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COMPARISON OF TWO NUMERICAL TECHNIQUES FOR AERODYNAMIC MODEL IDENTIFICATION

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Abstract

A new algorithm, called the Minimal Residual QR algorithm, is presented to solve subset regression problems. It is shown that this new scheme can be used as a numerically reliable implementation of the stepwise regression technique, which is widely used to identify an aerodynamic model from flight test data. This capability as well as the numerical superiority of this scheme over the stepwise regression technique is demonstrated in an experimental simulation study.

1. Introduction

This paper describes a study of the so-called subset regression problem (SRP). This problem occurs in the identification of a mathematical model representing the aerodynamic forces and moments which act on an aircraft as a function of 1) aircraft state quantities, such as angle of attack and Mach number and 2) aircraft input quantities, such as control surface deflections. This mathematical model is referred to as the aircraft's aerodynamic model. In Refs. 1-3, precise conditions have been stated which allow formulation of this identification problem as an SRP.

In this paper, we focus on the numerical techniques used to solve SRP. Generally, techniques for solving SRP are divided into two different classes:

1) A first class is formulated in a complete statistical framework. The techniques in this class are generally referred to as subset regression methods and are widely used by practicing engineers and econometrists.[4]

2) A second class has its basis mainly in numerical analysis. Currently the most widely used technique from this class is the Singular Value Decomposition (SVD) method.[5]

The SVD has clearly demonstrated its numerical superiority over the techniques from the first class.[6] However, it does not do the subset selection from the originally defined model parameters. This is a major drawback for the aerodynamic model identification problem, since the original model parameters have a physical interpretation.

In order to overcome this drawback, a new algorithm has been developed. This new technique is called the Minimal Residual QR (MRQR) algorithm. Although the MRQR scheme is derived from a numerical analysis point of view and therefore belongs in the second class mentioned above, it will be shown that it combines the advantages of techniques from both classes. On the one hand, it retains the precision of the original data such as the SVD and on the other hand can be used in a complete similar way as the stepwise regression technique (SRT), since it produces quantities such as sequential F-tests, etc.

The outline of the paper is as follows. In Section 2, the new scheme will be described. Its relationship with the existing SRT is indicated in Section 3, and Section 4 presents the result of a comparison study between the MRQR and the SRT. Finally, Section 5 presents the conclusions of this research.

2. The Minimal Residual QR Algorithm

This new scheme is originally proposed in Ref. 7. In this section, we review this scheme to reveal the relationship with the SRT.

The MRQR algorithm performs a QR factorization with column pivoting of the system matrix A in the considered SRP. If this SRP is denoted as:

$$\min_x \|Ax - b\|_2 \tag{1}$$

with $\|\cdot\|_2$ representing the Euclidean norm, and

$$A \in R^{m\times n}(m \geq n) \,, \quad b \in R^m \,, \quad x \in R^n$$

and

$$\text{rank}(A) = k \leq n$$

then we can write the result of MRQR as

$$\min_x \|Q^T A\pi(\pi^T x) - Q^T b\|_2 \tag{2}$$

where Q is an orthogonal transformation matrix, i.e., $Q^T Q = I$ and π is the column permutation matrix. The operation indicated by Eq. (2) results in

$$\min_x \left\| \begin{bmatrix} R_{11} & R_{12} \\ 0 & R_{22} \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \end{bmatrix} - \begin{bmatrix} b_1 \\ \epsilon \end{bmatrix} \right\|_2 \tag{3}$$

where R_{11} is $k\times k$ and upper triangular, R_{12} is $k\times(n-k)$ and $b_1 \in R^k$. From the decomposition of Eq. (3), the solution to Eq. (1), which now is no longer a minimal norm solution, becomes

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$$\hat{x}_1 = R_{11}^{-1} b_1 \qquad (4)$$

and the 2-norm of the residual is given by $\|\epsilon\|_2$.

The crucial part in this computational scheme [Eqs. (2-4)] is the generation of the column permutation matrix π. The way this is done in the MRQR scheme is now illustrated for the simple two-dimensional case.

2.1 The Two-Dimensional Case

In this case, the data of problem (1), i.e., the two-column vectors of A and the right hand side (rhs) b, are graphically represented in Fig. 1.

Just as with the SRT, the MRQR algorithm selects one column vector of A at a time. The measure initially proposed in Ref. 7 for this selection is "the distance of the rhs to the column vectors of A."

In Fig. 1, these distances are denoted by m_ℓ^1, where the superscript indicates that we are selecting the first column and the subscript indicates the corresponding column position in the A-matrix. The column that will be selected by the MRQR algorithm is the one "closest" to the rhs. This corresponds to finding the minimum of the sequence $\{\|m_\ell^1\|\}$, which for the case of Fig. 1 results in the selection of a_1^1. This column is then permuted to the first column position of A. In the next step, the same selection procedure is repeated for the components of the remaining columns of A and the component of the rhs that is orthogonal to a_1^1. These components are represented in Fig. 1 by a_2^2 and b^2, respectively. The selection in the orthogonal complement of a_1^1 is obvious now, since $\|m_2^2\| = 0$.

The quantities $\|m_\ell^i\|$ will be referred to as "residuals" because each $\|m_\ell^i\|$ is the residual of the least squares problem

$$\min_\xi \|a_\ell^i \xi - b^i\|_2$$

(Ref. 7). The selection of the minimum of that sequence $\{\|m_\ell^i\|\}$ gave rise to the algorithmic name, Minimal Residual QR algorithm.

2.2 Generalization of the Two-Dimensional Case

The generalization of the two-dimensional case, given in pseudo-programming-language form, is summarized in the following algorithm.

Algorithm 1:

Define:

$$A = A^1 = [a_1^1 \ldots a_j^1 \ldots a_n^1] \text{ and } b^1 = b \qquad \text{rank} = n \qquad (5)$$

DO i = 1:n,

1) Select column vector of A^i closest to b^i, based on the computed sequence of residuals $\{\|m_\ell^i\|\}$. This column is called a_j^i.

2) Interchange the jth and ith column vector of A^i by the column permutation matrix π_i.

3) Perform an orthogonal projection Q_i, such that

$$Q_i^T \left[\begin{array}{c|c} R_{11}^{(i-1)} & R_{12}^{(i-1)x(n-i+1)} \\ \hline 0 & a_i^i \ldots a_j^i \ldots a_n^i \end{array}\right] \pi_i = \left[\begin{array}{c|c} R_{11}^{(i)} & R_{12}^{(i)x(n-i)} \\ \hline 0 & a_{i+1}^{i+1} \ldots a_n^{i+1} \end{array}\right] = \left[\begin{array}{c|c} R_{11}^{(i)} & R_{12}^{(i)x(n-i)} \\ \hline 0 & A^{i+1} \end{array}\right] \qquad (6)$$

and

$$Q_i b^i = \begin{bmatrix} * \\ \hline b^{i+1} \end{bmatrix} \}i$$

where $R_{11}^{(i)}$ is an ixi upper-triangular matrix and $R_{12}^{(i)x(j)}$ is an ixj rectangular matrix.

4) Rank determination test.

END

The rank determination in the fourth step of the do-loop, i.e., rejecting one of the columns of A, can be done in various ways. In this paper, this determination will be based on the so-called partial F-test quantities, such as those used in SRT.[4] This is outlined in section 3.

2.3 Implementation Note on the MRQR Algorithm

An efficient way to calculate the residuals $\|m_\ell^i\|$ and to perform the orthogonal transformations Q_i in Eq. (6) is described in Ref. 7. Here we initially transform problem (1) to:

$$\min_x \|Q_h^T A x - Q_h^T b\|_2 = \min_x \left\| \begin{bmatrix} R \\ \hline 0 \end{bmatrix} x - \begin{bmatrix} b^1 \\ \hline 0 \end{bmatrix} \right\|_2 \qquad (7)$$

where R is an nxn upper-triangular matrix and $b^1 \in R^{n+1}$.

This compression of the data can be done sequentially, such as with the construction of the "normal" equations (see section 3). This sequential ability also makes this scheme attractive from the data storage point of view.

Next, Algorithm 1 is applied to the right-hand side of Eq. (7). A combination of these intermediate steps would then result in the following reformulation of problem (1).

$$\min_x \| [Q_k^T \cdots Q_1^T] Q_h^T A [\pi_1 \cdots \pi_k]([\pi_1 \cdots \pi_k]^T x) - [Q_k^T \cdots Q_1^T] Q_h^T b \|_2 \quad (8)$$

where $[Q_k^T \cdots Q_1^T]$ will be denoted as Q_g and $[\pi_1 \cdots \pi_k]$ as π.

Note: The robustness properties of the MRQR algorithm rely on the use of orthogonal transformations, denoted in Eq. (8) by Q_h, Q_g, and π, respectively. These do not modify the error pattern present on the original data.[6]

3. Relationship of the MRQR Algorithm with Stepwise Regression

The classical stepwise regression technique (SRT) solves problem (1) via the so-called normal equations.

$$[A^T A]x = A^T b \quad (9)$$

From this set of equations the partial F-test quantities, the partial correlation coefficients, and other quantities are computed.[4,8] These latter quantities form the crucial information source in adding (or subtracting) a new variable x_i in the regression model. The partial F-test quantities are defined next.

Definition 1. When in the construction of the regression model, we add parameter x_ℓ to the model already containing i-1 parameters, the F-test value for that parameter x_ℓ is:

$$F_{x_\ell} = \frac{\epsilon_{i-1}^T \epsilon_{i-1} - \epsilon_i^T \epsilon_i}{\epsilon_i^T \epsilon_i} (m - i) \quad (10)$$

where ϵ_{i-1}, ϵ_i are the residuals of regression models containing i-1 and i parameters, respectively.

The quantity defined in Eq. (10) can also be derived from the MRQR algorithm. In order to clarify this, let us focus at the ith stage of the do-loop of algorithm 1. The partial F-test value for each parameter x_ℓ not yet in the model becomes:

$$F_{x_\ell} = \frac{\|b^i\|_2^2 - \|m_\ell^i\|_2^2}{\|m_\ell^i\|_2^2} (m - i) \quad (11)$$

The difference of the two norms in Eq. (11) does not have to be computed explicitly; it becomes available during the computation of $\|m_\ell^i\|_2$. If we denote this difference by $\|d_\ell^i\|_2$, then Eq. (11) becomes

$$F_{x_\ell} = \frac{\|d_\ell^i\|_2^2}{\|m_\ell^i\|_2^2} (m - i) \quad (12)$$

The F-test quantities now allow us to make a decision about the statistical significance of adding parameter x_ℓ to the model. Statistical significance corresponds to the F-test values being above a threshold F_α, which is taken from statistical tables about the F-distribution (e.g., Ref. 4). From Eq. (12), we clearly see that we can impose this test directly on the elements of the derived residual sequence $\{\|m_\ell^i\|_2\}$ in the MRQR algorithm. Based on the desired threshold F_α, such a test would become

$$\|m_\ell^i\|_2 \le \frac{\|d_\ell^i\|_2 \sqrt{(m - i)}}{\sqrt{F_\alpha}} \quad (13)$$

This inequality precisely reveals the relationship between the MRQR and SRT. It indicates that the same parameter x_ℓ would be added to the model whether based on the $\{\|m_\ell^i\|_2\}$ sequence or the $\{F_{x_\ell}\}$ sequence.

This section summarizes the use of the MRQR algorithm as a robust implementation of the classical stepwise regression scheme. Furthermore, as demonstrated in Ref. 7, the MRQR algorithm gives rise to additional parameters, such as smallest singular values, that may be helpful in constructing the regression model. In this paper, such an additional feature is presented in section 4.2.

4. Experimental Evaluation

4.1 Demonstration of the Equivalence between MRQR and SRT

In this first example, the data taken from Ref. 9 (p. 647) and analyzed in Ref. 4 via the classical SRT are analyzed by the proposed MRQR algorithm. These data, summarized in Table 1, comprise four candidate solutions (i.e., the columns of the A matrix), and a right-hand side (the b-vector), from which 13 samples have been recorded. Hence, m = 13 and n = 4. In this section the candidate solutions will be referred to by their corresponding component of the x-vector, defined in Eq. (1). The results of the MRQR algorithm are summarized in Table 2. Comparison of the partial F-test values derived from MRQR, denoted by F_1, with those derived from SRT, denoted by F_2, clearly demonstrates the equivalence of both schemes. Furthermore, we also observe that using the $\{\|m_\ell^i\|_2\}$ sequence results in the same model as using the partial F-test values.

4.2 An Additional Feature of the MRQR

In analyzing real data, visual inspection of time-history plots of the available data is often used. Such plots of the candidate solutions and the rhs might allow (or influence) the decision of which parameter to include in the regression model or might help to judge the validity of the extracted model. If the same graphical information for all subsequent decision stages is desired, it is generally necessary to store the original A-matrix and explicitly compute the residuals of the columns not yet in the model.

In the ith decision stage, the matrix A is partitioned as

$$A = [A^{i-1} \mid A^{n-i+1}] \tag{14}$$

where A^{i-1} designates the already-selected columns, then the residuals of the columns in A^{n-i+1} become $[A^{i-1}x_{i-1} - A^{n-i+1}]$. These residuals can also be computed from the data available from the MRQR algorithm <u>without explicitly storing</u> the A-matrix. Using Eqs. (6-8) they become

$$Q_h[Q_1 \; \dots \; Q_{i-1}] \left| \begin{array}{c} 0 \;\; \dots \;\; 0 \;\; \dots \;\; 0 \\ \hline a_i^i \;\; \dots \;\; a_j^i \;\; \dots \;\; a_n^i \end{array} \right| \tag{15}$$

Here only $[Q_1 \; \dots \; Q_{i-1}]$ is stored explicitly, whereas the information to construct Q_h can be stored in the lower triangular part of $[R|0]^T$ in Eq. (7).[10] In this way, use of Eq. (15) becomes a very reliable and storage-efficient way to evaluate residuals in the regression analysis.

This feature is demonstrated using the following example taken from Ref. 11 (p. ID-4.7). Here the original plant was generated by the following static model.

$$y(t) = 1.2 \sin(t) + 2 \sin(.7t) + 3 \cos(t)$$

$$- 0.1[\sin(2t) + \delta] - 2 \sin(.99t) + e \tag{16}$$

where δ and e are zero-mean white noise sequences with standard deviations of 0.001 and 0.1, respectively.

In the regression analysis, the following model was postulated.

$$b(t) = x_1 \sin(t) + x_2 \sin(.7t) + x_3 \cos(t)$$

$$+ x_4 \cos(3t) \tag{17}$$

The four candidate solutions {sin(t), sin(.7t), cos(t), cos(3t)} and y(t) of Eq. (16) are shown in Fig. 2. Clearly, from this figure candidate solution x_3 appears most closely related to the rhs, as was also demonstrated by the F-test values or the $(\|m_\ell^i\|_2)$ residuals. After this selection, the remaining parts of the other candidate solutions as well as the remaining part of the rhs are shown in Fig. 3.

Next the candidate solution x_2 was selected. Again, Fig. 4 displays the residuals of the remaining columns and the rhs. From this figure the "high" correlation with x_1 becomes clear. The residuals of the remaining signals, i.e., rhs and x_4, are displayed in Fig. 4.

From this analysis, we clearly observe that the residuals of each of the candidate solutions after each selection are nearly the same as the candidate solutions before the selections. This is a clear indication of "orthogonality" of the candidate solutions, which is also attributed to the value of the condition number of the matrix containing the original four candidate solutions. The latter is ≈2, which clearly is close to 1.

This example demonstrates one feature of this "residual" analysis, but practical experience might provide additional uses.

4.3 <u>Numerical Superiority of the MRQR</u>

In the previous two examples, the condition number of the original A-matrix, as defined in Eq. (1), was very close to 1. For these cases, the F-tests computed from the MRQR algorithm or the SRT are completely identical. For cases where the condition number is larger, this numerical equivalence may be lost, as is demonstrated by the following least-squares problem:

$$\min_x \left\| \begin{bmatrix} 1 & 1 & 1 \\ \varepsilon & 0 & 0 \\ 0 & \varepsilon & 0 \\ 0 & 0 & \varepsilon \end{bmatrix} x - \begin{bmatrix} 3 \\ \varepsilon \\ \varepsilon \\ \varepsilon \end{bmatrix} \right\|_2 \tag{18}$$

where ε is a very small number. The normal equations are

$$\begin{bmatrix} 1+\varepsilon^2 & 1 & 1 \\ & 1+\varepsilon^2 & 1 \\ 1 & 1 & 1+\varepsilon^2 \end{bmatrix} x = \begin{bmatrix} 3+\varepsilon^2 \\ 3+\varepsilon^2 \\ 3+\varepsilon^2 \end{bmatrix} \tag{19}$$

When ε is taken, for example, equal to 10^{-8}, with a machine precision of 10^{-14}, the matrix A^TA in Eq. (19) becomes singular and clearly destroys the accuracy of the F-test values. With the use of the MRQR algorithm and the same machine precision of 10^{-14}, this does not occur.

5. <u>Conclusion</u>

A new technique to solve subset regression problems has been presented. The technique is called the minimal residual QR algorithm (MRQR). Basically, it performs a QR factorization with column pivoting. It has been shown analytically that the MRQR algorithm can be used as a numerically stable implementation of existing stepwise regression techniques. The numerical stability is demonstrated in an experimental evaluation study.

From this technique, a reliable solution of subset regression problems has been derived that allows the use of statistical parameters commonly used in classical solutions, such as the stepwise regression technique. This new technique should allow more "accurate" models to be constructed, especially when a high correlation exists between candidate solutions in the model.

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Table 1 Data taken from Ref. 9 (p. 647) in example 1.

x_1	x_2	x_3	x_4	RHS
7.0	26.0	6.0	60.0	78.5
1.0	29.0	15.0	52.0	74.3
11.0	56.0	8.0	20.0	104.3
11.0	31.0	8.0	47.0	87.6
7.0	52.0	6.0	33.0	95.9
11.0	55.0	9.0	22.0	109.2
3.0	71.0	17.0	6.0	102.7
1.0	31.0	22.0	44.0	72.5
2.0	54.0	18.0	22.0	93.1
21.0	47.0	4.0	26.0	115.9
1.0	40.0	23.0	34.0	83.8
11.0	66.0	9.0	12.0	113.3
10.0	68.0	8.0	12.0	109.4

Table 2 (Retyped) computer output of MRQR on the data of Ref. 9 (p. 647).

A. SELECTING THE FIRST COLUMN FOR ENTRY

SEL. #	CAND. SOL.	$\|m^1_{SEL.\#}\|$	F_1	F_2
1	x_1	35.5764914	12.6025177	12.6025150
2	x_2	30.1054204	21.9606047	21.9606160
3	x_3	44.3862470	4.4034168	4.4034159
4	x_4	29.7298993	22.7985202	22.7985270

WHICH SEL # [ENTER 0 TO QUIT SELECTION]: 3

DELETING A COLUMN FROM THE FIRST 1 SELECTED

SEL. #	CAND. SOL.	F_1	F_2
1	x_4	22.7985202	22.7985270

WHICH SEL # [ENTER 0 TO SKIP]: 0

B. SELECTING THE SECOND COLUMN FOR ENTRY

SEL. #	CAND. SOL.	$\|m^2_{SEL.\#}\|$	F_1	F_2
1	x_1	8.6465085	108.2239145	108.2238900
2	x_2	29.4767726	.1724839	.1724847
3	x_3	13.2566210	40.2945810	40.2945430

WHICH SEL # [ENTER 0 TO QUIT SELECTION]: 1

DELETING A COLUMN FROM THE FIRST 2 SELECTED

SEL. #	CAND. SOL.	F_1	F_2
1	x_4	159.2952101	159.2952400
2	x_1	108.2239145	108.2238900

WHICH SEL # [ENTER 0 TO SKIP]: 0

C. SELECTING THE THIRD COLUMN FOR ENTRY

SEL. #	CAND. SOL.	$\|m^3_{SEL.\#}\|$	F_1	F_2
1	x_2	6.9262346	5.0258650	5.0258974
2	x_3	7.1299448	4.2358460	4.2358519

WHICH SEL # [ENTER 0 TO QUIT SELECTION]: 1

DELETING A COLUMN FROM THE FIRST 3 SELECTED

SEL. #	CAND. SOL.	F_1	F_2
1	x_4	1.8632624	1.8632548
2	x_1	154.0076353	154.0080400
3	x_2	5.0258646	5.0258974

WHICH SEL # [ENTER 0 TO SKIP]: 0

D. SELECTING THE FOURTH COLUMN FOR ENTRY

SEL. #	CAND. SOL.	$\|m^4_{SEL.\#}\|$	F_1	F_2
1	x_3	6.9183549	.0182335	.0182345

WHICH SEL # [ENTER 0 TO QUIT SELECTION]: 0

SUMMARY

RANK ESTIMATE: 3

PIVOT INFORMATION:

ORIGINAL COLUMN 4 PIVOTED TO COLUMN 1
ORIGINAL COLUMN 1 PIVOTED TO COLUMN 2
ORIGINAL COLUMN 2 PIVOTED TO COLUMN 3
ORIGINAL COLUMN 3 DELETED

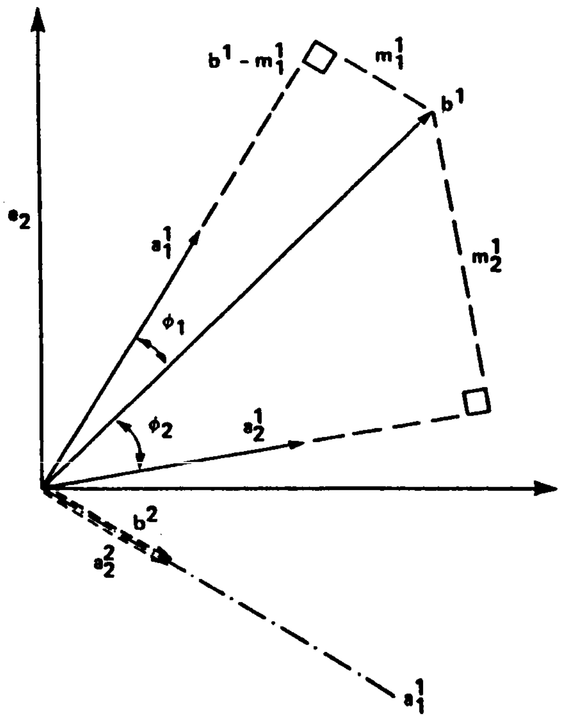

Fig. 1 The MRQR algorithm for the two-dimensional case (m = 2, n = 2).

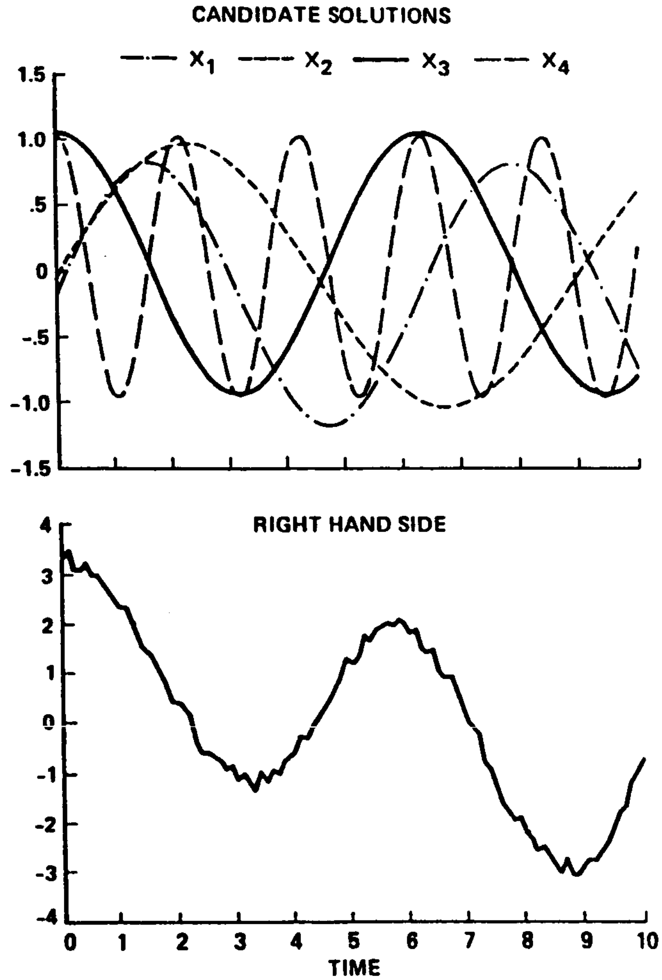

Fig. 2 Time-history plots of the candidate solutions and right-hand side before first selection in the MRQR.

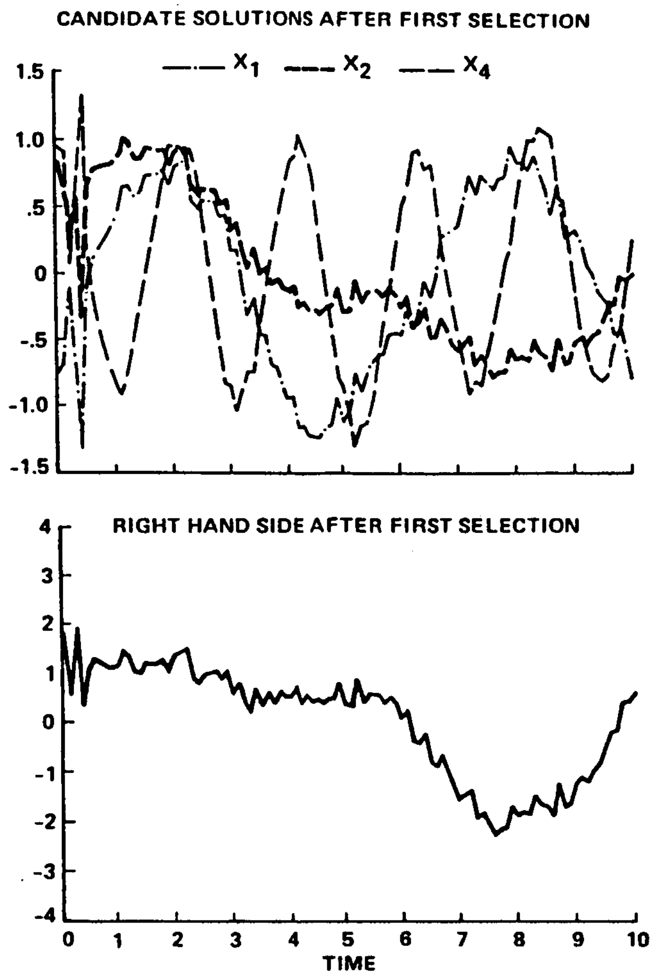

Fig. 3 Time-history plots of the candidate solutions and right-hand side after first selection ir the MRQR.

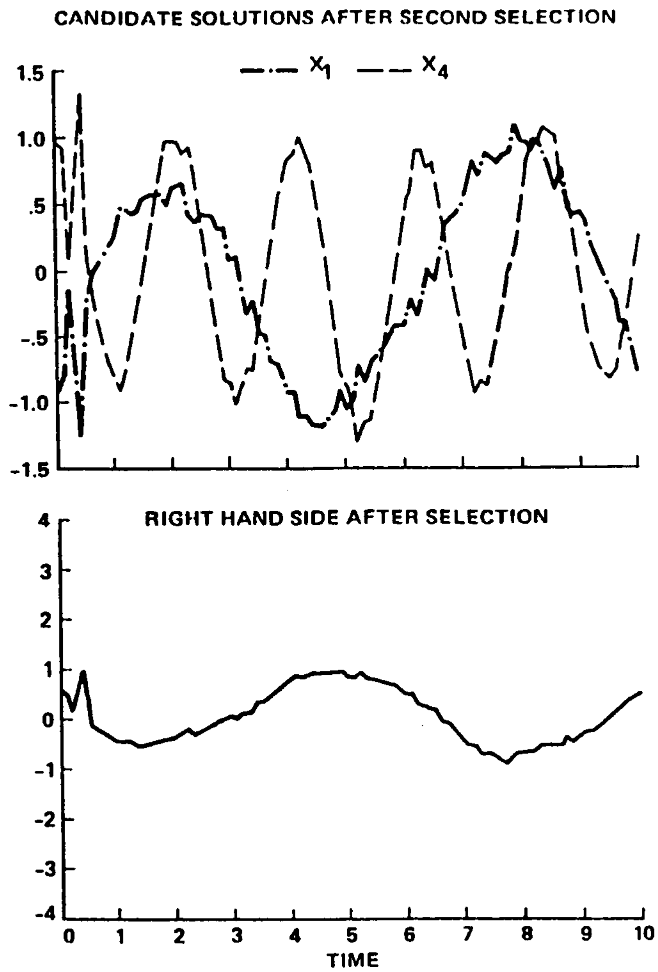

Fig. 4 Time-history plots of the candidate solutions and right-hand side after second selection in the MRQR.

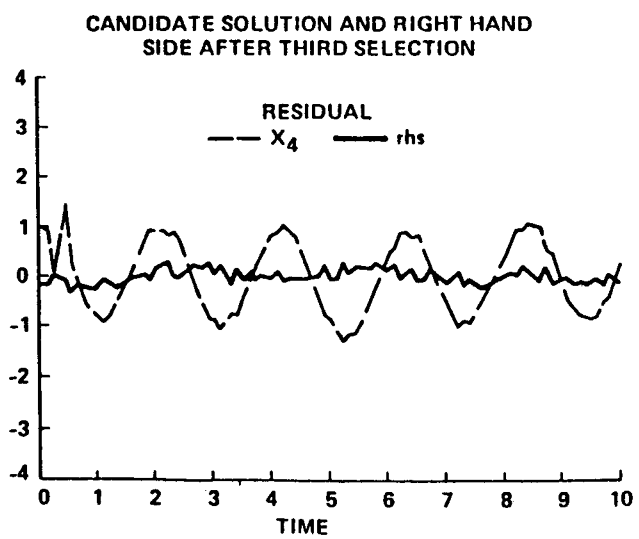

Fig. 5 Time-history plots of the candidate solutions and right-hand side after third selection in the MRQR.

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16. Abstract

A new algorithm, called the Minimal Residual QR algorithm, is presented to solve subset regression problems. It is shown that this new scheme can be used as a numerically reliable implementation of the stepwise regression technique, which is widely used to identify an aerodynamic model from flight test data. This capability as well as the numerical superiority of this scheme over the stepwise regression technique is demonstrated in an experimental simulation study.

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