# ERROR ANALYSIS OF LEAF AREA ESTIMATES MADE FROM ALLOMETRIC REGRESSION MODELS

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ABSTRACT

Biological net productivity, measured in terms of the change in biomass with time, affects global habitability and the quality of life through biochemical and hydrological cycles and by its effect on the overall energy balance (cf. Botkin (1982) or Botkin, et.al. (1984). Estimating leaf area for large ecosystems such as forests is one of the more important means of monitoring this productivity today.

For a particular forest plot, the leaf area is often estimated by a two-stage process. In the first stage, known as "dimension analysis", a small number of trees are felled ("sacrificed") so that their leaf areas can be measured as accurately as possible. The leaf areas of the sacrificed trees are then related to non-destructive, easily-measured features such as bole diameter or tree height, by using a regression model. In the second stage, the non-destructive features are measured for all or for a sample of trees in the plots and then used as input into the regression model to estimate the total leaf area.

Because both stages of the estimation process are subject to error, it is difficult to evaluate the accuracy of the final plot leaf area estimates; indeed, it is not even possible to establish a meaningful criterion of accuracy without resolving issues involving sampling and modeling. This paper illustrates how a complete error analysis can be made, using an example from a study made on aspen trees in northern Minnesota.

# 1. BACKGROUND

A joint study of leaf area was made in the Superior National Forest in northern Minnesota during the years 1983-4 by the National Aeronautics and Space Administration and the University of California at Santa Barbara. The main objective of this study known as COVER (Characterization of Vegetation with Remote Sensing) was to be able to relate the leaf area index (leaf area per unit ground area) of typical boreal forest plots to remotely-sensed multispectral scanner data, with the ultimate goal of being able to estimate biological productivity by remote sensing. Botkin and Running (1984) provide a discussion of COVER's basic approach. For a full description of the project the reader is referred to the forthcoming paper of Woods and Botkin (1986).

For a particular forest plot, the leaf area is often estimated by a two-stage process. In the first stage, known as "dimension analysis", a small number of trees are felled ("sacrificed") so that their leaf areas can be estimated as accurately as possible. In practice, leaf area for entire trees is not measured directly but is instead obtained from a ratio or regression model using leaf weight as an auxilliary variable, as in Watson (1937). The estimated leaf areas of the sacrificed trees are then related to non-destructive, easily-measured features such as bole diameter or tree height, by using a regression model. In the second stage, the non-destructive features are measured for all or for a sample of trees in the plots and then used as input into the regression model to estimate the total leaf area.

Because both stages of the estimation process involve errors, it is difficult to evaluate the accuracy of the final plot leaf area estimates. Some assessments of accuracy of dimension analysis regression fits have been found in the literature (cf. Pollard (1970 and 1972)), but they do not take into account errors in the leaf estimates of the sacrificed trees; in addition there do not appear to be any published estimates of accuracy at a plot level. In order to establish a meaningful criterion of accuracy, underlying sampling and modeling issues must first be resolved. For example, should finite-sampling or infinite-population models be assumed? How should model error variances be structured? Should sampling be random or should it be forced to cover the domain of the predictor variables?

This paper describes how an extensive error analysis of the COVER leaf area index (LAI) estimates was made, with the intent of pointing out some of the practical problems that arose and how they were handled. Results given here deal only with plots of aspen trees, although stands of other species were also included in the study.

The organization of the paper is as follows: Section 2 describes in a procedural sense how the LAI estimates were made, both at the tree level and at the plot level. Section 3 provides the mathematical setting for obtaining the LAI estimates and also addresses the problem of accuracy estimation for both stages of the estimation process; Section 4 provides some numerical results including an error analysis, and the fifth and concluding section

discusses the strengths and weaknesses of the COVER procedure, suggesting how it might have been improved.

## 2. THE COVER ESTIMATION PROCEDURE

The target areas for which it was desired to estimate the LAI, consisted of 32 circular plots of (preponderantly) aspen trees. Each plot had a radius of 30 meters and was sampled by five circular subplots, (usually of radius 8 meters) arranged in a quincunx. The 30-meter plots were roughly the areas "seen" by a multispectral scanner carried aboard a helicopter hovering overhead (Figure 1).

### 2.1 Leaf Area Estimation for Sacrificed Trees

In COVER, the selection of the sacrificed trees was made so as to best represent the range of variation both in size and in local environment, of aspen trees in the region. Tree sizes were quantified in terms of "diameter at breast height" (DBH), an ecological term which is defined to be $1/\pi$ times the perimeter of the bole 1.4 meters from the ground. Sacrificed trees were chosen near but not in, the plots. They were selected at random, subject to two restrictions: (1) a minimum number of trees from each of a predefined set of DBH strata had to be filled; (2) unusually deformed or diseased trees were not taken, although no effort was made to restrict the sample to trees in the best of condition.

The raw data for the sacrificed trees comprised various categories of measurements: those made for the entire tree,

those made for all branches, those made for selected branches, and those made only for small samples of foliage. Measurements made on the whole tree were non-destructive, consisting of DBH, H - the height of the tree and $H_1$ - the height above ground of the point where the lowest live branch joins the bole (see Figure 2).

Measurements made on branches and foliage were destructive and were taken for each of three crown strata. Branches were partitioned into crown strata ("upper", "middle" and "lower") by first computing the depth of crown of the tree, defined as DOC = $H-H_1$: then dividing the crown into three parts so that if h is the height at which a branch joins the bole, then the branch is in the lower stratum if $h < H_1+DOC/3$; the middle stratum if $H_1 + DOC/3 \leq h < H-DOC/3$; or the upper stratum if $h \geq H - DOC/3$. A leaf was considered to be in a particular stratum if its branch was in that stratum, regardless of the leaf's actual position in the crown.

Given a tree whose crown was partitioned into strata as above, the folowing destructive raw data was taken:

(1) Total leaf area for a sample of leaves from the i-th stratum (i = 1,2,3), where i = 1 refers to the upper stratum, i = 2 refers to the middle stratum and i = 3 refers to the lower stratum. For a given leaf, "leaf area" is defined as its one-sided surface area. This was measured in the field by an optical scanner.

(2) Total leaf fresh weight for each of the above samples.

(3) Branch dimensions - For each branch of the tree, (see Figure 3) measurements of:

(a) D = diameter at the base of the branch

(b) L = length of the branch (straight line distance from base to tip).

(c) $L_1$ = straight line distance from the base of the branch to the point where the first live sub-branch joins the branch.

(These measurements are analogous to DBH, H and $H_1$, which pertain to the entire tree.)

(4) Total foliage fresh weight for

(a) a stratified random sample of branches for which a foliage-weight-vs-branch-dimension regression model is appropriate. These will be called "regression" branches in the remainder of this paper.

(b) "special" branches for which a foliage-weight-vs-branch-dimension regression model is not appropriate. Examples of these are "leaders' - i.e. the very tops of trees, which contribute to the total leaf area, but are not true branches.

We will refer to all branches in (4) above as "sampled" branches, an important subset of which is the collection of regression branches (4a). Only data from the regression branches was used to fit the model (see Section 3.1.1).

A leaf area estimate for each sacrificed tree was made at the stratum level and then summed to obtain an estimate for the entire tree. For a given stratum the leaf area estimate was

obtained by applying an area-to-weight ratio to a leaf weight estimate using a regression model on branch dimensions. Because there were insufficient numbers of regression branches to allow separate models within strata, one model was used for all branches of each tree. The ratios were estimated by using a linear model to smooth sampling errors associated with raw ratios obtained from the data in (1) and (2) above.

## 2.2 Plot-level Estimation

Once leaf area estimates were available on the sacrificed trees, they were regressed on functions of the non-destructive tree-level measurements to obtain a "dimension analysis" prediction model. During the second stage of the LAI estimation process, this model was then used on each plot to predict its total leaf area which was then normalized to a LAI estimate.

# 3. MATHEMATICAL MODELS

In order to construct the LAI estimates and a measure of their accuracies, some underlying conditions and response models must be assumed. This section discusses the problem of model selection and shows how the LAI estimators were obtained. We begin with the estimation of leaf area for the sacrificed trees, (within-tree models) then discuss the dimension analysis (between-tree) model and finally, show how it is used to obtain the plot-level LAI estimates. While it would be possible to use one set of notation to identify relevant quantities in all stages of the estimation process, such notation would be

cumbersome. Consequently, the notation will be simplified, expanded or even changed according to the needs of each subsection below.

## 3.1 Within Trees

For a given sacrificed tree, let $\eta_{ij}$ be the total area of the foliage on the j-th branch in the i-th stratum; also let $\eta = \Sigma\Sigma\eta_{ij}$ be the total leaf area of the tree and let $\hat{\eta}$ be its estimate. The main contributions to the error in estimating $\eta$ come from the regression estimates of leaf weight and from ratioing to leaf area. Whether $\hat{\eta}$ is biased or not and how its precision should be estimated depends on (a) the sampling strategy, (b) what models are assumed and (c) the form of the estimator.

### 3.1.1 Foliage Weight Regression Model

Let N be the total number of branches on the tree and let $Y_j$ be the total leaf weight on the j-th branch, j=1,...,N (ignoring strata at present). The $Y_j$ can be considered to have arisen in one of two ways: as a set of fixed numbers, or as realizations of a random process through time. Under the first formulation, a function of the Y's such as the total leaf weight, is estimated as a population parameter with the sampling design playing an important role. In the second case, the function is random and is thus predicted by a model-based approach. Further discussion of this topic may be found in Royall(1970, 1985), Kalton(1983) and Hansen, Madow and Tepping(1983).

Preliminary studies indicated that a good auxiliary variable for predicting (estimating) $Y_j$ from the available branch measurements D,L, and $L_1$, was the branch depth of crown, $dc = L-L_1$. Plots of leaf weight vs dc for various trees suggested the regression function $E(Y_j) = \beta_1 + \beta_2 dc_j + \beta_3 dc_j^2$, where $dc_j$ is the value of dc for the j-th branch. Since the foliage of a tree may vary throughout a year (or years) with little change in the branch dimensions it seems appropriate here to use the random ("superpopulation") model and think in terms of predicting, rather than estimating the Y's.

Let $\underline{Y} = (Y_1,\ldots,Y_N)^T$ and let the N x 3 matrix $\underline{X} = (\underline{x}_1|\underline{x}_2|\ldots|\underline{x}_N)^T$ where $\underline{x}_j = (1, dc_j, dc_j^2)^T$. A superpopulation model for leaf weight, conditional on the $\underline{x}_j$ was then initially taken as

$$\underline{Y} = \underline{X}\ \underline{\beta} + \underline{\varepsilon} \qquad (3.1)$$

where $\underline{\beta} = (\beta_1,\beta_2,\beta_3)$ and $\underline{\varepsilon}$ is an error vector.

It was observed (see Figures 4a-c, for example) that (3.1) fit more accurately for small branches than for large. Examination of the pooled residuals from preliminary regressions of $\underline{Y}$ on $\underline{X}$ for all 32 trees showed that their variance was roughly proportional to the square of dc; hence it was decided to represent the variance of $\underline{\varepsilon}$ by

$$\mathrm{Var}(\underline{\varepsilon}|\underline{X}) = \sigma^2\underline{V}, \qquad (3.2)$$

where $\sigma^2$ is an unknown scalar, and $\underline{V} = \mathrm{diag}\,(v_1,v_2,\ldots v_N)$ with $v_j = dc_j^2$.

Empirical models such as (3.1) are only intended to

represent a local relationship; hence in order for one to have confidence in the regression predictor of $\underline{Y}$, not only should the fit of the model to the regression data be good, but also prediction should be avoided for x-values outside the range of the regression data.

Figures 4a-c which plot leaf weight against dc for trees #21, #24 and #27 respectively, illustrate typical situations which can occur. In these figures, vertical bars have been added above the dc-axis showing the distribution of dc values for the unsampled branches. In Figure 4c (Tree 27) the fit is good and the dc values for all the unsampled branches lie well within the domain of the regression points. In Figure 4a (Tree 21), the fit is not very good, but reasonable predictions can still be made for the unsampled branches because their dc-values lie within those of the regression branches. In Figure 4b (Tree 24), the fit is not bad, but there are three large branches with dc-values outside the domain of the regression, thus requiring extrapolation. In a case such as this it would have been better to not sample the branches randomly, but instead to have made sure that the largest branches were included in the regression, as suggested by Royall (1970).

Let $\underline{I}^* = \text{diag}\ (c_1,..,c_N)$ where $c_j = 1$ if the j-th branch is a regression branch, otherwise $c_j = 0$. Standard estimation of $\underline{\beta}$ with the model given by (3.1) and (3.2) consists of finding the vector $\underline{b}$ which minimizes $L(\underline{b}) = (\underline{Y}-\underline{X}\ \underline{b})^T\ \underline{I}^*\ \underline{V}^{-1}(\underline{Y}-\underline{X}\ \underline{b})$. Although the $\underline{b}$ obtained in this manner is unbiased and has desirable

necessarily produce useful predictions of $\underline{Y}$ for all dc values likely to be encountered. In particular, foliage weight predictions $\underline{b}^T\underline{x}_j$ were sometimes negative for small branches, and in one case, where a tree had many small branches, the estimated foliage weight for the entire tree was negative. To prevent this anomaly, the model was revised to constrain $\underline{\beta}$ to be nonnegative and $\underline{b}$ was then calculated as the solution to minimizing $L(\underline{b})$ subject to $\underline{b} \geq 0$. It is known - e.g. see Lawson and Hanson (1974), that the constrained $\underline{b}$ is the solution to one of the unconstrained subset regressions using some or all of the independent variables $dc^0=1$, dc and $dc^2$; in fact it is the solution to the subset regression which has the smallest value of L, given that the (unconstrained) regression coefficients are nonnegative.

Even when $\underline{\varepsilon}$ is assumed to have a multivariate normal distribution, the distribution of $\underline{b}$, or even its variance, is not known. In order to obtain an approximate expression for $Var(\underline{b})$, we pretend that the model (3.1) originally included only those independent variables whose subset regression produced the optimal nonnegative $\underline{b}$, so that $\underline{x}_j$, $\underline{\beta}$ and $\underline{b}$ are now understood to be p-dimensional vectors and $\underline{X}$ is a N x p matrix, where $1 \leq p \leq 3$, depending on the tree. In terms of the revised $\underline{X}$, we have

$$\underline{b} = (\underline{X}^T\underline{W}^*\underline{X})^{-1}\underline{X}^T\underline{W}^*\underline{Y} \qquad (3.3)$$

where $\underline{W}^* = \underline{I}^*\underline{V}^{-1}$.

### 3.1.2 The Ratio Model

In addition to the above model for describing the relationship between leaf weight and branch dimensions, another is required to relate leaf area to weight.

As described in Section 2.1, samples of foliage were taken from each stratum of each tree to obtain leaf area and leaf weight measurements. Let $y_{ik}$ and $x_{ik}$ denote the respective total area and total fresh weight of the leaves in the sample from the i-th stratum in the k-th tree and let $Y_{ik}$ and $X_{ik}$ be the corresponding population quantities (i=1,2,3; k=1,..,K).

To a first approximation, it can be assumed that the sample ratio $r_{ik} = y_{ik}/x_{ik}$ is an unbiased estimate of the population ratio $R_{ik} = Y_{ik}/X_{ik}$; i.e.

$$r_{ik} = R_{ik} + e_{ik} \tag{3.4}$$

where $e_{ik}$ is an error term. In general, one would expect $r_{ik}$ to be a more accurate estimate of $R_{ik}$ when the sample is large; i.e. when $x_{ik}$ is large. For this reason we assume Var $(e_{ik}|x_{ik}) = \omega^2/x_{ik}$ for some positive constant $\omega^2$.

Rather than base the estimation of leaf area of the sacrificed trees on the rik directly, it was decided to first smooth the raw ratios using an additive model for the true ratios which allow one to incorporate information about all sacrificed trees and strata to obtain the estimate of the ratio for a given tree and stratum. The true-ratio model is given by

$$R_{ik} = \mu + \alpha_i + \beta_k + d_{ik} \tag{3.5}$$

where $\mu$ is an overall mean, $\alpha_i$ is a stratum effect, $\beta_k$ is a tree effect with $\Sigma\alpha_i = \Sigma\beta_k = 0$, and the $d_{ik}$ are mutually independent random errors with $E(d_{ik}) = 0$ and $Var(d_{ik}) = \tau^2$.

Let $\rho_{ik} = E(R_{ik}) = \mu + \alpha_i + \beta_k$. The additive model for the $\rho_{ik}$ is partially justifiable by ecological considerations. It is a recognized fact (cf. Talbert and Holch (1957)) that for a given

tree, "sun" leaves (those high in the tree crown) tend to be thicker and smaller in area than "shade" leaves (those low in the crown); thus creating a tendency for the area-to-weight ratios to be ordered by strata with the lowest ratio for the upper stratum, and the highest ratio for the lower stratum. If $\alpha_1 < \alpha_2 < \alpha_3$, equation (3.5) imposes this theoretical ordering on the expected ratios for each tree.

Combining (3.4) and (3.5) gives

$$r_{ik} = \mu + \alpha_i + \beta_k + e'_{ik} \qquad (3.6)$$

where $e'_{ik} = e_{ik} + d_{ik}$. It is not unreasonable to assume the $e_{ik}$ and $d_{ik}$ to be uncorrelated in which case

$$\mathrm{Var}(e'_{ik} | x_{ik}) = \omega^2/x_{ik} + \tau^2 \qquad (3.7)$$

Let $\hat{\rho}_{ik} = \hat{\mu} + \hat{\alpha}_i + \hat{\beta}_k$ where $\hat{\mu}$, $\hat{\alpha}_i$ and $\hat{\beta}_k$ are obtained from fitting the model (3.6) by weighted least squares with weights proportional to $x_{ik}$. It is advantageous (in terms of a smaller prediction error) to predict $R_{ik}$ by $r_{ik}$ when the second term of (3.7) dominates, or by $\hat{\rho}_{ik}$ when the first term dominates. In this study, we assumed the latter case to be true because (a) the sample sizes of leaves were fairly small, and (b) it was felt that the $R_{ik}$ were well represented by the additive model - i.e. $\tau^2$ was relatively small.

Let $\hat{\rho}^*$ be a 3K x 1 vector of the $\hat{\rho}_{ik}$ and let $\underline{r}^*$ be a corresponding vector whose components are the $r_{ik}$. Then one may write $\hat{\rho}^* = \underline{U}\,\underline{r}^*$, where $\underline{U}$ is a function of the design matrix of the model (3.6) and the $x_{ik}$. For purposes of computing the estimated mean squared prediction error (MSPE) of $\hat{\eta}$ for a given

tree (say the k-th), it is assumed that the 3 x 1 vector $\hat{\rho}_k = (\hat{\rho}_{1k}, \hat{\rho}_{2k}, \hat{\rho}_{3k})^T$ is distributed as multivariate normal with mean $\underline{\rho}_k = (\rho_{1k}, \rho_{2k}, \rho_{3k})^T$ and covariance matrix $\underline{\Psi}_k$, where $\underline{\Psi}_k$ is the submatrix corresponding to the k-th tree, taken from the covariance matrix $\underline{\Psi}^*$ of $\hat{\underline{\rho}}^*$. The matrix $\underline{\Psi}^*$ is of the form $\omega^2 A_1 + \tau^2 A_2$, where $A_1$ and $A_2$ are functions of $\underline{U}$ and the $x_{ik}$; however $\underline{\Psi}^*$ cannot be estimated unbiasedly because there is no way of estimating $\omega^2 A_1 + \tau^2 A_2$ directly or $\omega^2$ and $\tau^2$ separately with the available data. An approximately unbiased estimate of $\underline{\Psi}^*$ is $\hat{\underline{\Psi}}^* = \hat{\omega}^2 \underline{U}\, \underline{D}^{-1}(x) \underline{U}^T$, where $\underline{D}(x) = \text{diag}(x_{ik})$ and $\hat{\omega}^2$ is the residual mean square given by

$$\hat{\omega}^2 = [\Sigma\Sigma x_{ik}(r_{ik} - \hat{\rho}_{ik})^2]/2(k-1) \qquad (3.8)$$

This estimate would be unbiased if $\tau^2$ were zero.

### 3.1.3 The Estimator of $\eta$ and its MSPE

Reverting to the discussion of the prediction of $\eta$ for a given tree, we drop the subscript "k" of Section 3.1.2, using $\hat{\underline{\rho}}$ to mean the 3 x 1 vector of smoothed ratio estimates and $\underline{\Psi}$ to mean the 3 x 3 covariance matrix of $\hat{\rho}$ for the tree. Likewise, "$\underline{R}$" will designate the 3 x 1 vector of true ratios for the tree with $E(\underline{R}) = \underline{\rho}$.

The foliage weight estimate for a stratum was the sum of two components: one being the sum of measured weights for sampled branches; the other being the sum of predicted weights for the unsampled branches using the model (3.1). Let $\underline{e}_{is}$ be the N x 1 membership vector for the sampled branches of the i-th stratum; i.e. the $\nu$-th element of $\underline{e}_{is}$ is zero or one, being unity if and

a sampled branch. Similarly, let $\underline{e}_{iu}$ be the membership vector for the unsampled branches in the i-th stratum. Then the estimated foliage weight for the stratum is

$$\begin{aligned}\hat{y}_i &= \underline{e}_{is}^T \underline{Y} + \underline{e}_{iu}^T \underline{X}\, \underline{b} \\ &= (\underline{e}_{is}^T + \underline{e}_{iu}^T \underline{P}\, \underline{W}^*)\underline{Y} \end{aligned} \tag{3.9}$$

where $\underline{P} = \underline{X}(\underline{X}^T \underline{W}^* \underline{X})^{-1} \underline{X}^T$. Let $\hat{\underline{y}} = (\hat{y}_1, \hat{y}_2, \hat{y}_3)^T$. The total leaf area estimate is given by

$$\hat{\eta} = \hat{\underline{y}}^T \hat{\underline{\rho}} \tag{3.10}$$

In terms of $\underline{Y}$, $y_i$, the actual foliage weight for the i-th stratum can be expressed by $y_i = (\underline{e}^T_{is} + \underline{e}^T_{iu})\underline{Y} = \underline{e}_i{}^T \underline{Y}$, where $\underline{e}_i = \underline{e}_{is} + \underline{e}_{iu}$. Let $\underline{y} = (y_1, y_2, y_3)^T$. The actual leaf area is thus equal to $\underline{y}^T \underline{R}$. To compute the MSPE of $\hat{\eta}$, let the 12 x 1 vector $\underline{z}$ be defined by

$$\underline{z} = (\hat{\underline{y}}^T, \underline{y}^T, \hat{\underline{\rho}}^T, \underline{R}^T)^T \tag{3.11}$$

and the 12 x 12 matrix A be given by

$$\underline{A} = (1/2) \begin{bmatrix} \underline{0} & \underline{0} & \underline{I} & \underline{0} \\ \underline{0} & \underline{0} & \underline{0} & -\underline{I} \\ \underline{I} & \underline{0} & \underline{0} & \underline{0} \\ \underline{0} & -\underline{I} & \underline{0} & \underline{0} \end{bmatrix} \tag{3.12}$$

where $\underline{I}$ is a 3 x 3 identity matrix. Then $\underline{z}^T \underline{A}\, \underline{z} = \hat{\underline{y}}^T \hat{\underline{\rho}} - \underline{y}^T \underline{R}$; i.e. it is the prediction error $\hat{\eta} - \eta$.

Let $\underline{\mu} = E(\underline{y}) = (\underline{e}_1^T \underline{X}\, \underline{\beta}, \underline{e}_2^T \underline{X}\, \underline{\beta}, \underline{e}_3^T \underline{X}\, \underline{\beta})^T$. From the revised version of (3.1) and from (3.9), it can be seen that

$E(\hat{\underline{y}}) = E(\underline{y}) = \underline{\mu}$. From previous assumptions we also have $E(\hat{\underline{\rho}}) = E(\underline{R}) = \underline{\rho}$. Assuming further that $\hat{\underline{y}}$ is independent of $\hat{\underline{\rho}}$ yields $E(\underline{z}^T \underline{A}\ \underline{z}) = \underline{\mu}^T \underline{\rho} - \underline{\mu}^T \underline{\rho} = 0$, so that the MSPE of $\hat{\eta}$ is the variance of $\underline{z}^T \underline{A}\ \underline{z}$.

Let $E(\underline{z}) = \underline{\zeta}$ and $Var(\underline{z}) = \underline{\Sigma}$. If $\underline{z}$ is assumed to be distributed multivariate normal, then

$$MSPE(\hat{\eta}) = Var(\underline{z}^T \underline{A}\ \underline{z}) = 4\ \underline{\zeta}^T \underline{A}\ \underline{\Sigma}\ \underline{A}\ \underline{\zeta} + 2\ tr(\underline{\Sigma}\ \underline{A}\ \underline{\Sigma}\ \underline{A}) \qquad (3.13)$$

We have already seen that $\underline{\zeta} = (\underline{\mu}^T, \underline{\mu}^T, \underline{\rho}^T, \underline{\rho}^T)^T$. To evaluate (3.13) it remains to specify $\underline{\Sigma}$ which is of the form

$$\underline{\Sigma} = \begin{bmatrix} \underline{\Sigma}_{11} & \underline{\Sigma}_{12} & \underline{0} & \underline{0} \\ \underline{\Sigma}_{21} & \underline{\Sigma}_{22} & \underline{0} & \underline{0} \\ \underline{0} & \underline{0} & \underline{\Psi} & \tau^2 \underline{U} \\ \underline{0} & \underline{0} & \tau^2 \underline{U}^T & \tau^2 \underline{I} \end{bmatrix} \qquad (3.14)$$

where $\underline{\Sigma}_{11}$, $\underline{\Sigma}_{22}$ and $\underline{\Sigma}_{12}$ are the 3 x 3 matrices Var $(\hat{\underline{y}})$, Var $(\underline{y})$ and Cov $(\underline{y}, \hat{\underline{y}})$, respectively, and $\underline{\Sigma}_{21} = \underline{\Sigma}_{12}^T$. In terms of (3.12) and (3.14), equation (3.13) becomes

$$MSPE(\hat{\eta}) = \underline{\mu}^T [\underline{\Psi} + \tau^2 (\underline{I} - \underline{U} - \underline{U}^T)] \underline{\mu} + \underline{\rho}^T (\underline{\Sigma}_{11} - \underline{\Sigma}_{12} - \underline{\Sigma}_{21} + \underline{\Sigma}_{22}) \underline{\rho}$$
$$+ tr\{\underline{\Sigma}_{11} \underline{\Psi} + \tau^2 (\underline{\Sigma}_{22} - \underline{U}\ \underline{\Sigma}_{21} - \underline{U}^T \underline{\Sigma}_{12})\} \qquad (3.15)$$

The elements of $\underline{\Sigma}_{11}$, $\underline{\Sigma}_{12}$ and $\underline{\Sigma}_{22}$ are derived from the model given by (3.1) and (3.2), and from the definition of $\hat{\underline{y}}$ in (3.9).

Specifically, it can be shown that

$$\mathrm{Cov}(\hat{y}_i, \hat{y}_{i'}) = \sigma^2[\underline{e}_{is}^T \underline{V}\, \underline{e}_{i's} + \underline{e}_{iu}^T \underline{P}\, \underline{e}_{i'r} + \underline{e}_{ir}^T \underline{P}\, \underline{e}_{i'u} + \underline{e}_{iu}^T \underline{P}\, \underline{e}_{i'u}] \quad (3.16)$$

where $\underline{e}_{ir} = \underline{I}^* \underline{e}_{is} = \underline{I}^* \underline{e}_i$. It can be similarly shown that

$$\mathrm{Cov}(\hat{y}_i, y_{i'}) = \sigma^2[\underline{e}_{is}^T \underline{V}\, \underline{e}_{i's} + \underline{e}_{iu}^T \underline{P}\, \underline{e}_{i'r}] \quad (3.17)$$

and

$$\mathrm{Cov}(y_i, y_{i'}) = \sigma^2 \underline{e}_i^T \underline{V}\, \underline{e}_{i'}. \quad (3.18)$$

The MSPE of $\hat{\eta}$ is estimated by substituting estimates for the unknown parameters in (3.15); i.e. $\hat{\underline{y}}$ for $\underline{\mu}$, $\hat{\rho}$ for $\rho$, $\hat{\Psi}$ for $\underline{\Psi}$, and $\hat{\underline{\Sigma}}_{ij}$ for $\underline{\Sigma}_{ij}$ $(i,j = 1,2)$. The matrix $\hat{\underline{\Psi}}$ is the appropriate submatrix of $\hat{\underline{\Psi}}^*$ of Section 3.1.2, while the $\hat{\underline{\Sigma}}_{ij}$ are estimated by substituting $\hat{\sigma}^2$ for $\sigma^2$ in (3.12a-c), where $\hat{\sigma}^2$ is the residual mean square given by

$$\hat{\sigma}^2 = (\underline{Y} - \underline{X}\, \underline{b})^T \underline{W}^* (\underline{Y} - \underline{X}\, \underline{b})/(n-p), \quad (3.19)$$

and $n = \Sigma c_j$ is the number of observations in the foliage weight regression. In this analysis, terms of (3.15) involving $\tau^2$ are neglected; however (1) $\tau^2$ is presumed small and (2) the effect of $\tau^2$ is absorbed into $\hat{\omega}^2$ and hence $\hat{\underline{\Psi}}$. Ignoring the other blocks of $\underline{\Sigma}$ involving $\tau^2$ is equivalent to treating $\underline{R}$ as a fixed, rather than a random quantity, thus making (3.15) more like a "variance" rather than a MSPE as far as the ratios are concerned. If multiple observations of area-to-weight ratios had been taken within strata, it would have been possible to estimate $\tau^2$. It is suggested that this be done in future studies of this type.

## 3.2 Between Trees - The Dimension Analysis Model

Given estimates of leaf area for the sacrificed trees, the next step in the LAI estimation process is to relate these

estimates to the nondestructive measurements L, $L_1$ and DBH.

Let $\underline{x}$ be a p x 1 vector of functions of L, $L_1$, and DBH which predict leaf area, $\eta$ through a model

$$\eta = \underline{\gamma}^T \underline{x} + d \tag{3.20}$$

where d is an error term, independent of $\underline{x}$, with a zero mean and a variance $v(\underline{x})$ (conditional on $\underline{x}$) which is an increasing function of $E(\eta) = \underline{\gamma}^T \underline{x}$. (The actual functions of L, $L_1$ and DBH producing the components of $\underline{x}$ are described in Section 4.2.) The leaf area for a new tree with given x-vector $\underline{x}'$, may then be predicted by

$$\hat{\eta} = \underline{g}^T \underline{x}' \tag{3.21}$$

where $\underline{g}$ is an estimate of $\underline{\gamma}$ obtained from the sacrificed tree data.

### 3.2.1 Estimation of Model Coefficients

Let $\eta_k$, $x_k$ and $d_k$ play the respective roles of $\eta$, x and d in the model (3.20) for the k-th sacrificed tree (k = 1,..,K). Also for the k-th sacrificed tree, let $\hat{\eta}_k$ be given by (3.10). To a first approximation $\hat{\eta}_k$ is an unbiased predictor of $\eta_k$ with MSPE given by (3.15). If the within-tree estimation errors are independent of the between-tree errors $d_k$, then the $\hat{\eta}_k$ follow a revised model

$$\hat{\eta}_k = \underline{\gamma}^T \underline{x}_k + d_k' \tag{3.22}$$

with $d_k'$ being similar to $d_k$, except that $\mathrm{Var}(d_k' | \underline{x}_k) = v(\underline{x}_k) + v_{0k}$, where $v_{0k}$ is the within-tree MSPE given by (3.15) for the k-th tree.

Ideally, it would be best to use the weighted least-squares estimate $g_w = (\underline{X}^T \underline{V}^{-1} \underline{X})^{-1} \underline{X}^T \underline{V}^{-1} \hat{\eta}$ for g in (3.21),

where $\eta = (\eta_1, \eta_2, \ldots, \eta_K)'$, $\underline{X} = (\underline{x}_1 | \underline{x}_2 | \ldots | \underline{x}_K)'$ and $\underline{V} = \text{diag}\{v(\underline{x}_k) + v_{0k}\}$; but the function $v(.)$ is not known and can only be roughly estimated from the data. Unlike the within-tree situation for which there were many observations of residuals available for estimating a weighting function, there were only 32 here. With such a small sample size, estimating the weights from the data could seriously bias the estimate of $\underline{\gamma}$. As a consequence, the unweighted quantity

$$\underline{g} = (\underline{X}^T \underline{X})^{-1} \underline{X}^T \hat{\eta} \tag{3.23}$$

was used as an estimate of $\underline{\gamma}$. Of course, $\underline{g}$ is still an unbiased estimator of $\underline{\gamma}$ under the model (3.22). Its covariance matrix is given by

$$\underline{W} = (\underline{X}^T \underline{X})^{-1} \underline{X}^T \underline{V} \underline{X} (\underline{X}^T \underline{X})^{-1}. \tag{3.24}$$

3.2.2 Estimation of the Model Variance Function

For purposes of estimating the precision of the plot LAI estimate, it was necessary to have at least rough estimates of the function $v(.)$ and the matrix $\underline{W}$. These estimates were obtained through a parametric model using the residuals of the unweighted regression as observations. Let $\underline{e} = \hat{\eta} - \underline{X}\,\underline{g}$ be the K x 1 vector of residuals. Under the model (3.22) the covariance matrix of $\underline{e}$ is

$$\begin{aligned} \text{Var}(\underline{e}|\underline{X}) = {} & \underline{V} - \underline{X} (\underline{X}^T \underline{X})^{-1} \underline{X}^T \underline{V} - \underline{V} \underline{X} (\underline{X}^T \underline{X})^{-1} \underline{X}^T \\ & + \underline{X} (\underline{X}^T \underline{X})^{-1} (\underline{X}^T \underline{V} \underline{X}) (\underline{X}^T \underline{X})^{-1} \underline{X}^T \end{aligned} \tag{3.25}$$

and the residual sum of squares $\underline{e}^T \underline{e}$ has expectation

$$E(\underline{e}^T \underline{e}) = \text{tr}\{\underline{V}\} - \text{tr}\{(\underline{X}^T \underline{X})^{-1} (\underline{X}^T \underline{V} \underline{X})\}. \tag{3.26}$$

The non-leading terms of both (3.25) and (3.26) are of order 1/K. Neglecting them is equivalent to treating the residuals as actual observations of the $d_k$' in (3.22). Assuming that this is a

reasonable approximation gives

$$E(\underline{e}^T\underline{e}) \doteq \sum_k v(\underline{x}_k) + \sum_k v_{0k} \tag{3.27}$$

and

$$\mathrm{Var}(e_k) \doteq v(\underline{x}_k) + v_{0k} \tag{3.28}$$

where $e_k$ is the k-th component of $\underline{e}$ .

Suppose now that $v(\underline{x})$ can be modeled by

$$v(\underline{x}) = \theta_0(\underline{\gamma}^T\underline{x})^{\theta_1} \tag{3.29}$$

where $\theta_0$ and $\theta_1$ are parameters. From (3.27), we have $E(\underline{e}^T\underline{e}) = \theta_0 \sum_k (\underline{\gamma}^T\underline{x}_k)^{\theta_1} + \sum_k v_{0k}$. Given an estimate of $\theta_1$, say $\hat{\theta}_1$, $\theta_0$ can be estimated by setting $\underline{e}^T\underline{e}$ equal to its estimated expectation; i.e.

$$\hat{\theta}_0 = (\underline{e}^T\underline{e} - \sum_k v_{0k}) / \sum_k (\underline{g}^T\underline{x}_k)^{\hat{\theta}_1} \tag{3.30}$$

$$= G(\hat{\theta}_1), \text{ say,}$$

where $\hat{\theta}_0$ is the estimate of $\theta_0$.

Consider the normalized residuals $z_k = e_k / [G(\tilde{\theta}_1)(\underline{g}^T\underline{x}_k)^{\tilde{\theta}_1}]^{1/2}$, (k=1,..,K) where $\tilde{\theta}_1$ is a trial value of $\theta_1$. Given the assumptions of this section, if $\tilde{\theta}_1$ is close to $\theta_1$, the $z_k$ should have approximately unit variance; in fact, if the residuals are normally distributed, the $z_k$ should resemble a random sample from a N(0,1) population. We therefore choose $\hat{\theta}_1$ to be the value of $\tilde{\theta}_1$ which minimizes the Kolmogorov-Smirnov statistic D, which is the maximum absolute deviation between the empirical distribution function of the $z_k$ and that of a standard normal distribution. Once $\hat{\theta}_1$ is found, $\hat{\theta}_0$ is already available as $G(\hat{\theta}_0)$. Let $\hat{v}_{ok}$ be the sample analog of $v_{0k}$. For an arbitrary $\underline{x}$, $v(\underline{x})$ is then estimated by

$$\hat{v}(x) = \hat{\theta}_0(\underline{g}^T\underline{x})^{\hat{\theta}_1} \qquad (3.31)$$

and W is estimated by using $\hat{v} = \text{diag}\,(\hat{v}(\underline{x}_k)+\hat{v}_{0k})$, for $\underline{W}$ in (3.24).

## 3.3 The LAI Estimate and its Precision

Given $\underline{g}$, the total leaf area of all trees in the subplots within a plot can be estimated by the application of (3.21) since $\underline{x}$ is known for these trees. The total leaf area is then normalized by the total subplot area to obtain the LAI estimate. There are two main independent contributions of error in this estimate. The first is a between-subplot sampling error due to the nonhomogeneity of the LAI throughout the plot. The second error is the discrepancy between the model-predicted leaf area and the actual leaf area within the subplots. It is not unreasonable to assume these two errors independent, in which case the overall MSPE of the LAI estimate is

$$E(\hat{\lambda} - \Lambda)^2 = E(\lambda - \Lambda)^2 + E(\hat{\lambda} - \lambda)^2 \qquad (3.32)$$

where $\hat{\lambda}$ is the estimated LAI over the subplots (and also is the LAI estimate for the whole plot), $\Lambda$ is the true LAI of the plot and $\lambda$ is the true LAI over the subplots.

For purposes of evaluating the first term of (3.32), the leaf area pattern for the whole plot is considered fixed so that the LAI's of the subplots become fixed numbers once the subplots are chosen. For the second term, we shall condition on the observed x-values for both the sacrificed and the plot trees. We thus use

$$Q = E\{(\lambda - \Lambda)^2 | L\} + E\{(\hat{\lambda} - \Lambda)^2 | \underline{x}\}$$

$$= Q_1 + Q_2 \qquad (3.33)$$

instead of (3.32) as a measure of precision of $\hat{\lambda}$. Here, "..|L}" denotes conditioning on a fixed plot leaf area pattern.

3.3.1 Between-Subplot Sampling Variance

The variance of the between-subplot sampling error is difficult to estimate without knowledge of a leaf area intensity function over the whole plot; i.e. the leaf area "pattern". As a practical alternative, we pretend that the whole plot consists of M subplots, of which we have selected m at random. Let f denote the sampling fraction m/M and let $\Lambda_i$ and $A_i$ denote the respective true LAI and area of the i-th subplot (i = 1,..M). For the i-th sampled subplot let $\lambda_i$ and $a_i$ be the respective LAI and area (i = 1,..,m), and let $\hat{\lambda}_i$ be the estimate of $\lambda_i$ obtained from the dimension analysis model.

We shall now condition on the leaf area pattern for the plot so that the $\Lambda_i$ may be regarded as fixed numbers, as opposed to the random variables they would be under the model (3.20). With this restriction, the $\lambda_i$ are random only because of the subplot sampling process. The LAI over the m sampled subplots is then $\lambda = \sum^{m}(a_i/a.)\lambda_i$, where $a. = \sum^{m} a_i$. For the whole plot, the LAI is $\Lambda = \sum^{M}(A_i/A.)\Lambda_i$ where $A. = \sum^{M} A_i$. The difference $\lambda - \Lambda$ is the error due to sampling the plot. When the areas of all M subplots are equal, $\lambda$ is an unbiased estimator of $\Lambda$; however when the areas are unequal, $\lambda$ becomes a ratio estimator and is only approximately unbiased. Assuming the bias is neglibible, $Q_1 = E\{(\lambda - \Lambda)^2 | L\} = \text{Var}\{\lambda | L\}$. In the general case of unequal $A_i$, we have the approximation (cf. Cochran, 1977, p.155)

$$Q_1 = [(1-f)/m]M^2 \sum_i^m (A_i/A.)^2/(\lambda_i - \lambda)^2/(M-1). \qquad (3.34)$$

If the $\lambda_i$ were known, $Q_1$ would be estimated by

$$q_1 = (1 - \hat{f})m \sum_i^m (a_i/a.)^2(\lambda_i - \lambda)^2/(m-1) \qquad (3.35)$$

where $\hat{f} = a./A.$ replaces f in (3.34). Since the $\lambda_i$ are not known, we replace them with their estimates $\hat{\lambda}_i$ to obtain

$$\hat{Q}_1 = (1-\hat{f})m \sum_i^m (a_i/a.)^2(\hat{\lambda}_i - \hat{\lambda})^2/(m-1) \qquad (3.36)$$

as the estimate of $Q_1$.

3.3.2 Within-Subplot Prediction Error

Let $\eta_{ij}$ be the true leaf area of the j-th tree in the i-th subplot and let $\underline{x}_{ij}$ be the vector of leaf area predictors for that tree, with $\eta.. = \Sigma\Sigma\eta_{ij}$ and $\underline{x}.. = \Sigma\Sigma\underline{x}_{ij}$. The estimate of the LAI for the m sampled subplots is $\hat{\lambda} = \underline{g}^T\underline{x}../a.$ and the actual LAI is $\lambda = \sum_i^m (a_i/a.)\ \lambda_i = \eta../a.$ From (3.20), $\eta_{ij} = \underline{\gamma}^T\underline{x}_{ij} + d_{ij}$ where $E[d_{ij}|\underline{x}_{ij}] = 0$, and $Var[d_{ij}|\underline{x}_{ij}] = v(\underline{x}_{ij})$. Since $\underline{g}$ is an unbiased estimator of $\underline{\gamma}$, the conditional mean prediction error of $\hat{\lambda} - \lambda$ is zero; i.e.

$$E(\hat{\lambda} - \lambda|\underline{x}) = E(\underline{g}^T\underline{x}../a.|\underline{x}) - E(\eta../a.|\underline{x}) \qquad (3.37)$$
$$= (\underline{\gamma}^T\underline{x}..-\underline{\gamma}^T\underline{x}..)/a. = 0$$

The second term ($Q_2$) in (3.33) is thus the conditonal variance of $\hat{\lambda} - \lambda$, so that

$$Q_2 = E\{(\hat{\lambda} - \lambda)^2|x\} = Var\{(\hat{\lambda} - \lambda)|\underline{x}\}$$
$$= Var\{(\underline{g}^T\underline{x}../a. - \eta../a.)|\underline{x}\}$$
$$= (1/a.^2)[\underline{x}^T..\underline{W}x.. + \sum_{ij}\sum v(x_{ij})] \qquad (3.38)$$

Its estimate, $\hat{Q}_2$ is obtained by direct substitution; i.e.

$$\hat{Q}_2 = (1/a.^2)[\underline{x}^T..\hat{\underline{W}}\ \underline{x}.. + \sum_{ij}\sum \hat{v}(x_{ij})] \qquad (3.39)$$

where $\hat{\underline{W}}$ and $\hat{v}(.)$ are the respective estimates of the matrix $\underline{W}$ and

the function v(.) from Section 3.2.2. The term $\underline{x}_{..}^{T}\underline{W}\,\underline{x}_{..}$ reflects the fact that $\underline{\gamma}$ is estimated, while the other term $\Sigma\Sigma v(x_{ij})$ reflects the actual model error. By increasing the number of sacrificed trees, the former term can be made arbitrarily small, but not the latter. In Section 4.3, the effect of each of these terms on $\hat{Q}_2$ is evaluated numerically to give a feeling for whether the sample size of sacrificed trees was adequate.

## 4.0 NUMERICAL RESULTS

In this section, numerical results from the COVER experiment are given. For the most part these results are organized along the lines of the theoretical development in Section 3 so that a subsection numbered "4.x...y" consists of a numerical illustration of the topic covered under Section 3.x...y.

### 4.1 Within Trees

Recall that in the COVER experiment 32 trees were destructively sampled so as to best estimate their leaf area; that for a given tree, foliage weights were estimated with the aid of a regression model and then ratioed to leaf areas for each of three crown strata. These trees are identified by a number ranging from 1 to 32 which corresponds to the order of their selection. Table 1 lists the trees in order of size (i.e. DBH) showing the Tree Number, the DBH in cm, H in meters, the total number of branches (N), the number of sampled branches (NS) and the foliage weight regression sample size (n).

#### 4.1.1 Foliage Weight Regressions

From Table 1 it can be seen that the foliage weight was

measured for anywhere between 20 and 100 percent of the branches. For purposes of measuring foliage weights, it was originally intended that a census of branches on the sacrificed trees be taken. For the first five trees, a census or near-census was made, but the excessive amount of work encountered soon made it clear that some sort of sampling was necessary. For this reason the sampling fractions for the other trees were considerably smaller (except Tree 11 which had only 9 branches, all of which were sampled).

While the relationship between leaf weight and branch dimensions was not overly strong, it was usually good enough to justify the use of regression to estimate the foliage weight totals within strata for trees in which branches were sampled. For each tree, Table 2 gives the dimension (p) of the revised regression model of Section 3.1, the regression coefficients given by (3.3) and the t-values associated with the coefficients, assuming the p-dimensional model. The coefficients $b_i$ are scaled to obtain the predicted foliage weight ($\hat{y}$) in grams, from the branch depth-of-crown (dc) in meters, from the equation $\hat{y} = b_o + b_1(dc) + b_2(dc)^2$. Terms deleted from the model by the constrained least-squares algorithm are indicated by a dash "--". In only 7 of the 32 regressions were all three possible terms included in the revised model, while two terms were used in 24 cases and one term in one case.

For many trees, more terms could have been deleted because they did not contribute significantly to the regression fit; however for simplicity, any terms with nonzero coefficients as

found by the algorithm, were retained. From Table 2 it can be seen that in most cases where the intercept term was included in a model, it was found to have a small t-value, but that the linear term was usually significant and the quadratic term was almost always significant when included in a revised model.

4.1.2 Estimation of Area-to-Weight Ratios

The average raw ratio of leaf area to leaf weight was about 4.9 $m^2$/kg. As predicted, these ratios tended to be greatest in the lower crown stratum and smallest in the upper crown stratum, although the magnitude of the difference was fairly small (generally about 10 percent). For 26 of the 32 trees, the upper-stratum ratio was the lowest of the three, while in 21 cases the lower stratum ratio was the highest. For 19 trees, the three ratios ranked exactly as expected; i.e. inversely with the crown position. The tree effect was even more noticeable, with the average raw ratio ranging from about 4.3 $m^2$/kg for Tree 29 to 7.3 $m^2$/kg for Tree 30. In general, the largest ratios were observed on small trees.

As explained in Section 3.1.2, it was decided to replace the raw ratios ($r_{ik}$) by smoothed ones ($\hat{\rho}_{ik}$) which were felt to be better predictors of the actual ratios. Figure 5 shows the $r_{ik}$ plotted against the $\hat{\rho}_{ik}$ for i = 1, 2, 3; k = 1,2,...,32. Points lying between the two lines are those cases (73 of 96) for which $\hat{\rho}_{ik}$ was within 5% of $r_{ik}$. Without replicated samples, it was not possible to formally test whether the additive model (3.6) was correct, but it appeared to fit the data well. The $\hat{\alpha}_i$ were in the "correct" biological order (decreasing with i), thus so were the $\hat{\rho}_{ik}$ for each k. Estimated coefficients of variation of the

$\hat{\rho}_{ik}$ ranged from about 4 to 12 percent. It was not possible to calculate a comparable quantity for the raw ratios.

4.1.3 Leaf Area Estimates for the Sacrificed Trees

For each sacrificed tree, (3.10) was used to obtain $\hat{\eta}$ the estimated leaf area, and the sample analog of (3.15) was used to estimate its MSPE. For the sacrificed trees arranged in order of increasing DBH, Table 3 gives $\hat{\eta}$, its standard error SE = $(\text{MSPE})^{1/2}$ and proportional contributions to the MSPE due to branch sampling and to area-to-weight ratio estimation. These components were obtained by evaluating (3.15) with certain changes to $\underline{\Sigma}$. Specifically, if a census of branches were taken, thus making the leaf weights of all branches known (assuming negligible measurement errors), then in the expresion for $\underline{\Sigma}$ given by (3.14), $\underline{\Sigma}_{11}$ and $\underline{\Sigma}_{12}$ would both be equal to $\underline{\Sigma}_{22}$ because $\hat{y}$ would be equal to y. As a result, the middle term of (3.15) would vanish. Let MSPE1 denote (3.15) evaluated with $\underline{\Sigma}$ changed in this way. The normalized difference (MSPE-MSPE1)/MSPE is then shown in Table 3 under the heading "BRANCH SAMPLING" as the proportional contribution of branch sampling to the MSPE.

Similarly, if the ratios of leaf area to leaf weight were already known on the stratum level, one would replace $\hat{\underline{\rho}}$ with $\underline{R}$ so that in $\underline{\Sigma}$, $\underline{\Psi}$ would be replaced by $\tau^2\underline{I}$ and U=I. Let MSPE2 denote (3.15) evaluated with $\underline{\Psi} = \tau^2 I$ and U=I. The first term vanishes regardless of the value of $\tau^2$. We have also neglected the third term under the assumption that $\tau^2$ is negligible . The proportion of MSPE due to ratio estimation

then equal to 100(MSPE - MSPE2)/MSPE and is shown in Table 3 in the last column entitled "RATIO ESTIMATION". A perusal of the two reductions (ignoring those trees for which a census had already been taken), reveals that neither was consistently greater, thus indicating that the manpower expended was well-allocated between the tasks of collecting total foliage weights for the sampled branches and measuring areas and weights for the small foliage samples.

To quantify the effect on the overall plot LAI estimates, of errors in the leaf areas of the sacrificed trees, the MSPE of the plot estimates was evaluated with and without the effect of these errors, and then compared. (See Section 4.3).

## 4.2 Between Trees

### 4.2.1 Fitting the Dimension Analysis Model

Previous work on non-destructive dimension analysis models for aspen trees (cf. Peterson, et al (1970) or Pollard (1970 and 1972)) has utilized DBH as the main predictor of leaf area. The additional inclusion of tree height (H) to linear or loglinear models was at best marginally better. For this study, Woods and Botkin (1986) obtained the measurement $H_1$ which made it possible to use information about the size of the tree crown, say its volume, as a predictor. This was done by modeling the crown as a cylinder or cone which enables one to approximate the crown volume by $c(H-H_1)w^2$, where w is the crown width at its base and c is a constant. The crown width was not directly measured in this study, so DBH was used as a surrogate. As a result, functions of

the dimension analysis model. Figure 6 illustrates the relationship between leaf area and VOC by plotting the former against $(VOC)^{1/2}$ for the 32 sacrificed trees.

The DBH values for the trees encountered in the subplots ranged from less than 1 cm to about 40 cm. Preliminary studies indicated that the leaf area was roughly proportional to $(VOC)^{1/2}$ for small trees and to $(VOC)^2$ for large ones. Rather than suffer the awkwardness of working with two separate models, it was decided to use both powers of VOC as predictors, so that in (3.20) the vector $\underline{x} = [VOC^{1/2}, VOC^2]^T$. The prediction equation (3.21) thus is

$$\hat{\eta} = g_1(VOC)^{1/2} + g_2(VOC)^2 \qquad (4.1)$$

where $\hat{\eta}$ is the predicted leaf area and the coefficients $g_1$ and $g_2$ are given by (3.23). For leaf area measured in $m^2$, H and $H_1$ in m, and DBH in cm, $g_1$ and $g_2$ were calculated as 0.39476 and $2.94887 \times 10^{-7}$ respectively.

4.2.2 The Error Variance Function

Using the methods of Section 3.2.2, the estimated variance function $\hat{v}(\underline{x})$ given by (3.31) was calculated as

$$\hat{v}(\underline{x}) = .35858\ (\hat{\eta})^{1.4} \qquad (4.2)$$

Recall that $\hat{v}(\underline{x})$ estimates $v(\underline{x})$, the conditional variance of a tree's leaf area given $\underline{x}$. Taking values of $\underline{x}$ observed on the sacrificed trees as typical, the estimated coefficient of variation $[\hat{v}(\underline{x})]^{1/2}/\hat{\eta}$ ranges from about 0.6 for a small tree (DBH about 2 cm.) to about 0.15 for the largest trees (DBH about 35 cm.).

## 4.3 Plot-Level Results - Error Analysis

The leaf area index was estimated for the 32 plots using (4.1) to obtain an estimated leaf area for each tree in the subplots, then summing and normalizing by the sum of the subplot areas. Standard errors were estimated as $[\hat{Q}_1 + \hat{Q}_2]^{1/2}$ where $\hat{Q}_1$ and $\hat{Q}_2$ are given by (3.36) and (3.39) respectively. Table 4 shows the sampling fraction ($\hat{f}$), estimated LAI ($\hat{LAI}$), its standard error (SE) and its coefficient of variation (CV) for each plot. In this table, the plots have been ordered by their LAI estimates.

By looking at $\hat{Q}_1$ and $\hat{Q}_2$ separately it can be seen whether large standard errors in $\hat{LAI}$ can be attributed to the heterogeneity of a given plot (as quantified by $Q_1$) or to the use of the dimension analysis model (as quantified by $Q_2$). A further breakdown can be made by separating the two terms of $\hat{Q}_2$. The first term, $\hat{Q}_2^{(1)} = (a.)^{-2} \underline{x}_{..}^T \hat{\underline{W}} \underline{x}_{..}$, reflects the uncertainty in the estimates of the regression coefficients and could be made arbitrarily small by taking a sufficiently large sample of sacrificed trees. The second term $\hat{Q}_2^{(2)} = (a.)^{-2} \Sigma\Sigma \hat{v}(x_{ij})$, represents the true model error and/or lack of fit. This term cannot be reduced by increasing the number of sacrificed trees.

In Table 4, the three components of the estimated MSPE - $\hat{Q}2^{(1)}$, $\hat{Q}_2^{(2)}$ and $\hat{Q}_1$ are shown for each plot under the respective headings "COEF EST VAR", "MODEL VAR" and "SAMP VAR". In every case, the dimension analysis model was by far the smallest contributor to the MSPE. The ratio of the sampling component ($\hat{Q}_1$) to the coefficient estimation component ($\hat{Q}_2^{(1)}$), is shown under the column headed "RATIO". An examination of this ratio

shows that for 20 of the 32 plots, the coefficient estimation component exceeded the sampling component, thus suggesting that it would have been worthwhile to estimate the regression coefficients more accurately by taking a larger sample of sacrificed trees.

In a few plots (e.g. Nos. 20, 21 and 89) the sampling component especially predominated. An examination of the nondestructive data for these plots showed that a few very large trees were heavily influencing the LAI estimate. As an example, Plot #21 contained a tree with a DBH of 45 cm - considerably larger than the greatest DBH (35.4) in the sample of sacrificed trees, thus resulting in an extrapolated (and probably spurious) leaf area estimate which contributed about 30% of the entire plot's leaf area estimate!

The effect of estimating the leaf areas of the sacrificed trees by sampling and ratioing manifests itself in the $v_{0k}$ of Section 3.2.1. By setting $\hat{v}_{0k}$ equal to zero in the evaluation of $\underline{\hat{W}}$ in $\hat{Q}_2^{(1)}$, one can see what the MSPE of the plot LAI estimates would have been had the leaf areas of the sacrificed trees been known. On the average, the standard errors (SE) in Table 4, would have been reduced by about 7% for the various plots if the $v_0k$ were zero. The smallest reduction was on Plot#98 (2%), while the largest reduction was 13% on Plot 71.

## 5.0 CONCLUSIONS AND RECOMMENDATIONS

In this paper, we have shown how various components of error may be identified and quantified in a typical ecological estimation problem. While it is true that many of the models

employed were approximate, it is felt that they represented reality well enough to provide a viable means of assessing the relative contributions of the many sampling and estimation errors encountered, as long as extrapolation was avoided. The results of this analysis can be used to design future surveys of this type more efficiently in terms of both sampling strategy and determination of sample size. Below, we list some improvements that could be made to the COVER experiment that have been identified as a direct result of this analysis.

1. Number of Sacrificed Trees - Given the same sampling strategy on the plots, the number of sacrificed trees should be at least doubled. Doing so for this study would have reduced the number of plots from 20 to 11 for which the dimension analysis coefficient estimation error component was dominant. Tripling the number of sacrificed trees would leave only five such plots.

2. Branch Sampling and Regression Within Sacrificed Trees - Care should be taken to include all non-representative and particularly the largest branches in the sample for which foliage weights were measured. The range of the predictor variable (e.g. branch depth of crown) for the unsampled branches should be well-covered by the values of this variable for the sampled branches in the regression.

3. Area-to-Weight Ratios - Multiple samples of foliage should be taken from each crown stratum to provide a means of corroborating the additive model (3.6) and for variance estimation. At least rough estimates of sampling fractions for

this data would be useful in the estimation of the variance of the ratios.

4. Dimension Analysis - With a larger number of sacrificed trees, it would be better to fit separate models for small and large trees. For the few extremely large plot trees which are bigger than the largest sacrificed trees, extra effort should be made to estimate their leaf area by intense non-destructive measurements to avoid the sort of extrapolation that occured on Plot #21. For example, even a rough assessment such as "twice the leaf area of a neighboring smaller tree" would probably be more accurate than a gross extrapolation of the fitted dimension analysis model.

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TABLE 1. Sacrificed Tree Data

| Tree No. | DBH | H | N | NS | n |
|---|---|---|---|---|---|
| 11 | 0.90 | 2.20 | 9 | 9 | 8 |
| 8 | 1.20 | 2.80 | 18 | 14 | 13 |
| 10 | 1.45 | 3.20 | 27 | 17 | 17 |
| 7 | 1.80 | 3.78 | 37 | 16 | 15 |
| 6 | 2.00 | 4.60 | 34 | 16 | 15 |
| 9 | 2.20 | 3.10 | 17 | 13 | 12 |
| 31 | 3.40 | 5.35 | 28 | 16 | 16 |
| 32 | 3.40 | 5.70 | 25 | 19 | 18 |
| 30 | 3.50 | 5.35 | 28 | 19 | 18 |
| 17 | 7.30 | 9.20 | 52 | 20 | 18 |
| 16 | 9.10 | 9.40 | 35 | 19 | 17 |
| 15 | 10.50 | 11.50 | 51 | 20 | 18 |
| 29 | 13.00 | 16.10 | 20 | 13 | 12 |
| 28 | 13.70 | 15.90 | 20 | 13 | 12 |
| 13 | 15.10 | 16.70 | 49 | 18 | 16 |
| 27 | 15.40 | 17.40 | 24 | 14 | 13 |
| 3 | 15.80 | 15.60 | 63 | 63 | 59 |
| 5 | 17.30 | 15.50 | 67 | 67 | 62 |
| 1 | 19.40 | 23.00 | 40 | 40 | 37 |
| 21 | 19.50 | 19.35 | 20 | 13 | 12 |
| 18 | 21.50 | 23.10 | 66 | 15 | 12 |
| 24 | 22.50 | 22.50 | 59 | 13 | 11 |
| 4 | 22.60 | 18.10 | 60 | 59 | 55 |
| 25 | 22.80 | 22.40 | 33 | 12 | 11 |
| 2 | 23.00 | 22.50 | 47 | 47 | 43 |
| 19 | 25.10 | 23.80 | 27 | 14 | 13 |
| 26 | 25.20 | 22.50 | 49 | 10 | 9 |
| 12 | 27.80 | 23.50 | 42 | 15 | 13 |
| 22 | 30.20 | 23.50 | 40 | 16 | 15 |
| 20 | 32.10 | 23.80 | 64 | 13 | 12 |
| 14 | 32.40 | 23.50 | 36 | 13 | 12 |
| 23 | 35.40 | 22.50 | 48 | 13 | 12 |

TABLE 2. Within-tree Regressions: Foliage Weight = $b_0 + b_1 dc + b_2 dc^2$

| TREE | DBH | p | COEFFICIENTS $b_0$ | $b_1$ | $b_2$ | t - VALUES $b_0$ | $b_1$ | $b_2$ |
|---|---|---|---|---|---|---|---|---|
| 11 | 0.90 | 2 | 0.13 | -- | 35.53 | 0.12 | -- | 2.3 |
| 8 | 1.20 | 2 | 0.47 | 3.22 | -- | 1.86 | 1.98 | -- |
| 10 | 1.45 | 2 | 0.56 | 6.04 | -- | 1.06 | 2.27 | -- |
| 7 | 1.80 | 3 | 0.35 | 3.54 | 8.78 | 0.38 | 0.50 | 0.8 |
| 6 | 2.00 | 2 | 1.26 | -- | 19.59 | 7.60 | -- | 7.3 |
| 9 | 2.20 | 2 | 4.06 | -- | 23.87 | 2.62 | -- | 1.0 |
| 31 | 3.40 | 2 | 1.55 | -- | 30.48 | 3.26 | -- | 7.8 |
| 32 | 3.40 | 2 | 0.93 | -- | 22.83 | 2.79 | -- | 9.6 |
| 30 | 3.50 | 2 | -- | 4.64 | 11.39 | -- | 1.88 | 2.8 |
| 17 | 7.30 | | -- | 37.41 | 14.03 | -- | 1.84 | 0.6 |
| 16 | 9.10 | 2 | -- | 28.79 | 36.97 | -- | 2.57 | 2.2 |
| 15 | 10.50 | 2 | -- | 33.79 | 49.06 | -- | 1.71 | 2.4 |
| 29 | 13.00 | 2 | -- | 49.69 | 54.58 | -- | 1.76 | 2.1 |
| 28 | 13.70 | 2 | -- | 29.31 | 91.48 | -- | 1.64 | 5.1 |
| 13 | 15.10 | 3 | 0.50 | 47.47 | 29.72 | 0.40 | 4.84 | 3.9 |
| 27 | 15.40 | 3 | 0.32 | 59.62 | 56.82 | 0.11 | 2.61 | 3.9 |
| 3 | 15.80 | 2 | -- | 31.64 | 34.17 | -- | 6.23 | 7.3 |
| 5 | 17.30 | 2 | -- | 32.18 | 22.59 | -- | 6.55 | 6.2 |
| 1 | 19.40 | 2 | -- | 43.97 | 28.60 | -- | 3.72 | 4.5 |
| 21 | 19.50 | 2 | -- | 36.23 | 80.81 | -- | 1.15 | 4.2 |
| 18 | 21.50 | 2 | -- | 62.01 | 33.31 | -- | 2.33 | 1.8 |
| 24 | 22.50 | 3 | 1.72 | 12.38 | 113.67 | 0.54 | 0.31 | 5.1 |
| 4 | 22.60 | 3 | 0.93 | 27.61 | 34.62 | 0.74 | 2.75 | 6.1 |
| 25 | 22.80 | 3 | 0.71 | 54.08 | 78.40 | 0.36 | 2.07 | 7.3 |
| 2 | 23.00 | 2 | -- | 26.29 | 37.82 | -- | 2.15 | 5.9 |
| 19 | 25.10 | 1 | -- | 140.56 | -- | -- | 9.10 | -- |
| 26 | 25.20 | 2 | -- | 50.61 | 20.55 | -- | 2.14 | 1.5 |
| 12 | 27.80 | 2 | -- | 28.81 | 42.86 | -- | 0.56 | 2.9 |
| 22 | 30.20 | 2 | 4.14 | -- | 89.18 | 0.74 | -- | 10.4 |
| 20 | 32.10 | 3 | 2.47 | 61.71 | 46.91 | 0.60 | 1.15 | 2.2 |
| 14 | 32.40 | 2 | -- | 16.16 | 71.00 | -- | 0.74 | 7.7 |
| 23 | 35.40 | 2 | -- | 77.40 | 82.64 | -- | 1.95 | 5.8 |

Table 3. Within-Tree Leaf Area Estimation: Error Analysis

| Tree | DBH | $\hat{\bar{Y}}_m$ | SE | PROP MSPE BRANCH SAMPLING | PROP MSPE RATIO EST |
|---|---|---|---|---|---|
| 11 | 0.90 | 0.43 | 0.05 | .000 | 1.000 |
| 8 | 1.20 | 0.18 | 0.03 | .449 | .551 |
| 10 | 1.45 | 0.37 | 0.05 | .460 | .540 |
| 7 | 1.80 | 0.91 | 0.17 | .793 | .207 |
| 6 | 2.00 | 0.85 | 0.10 | .204 | .796 |
| 9 | 2.20 | 1.12 | 0.22 | .737 | .263 |
| 31 | 3.40 | 3.19 | 0.42 | .435 | .565 |
| 32 | 3.40 | 2.03 | 0.15 | .225 | .775 |
| 30 | 3.50 | 1.41 | 0.11 | .435 | .565 |
| 17 | 7.30 | 10.41 | 1.88 | .875 | .125 |
| 16 | 9.10 | 8.31 | 1.15 | .583 | .417 |
| 15 | 10.50 | 14.32 | 1.47 | .412 | .588 |
| 29 | 13.00 | 11.01 | 1.28 | .772 | .228 |
| 28 | 13.70 | 10.97 | 1.23 | .270 | .731 |
| 13 | 15.10 | 8.79 | 0.82 | .252 | .748 |
| 27 | 15.40 | 13.94 | 1.00 | .448 | .552 |
| 3 | 15.80 | 19.39 | 1.55 | .000 | 1.000 |
| 5 | 17.30 | 21.44 | 1.61 | .000 | 1.000 |
| 1 | 19.40 | 31.44 | 2.24 | .000 | 1.000 |
| 21 | 19.50 | 17.46 | 1.53 | .260 | .740 |
| 18 | 21.50 | 18.38 | 2.24 | .888 | .112 |
| 24 | 22.50 | 49.93 | 5.53 | .487 | .513 |
| 4 | 22.60 | 28.71 | 2.06 | .000 | 1.000 |
| 25 | 22.80 | 41.60 | 3.92 | .212 | .788 |
| 2 | 23.00 | 38.67 | 2.49 | .000 | 1.000 |
| 19 | 25.10 | 27.20 | 2.85 | .463 | .538 |
| 26 | 25.20 | 23.71 | 4.86 | .818 | .182 |
| 12 | 27.80 | 72.29 | 7.95 | .668 | .333 |
| 22 | 30.20 | 74.20 | 8.35 | .352 | .648 |
| 20 | 32.10 | 52.49 | 8.01 | .795 | .205 |
| 14 | 32.40 | 102.01 | 10.75 | .443 | .557 |
| 23 | 35.40 | 120.80 | 13.29 | .416 | .584 |

TABLE 4. Plot-Level LAI Estimates and Error Analysis

| PLOT | NO. OF TREES | $\hat{f}$ | $\hat{LAI}$ | SE | CV | SAMP VAR | COFF EST VAR | MODEL VAR | RA |
|---|---|---|---|---|---|---|---|---|---|
| 99 | 319 | 0.0889 | 1.293 | 0.193 | 0.1493 | 0.0135 | 0.0218 | 0.0020 | 0.6 |
| 86 | 301 | 0.0678 | 1.593 | 0.315 | 0.1977 | 0.0626 | 0.0331 | 0.0032 | 1.8 |
| 94 | 458 | 0.0889 | 1.642 | 0.227 | 0.1382 | 0.0140 | 0.0352 | 0.0024 | 0.3 |
| 36 | 236 | 0.3556 | 2.047 | 0.225 | 0.1099 | 0.0321 | 0.0157 | 0.0030 | 2.0 |
| 95 | 569 | 0.0889 | 2.058 | 0.439 | 0.2133 | 0.1344 | 0.0553 | 0.0031 | 2.4 |
| 83 | 175 | 0.3556 | 2.323 | 0.211 | 0.0908 | 0.0204 | 0.0206 | 0.0034 | 0.9 |
| 16 | 100 | 0.3556 | 2.413 | 0.223 | 0.0924 | 0.0269 | 0.0190 | 0.0038 | 1.4 |
| 20 | 231 | 0.3556 | 2.416 | 0.429 | 0.1776 | 0.1507 | 0.0304 | 0.0031 | 4.9 |
| 89 | 297 | 0.0756 | 2.429 | 0.598 | 0.2462 | 0.2750 | 0.0769 | 0.0058 | 3.5 |
| 80 | 369 | 0.3556 | 2.451 | 0.240 | 0.0979 | 0.0337 | 0.0202 | 0.0037 | 1.6 |
| 82 | 202 | 0.3556 | 2.496 | 0.255 | 0.1022 | 0.0281 | 0.0336 | 0.0033 | 0.8 |
| 3 | 95 | 0.3556 | 2.507 | 0.244 | 0.0973 | 0.0362 | 0.0193 | 0.0041 | 1.8 |
| 93 | 86 | 0.3556 | 2.536 | 0.246 | 0.0970 | 0.0349 | 0.0215 | 0.0039 | 1.6 |
| 87 | 194 | 0.0656 | 2.591 | 0.431 | 0.1663 | 0.0907 | 0.0872 | 0.0080 | 1.0 |
| 84 | 644 | 0.0733 | 2.634 | 0.403 | 0.1530 | 0.0669 | 0.0906 | 0.0048 | 0.7 |
| 69 | 377 | 0.0889 | 2.789 | 0.406 | 0.1456 | 0.0575 | 0.1013 | 0.0059 | 0.5 |
| 81 | 145 | 0.3556 | 2.815 | 0.212 | 0.0753 | 0.0135 | 0.0271 | 0.0043 | 0.4 |
| 85 | 230 | 0.3556 | 2.815 | 0.294 | 0.1044 | 0.0137 | 0.0699 | 0.0031 | 0.1 |
| 71 | 498 | 0.0889 | 2.835 | 0.354 | 0.1249 | 0.0148 | 0.1048 | 0.0054 | 0.1 |
| 75 | 161 | 0.3556 | 2.911 | 0.288 | 0.0989 | 0.0200 | 0.0592 | 0.0036 | 0.3 |
| 98 | 262 | 0.3556 | 2.924 | 0.370 | 0.1265 | 0.1041 | 0.0281 | 0.0045 | 3.70 |
| 72 | 167 | 0.3556 | 2.933 | 0.297 | 0.1013 | 0.0243 | 0.0601 | 0.0036 | 0.40 |
| 96 | 173 | 0.3556 | 3.033 | 0.253 | 0.0834 | 0.0146 | 0.0454 | 0.0041 | 0.32 |
| 73 | 107 | 0.3556 | 3.034 | 0.195 | 0.0643 | 0.0035 | 0.0299 | 0.0048 | 0.11 |
| 92 | 108 | 0.3556 | 3.071 | 0.236 | 0.0768 | 0.0150 | 0.0361 | 0.0045 | 0.41 |
| 21 | 144 | 0.3556 | 3.082 | 0.799 | 0.2592 | 0.5574 | 0.0724 | 0.0090 | 7.69 |
| 74 | 142 | 0.3556 | 3.134 | 0.238 | 0.0759 | 0.0088 | 0.0437 | 0.0044 | 0.20 |
| 90 | 186 | 0.3556 | 3.167 | 0.248 | 0.0783 | 0.0153 | 0.0416 | 0.0046 | 0.36 |
| 97 | 167 | 0.3556 | 3.264 | 0.305 | 0.0934 | 0.0247 | 0.0642 | 0.0042 | 0.38 |
| 77 | 105 | 0.3556 | 3.322 | 0.263 | 0.0792 | 0.0297 | 0.0339 | 0.0055 | 0.87 |
| 88 | 370 | 0.0811 | 3.378 | 0.533 | 0.1578 | 0.1279 | 0.1482 | 0.0083 | 0.86 |
| 79 | 283 | 0.3556 | 3.945 | 0.291 | 0.0738 | 0.0298 | 0.0480 | 0.0067 | 0.62 |

* "RATIO" = (SAMP VAR)/(COEFF EST VAR)

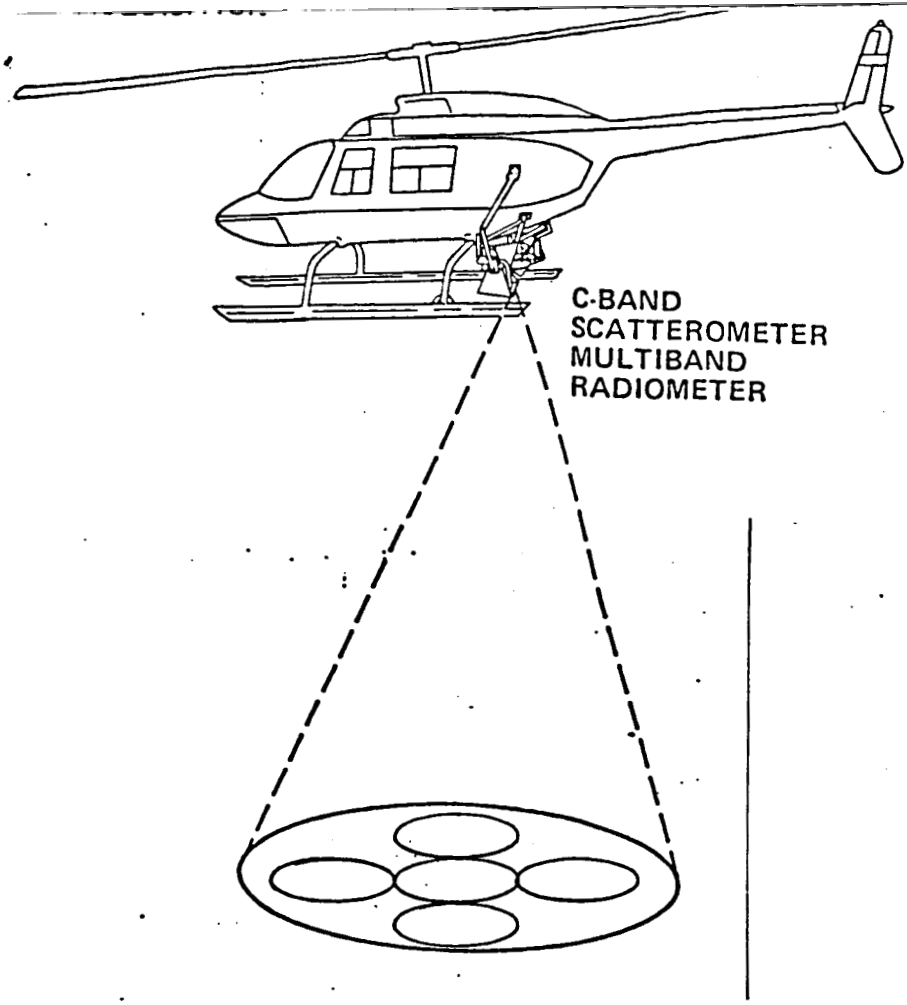


Figure 1: Plot Spectral Measurements

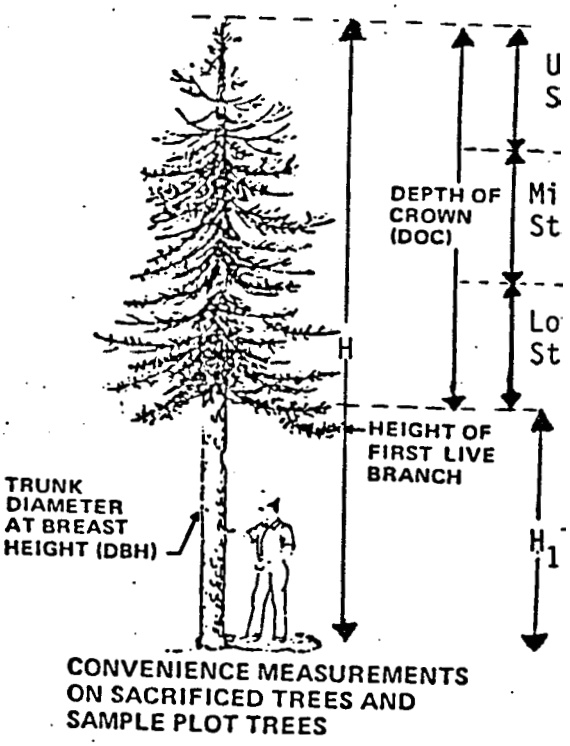


Figure 2: Tree Dimensions

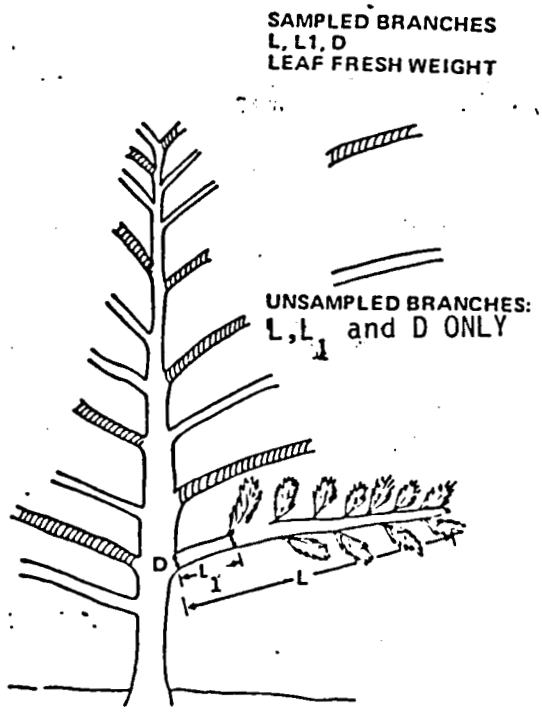


Figure 3: Branch Dimensions

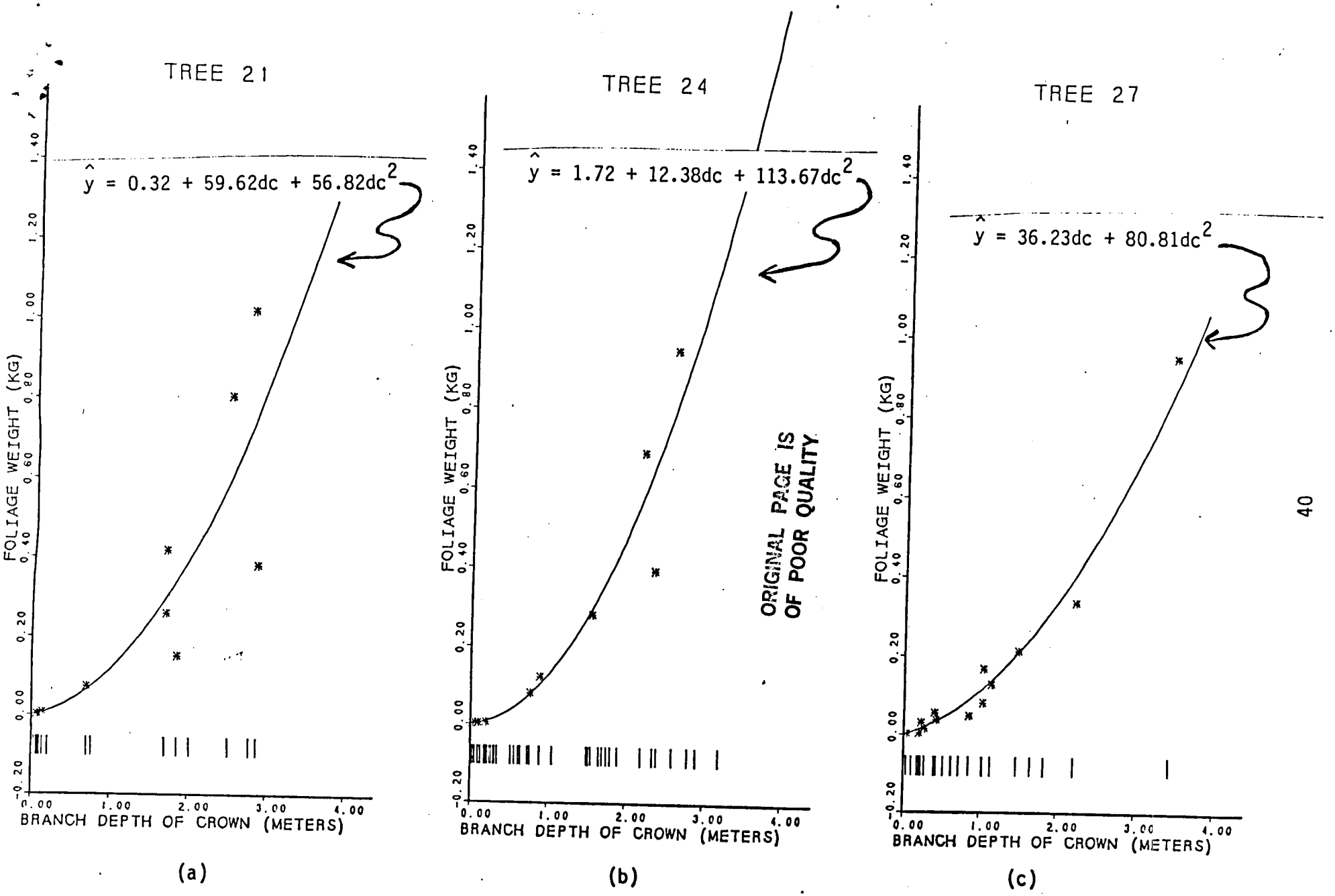






Figure 4: Branch foliage weight as a function of branch depth of crown

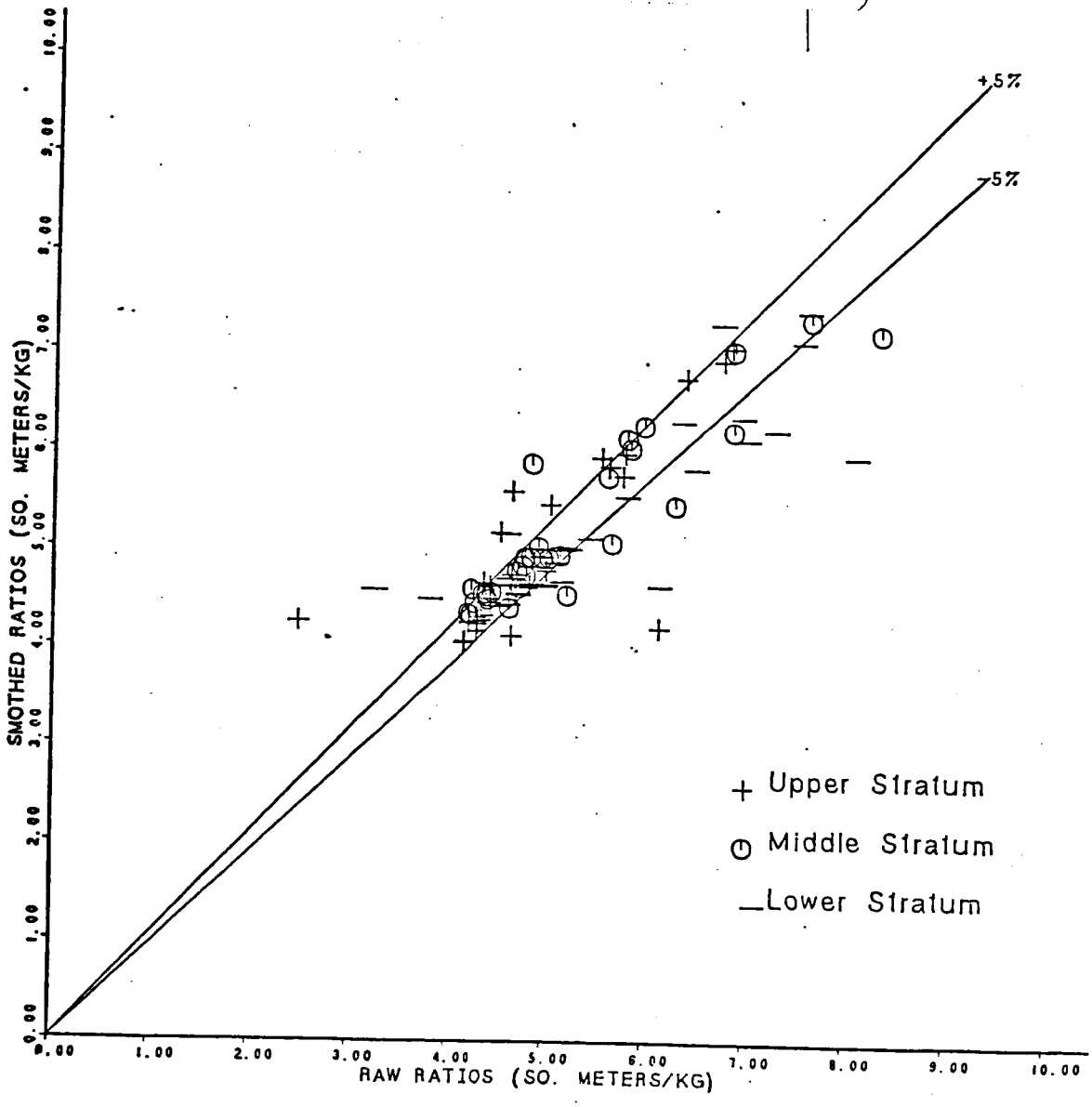


Figure 5: Comparison of smoothed to raw ratios: Leaf area to leaf weight

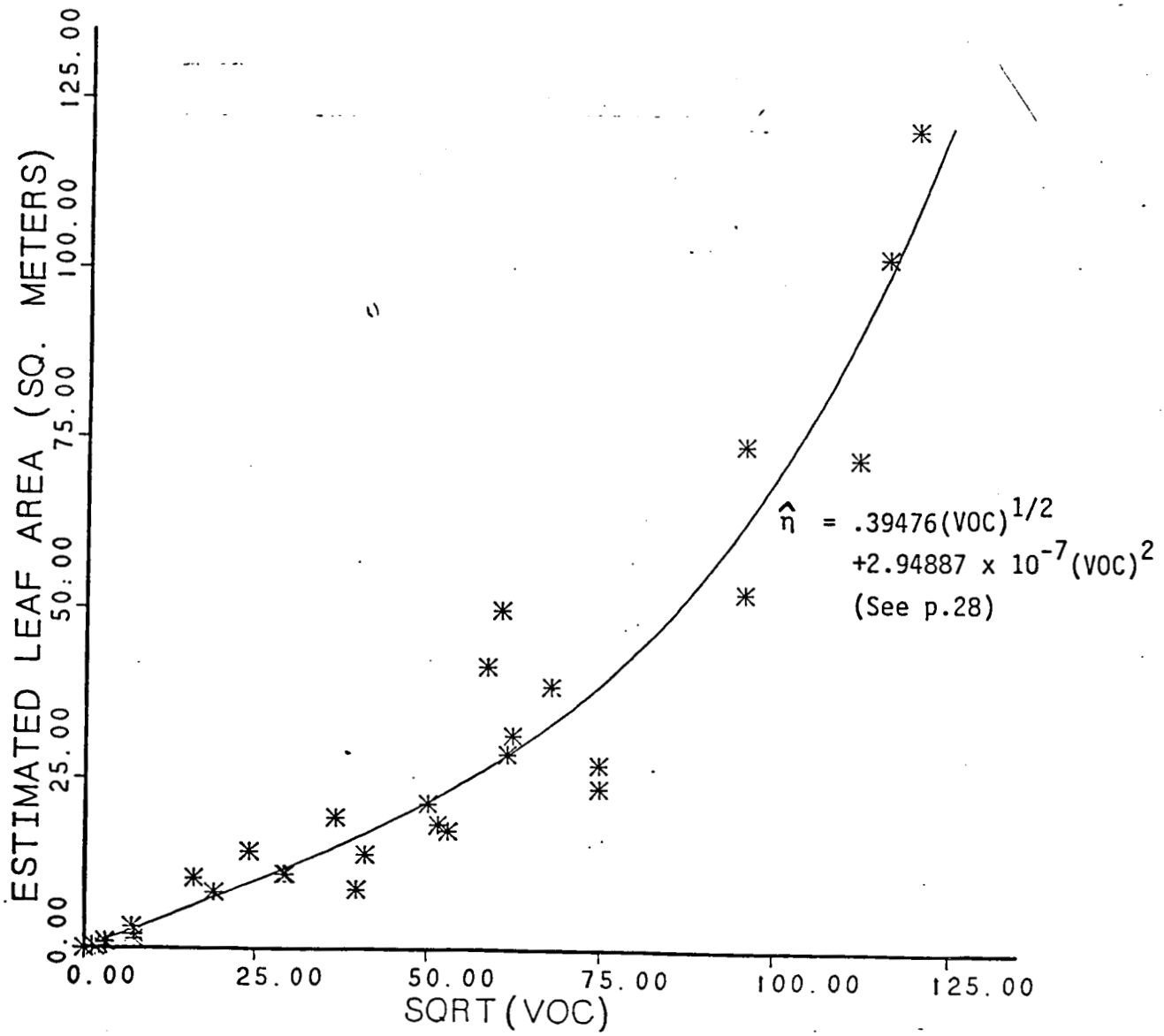


Figure 6: Dimension analysis model