TOWARD A MODEL FOR LEVEED LAVA FLOWS

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Many lava flows have two distinct volumetric components during emplacement. First, there is a component actively flowing in accordance with Newtonian or other constitutive relations. Second, there may be an inactive, stationary component that is no longer participating in the forward movement of the flow. Such passive components may take the form of flow-confining levees, solidified lateral margins, overspills, plating, small ponds and sidestreams, or a lava tube. To describe the conservation of flow volume for the active component, the governing equation is taken as,

$$w \frac{\partial h}{\partial t} + \frac{\partial Q}{\partial x} = -\lambda w h \tag{1}$$

where $h = h(x,t)$ is the depth of the flow, $w = w(x)$ is the width, $Q = Q(x,t)$ is the local flowrate, x and t represent the distance from the source and time, and λ is a rate constant for the volumetric loss to levees or other stationary constructs. "Global" volume conservation is described by,

$$\int_0^t Q(t')\,dt' = \int_0^{L(t)} w(x)\,h(x,\,t)\,dx + \lambda \int_0^t \int_0^{L(t')} w(x')\,h(x',t')\,dx'\,dt'$$

$$\text{Discharge} \quad = \quad \text{Active} \quad + \quad \text{Passive} \tag{2}$$

where $Q(t) = Q(0,t)$ is the effusion rate and $L(t)$ is the length of the flow. Eqs. (1) and (2) with λ=0 have been studied by Baloga and Pieri [1986] and Baloga [1986]. Eq. (2) accounts for the entire discharge by distributing it dynamically between the active and passive components. From eq. (2), the active volume of the flow, $V(t)$, is given by,

$$V(t) = \int_0^{L(t)} w(x)\,h(x,\,t)\,dx = \int_0^t e^{-\lambda(t-t')}\,Q(t')\,dt' \tag{3}$$

The growth of the stationary volume fraction is determined solely by the effusion rate and the rate constant. Because $h(x,t)$ must satisfy eq. (1), we have in eq. (3) a key relationship between levee production, time-dependent source behavior, the advance of the flow front, and viscous changes along the flow path. Studies of these interactions are in progress.

To illustrate some of the consequences of eq. (1) alone, we will choose a highly specialized flowrate,

$$Q(x) = \frac{g \sin \Theta\, h(x)^3 w(x)}{3\,\nu\,(x)}, \tag{4}$$

where g is gravity, Θ is the slope and ν is the viscosity, and use data from the 1951 eruption of the Mihara volcano in Japan [Minikami, 1951]. Figure 1 shows the relevant geometrical data for this flow and appropriately fitted lines. Although the width and depth of the flow increase appreciably downstream, the product of the flow width and the slope measurements are approximately constant. The theoretical formalism simplifies considerably if $w(x) \sin \Theta$ is taken as its average value, $\langle w \sin \Theta \rangle$. In Figure 1, the average is indicated by the dashed line. From eqs. (1) and (4), with the boundary condition $h(0) = ho$,

$$h(x) = ho \left[\frac{\nu(x)}{\nu o}\right]^{1/3} \left[1 - \frac{2\lambda \, \nu o^{2/3}}{h_o^2 g \langle w \sin \Theta \rangle} \int_o^x \nu(x')^{1/3} w(x') \, dx\right]^{1/2} \quad (5)$$

This solution indicates that the flowdepth is affected by both the local viscosity and its cumulative behavior along the path of the flow. Because depth and width variables are, unlike viscosity, more amenable to direct measurement while the flow is active, a more useful result is obtained by inverting eq. (5) for the viscosity in terms of the depth and width. One can show that,

$$\frac{\nu(x)}{\nu o} = \left(\frac{h(x)}{ho}\right)^3 \left[1 - \frac{\lambda}{Qo} \int_o^x h(x') \, w(x') dx'\right]^{-1} \quad (6)$$

where $Qo = g\langle w \sin \Theta \rangle ho^3 / 3 \nu o$. This interesting result shows that the viscosity has a simple power law dependence on the local depth of the flow unless significant flowrate losses are occurring. Estimated lava viscosities were computed by Minikami using a form of the Jeffreys' equation [Williams and McBirney, 1979]. The particular formula used by Minikami does not account for changes in width or flowrate losses, but does have a slope dependence. Minikami also attempted to correct the viscosity estimates for effects from the sides of the channel. His values are shown in Figure 2, where a constant lava density of 2.5 gm/cm^3 has been assumed. Figure 2 also shows results computed from eq. (6) using identical parameters and the linear fits to the depth and width data. When $\lambda = 0$, the volumetric flowrate is conserved in the channel and the computed viscosities are significantly higher than the conventional Jeffreys' equation results. Even for a small λ, the effect of a small flowrate loss eventually accumulates and produces a significant discrepancy between methods of estimation. The flowrate can also be recast directly in terms of depth and width measurements without resorting to intermediate viscosity computations. Eqs. (4)-(6) imply,

$$\frac{Q(x)}{Qo} = \left[1 - \frac{2\lambda}{3\,Qo} \int_o^x h(x') \, w(x') \, dx' \left\{1 - \frac{\lambda}{Qo} \int_o^{x'} h(x'') \, w(x'') \, dx''\right\}^{-1/3}\right]^{3/2} \quad (7)$$

In principle, measurement of the flowrate and the flow geometry, i.e., both sides of eq. (7), provides a mechanism for testing the validity of the theory. Flowrates computed from the Mihara flow data are shown in Figure 3. Although a flowrate loss seems clearly evident, Minikami discusses a variety of errors that could easily account for the flowrate variations depicted. Figure 3 also shows typical results from eq. (7), illustrating the form of the flowrate loss associated with linear fits to the flow depth and width data. There is enough uncertainty in the data to preclude these results from being considered as actual improvements over Minikami's analysis. Efforts are underway to apply eqs. (1) and (2) to lava flows at Alba Patera, Mars, where high resolution Viking images clearly indicate the presence of levees and other passive components and dimensional data has been compiled

[Pieri et al., 1986]. A model that describes the emplacement of leveed lava flows is expected to provide interesting inferences about the nature of the eruptions and possibly the compositions involved.

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