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Introduction

This paper develops a technique for estimating software reliability using the Moranda geometric de-eutrophication model (ref. 1). The major premise of Moranda's model with respect to software reliability is that the failure rate of the software decreases in a geometric progression as bugs (design flaws and coding errors) are removed from the software, and this decreasing failure rate implies growth in the reliability. Work in the area of software reliability-growth models has largely emphasized estimation of only the model parameters by least-squares estimation (ref. 2) or maximum-likelihood techniques. However, only single point estimates and asymptotic approximations for reliability are directly attainable from the estimates of the model parameters. The emphasis of this work is on extending the estimation procedures for a software reliability-growth model. The estimation procedures are extended by developing confidence limits for reliability and prediction limits for the time to the next failure based on Moranda's model.

Since confidence and prediction limits for reliability are not directly obtainable from the estimates of the model parameters, a technique called the pivotal method is utilized. The pivotal method allows a straightforward construction of exact bounds with an associated degree of statistical confidence about a desired quantity. In this case, the desired quantity is the reliability of the software. The confidence limits derived in this paper provide a precise means of assessing the quality of software. The limits take into account the number of bugs found while testing and the effects of sampling variation associated with the random order of discovering bugs.

To use the geometric de-eutrophication model and the pivotal method to construct the limits, the environment for gathering the necessary software failure data for the model must be specified. The testing and development process assumed in this paper consists of inputting a series of randomly selected test cases to the software and correcting bugs as they occur. No assumptions beyond proper repair of the identified bug are made as to whether new bugs are introduced during repair. Interest is primarily focused on the times $X_1, X_2, \ldots, X_n$ between detecting bugs where n is the total number of bugs found. In general, as more bugs are repaired, the interfailure times $X_1, X_2, \ldots$ are expected to increase.

Moranda's geometric de-eutrophication model is used here to model the interfailure times resulting from the described testing process. The model is fully defined in the next section and the model parameters are estimated using maximum-likelihood techniques. Then, the pivotal approach to statistical estimation is presented and used to derive the equations for the confidence limits for reliability and the prediction limits for the time to the next failure. The accuracy of asymptotic approximations to both the confidence and prediction limits is also examined. Further, the effect of departures from the assumed exponentially distributed interfailure times in the model is investigated by simulating interfailure times from Pareto, Weibull, and gamma distributions. Through this simulation the sensitivity of the model to the interfailure-time distribution will be demonstrated.

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Symbols

$\mathbf{A}$	2×2 covariance matrix for a limiting normal distribution
a_{ij}	element (i,j) of $\mathbf{A}$ matrix $(i,j = 1,2)$
$F()$	cumulative distribution function
$f()$	probability density function
L_c	lower confidence limit for reliability
L_p	lower prediction limit for time to next failure
$M_Z()$	moment-generating function for random variable Z
n	number of bugs found in software
$P_r()$	probability function
Q	pivotal function used to define S and T
$R_n(y)$	reliability at time y after n bugs have been found in software
$\hat{R}_n(y)$	estimator of $R_n(y)$
S	pivotal function for estimating time to next failure
s_i	percentage point of S distribution
T	pivotal function for estimating reliability
t_i	percentage point of T distribution
U_c	upper confidence limit for reliability
U_p	upper prediction limit for time to next failure
v	dummy variable
W	pivotal function used to define S and T
X_i	random variable for time between detecting the $(i - 1)$st and ith bugs

x_i	specific time between detecting the $(i-1)$st and ith bugs
y	operational time of software
Z_i	random variable from standard exponential distribution
z_i	specific value for Z_i
$1-\alpha$	confidence coefficient for confidence and prediction limits
β	parameter of geometric de-eutrophication model
$\hat{\beta}$	maximum-likelihood estimator of β
γ	parameter of geometric de-eutrophication model
$\hat{\gamma}$	maximum-likelihood estimator of γ
θ	parameter for Pareto distribution
λ_i	failure rate of software after $i-1$ bugs have been found
$\sigma^2()$	statistical variance of a variable
ϕ	parameter for Weibull distribution
ψ	parameter for gamma distribution

Geometric De-Eutrophication Model and Parameter Estimation

The Moranda geometric de-eutrophication model, which has been studied previously in the context of software reliability, is described in this section. The geometric de-eutrophication model is a relatively simple reliability-growth model that has been popular for describing the failure rate of software. The principal assumptions of the model are that the interfailure times are exponentially distributed and the failure rate of the software decreases in a geometric progression with each failure. Intuitively, one would expect the failure rate of the software to decrease as the testing process continues. Correspondingly, the reliability of the software is expected to increase as the rate at which failures occur diminishes. The geometric de-eutrophication model uses the interfailure times from the testing process to determine the failure rate of the software and, consequently, the growth in the reliability.

The interfailure times $X_1, X_2, \ldots$ are modeled using the assumption that the failure rate λ_i of the software after $i-1$ bugs are removed is

$$\lambda_i = e^{\gamma-\beta(i-1)} \qquad (i = 1,\ 2,\ \ldots)$$

The model is completely defined by adding the assumption that $X_1, X_2, \ldots, X_n$ are independent and exponentially distributed. The β parameter governs the rate at which λ_i increases or decreases. Note that as i increases, $\beta(i-1)$ becomes the dominating term in the failure rate function. For $\beta > 0$, the failure rate decreases and reliability growth is implied. Using the above assumptions, the probability density function of each interfailure time X_i is given as

$$f(x_i) = \lambda_i e^{-x_i \lambda_i}$$

Maximum-likelihood techniques are then used to estimate the model parameters. From the assumption of independent and exponentially distributed interfailure times, the likelihood function given a set of interfailure times $x_1, x_2, \ldots, x_n$ is

$$\prod_{i=1}^{n} e^{\gamma-\beta(i-1)} e^{-x_i e^{\gamma-\beta(i-1)}} \tag{1}$$

The maximum-likelihood estimators $\hat{\gamma}$ and $\hat{\beta}$ (given in a slightly different form by Moranda) are the solutions to

$$\left.\begin{aligned} \hat{\gamma} &= -\ln\left[n^{-1}\sum_{i=1}^{n} x_i e^{-\hat{\beta}(i-1)}\right] \\ \sum_{i=1}^{n} x_i(i-1-(n-1)/2)&e^{-\hat{\beta}(i-1)} = 0 \end{aligned}\right\} \tag{2}$$

Estimates of these parameters can be evaluated numerically from equations (2) by knowing only the interfailure times. If $\hat{\beta} > 0$, reliability growth in the software is implied. From these estimates of the model parameters, single point estimates of reliability are possible. However, estimators that possess a degree of statistical confidence are desired, and these estimators are obtained by utilizing a pivotal function approach.

Pivotal Functions and Confidence Limits

Confidence and prediction limits are developed in this section to provide interval estimators of software reliability and the time to the next failure. Confidence limits are developed since they give a sense of assurance in the accuracy of estimating the reliability and the time to the next failure as opposed to single point estimators. The confidence coefficient associated with the limits, denoted by $1-\alpha$, gives the percentage of time, in repeated sampling, that the constructed limits will contain the parameter of interest. If the confidence coefficient is high, one can be highly confident that the limits contain the true parameter of interest.

Since exact confidence and prediction limits are not direct results from the estimation of the model parameters, the pivotal method is used to construct both the confidence and prediction limits. The pivotal method hinges on defining a function that possesses the following two properties: (1) the function is an association of the sample observations (interfailure times in this case) with an unknown quantity of interest, say θ, where θ is the only unknown quantity (here, θ represents reliability and the time to the next failure); and (2) the function has a probability distribution that is independent of θ (ref. 3). If the distribution of the pivotal function can be obtained by using analytic derivation, simulation, or any other means, the limits can be constructed from that distribution. The Student's t statistic is a familiar example of a pivotal function.

In this section two pivotal functions, S and T, based on the maximum-likelihood estimators of the model parameters, are defined from which the confidence and prediction limits will be derived. The distributions of these pivotals seem analytically intractable, so no attempt is made to derive them. However, the distributions can be ascertained by simulation since the pivotal functions used here can be represented as implicit functions of independent standard exponential variates that can be randomly generated. The simulation will be discussed in further detail after the theoretical development of the pivotal functions.

Using the estimators of the model parameters γ and β in equations (2), the quantities $W = \hat{\gamma} - \gamma$ and $Q = \hat{\beta} - \beta$ can be defined; these functions provide the foundation for generating the pivotal functions S and T needed to construct the confidence limits. The quantities W and Q are also pivotal functions since their sampling distributions are independent of γ and β. To see this, let $Z_1, Z_2, \ldots, Z_n$ be independent random variables having a standard exponential distribution with a probability density function e^{-z} where $z \geq 0$. Using moment-generating functions, it can be shown that the quantity $Z_i e^{-\gamma+\beta(i-1)}$ where $i = 1, 2, \ldots, n$ has a distribution identical to that of X_i. It follows from the moment-generating function of Z_i, that is,

$$M_{Z_i}(t) = (1-t)^{-1}$$

that the moment-generating function of $Z_i e^{-\gamma+\beta(i-1)}$ is given as $\left[1 - te^{-\gamma+\beta(i-1)}\right]^{-1}$. Since each distribution has a unique moment-generating function, it follows that the probability density function of the quantity $Z_i e^{-\gamma+\beta(i-1)}$ must be

$$e^{\gamma-\beta(i-1)} e^{-z_i e^{\gamma-\beta(i-1)}}$$

which is equivalent to the density of X_i. Substituting for each x_i in equations (2) yields the following representations:

$$\left.\begin{aligned} W &= -\ln\left[n^{-1}\sum_{i=1}^{n} z_i e^{-Q(i-1)}\right] \\ &\sum_{i=1}^{n} z_i\left[i-1-(n-1)/2\right]e^{-Q(i-1)} = 0 \end{aligned}\right\} \tag{3}$$

Although inference about γ and β is now possible on the basis of W and Q, the estimation effort is extended by using W and Q as building blocks of other pivotal functions germane to reliability and the time to the next failure.

Given W and Q, consider the following pivotal which is a function of the time to the $(n+1)$st failure and the estimated failure rate after n bugs have been removed from the software:

$$S = x_{n+1} e^{\hat{\gamma}-\hat{\beta}n} \tag{4}$$

Note that S is composed of the quantity of interest (the time to the next failure x_{n+1}) and a function of the sample observations (the failure rate after n bugs have been removed from the software λ_{n+1}, which can be calculated using $\hat{\gamma}$ and $\hat{\beta}$ from equations (3)). The distribution of S can be written in terms of standard exponential variates as in the following equation so that the distribution can be easily simulated:

$$S = z_{n+1} e^{W-nQ} \tag{5}$$

Once the percentage points of S are determined, prediction limits for the time to the $(n+1)$st failure can be obtained. From the S distribution, percentage points s_1 and s_2 can be identified such that

$$P_r(s_1 < S < s_2) = 1 - \alpha \qquad (0 < \alpha < 1)$$

It follows that

$$P_r\left(s_1 < x_{n+1} e^{\hat{\gamma}-\hat{\beta}n} < s_2\right) = 1 - \alpha$$

From this probability statement the general equations for the lower and upper prediction limits, denoted respectively, by L_p and U_p, for X_{n+1} are defined as

$$\left.\begin{aligned} L_p &= s_1 e^{-\hat{\gamma}+\hat{\beta}n} \\ U_p &= s_2 e^{-\hat{\gamma}+\hat{\beta}n} \end{aligned}\right\} \tag{6}$$

where $s_1 < s_2$ are percentage points of S, corresponding to some confidence level $1 - \alpha$.

The same technique is used to generate confidence limits for reliability at a specified operational time y. In accordance with Musa (ref. 4), the reliability of software at a given time $y \geq 0$ after n bugs have been found, denoted by $R_n(y)$, is the probability of failure-free execution of the software for some time interval of length y. In terms of the model, reliability is defined as

$$R_n(y) = e^{-ye^{\gamma - \beta n}} \tag{7}$$

Now, consider a function T defined in terms of the maximum-likelihood estimator $\hat{R}_n(y)$ (obtained by replacing γ and β in eq. (7) by $\hat{\gamma}$ and $\hat{\beta}$) where

$$T = \ln\left[\ln \hat{R}_n(y)/\ln R_n(y)\right] \tag{8}$$

By manipulating equation (8), T can be written as $\hat{\gamma} - \gamma - (\hat{\beta} - \beta)n$. Then, substituting the pivotal functions W and Q as before gives

$$T = W - nQ \tag{9}$$

The sampling distribution of T is, thus, independent of the unknown parameters (γ and β), which indicates that T is a pivotal function. Given that the distribution of T can be determined, percentage points t_1 and t_2 of the T distribution can be identified such that

$$P_r\,(t_1 < T < t_2) = 1 - \alpha \qquad (0 < \alpha < 1)$$

Substituting the form of T in equation (8) into this probability statement gives

$$P_r\left\{t_1 < \ln\left[\ln \hat{R}_n(y)/\ln R_n(y)\right] < t_2\right\} = 1 - \alpha$$

After some manipulation, lower and upper confidence limits, denoted by L_c and U_c, respectively, for $R_n(y)$ are given as

$$\left.\begin{aligned} L_c &= e^{-ye^{\hat{\gamma} - \hat{\beta}n - t_1}} \\ U_c &= e^{-ye^{\hat{\gamma} - \hat{\beta}n - t_2}} \end{aligned}\right\} \tag{10}$$

where $t_1 < t_2$ are percentage points of the pivotal T distribution chosen for a level of confidence $1 - \alpha$.

Simulation and Large Sample Approximation

As mentioned earlier, analytic derivation of the exact distributions of pivotal S and pivotal T seems improbable. Hence, Monte Carlo techniques were employed to obtain the desired distributions. The simulation was implemented with the following algorithm for $n = 5$ to 100 (as shown in tables I and II): (1) randomly generate $n + 1$ of the standard exponentially distributed interfailure times, denoted earlier as $Z_1, Z_2, \ldots, Z_{n+1}$, (2) solve for W and Q in equations (3), (3) calculate S and T (eqs. (5) and (9), respectively), (4) repeat steps (1) to (3) 100 000 times to realize the two distributions, and (5) determine the percentiles of each distribution. A general purpose bisection method with a tolerance of 10^{-6} was invoked to solve for W and Q in equations (3). Tables I and II contain the simulated percentage points of the pivotal S and pivotal T distributions, respectively.

Theoretically, asymptotic normality of the estimators of the model parameters could be used to construct asymptotically distribution-free confidence limits for reliability. Using asymptotic approximations to estimate reliability would eliminate the need to construct pivotal functions and generate their distributions as long as the asymptotic approximations are close to the true limits. The asymptotic approximations are often assumed to be adequate for estimating reliability. However, the accuracy of the approximated limits is not assured in this case since the interfailure times are not identically distributed and n may not be large enough to justify the use of limiting distributions.

By comparing the limiting distributions of each pivotal to its simulated sampling distribution, the adequacy of asymptotic approximations for this problem can be studied. Using a central limit theorem and a weak law of large numbers, it is shown by Cox and Hinkley (ref. 5) that $\hat{\gamma}$ and $\hat{\beta}$, as in equations (2), have limiting ($n \to \infty$) normal distributions. In particular, $\left(n^{1/2}(\hat{\gamma} - \gamma),\ n^{3/2}(\hat{\beta} - \beta)\right)$ converges in distribution as $n \to \infty$ to a bivariate normal distribution with a mean vector $\mathbf{0}$ and a 2×2 covariance matrix $\mathbf{A}$ where $a_{11} = 4$ and $a_{12} = a_{21} = a_{22} = 6$. For large n, the $\mathbf{A}$ matrix is defined by

$$\begin{aligned} a_{11} &= 2(2n - 1)/(n + 1) \\ a_{12} &= a_{21} = 6n/(n + 1) \\ a_{22} &= 12n^2/[(n - 1)(n + 1)] \end{aligned}$$

Consequently, the asymptotic variances for large n are

$$\sigma^2(\hat{\gamma}) = 2(2n - 1)/[n(n + 1)] \approx 4n^{-1}$$

$$\sigma^2(\hat{\beta}) = 12/[n(n + 1)(n - 1)] \approx 12/n^3$$

Now, consider the sampling distribution for pivotal T given in equation (9). Using the multivariate δ method as described in reference 6 (p. 493), it follows that

$$n^{1/2}(W - nQ) = n^{1/2}T$$

converges in distribution to the normal with a mean of 0 and a variance of approximately 4. The convergence of $n^{1/2}T$ also implies that T converges in probability to 0; hence, e^{W-nQ} converges in probability to 1. Therefore, because Z_{n+1} is a standard exponential variable

$$S = Z_{n+1}e^{W-nQ}$$

has a limiting exponential distribution. So, a normal distribution and a standard exponential distribution are the asymptotic approximations for the T and S distributions, respectively. The convergence of T and S to these approximations is apparent when observing the percentage points in tables I and II.

Using tables I and II and the asymptotic results above, the percentage points of the pivotal distributions can be compared with their limiting distributions to realize how close the approximate distributions are to the true distributions. For $n = 5$, the 95th percentile is 2.332 for the pivotal T distribution and is 1.471 for the asymptotic approximation. However, for $n = 100$, the 95th percentile is 0.355 for the pivotal T as compared with 0.329 for the limiting normal. For $n = 5$, the 95th percentile point is 10.948 for the pivotal S compared with 2.996 from the standard exponential distribution; and, for $n = 100$, the 95th percentile is 3.137 for the pivotal S as compared again with 2.996. Thus, it is obvious that for small values of n, the limiting distributions provide poor approximations to the true distributions. However, as n increases, the difference between the percentage points of the limiting and pivotal distributions decreases along with the change in the limits.

Because of the inadequacy of the asymptotic approximations for small n, confidence and prediction limits based on the limiting distributions would be poor approximations for the true limits for small n. The inadequacy of the approximations when n is small is particularly significant since existing software testing data largely consist of relatively small sample sizes, with $n \geq 50$ being uncommon. Unless other approaches are considered, such as estimating the percentage points of the pivotal distributions by higher order approximation to their moments (see ref. 5, p. 282), the use of the simulated pivotal distributions provides the best way of computing the confidence and prediction limits, especially for real data where n is small.

Robustness

In computing confidence and prediction limits based on the geometric de-eutrophication model, the interfailure times are assumed to be exponentially distributed. It has yet to be shown, though, that the interfailure times from a software testing process are exponentially distributed. Robustness, as considered here, is concerned with how well the limits cover the parameter of interest when the interfailure times are not exponentially distributed. The Pareto distribution, whose distributional shape is similar to the exponential but has a longer tail, has been suggested as a more realistic distribution for describing the behavior of the interfailure times (ref. 7). The Weibull and gamma distributions are also candidates since both take on a wide variety of shapes, including the exponential, by varying their parameter values. This investigation of robustness is limited to studying the effect of the different distributions on the prediction limits and considers interfailure times from the Pareto, Weibull, and gamma distributions.

Monte Carlo techniques are again used to produce interfailure times from the different distributions. The simulation consisted of generating $n + 1$ random variables representing the interfailure times $X_1, X_2, \ldots, X_{n+1}$ according to each of the three distributions and then scaling each X_i by a factor of $e^{\gamma-\beta(i-1)}$ where $i = 1, 2, \ldots, n+1$. Prediction limits for the time to the $(n+1)$st failure were computed from the first n variates using equations (6) and the percentage points from the S distribution. To realize the actual confidence level for these limits, this process was repeated 10 000 times checking at each iterate to see if the $(n + 1)$st variate was contained in the prediction limits.

The results of this simulation, found in table III, show that the distributional form of the interfailure times significantly affects the actual level of confidence of the prediction limits. Confidence levels for the limits in the case of Pareto (θ)-distributed interfailure times appear to converge to the expected 90-percent level as θ increases. Confidence levels in the Weibull (ϕ) and gamma (ψ) cases, though, fall below the expected 90-percent level for ϕ and $\psi < 1$ and exceed the 90-percent level for ϕ and $\psi > 1$. Hence, the method appears robust with respect to Pareto (θ)-distributed interfailure times where θ is large, but it does not appear robust for Weibull- and gamma-distributed interfailure times if their parameter values are not close to 1. This result indicates that the model should be applied to cases where the interfailure times are determined to be exponentially distributed.

Concluding Remarks

Pivotal functions are effective tools for determining confidence limits for software reliability and prediction limits for the time to the next failure. The method of pivotal functions produces exact confidence and prediction limits with corresponding degrees of statistical assurance in the quality of the reliability estimates. The usual application of asymptotic results for estimating the limits is inadequate as compared with the pivotal approach, especially when only a small number of bugs have been found during the testing process. Furthermore, the distributional form of the interfailure times does influence the confidence level of the prediction limits, but the limits derived by the method of pivotal functions appear robust for a special case of Pareto-distributed interfailure times. Because of the sensitivity to the interfailure-time distribution, use of the Moranda model should be restricted to cases in which the interfailure times are exponentially distributed.

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References

1. Moranda, P. B.: Prediction of Software Reliability During Debugging. *Proceedings 1975 Annual Reliability and Maintainability Symposium*, IEEE Catalog Number 75 CHO 918-3 RQC, Jan. 1975, pp. 327–332.
2. Miller, Douglas R.; and Sofer, Ariela: *Completely Monotone Regression Estimates of Software Failure Rates.* GWU/IMSE/Serial T-497/85 (Grant NAG-1-179), George Washington Univ., Jan. 18, 1985. (Available as NASA CR-174319.)
3. Mendenhall, William; Scheaffer, Richard L.; and Wackerly, Dennis D.: *Mathematical Statistics With Applications*, Second ed. Duxbury Press, c.1981.
4. Musa, John D.: Software Reliability. *Handbook of Software Engineering*, C. R. Vick and C. V. Ramamoorthy, eds., Van Nostrand Reinhold Co., c.1984, pp. 392–412.
5. Cox, D. R.; and Hinkley, D. V.: *Theoretical Statistics.* Chapman & Hall, Ltd., 1974.
6. Bishop, Yvonne M. M.; Fienberg, Stephen E.; and Holland, Paul W.: *Discrete Multivariate Analysis: Theory and Practice.* MIT Press, c.1975.
7. Littlewood, B.; and Verrall, J. L.: Bayesian Reliability Growth Model for Computer Software. *J. Royal Stat. Soc.*, vol. 22, no. 3, 1973, pp. 332–346.

Table I. Percentage Points of Pivotal S Distribution

n	$s_{0.01}$	$s_{0.05}$	$s_{0.10}$	$s_{0.90}$	$s_{0.95}$	$s_{0.99}$
5	0.0065	0.0347	0.0753	5.864	10.948	39.417
6	.0071	.0381	.0814	4.889	8.469	25.470
7	.0080	.0412	.0866	4.340	7.149	19.417
8	.0082	.0429	.0887	3.935	6.320	15.911
9	.0082	.0431	.0910	3.669	5.797	13.641
10	.0085	.0453	.0946	3.488	5.288	11.725
11	.0084	.0450	.0942	3.337	5.018	10.949
12	.0087	.0449	.0943	3.241	4.767	9.842
13	.0092	.0471	.0968	3.121	4.602	9.508
14	.0094	.0467	.0976	3.072	4.452	8.663
15	0.0095	0.0474	0.0989	2.992	4.288	8.387
20	.0092	.0481	.0993	2.790	3.896	7.147
25	.0093	.0484	.1013	2.694	3.732	6.606
30	.0096	.0504	.1039	2.622	3.597	6.117
35	.0095	.0498	.1024	2.568	3.484	5.919
40	.0102	.0516	.1047	2.516	3.396	5.661
45	.0106	.0506	.1049	2.520	3.387	5.621
50	0.0096	0.0509	0.1032	2.484	3.346	5.417
60	.0098	.0507	.1044	2.438	3.254	5.250
70	.0104	.0503	.1051	2.425	3.210	5.165
100	.0101	.0511	.1050	2.376	3.137	4.950

Table II. Percentage Points of Pivotal T Distribution

n	$t_{0.01}$	$t_{0.05}$	$t_{0.10}$	$t_{0.90}$	$t_{0.95}$	$t_{0.99}$
5	−2.351	−1.613	−1.241	1.772	2.332	3.522
6	−2.049	−1.419	−1.095	1.537	2.021	3.045
7	−1.821	−1.267	−.983	1.369	1.793	2.676
8	−1.685	−1.183	−.916	1.241	1.624	2.431
9	−1.556	−1.093	−.847	1.143	1.488	2.219
10	−1.461	−1.030	−0.799	1.057	1.381	2.046
11	−1.390	−.972	−.753	.992	1.296	1.900
12	−1.322	−.931	−.727	.935	1.221	1.789
13	−1.254	−.884	−.687	.890	1.153	1.699
14	−1.204	−.850	−.663	.849	1.105	1.617
15	−1.164	−0.821	−0.635	0.807	1.049	1.534
20	−.993	−.708	−.553	.680	.879	1.260
25	−.881	−.626	−.488	.595	.770	1.123
30	−.808	−.574	−.448	.534	.691	.997
35	−.749	−.532	−.415	.487	.631	.907
40	−.701	−.496	−.386	.455	.588	.840
45	−.657	−.468	−.365	.424	.547	.789
50	−0.623	−0.445	−0.346	0.398	0.512	0.736
60	−.573	−.407	−.318	.360	.465	.666
70	−.532	−.378	−.293	.328	.424	.606
100	−.445	−.313	−.246	.275	.355	.505

Table III. Actual Confidence Levels of Prediction Limits for Time to Next Software Failure Given Pareto-, Weibull-, and Gamma-Distributed Interfailure Times

Distribution of interfailure times	Parameters	Actual confidence level of 90-percent prediction limits for time to the $(n+1)$st failure		
		$n = 5$	$n = 15$	$n = 30$
Modified Pareto (θ):	$\theta = 1.0$	0.702	0.680	0.680
$F(x) = 1 - (1+x)^{-\theta}$	$\theta = 2.0$	.812	.809	.818
(where $x \geq 0$)	$\theta = 5.0$	.873	.869	.870
	$\theta = 15.0$	.890	.889	.891
Weibull (ϕ):	$\phi = 0.8$	0.818	0.825	0.834
$F(x) = 1 - e^{-x^{\phi}}$	$\phi = 1.0$	.900	.900	.900
(where $x \geq 0$)	$\phi = 1.4$	.970	.968	.970
	$\phi = 2.0$	.996	.996	.996
Gamma (ψ):	$\psi = 0.5$	0.712	0.728	0.736
$F(x) =^{*} \Gamma(\psi)^{-1} \int e^{\psi-1} e^{-t} dt$	$\psi = 1.0$	.900	.900	.900
(where $x > 0$)	$\psi = 2.0$	.984	.980	.980

$^{*}\Gamma(y) = \int_0^{\infty} v^{y-1} e^{-v}\, dv$ where $y > 0$.

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16. Abstract

This paper establishes the utility of pivotal functions for assessing software reliability. Based on the Moranda geometric de-eutrophication model of reliability growth, confidence limits for attained reliability and prediction limits for the time to the next failure are derived using a pivotal function approach. Asymptotic approximations to the confidence and prediction limits are considered and are shown to be inadequate in cases where only a few bugs are found in the software. Departures from the assumed exponentially distributed interfailure times in the model are also investigated. The effect of these departures is discussed relative to restricting the use of the Moranda model.

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