

# ION PHASE-SPACE VORTICES AND THEIR RELATION TO SMALL AMPLITUDE DOUBLE LAYERS

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## ABSTRACT

The properties of ion phase-space vortices are reviewed with particular attention to their role in the formation of small amplitude double layers in current-carrying plasmas. In a one-dimensional analysis, many such double layers simply add up to produce a large voltage drop. A laboratory experiment is carried out in order to investigate the properties of ion phase-space vortices in three dimensions. Their lifetime is significantly reduced as compared with similar results from one-dimensional numerical simulations of the problem.

## I. INTRODUCTION

A plasma can support a large variety of stationary (or quasi-stationary) double-layer-like structures. The proceedings (Michelsen and Rasmussen, 1982; Schrittwieser and Eder, 1984) of the first two double layer symposia at Riso and in Innsbruck contain an extensive summary of theoretical, numerical, and experimental investigations. A number of these investigations, however, refer to conditions with very carefully chosen initial or boundary conditions imposed on the plasma. These conditions may often be highly idealized, or even unrealistic representations of those met in, for example, ionospheric conditions. However, not all examples have this shortcoming. One of these seems to be small amplitude double layers occurring in current-carrying plasmas. One possible mechanism for their generation is reflection of electrons by a negative potential dip associated with an ion plasma-space vortex, which consequently acts as a "seed" for the double layer (Sato and Okuda, 1980; Hasegawa and Sato, 1982; Nishihara et al., 1982; Berman et al., 1985; Pécseli, 1984). This is a spatially localized process and thus independent of any boundary conditions. The potential drop associated with one such double layer will be rather small. If, however, the system is large and the ion phase-space vortices sufficiently frequently occur, many of these small double layers may be generated and will eventually add up to a significant potential drop. With this scenario in mind, we found it worthwhile to investigate the properties of the ion phase-space vortices in detail. These properties will be summarized in the following paragraphs.

## II. ION PHASE-SPACE VORTICES IN ONE-DIMENSIONAL SYSTEMS

The properties of ion phase-space vortices are discussed in some detail in Berman et al., 1985, Pécseli, 1984, Burjarbarua and Schamel, 1981, Pécseli et al., 1984, 1984, and Trulsen, 1980. They represent one particular type of Bernstein-Green-Kruskal (BGK) equilibria (Bernstein et al., 1957) which appear to be very stable. An ion phase-space vortex thus represents a careful balance between trapped and untrapped particles maintaining a local potential dip, resulting in a corresponding plasma density depletion. It was demonstrated (Burjarbarua and Schamel, 1981) that a simple analytical model, characterized by only three parameters, can be constructed for the ion velocity distribution function. The electrons were assumed to be Boltzmann-distributed. In spite of its simplicity, this model accounts very well for the properties of ion vortices. The analysis is formally very similar to that of electron holes

(Bujarbarua and Schamel, 1981; Lynov et al., 1979, 1985) especially if electron modes in a strongly magnetized plasma waveguide are considered (Pécseľi, 1984). A particularly important result of the analysis (Bujarbarua and Schamel, 1981) predicts that ion vortices cease to exist (i.e., their amplitude goes to zero) as the electron-to-ion temperature ratio  $T_e/T_i$  becomes smaller than  $\sim 3.5$ . This result was confirmed in a numerical particle simulation (Pécseľi et al., 1981, 1984; Trulsen, 1980).

The dynamic properties of ion phase-space vortices can be most conveniently accounted for by considering them as quasi-particles. A simplified analysis demonstrates that an ion vortex can be assigned a negative charge and a negative mass (Pécseľi, 1984; Dupree, 1983). Numerical simulations (Pécseľi, 1984; Bujarbarua and Schamel, 1981; Pécseľi et al., 1981, 1984; Trulsen, 1980) demonstrated that two ion vortices may coalesce into one when they are sufficiently close in phase space, very much like electron vortices (Lynov et al., 1979, 1980). A detailed parameter study of this process remains to be carried out. Isolated ion holes, on the other hand, appeared to be very stable (Pécseľi et al., 1981, 1984; Trulsen, 1980) in a description where the electron component is assumed to be in Boltzmann equilibrium at all times. This simplification becomes inappropriate in current-carrying plasmas, where the interaction between ion vortices and individual electrons becomes important. In this case, the reflected electrons give up a net momentum to the ion vortex, which consequently decelerates, since its effective mass is negative. However, as its velocity is decreased, it can move into regions of increasing ion phase-space density. The result is a slow increase in amplitude of the phase-space vortex, which consequently becomes more efficient in reflecting electrons. The process is thus accelerated. The charge distribution of the reflected electrons gives rise to localized double-layer-like structures. Eventually, the phase-space vortex is destroyed. The very simplified physical picture outlined here is elaborated in more detail by Berman et al. (1985) and Dupree (1983) and also by Nishihara et al., (1982) and Pécseľi (1984). In particular, Berman et al. (1985) describe very spectacular one-dimensional numerical particle simulations, showing the slow time evolution of ion phase-space vortices under conditions like those discussed here. It is important to emphasize that the unstable growth of the ion vortices is due to a slowly growing nonlinear instability, which can be excited for bulk electron flow velocities well below those giving the linear two-stream instability. The only criterion for the nonlinear instability seems to be that long-lived ion vortices are formed. In the simulations reported in Berman et al. (1985), this formation occurred for a rather wide class of initial phase-space distributions of simulation particles. The formation of large ion vortices was investigated by Pécseľi et al. (1981, 1984) and Trulsen (1980). It could be analytically demonstrated that such vortices are formed in the saturated stage of the one-dimensional ion-ion, two-stream instability of Pécseľi and Trulsen (1982). Alternatively, the formation could be due to ion bursts (which after all can be considered as a segment of an ion beam).

### III. ION PHASE-SPACE VORTICES IN THREE DIMENSIONS

The properties of ion phase-space vortices described in the previous section referred mainly to one-dimensional investigations. The experimental investigation reported in Pécseľi et al. (1981, 1984) and Trulsen (1980) is of course three-dimensional, but it refers to very carefully chosen initial and boundary conditions. Numerical investigations (Morse and Nielson, 1969) of electron phase-space vortices demonstrated that an ensemble of these was very stable in one dimension, while the phase-space structures were very rapidly eroded in two or, in particular, three spatial dimensions. In order to investigate the properties of ion vortices in three dimensions, we performed a laboratory experiment where the vortices were generated by the ion-ion beam instability, which gives linear instability for wave directions in a cone around the beam velocity.

Our investigations were carried out in the double-plasma device at the University of Tromsø (Johnsen, 1986; Johnsen et al., 1985). The vacuum vessel has an inner diameter of 60 cm and is divided into source and target parts (length 40 cm and 80 cm, respectively) by a fine meshed grid. The device was operated at a typical neutral argon pressure of  $1.5 \times 10^{-4}$  Torr, with plasma densities in the range 2 to  $10 \times 10^8 \text{ cm}^{-3}$ . The electron temperature was  $T_e \approx 2.5 \text{ eV}$ , while  $T_i \approx 0.15 \text{ eV}$  in the absence of a beam. By adjusting the bias of the source, an ion beam was injected into the target plasma. Typical beam energies were 4-8 eV. The density ratio between beam and background ions is

adjustable in the setup and was chosen to be around one. The fluctuation level increases along the direction of beam propagation ( $Z$  axis in the following, with  $Z = 0$  corresponding to the position of the separating grid) and saturates roughly at a distance of  $Z = 9$  cm, with a density fluctuation level of  $\tilde{n}/n_0 \sim 1$ -5 percent. The increase in noise level is accompanied by a significant scattering of the incoming ion beam, as observed by using both conventional three-grid and the novel directional electrostatic energy analyzers. It is not experimentally possible to obtain information about spontaneously generated individual ion vortices. Instead, we performed a statistical analysis of the experimental data. The turbulent plasma fluctuations in the frequency range 10 kHz to 1 MHz were investigated by the fluctuations in electron saturation current to two movable Langmuir probes with an exposed spherical tip of 1 mm in diameter. Realtime signal sequences of 800  $\mu$ s duration were recorded with a sample rate of 2.5 MHz. At each combination of probe positions, five such sequences formed the basis of a statistical analysis. With the realtime signals available, we thus performed a statistical analysis on a conditional basis. The signal  $n_A$  from the fixed probe (A) at position  $\vec{r}'$  was chosen as a reference. Choosing a certain value of the density perturbation, say  $n_i$ , the corresponding time records are subsequently searched for times  $t'$ , where the signal takes a value within the narrow interval  $(n_i, n_i + \Delta)$ , where  $\Delta$  is taken as the minimum amplitude resolution of the record. Each time this condition on signal A is satisfied, the signal from the movable probe B is recorded in a certain prescribed time interval  $(t' - \tau, t' + \tau)$ . These conditionally chosen time series are then considered as independent realizations for the ensuring statistical analysis. The analysis is repeated for varying positions  $\vec{r}$  of probe B. The result is most conveniently expressed in terms of the electrostatic potential by the relation  $\tilde{n}/n_0 \sim e\tilde{\phi}/T_e$ , which is adequate for the relatively low fluctuation level in the experiment. A record of 800  $\mu$ s duration is sufficiently long to give an adequate representation of many realizations in the ensemble. By the procedure outlined above, we thus obtained the conditional ensemble average, where  $t'$  is just a dummy variable for time stationary turbulence

$$\bar{\phi} = \langle \phi(\vec{r}, t+t') | \phi(\vec{r}', t') = \phi_1 \rangle. \quad (1)$$

This quantity has the following rather self-evident physical interpretation: given that a particle is located in a potential  $\phi_1$  at a position  $\vec{r}'$  at time  $t'$ , then  $\bar{\phi} = \bar{\phi}(\vec{r}, t+t')$  is the average potential variation it will experience in the vicinity of  $\vec{r}$  at the same or at different times.

One important question to be discussed in the following is the lifetime  $\tau_L$  of a conditional structure (or eddy for simplicity) described by equation (1), compared to the average bounce time  $\tau_B$  of a charged particle derived from  $\bar{\phi}$ . Thus, if  $\tau_B \lesssim \tau_L$ , a small cloud of test particles released at  $(\vec{r}', t')$  will be likely to stay together with the trajectories being correlated for a substantial time. Ions with velocities close to that of the eddy will, if  $\phi_1 < 0$ , be trapped, on average, by the (average) potential, thus exhibiting the features of three-dimensional ion phase-space vortices. On the other hand, if  $\tau_L$  is very short, the particles will disperse rapidly with a large probability, and vortex or "clump-like" features will be immaterial for the description of the turbulent fluctuations in question. In our case, we find  $\tau_L \approx \tau_B$ . In Figures 1a,b we show equipotential contours for  $\phi$  in a rectangular cross-section of the plasma for two different values of the reference potential  $\phi_1$ . The position of the reference probe is indicated by  $\bullet$ . The full spatial variation is obtained by rotating the figure around the  $Z$  axis. This symmetry was explicitly verified in the experiment. For the region of measurements, we may consider the turbulence to be homogeneous and isotropic in the plane perpendicular to the axis of the device. In particular we note that since full time records are available, it is perfectly feasible to let  $t$  be negative, i.e., to consider the formation of the conditional eddy. Evidently the eddy rapidly assumes a roughly spherical shape and propagates in the direction of the ion beam. A lifetime of 60  $\mu$ s for the eddy is estimated for the present plasma conditions. By fitting a parabola to the local minimum of the conditional spatial potential profile, we obtain an inverse angular ion bounce frequency  $\omega_B^{-1} \approx 8 \mu$ s for the largest eddy, indicating that the trapping of ions is a significant dynamic process. The observed structures corresponding to large negative values of  $\phi_1$  can thus be considered as evidence for quasi-static three-dimensional ion holes. Using the electrostatic energy analyzer, we verified (Johnsen et al., 1985) that there was indeed a significant number of ions in the velocity range where they can be trapped by the conditional eddy. From measurements such as those summarized in Figure 1, it is easy to deduce the eddy velocity.

An eddy described by equation (1) and shown in Figure 1 is an average quantity. In each individual realization we may find eddies which may deviate significantly from the average. However, we expect these to have little statistical weight. This statement can be given support by a theoretical analysis.

Being particularly interested in ion-hole formation, we concentrated on negative values for  $\phi_1$  in the present summary of our results. Of course, positive values of  $\phi_1$  can be chosen as well, where now the electrons can be trapped. We found that the evolution of conditional structures corresponding to  $\phi_1 > 0$  was somewhat similar to the overall features given in Figure 1, with some deviations in the actual shapes and velocities. A more general account of these results is in preparation.

#### IV. CONCLUSIONS

In this work we discussed experimental observations of conditional structures in ion beam driven turbulence, presenting the actual variation of the average potential deduced from a conditional analysis of measured fluctuations. Given the propagation velocity and lifetime of these structures, we obtained evidence for the formation of quasi-stationary, ion phase-space vortices. We find it worthwhile to emphasize that the conditionally averaged potential need not coincide with the most probably conditional potential variation. An analysis of this problem requires investigations of the conditional amplitude probability distribution of potential in each spatial point as a function of time. This (rather lengthy) investigation was also carried out. However, the differences between the resulting spatial potential variations and those shown in Figure 1 were not sufficiently pronounced to necessitate a separate figure here. Although we have obtained evidence for the formation of three-dimensional ion phase-space vortices, it seems conclusive that their lifetime is much shorter than for those found in one-dimensional numerical simulations (Pécsele et al., 1981, 1984; Trulsen, 1980; Pécsele et al., 1982). In particular, we find that the vortex lifetime is too short to manifest coalescence of two vortices, which is a relatively slow process in units of bounce time. Several reasons for this difference between one and higher dimensions can be found. First of all, a stability analysis (Schamel, 1982) has demonstrated that one isolated vortex is unstable with respect to transverse perturbations in three dimensions, although the growth rate of this instability is rather small for realistic conditions. Probably more important, however, is the possibility of two or many such vortices colliding at an angle in three dimensions, thus destroying the simple trapped particle orbits. Finally, the interaction between ions and potential structures is rather different in one and in higher spatial dimensions, as illustrated in Figure 2. Thus, in one dimension (Fig. 2a), an ion coming in from infinity may give up momentum to an isolated positive quasi-stationary potential structure (top trace) while it only gives a transient perturbation to a negative potential variation (lower trace). In two or three dimensions, an ion may give up momentum to both polarities of a potential variation as indicated in Figure 2b. We see no obvious method to discriminate between these effects in our experiment. Numerical simulations such as those reported in, for example, DeGroot et al. (1977) and Barnes et al. (1985) may provide some insight into these features. It is rather evident that the experimental conditions discussed here do not exactly match those met in current-carrying plasmas. It seems fair, however, to assume that the properties of ion phase-space vortices are, at least in a first approximation, independent of a small electron drift. The conclusion based on the results summarized here will consequently be that the lifetime of ion vortices in three-dimensional unmagnetized systems is not sufficiently long to allow an analysis in terms of quasi-particles interacting with individual electrons, in contrast to the one-dimensional investigations discussed in Berman et al. (1985) and Dupree (1983). The growth of very small vortices, or holes, from an initial low-level noise is thus improbable for a small electron drift. If, however, the electron drift exceeds the threshold for the linear current-driven instability, a rapid growth of negative potential spikes may occur (Barnes et al., 1985) which subsequently form ion vortices by particle trapping (Nishihara et al., 1982). The instability may then evolve nonlinearly as described in Section II. Although the ion vortices have a relatively short lifetime, they have in this case a large amplitude and are thus effective local barriers for the slow electrons. One might expect that these conclusions should be modified for magnetized plasmas with electron drifts

along B-field lines. However, the two-dimensional numerical simulations in Barnes et al. (1985) do not reveal any particular variations of the results with the intensity of an externally applied magnetic field. Unfortunately, practical limitations imply that most numerical simulations are restricted to at most two spatial dimensions.

Although ion phase-space vortices were discussed here with reference to one particular plasma phenomenon, it may be worth mentioning that they present a nonlinear plasma mode which may be interesting also in a different context [see, for instance, the discussion by Hershkowitz (1984)].

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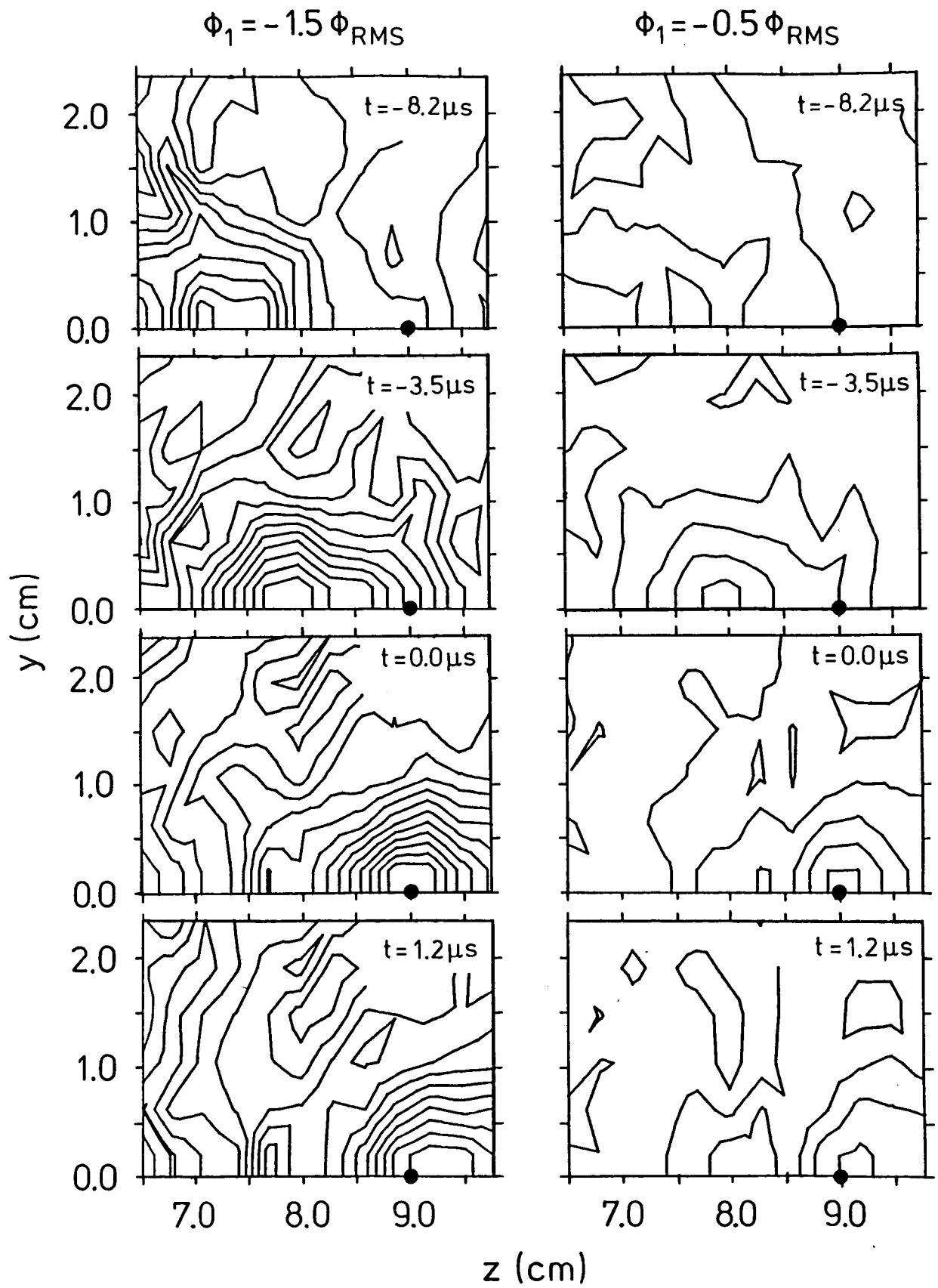
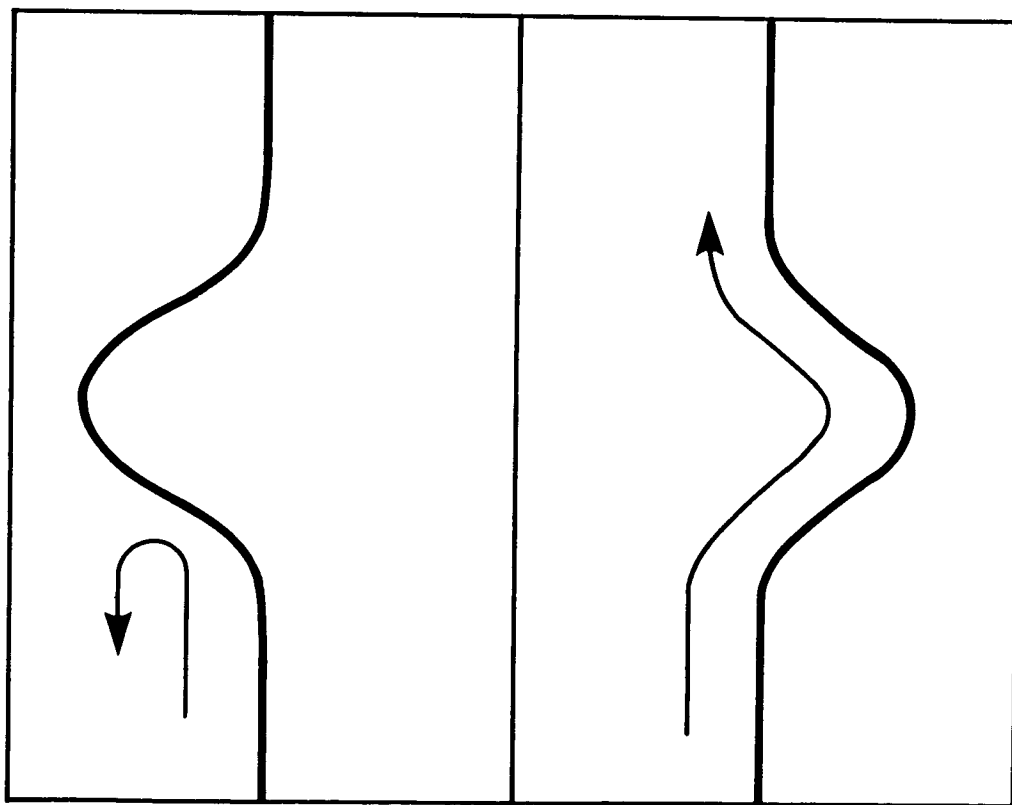


Figure 1. Contour plots of conditional eddies for two different reference values  $\phi_1$  in equation (1) measured in units of the rms value of the potential fluctuations  $\phi_{\text{rms}}$ . The position of the reference probe is  $Z = 9$  cm measured from the separating grid of the double-plasma device. The spacing between contours is  $0.1 \phi_{\text{rms}}$ .

a)



b)

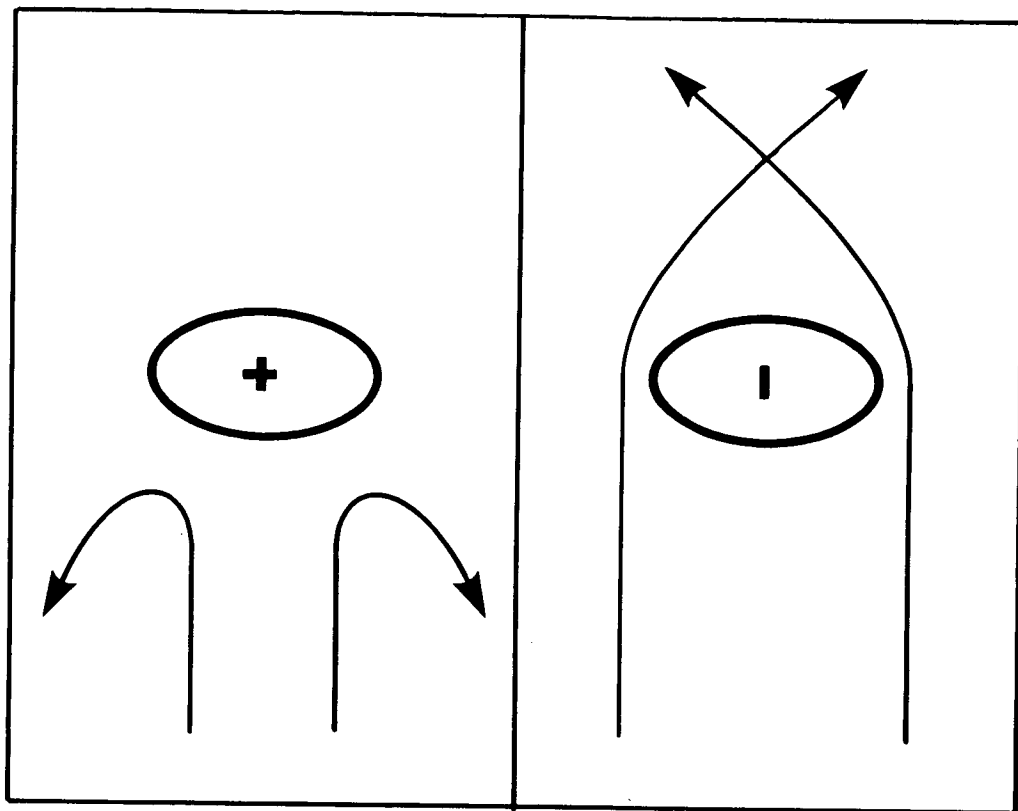


Figure 2. Schematic diagram for discussing the difference between particles interacting with localized potential variations in one and in higher dimensions.