic speeds depend only on the coefficients at the highest (second order) derivatives in the governing equation of structural members which significantly simplifies the analysis of characteristic waves.

Thus, it appears that the application of the characteristic wave approach is the most beneficial in the domains where spectral methods fail. That is why it can be used as a supplement to modal methods for linear analysis of controlled structures when loads can be decomposed in to "smooth" and impulsive components.

In this article, some aspects of characteristic wave propagation, reflection and transmission in structures with one-dimensional structural members as well as possible engineering tools for their analysis are discussed.

PROPAGATION OF IMPULSIVE LOADS IN ONE-DIMENSIONAL STRUCTURAL MEMBERS

We will start with a one-dimensional structural member subjected to a concentrated or impulsive load assuming that

$$\Delta \ell \ll L, \text{ and } \Delta t \ll \frac{L}{C} \tag{1}$$

in which L is the length of the structural member, $\Delta\ell$ is the width of the contact zone of the impulse, Δt is the duration of the concentrated load, and C is the characteristic speed of wave propagation, while

$$C_B^2 = \frac{E}{\rho},\ C_T^2 = \frac{G}{\rho},\ C_V^2 = \frac{GA_s}{\rho A},\ C_M^2 = \frac{EI}{\rho I},\ C_S^2 = \frac{T}{\rho} \tag{2}$$

Here C_B, C_T, C_V, C_M, and C_S are the characteristic speeds for longitudinal, torsional, shear, bending and transverse string waves, respectively. E is the Young modulus, G is the shear modulus, A is the cross-sectional area, A_s is an effective shear area of a Timoshenko beam, EI is the cross-sectional area moment of inertia, ρI is the rotatory inertia per unit length, ρ is the mass per unit length, T is the string tension, W. Flugge, 1962.

For homogenious structural members all the characteristic speeds (2) are constant, and consequently, the width $\Delta\ell$ as well as the duration Δt of the impulse will be constant too. However, the initial configuration of the impulse will be preserved only for the simple wave equation without damping.

In all other cases due to the dispersion phenomenon this configuration, strictly speaking, will not be preserved. Nevertheless, the dispersion can be ignored if the conditions (1) are satisfied. For further convenience we will introduce an equivalent rectangular impulse of the same length and energy. For such a rectangular impulse, all the waves listed in (2) are decoupled even if they propagate simultaneously in a structural member, and this is the most important advantage of the characteristic wave approach to propagation of impulsive loads.

As follows from the energy conservation law the height of the rectangular impulse expressed in terms of velocity, strain or stress will be constant if there is no material damping. If material damping is proportional to the velocity being characterized by the damping coefficient ζ then the height h of the rectangular impulse will exponentially decrease:

$$h^2 = h_0^2 e^{-\zeta t} \tag{3}$$

NON-HOMOGENEITY EFFECTS

The situation becomes more complicated even for a rectangular impulse if the speed of propagation is not constant. This effect can be caused by non-linear material properties (if the impulse is large enough to generate finite strains) or by non-homogenious properties of the structural member.

In the first case the speed becomes non-characteristic since it depends on the magnitude of the transmitted parameters, i.e., for a rectangular impulse:

$$c = c(h) \tag{4}$$

This dependence may lead to a qualitatively new effects such as shock wave formation.

In the second case the speed remains characteristic, but it depends on the space coordinate:

$$c = c(x) \tag{5}$$

Although the governing equations for wave propagations remain linear (but with variable coefficients) the dependence (5) may lead to some surprising effects. In order to describe them, we will start with the energy balance. The energy E of a propagating impulse consist of potential and kinetic components:

$$E = \frac{1}{2}\int_{x_1}^{x_2} \rho(C^2[\varepsilon]^2 + [v]^2)\, dx \tag{6}$$

in which x_1 and x_2 are the space coordinates of the trailing and leading fronts of the propagating wave, $[v]$ and $[\varepsilon]$ are the impulsive velocity and strain, respectively, while

$$[\varepsilon] = -c[v] \tag{7}$$

as follows from the kinematical condition at the front of a discontinuity, Miklowitz, (1984). Hence, for a rectangular impulse:

$$E = \rho[v]^2\,(x_2 - x_1) \tag{8}$$

and instead of Eq. (3) now one obtains:

$$h^2(x_2 - x_1) = h_o^2\,\Delta\ell e^{-\zeta t} \tag{9}$$

The first effect which can be found from Eq. (9) is associated with the specific energy cumulation, i.e., with the unbounded growth of h due to shrinking width $(x_2 - x_1)$ of the propagating impulse. This effect was first described and explained by M. Zak, 1983.

The second effect is associated with a trapping of a propagating impulse within a localized area of a structural member. Similar effect of normal mode localization was predicted by C.H. Hodges, 1982, in connection with a system of coupled linear oscillators with damping. Thus, the trapping effect of a propagating impulse can be considered as a "continuum version" of the normal mode localization. A mathematical treatment of this effect is presented below.

Consider a function (5) in the following form:

$$C = \begin{cases} C_o \text{ at } x < x_* \text{ and } x > x_{**} \\ \gamma C_o \text{ at } x_* > x > x_{**},\ 0 < \gamma < 1 \end{cases} \tag{10}$$

Such a discontinuity of the characteristic speed C within a small segment $(x_{**} - x_*) < \Delta\ell$ can be caused by some structural irregularities (such as material inclusions, joints, etc.)

As follows from Eq. (10)

$$x_2 - x_1 = \begin{cases} \Delta\ell \text{ at } x_1 = x_* - \Delta\ell,\ x_2 = x_* \\ \Delta\ell - C_o(1-\gamma)t \text{ at } x_1 < x_* < x_2 < x_{**} \\ \Delta\ell - (x_{**} - x_*) \text{ at } x_1 < x_*,\ x_2 > x_{**} \\ \Delta\ell - (x_{**} - x_*) + C_o(1-\gamma)t \text{ at } x_* < x_1 < x_{**}, x_2 > x_{**} \\ \Delta\ell \text{ at } x_1,\ x_2 > x_{**} \end{cases} \tag{11}$$

Substituting (11) into (9) one finds:

$$h^2 = \begin{cases} h_o^2 e^{-\zeta t} \text{ at } x_1 = x_* - \Delta\ell,\ x_2 = x_* \\ h_o^2 e^{-\zeta t}/(1-\xi t) \text{ at } x_1 < x_* < x_2 < x_{**} \\ h_1^2 e^{-\zeta t}/(1-\lambda) \text{ at } x_1 < x_{*1} < x_2 < x_{**} \\ h_o^2 e^{-\zeta t}/(1-\lambda+\xi t) \text{ at } x_* < x_1 < x_{**1} x_2 > x_{**} \\ h_3^2 e^{-\zeta t} \text{ at } x_i,\ x_2 > x_{**} \end{cases} \tag{12}$$

in which

$$\xi = \frac{C_o(1-\gamma)}{\Delta\ell},\quad \lambda = \frac{x_{**} - x_*}{\Delta\ell}$$

$$h_1^2 = h_0^2 e^{-\zeta t},\quad h_2^2 = h_0^2 e^{\zeta(t_1+t_2)}$$

$$h_3^2 = h_0^2 e^{-\zeta(t_1+t_2+t_3)} \tag{13}$$

$$t_1 = \frac{\lambda}{\xi},\quad t_2 = \frac{1-\gamma-\lambda}{\xi},\quad t_3 = \frac{\lambda}{\xi}$$

Simple analysis of Eq. (12) shows that the function h(x) has maximum at $x_1 < x_* < x_2 < x_{**}$ if

$$\xi < \zeta < \frac{\xi}{1-\lambda} \tag{14}$$

This maximum is:

$$h^2_{max} = h_0^2\,\frac{\zeta}{\xi}\,e^{\left(1 - \frac{\zeta}{\xi}\right)} \tag{15}$$

The maximum of h_{max} as a function of ζ/ξ will be at

$$\frac{\zeta}{\xi} = \frac{1}{2}\,(1 + \sqrt{5}) \tag{16}$$

and therefore:

$$h^2{}_{max,max} = 8.15\ h^2{}_o \tag{17}$$

Substituting (16) into (14) one obtains:

$$\lambda < 0.382 \tag{18}$$

But, as follows from (13):

$$h_3^2 = h_o^2 e^{-\frac{\zeta}{\xi}(1+\gamma+\lambda)t} \tag{19}$$

Hence, the trapping effect will be the strongest if

$$\lambda = 0.382 \tag{20}$$

Indeed, in this case the function (12) has the sharpest shape since Eq. (16) provides the largest maximum (17), while Eq. (20) leads to the highest degree of dissipation after this maximum. Thus, the conditions (16) and (20) can be used as the key for structural implementation of the trapping effect.

Governing Equations

As shown by Zak, M. 1985, the governing equations for a propagation, reflection and transmission of an initial impulse in a two-member structure with isolated ends at $x = 1$, and $x = 3$, and a joint at $x = 2$ form a system of difference equations:

$$h_{k+1} = A h_k \tag{21}$$

where

$$A = \begin{pmatrix} 0 & \zeta_{12}\zeta_1 & 0 & 0 \\ -\zeta_{12}\zeta_2 z_2 & 0 & 0 & -\zeta_{12}\zeta_2(1-z_2) \\ -\zeta_{23}\zeta_2(1-z_2) & 0 & 0 & -\zeta_{23}\zeta_2 z_2 \\ 0 & 0 & -\zeta_{23}\zeta_3 & 0 \end{pmatrix}, \quad h = \begin{pmatrix} h_{12} \\ h_{21} \\ h_{23} \\ h_{32} \end{pmatrix} \tag{22}$$

in which h_{12} is the wave at $x = 2$ coming from $x = 1$, etc. ζ_{12} and ζ_{13} are the damping coefficients in the corresponding structural members, ζ_1, ζ_2, ζ_3 are the damping coefficients at $x = 1, 2, 3$, respectively, ℓ_2 is the reflection coefficient at $X = 2$, while $\ell_1 = \ell_3 = 1$.

The same procedure can be applied to multi-member structural systems, while the matrix A will attain new submatrices corresponding to additional structural members (like a stiffness matrix in finite element methods).

Thus, any structure with n identical structural members subjected to impulsive loads is governed by the matrix difference equation (21) while the order of this system is $2n$.

Analysis of Solutions

In the case of n-member structure the solution to the governing equation (21) where the matrix A is of the order $2n$ can be written in the following form:

$$h_k = A^k h_o, \quad k = 0, 1, 2, \dots \tag{23}$$

All the qualitative properties of this solution are defined by the eigenvalues of the matrix A, i.e., by the roots of the characteristic polynomial:

$$|A - \lambda I| = 0 \tag{24}$$

For instance, the solution is stable if

$$|\lambda_i| < 1, \quad i = 1, 2, \dots 2n \tag{25}$$

By applying a linear fractional transformation

$$\lambda = \frac{\bar{\lambda}+1}{\bar{\lambda}-1} \tag{26}$$

to Eq. (25) one reduces the stability analysis to conventional methods.

Transition to Ergodicity

So far the only structures with identical members (characterized by the dimentionless time delay ℓ) were considered. It was demonstrated that there exists a formal analogy between the matrix techniques for treating these structures under impulsive loads and for conventional modal analysis (although the matrices have different physical nature). However, in reality the identicalness of structural members even in periodic structures is an exception rather than a rule. Indeed, different time delays for different structural members can be caused not only by different lengths, but also by different characteristic speeds. In turn, different characteristic speeds may occur if joints convert one type of deformation into another (see Eq. (2)). Another source of different time delays is associated with external forces if they applied not to joints. In this case, the points of their application must be considered as additional joints, but without reflection or damping, and this will lead to additional structural members with different time delays. As will be shown below, different time delays lead to new qualitative effects which do not occur in modal methods.

For simplicity, we will start with the two-member structure and assume the following (dimensionless) lengths:

$$AB = 4, \quad BC = 6 \tag{27}$$

Then, the characteristic equation for this two-member system obviously has the order 24 (which is the least common multiple of 2x4 and 2x6)

But in the case

$$AB = 1, \; BC = \sqrt{2} \tag{28}$$

the ratio of the time delays is irrational, and therefore, for successive rational approximations of $\sqrt{2}$ the order of the governing difference equation tends to infinity as:

$$4, \; 14, \; 282, \; 1414, \; \dots \text{ etc.} \tag{29}$$

Now it is easy to deduce, that if in an n-member structure the time delays $\tau_1, \tau_2, \dots \tau_n$ are commensurate, then the order of the governing difference equation will be finite and equal to the least common multiple of $2\tau_1, 2\tau_2, \dots 2\tau_n$.

If at least two time delays are not commersurate, this order will tend to infinity. Obviously, this effect does not have an analogy in modal approach where the order of the governing differential equation depends only on the number of modes (or finite elements) considered.

In order to clarify the physical meaning of such a phenomenon let us start with the following question: during what time interval T an initial impulse will return to its original location in "one piece"? Simple geometrical consideration show that

$$\begin{aligned} T &= 2 \text{ if } AB = 1, \; BC = 1 \\ T &= 24 \text{ if } AB = 1, \; BC = 6 \end{aligned} \tag{30}$$

$$T = \infty \text{ if } AB = 1, \; BC = \sqrt{2}$$

In other words, this (dimensionless) interval is equal to the order of the governing difference equation.

Thus, if at least two time delays in an n-member structure are not commensurate, the system will never return to its initial position, i.e., the motion will lose its periodicity. In classical mechanics such systems are known as ergodic systems. For infinite number of times they pass through every state of motion (which is consistent with constraints) spending equal time intervals near ech state.

As however, the rational numbers are a set of measure zero, practically every motion of this type sooner or later become ergodic.

Nevertheless, in engineering applications there always can be found such a characteristic time interval within which the motion is approximately periodic, while the transition to ergodicity can be ignored due to damping.

It is worth emphasizing that the transition to ergodicity is not "inevitable" if one takes into account non-linear properties of real structures. Non-linearities may provide some mechanisms (such as dynamical synchronization effects) which depress the disorder and lead to periodical motion. In this connection, it is relevant to mention the experiment with coupled chain of harmonis oscillators performed by Fermi, Pasta and Ulam. Instead of ergodicity which was expected they found periodic oscillations. However, if dynamical synchronization effects do not depress ergodicity and if the characteristic time during which the motion can be approximated as periodic is too short one has to apply methods of statistical mechanics.

ACKNOWLEDGEMENTS

The research described in this paper was carried out by the Jet Propulsion Laboratory, California Institute of Technology, under NASA Contract No. NAS7-918. The effort was supported by Dr. A. Amos, Air Force Office of Scientific Research.

REFERENCES

W. Flugge, (1962). Handbook of Engineering Mechanics, McGraw Hill Co., New York, pp 64-11.

J. Miklowitz, (1986). The theory of elastic waves and waveguides, North Hollywood, New York, pp 71.

M. Zak, (1983). Wrinking phenomenon in structures, Part II, Solid mechanics archives, 8, pp. 279-311.

C. H. Hodges, (1982). Confinement of vibrations by structural irregularities, J. Sound and vibrations, 82(3), 411-424.

M. Zak, (1986). Characteristic wave approach in controlled large space structures, AIAA/ASME/ASCE/AMS 27th Structures, Structural Dynamics and Material Conference, San Antonio, TX.

Modeling of Controlled Structures with Impulsive Loads

by Michail Zak

Jet Propulsion Laboratory
Caltech

Spectral Methods

1. Modal Approach

$$u(x,t) = \sum_{m=1}^{\infty} X_m(x)\, T_m(t)$$

$$X_m = A_m' \cos \lambda_m x + A_m'' \sin \lambda_m x \quad \Big| \quad \text{e. f.}$$

$$T_m = B_m' \cos \omega_m t + B_m'' \sin \omega_m t \quad \Big| \quad \text{time f.}$$

λ_m - shape parameter, ω - frequency

2. Travelling Wave Approach

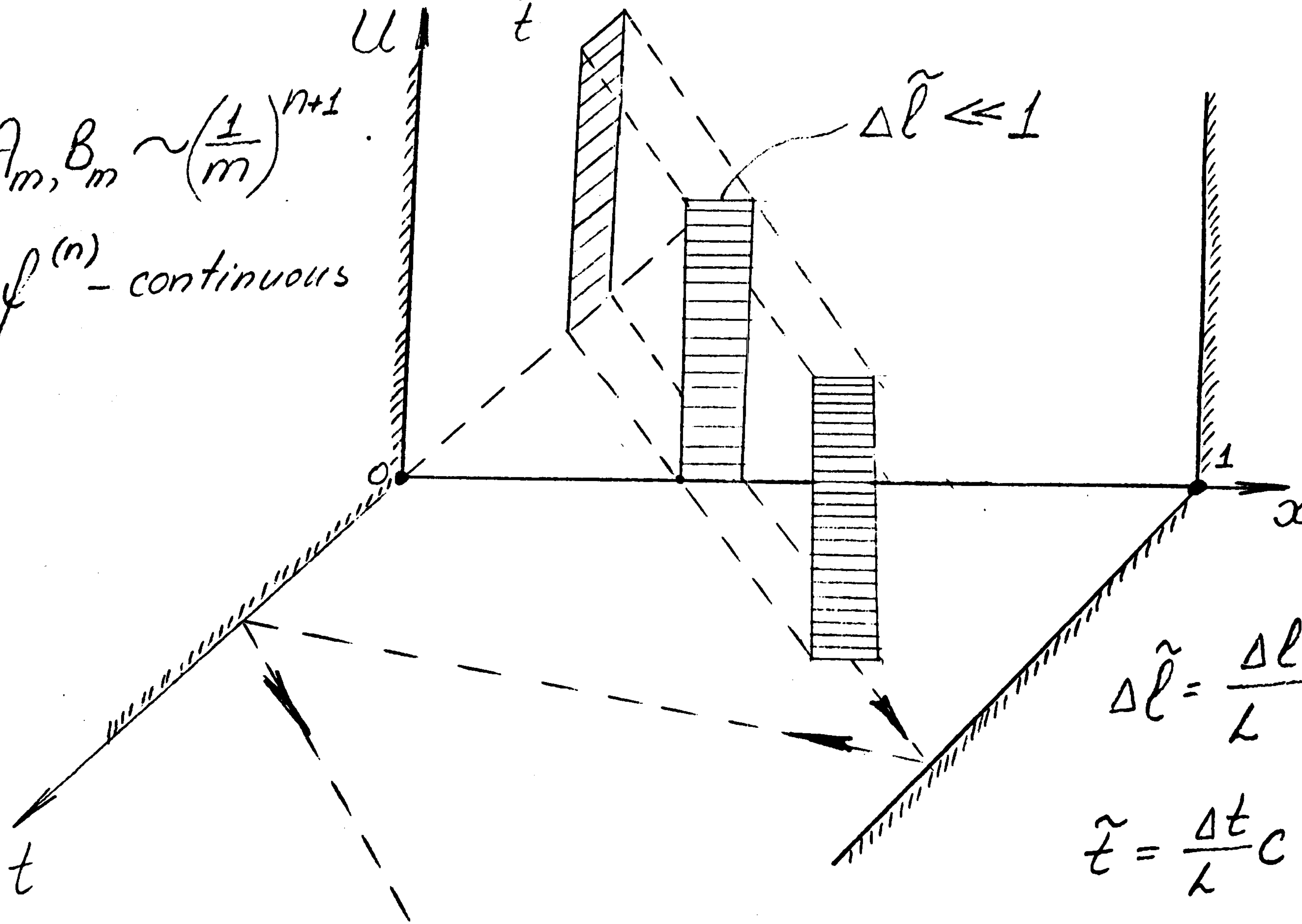
$$u(x,t) = \sum_{m=1}^{\infty} A_m \cos(kx-\omega t) + B_m \sin(kx-\omega t)$$

$$C = \frac{\omega}{k}$$ - speed of wave propagation

Characteristic Wave Approach

$A_m, B_m \sim \left(\frac{1}{m}\right)^{n+1}$.

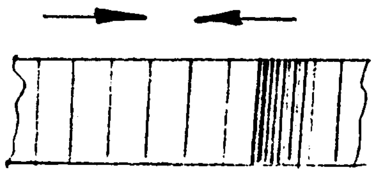
$f^{(n)}$ - continuous



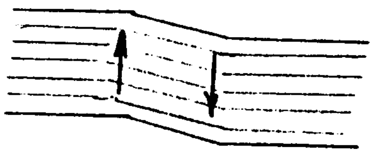
$\Delta\tilde{l} = \frac{\Delta l}{L}$

$\tilde{t} = \frac{\Delta t}{L} c$

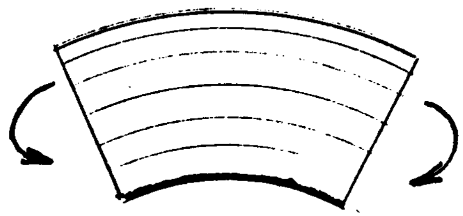
Characteristic Speeds



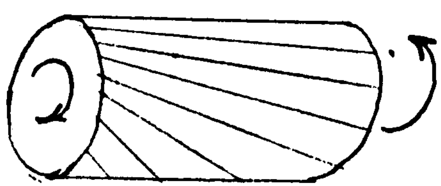
$$C_B^2 = \frac{E}{\rho} \text{ - longitudinal wave}$$



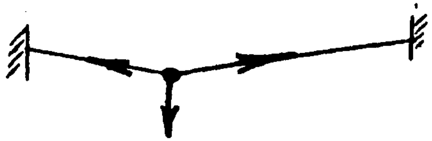
$$C_V^2 = \frac{GA_s}{\rho A} \text{ - shear wave}$$



$$C_M^2 = \frac{EI}{\rho I} \text{ - bending wave}$$

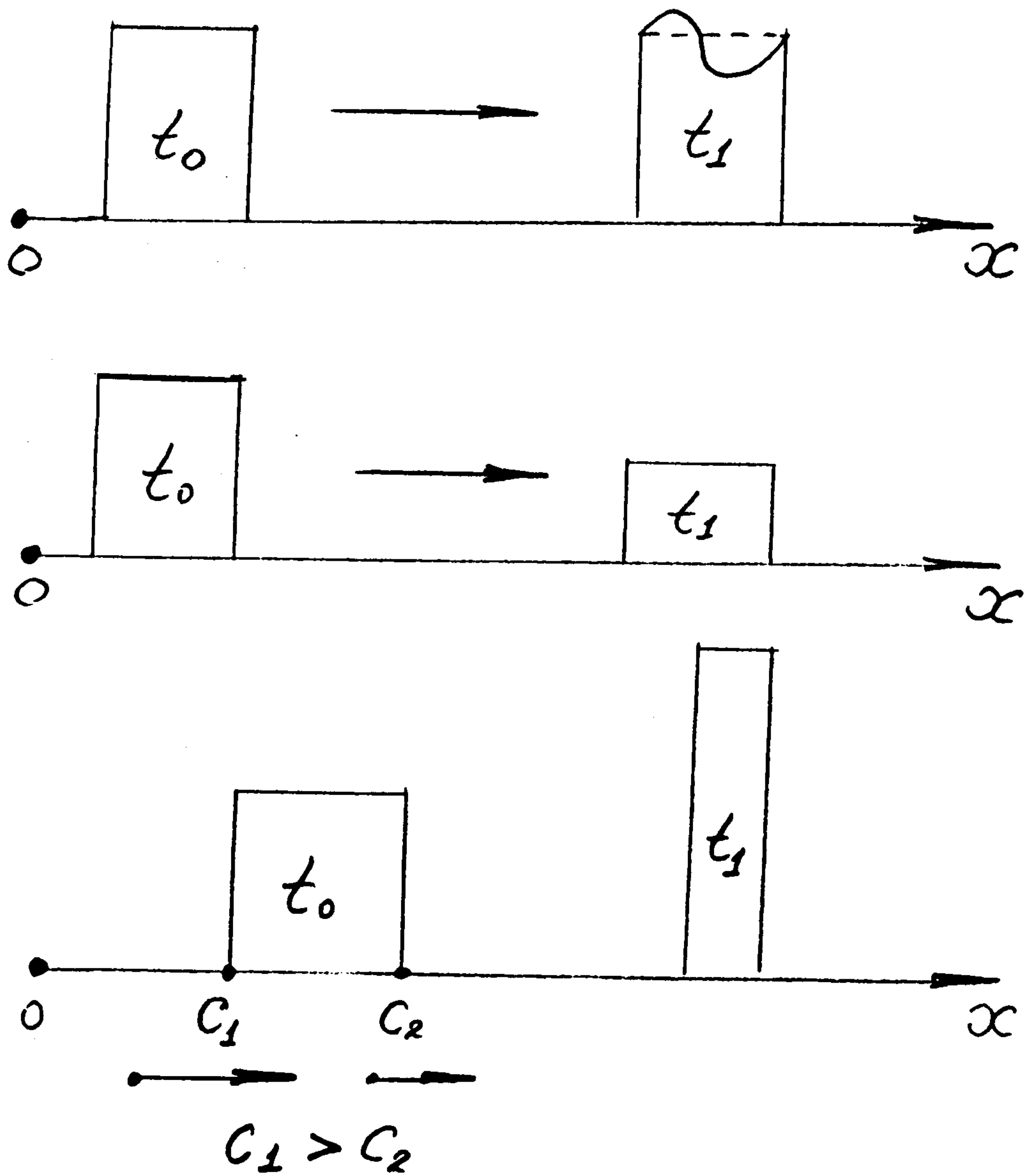


$$C_T^2 = \frac{G}{\rho} \text{ - torsion wave}$$



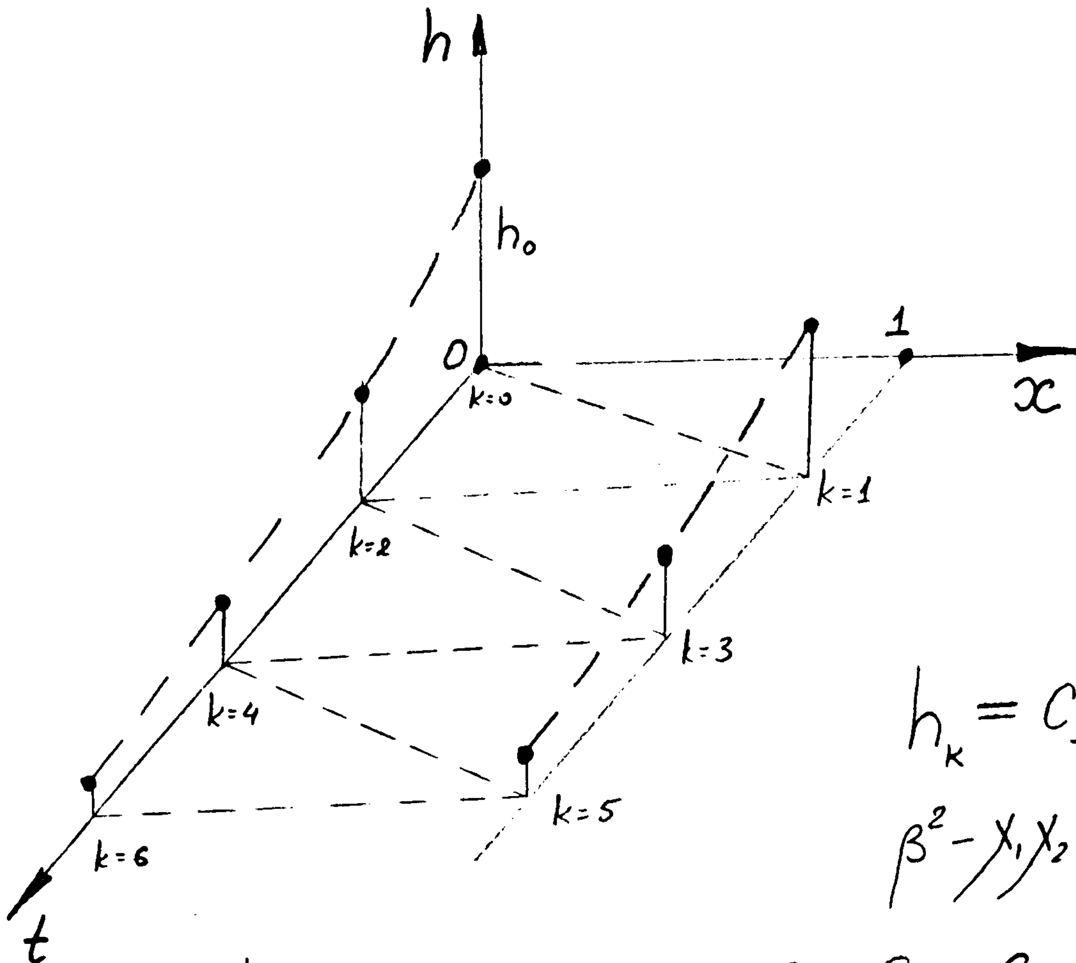
$$C_S^2 = \frac{T}{\rho} \text{ - shear string wave}$$

Linear Effects



- Dispersion
- Damping
- Cumulative effect

Periodic Structures: reflections



$$\left.\begin{aligned} h(1, t+1) &= \chi_1 h(0, t) \\ h(0, t+2) &= \chi_2 h(0, t+1) \end{aligned}\right\} \longrightarrow$$

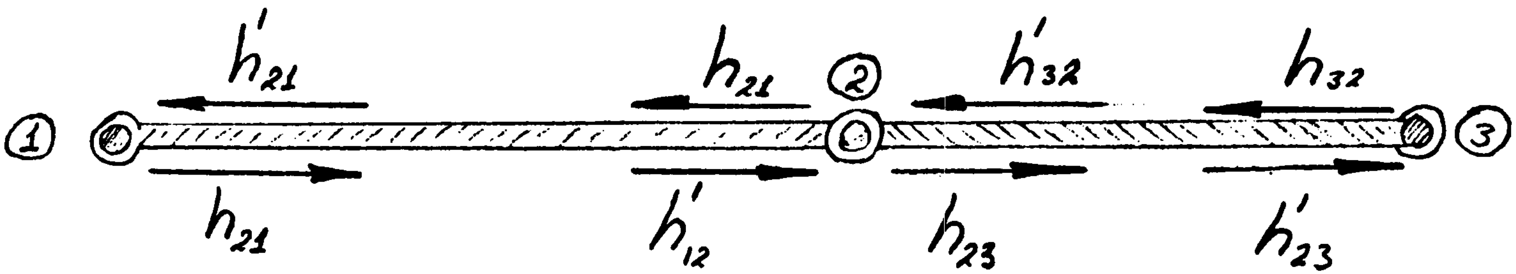
$$h_{k+2} - \chi_1 \chi_2 h_k = 0$$

$$h_k = C_1 e^{r_1 k} + C_2 e^{r_2 k}, \quad r_m = \log \beta_m$$

$$\beta^2 - \chi_1 \chi_2 = 0, \quad \beta_1 = \sqrt{\chi_1 \chi_2}, \quad \beta_2 = -\sqrt{\chi_1 \chi_2}$$

$$\left.\begin{aligned} h_0 &= C_1 + C_2 \\ h_1 &= (C_1 - C_2)\sqrt{\chi_1 \chi_2} = 0 \end{aligned}\right\} \quad \begin{aligned} C_1 &= C_2 = \tfrac{1}{2} h_0 \\ h_{2n} &= h_0 (\chi_1 \chi_2)^n, \end{aligned}$$

Periodic Structures: transmission and reflection



Propagation	Reflection-transmission
$h'_{12}(t+1) = \zeta_{12}\, h_{12}(t)$	$h_{12} = \zeta_1\, h_{21}$
$h'_{23}(t+1) = \zeta_{23}\, h_{23}(t)$	$h_{21} = \zeta_2 \left[\eta_2\, h'_{12} + (1-\eta_2) h'_{32}\right]$
$h'_{32}(t+1) = \zeta_{23}\, h_{32}(t)$	$h_{32} = \zeta_3\, h'_{23}$
$h'_{21}(t+1) = \zeta_{12}\, h_{21}(t)$	$h_{23} = \zeta_2 \left[\eta_2\, h'_{32} + (1-\eta_2) h'_{12}\right]$

$$0 \leq \zeta_{ij} \leq 1, \quad 0 \leq \zeta_k \leq 1, \quad -1 \leq \eta \leq 1$$

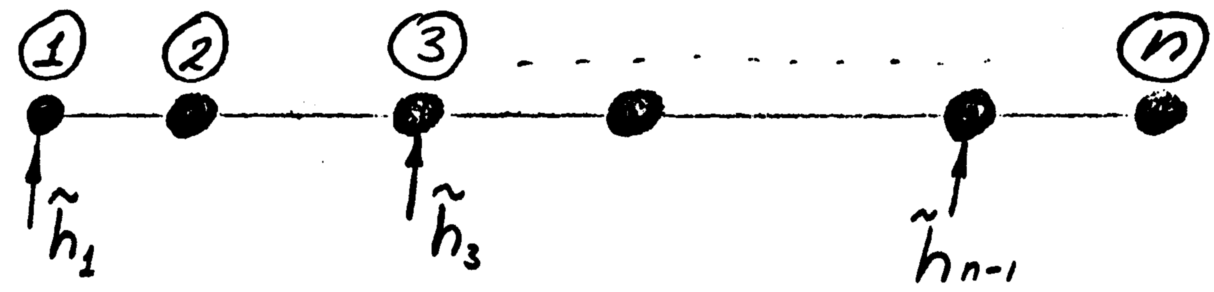
Governing Equations

$$h_{k+1} = A\,h_k ,$$

$$A = \begin{pmatrix} 0 & -\zeta_{12}\zeta_1 & 0 & 0 \\ -\zeta_{12}\zeta_2 q_2 & 0 & 0 & -\zeta_{12}\zeta_2(1-q_2) \\ -\zeta_{23}\zeta_2(1-q_2) & 0 & 0 & -\zeta_{23}\zeta_2 q_{12} \\ 0 & 0 & -\zeta_{23}\zeta_3 & 0 \end{pmatrix}$$

$$h_k = \begin{pmatrix} h'_{12} \\ h'_{21} \\ h'_{23} \\ h'_{32} \end{pmatrix}$$

Multi-member Structures



$$\mathbf{h}_{k+1} = A\,\mathbf{h}_k + \tilde{\mathbf{h}}_k$$

$$\tilde{\mathbf{h}}_k = \begin{Bmatrix} \tilde{h}_1 \\ \tilde{h}_2 \\ \vdots \\ \tilde{h}_n \end{Bmatrix}, \qquad A = \begin{pmatrix} A_{11} & A_{12} & \cdots & A_{1n} \\ A_{21} & \cdots & \cdots & \cdots \\ \vdots & & & \\ A_{n1} & \cdots & & A_{nn} \end{pmatrix}$$

Analysis of Solutions

$$\boldsymbol{h}_k = A^k \boldsymbol{h}_0 + \sum_{j=0}^{k-1} A^j \tilde{\boldsymbol{h}}_{k-j-1}, \quad (k = 0, 1, 2, \ldots)$$

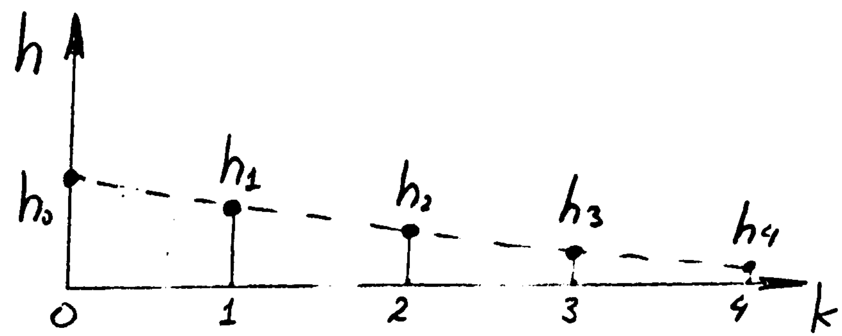
$$|A - \lambda \bar{I}| = 0, \quad A^m = \sum_j^{2n} \lambda_j^m Z_j(A)$$

$$Z_r(A) = \left[\lambda_1^m \bar{I} + m \lambda_1^{m-1} (A - \lambda_1 \bar{I})\right] \frac{\prod_{p=3}^{2n} (A - \lambda_p \bar{I})}{\prod_{p=3}^{2n} (\lambda_1 - \lambda_p)}$$

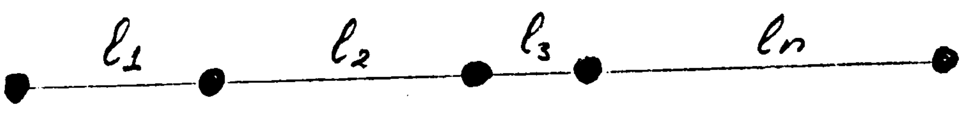
Conditions of Stability

$$|\lambda_i| < 1, \quad i = 1, 2, \ldots 2n,$$

$$\lambda = \frac{\tilde{\lambda} + 1}{\tilde{\lambda} - 1}$$

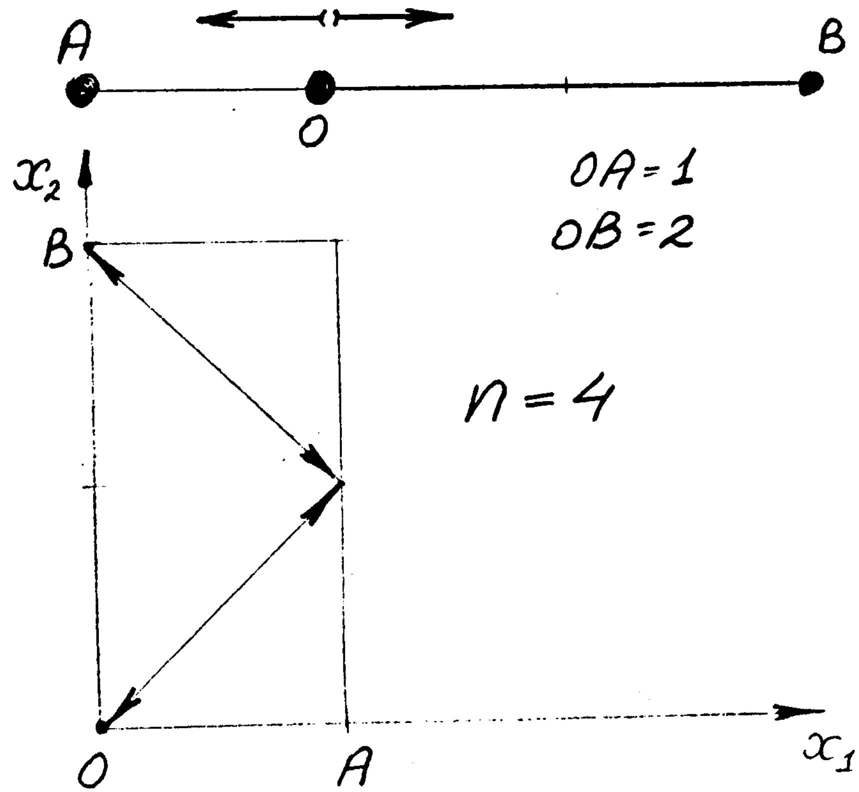


Transition to ergodicity



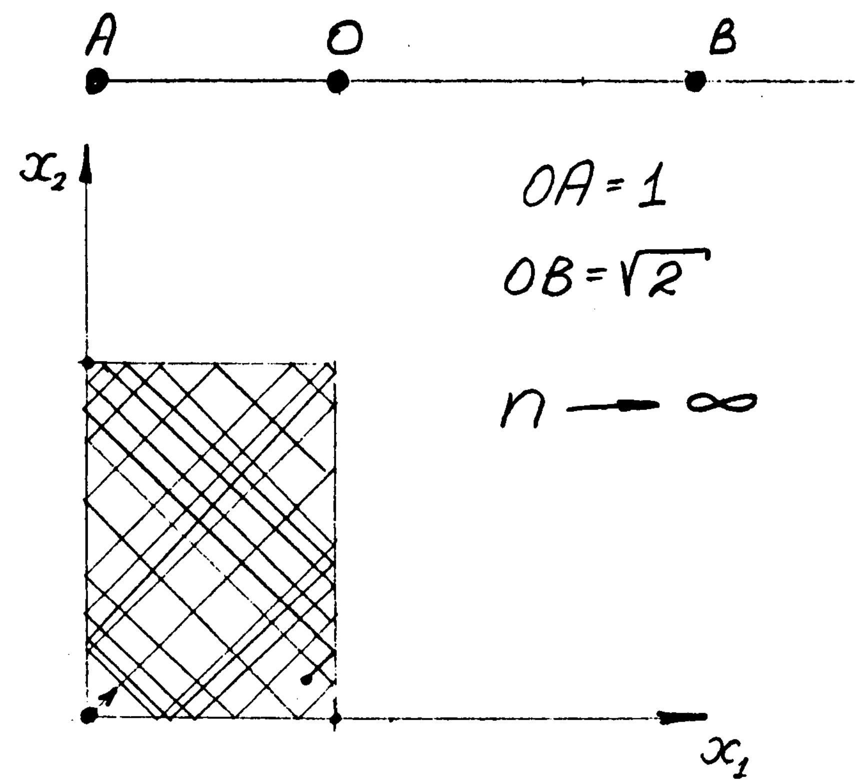
$$\lambda^n + \alpha_1 \lambda^{n-1} + \ldots = 0$$

n is the least common multiple of $2l_1, 2l_2, \ldots 2l_n$



$OA = 1$
$OB = 2$

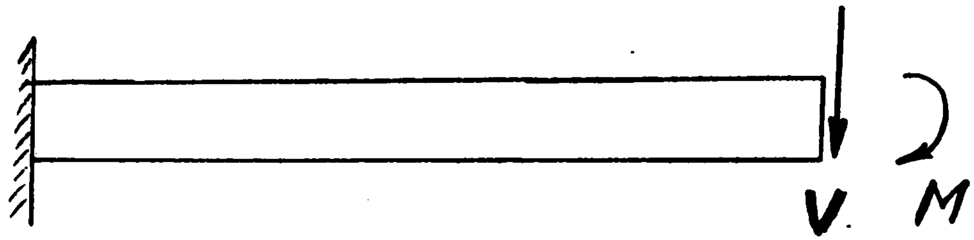
$n = 4$



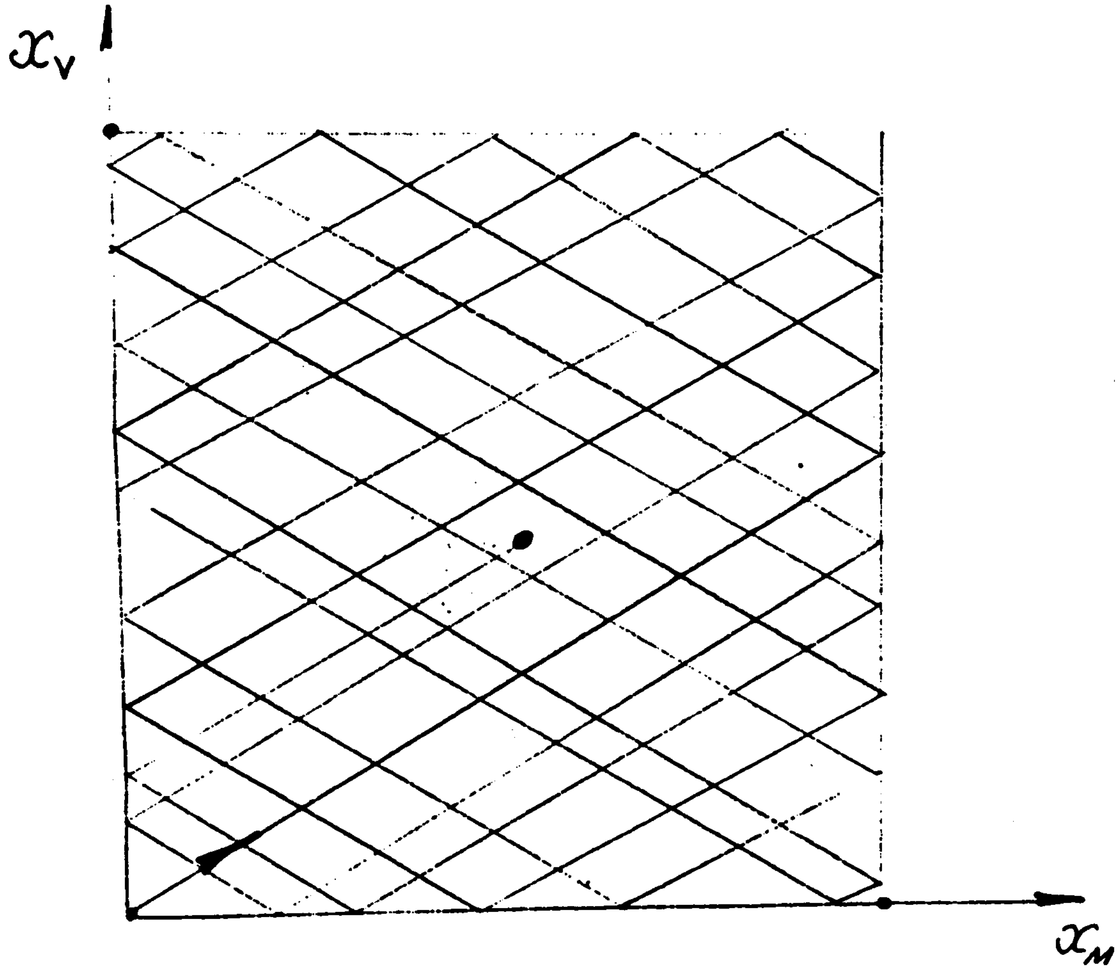
$OA = 1$
$OB = \sqrt{2}$

$n \to \infty$

Ergodicity in a Timoshenko Beam



$$\frac{C_M}{C_V} = 2.8\sqrt{2}$$



Thursday, April 24, 1986

SESSION 5

Critical Issues Forum

Control	Structures	Integrated
S. Seltzer	B. Wada	L. Pinson

(Concurrent Sessions on Integrated and Structures/Integrated)

Structures Integrated Session 5 - Carleton Moore, Chairman

Notes on Implementation of Coulomb Friction in Coupled Dynamical Simulations	R. J. VanderVoort, R. P. Singh, DYNACS
On the Control of Structures by Applied Thermal Gradients	D. R. Edberg and J. C. Chen, JPL
Finite Element Models of Wire Rope for Vibration Analysis	J. E. Cochran, Jr. N. G. Fitz-Coy, Auburn Univ.
Experimental Characterization of Deployable Trusses and Joints	R. Ikegami and S. Church, Boeing D. Kienholz and B. Fowler, CSA Eng.

Integrated Session 5 - Don McCutcheon, Chairman

System Identification for Large Space Structure Damage Assessment	J. C. Chen and J. A. Garba, JPL
Space Station Structures and Dynamics Test Program (Proposed)	E. W. Ivey, C. J. Moore, J. S. Townsend, MSFC
Analytical Determination of Space Station Response to Crew Motion and Design of Suspension System for Microgravity Experiments	F. C. Liu, UAH
Space Station Structural Dynamics/ Reaction Control System Interaction Study	M. Pinnamaneni, MMC