JAWS MULTIPLE DOPPLER DERIVED WINDS

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INTRODUCTION

This paper is designed to give an elementary working knowledge of the advantages and limitations of the multiple Doppler radar analyses that have recently become available from the Joint Airport Weather Studies (JAWS) Project. The emphasis is specifically directed towards engineers and other technical specialists working in aviation-related systems rather than research institutes. The paper addresses what Doppler radar is and what it does and describes the way Doppler radars were used in the JAWS Project to gather wind shear data. The working definition of wind shear used here is "winds that affect aircraft flight over a span of 15-45 seconds," whereas turbulence is defined as "air motions that cause abrupt (several seconds or less) aircraft motions." The JAWS data currently available contain no turbulence data.

The concept of multiple Doppler analysis and the geometry of how it works are described, followed by an explanation of how data gathered in radar space are interpolated to a common Cartesian coordinate system and the limitations involved. This section includes a discussion of the analysis grid and how it was constructed. What the user actually gets (quasi-horizontal wind components) is discussed, followed by a discussion of the expected errors in the three orthogonal wind components. The paper concludes with a discussion of why JAWS data are significant.

Although this paper is not intended to be an exhaustive treatment of Doppler radar technology and techniques, it will focus, in a very basic way, on the concepts needed to understand what JAWS can and cannot provide in the area of observed wind shear data.

DOPPLER RADAR: WHAT IS IT?

Like a Doppler radar, a standard, or incoherent, weather radar transmits a very short (about a microsecond long) pulse of electromagnetic energy and then listens for a relatively long period (roughly a millisecond) for any echoes. By carefully timing how long it takes to receive any echoes, the range to the echo can be determined very precisely. The direction of the echo from the radar is established by where the antenna is pointing when the echo is received as the antenna doesn't move a significant amount from the time of transmission to the time of echo reception. Some idea of the size and number of echo-producers--targets--can be determined if something is known about their physical makeup, e.g., liquid water, wet ice (as in hail), snow, or, in the case of non-weather radar, metal. For rain, the stronger the signal returned, the larger the raindrops.

A lot can be learned about targets using an incoherent radar, but target motion relative to the radar cannot be obtained directly. If it's a large single-point target, like an airplane, we can determine which way it's going and how fast after a minimum of only two scans by the radar--we can watch it move. But in the case of meteorological echoes, which are made of thousands of targets far smaller than the radar can resolve individually, we cannot know whether the particles are moving relative to the storm they make up. We can only know whether the entire storm is moving.

A Doppler, or coherent, radar does exactly what an incoherent radar does plus one other function: it measures how fast a target is moving toward or away from the radar by measuring the Doppler phase shift of the received signal. The speed radar used by police is a very simple version of the kind of radar used in the JAWS Project. The term coherent indicates that the phase of the transmitted radar signal is coherent from one pulse to the next; and so any phase shift in the returned signal can be measured and converted to engineering units of meters per second or knots. Since Doppler frequency shifts are so small at the speeds meteorological knots move and at the frequencies meteorological radars operate, phase shifts rather than frequency shifts are measured. Regardless of whether phase or frequency shifts are measured, the Doppler concept remains valid. Because a Doppler shift relative to the radar is used to measure velocity, we see that the target must have some component of motion toward or away from the radar to register a non-zero velocity. A Doppler radar can only measure the radial component of motion toward or away from it--any tangential component cannot be observed.

Data collected by a pulsed Doppler radar are described in terms of beams and gates (gates are often interchangeably referred to as pulse volumes). A Doppler radar operating with a stationary antenna maintaining a constant azimuth and elevation angle transmits a pulse of energy which, as it travels out, traces a beam. Due to the nature of the antennae currently in use on the JAWS radars, the beam is not perfectly collimated like a laser; it spreads out, getting wider the further it gets from the radar. This spread is called the "beam-width." For the JAWS radars, the beam-width is very nearly 1 degree; and at ranges greater than about 10 km becomes the limiting factor, restricting what spatial scales the radar can resolve.

A receiver can be gated so that the waiting time for echo return is at approximately one-microsecond intervals. The pulse has time to travel out to and return from a range of 150 m (total distance of 300 m) in the first microsecond, 300 m in the second, 450 m in the third, etc. A beam gated into discrete 150 m segments effectively defines a string of volumes that look like segments of a 1-degree cone, each segment 150 m long. These segments are what radar meteorologists refer to as "gates" and/or "pulse volumes."

Finally, the reflectivity and velocity data that a Doppler radar gathers are the ensemble average of what's in each gate. Thus, we measure what raindrops are doing on the average within each gate--there could be a tornado completely contained within a gate and the velocity data gathered by a Doppler radar would still reflect the average velocity within that gate.

MULTIPLE DOPPLER

Assume that we have two Doppler radars with beams oriented in a fashion similar to Figure 1, and that the antennae are pointing locally parallel to the earth's surface. At the point where the two beams intersect, each Doppler radar is measuring the radial velocity towards it. Figure 2 shows what radar A would measure in the gate coincident with the intersection of the two radar beams. For the sake of this example, assume further that radar B also has a gate coincident with the intersection of the two radar beams; Figure 2 also shows what radar B measures in the gate coincident with the intersection of the two radar beams. Obviously, only simple geometry is required to resolve the two radial components measured by the two radars into two orthogonal components, as shown by the inset. Quite simply, this is how two Doppler radars are used to define the quasi-horizontal wind at the surface. But in reality, the process is much more complicated.

INTERPOLATION FROM RADAR TO CARTESIAN SPACE

A radar gathers data in a spherical coordinate system defined by azimuth, elevation, and range with the radar at the coordinate system origin. Since each radar works in its own coordinate system, a coordinate system common to both radars is required. The common coordinate system used is standard three-dimensional Cartesian space. For JAWS data, the x-direction is always positive towards the east, the y-direction, positive towards the north, and the z-direction, positive upward. As an example, a positive x wind component indicates that the wind is blowing from the west.

The process of mapping radar data onto a Cartesian coordinate system is called objective analysis. Figure 3 shows a two-dimensional schematic view (not to scale) of radar data overlaid by a regular Cartesian grid. Each little square box signifies a gate of radar data and each plus-sign symbol signifies a Cartesian grid point or "node." There are many ways to perform an objective analysis, but all address the question of how best to derive a value at some grid point that is most representative of the surrounding data. We utilize a standard method called Cressman analysis that uses a distance-weighted mean computation.

Figure 4 shows a close-up of nine grid points where the center grid point has been surrounded by a circle of influence whose radius equals the Cartesian grid spacing. All radar gates within this radius of influence contribute to the final value that is ultimately applied to a particular grid point and is representative of the data around it. Note that in using this method some gates will affect as many as four different grid points.

The weighting function to determine how much a given datum affects its associated grid point is given by

$$g_i = \begin{cases} \dfrac{R_r^2 - d_i^2}{R_r^2 + d_i^2}, & d_i \leq R_r, \\ 0, & d_i > R_r \end{cases} \tag{1}$$
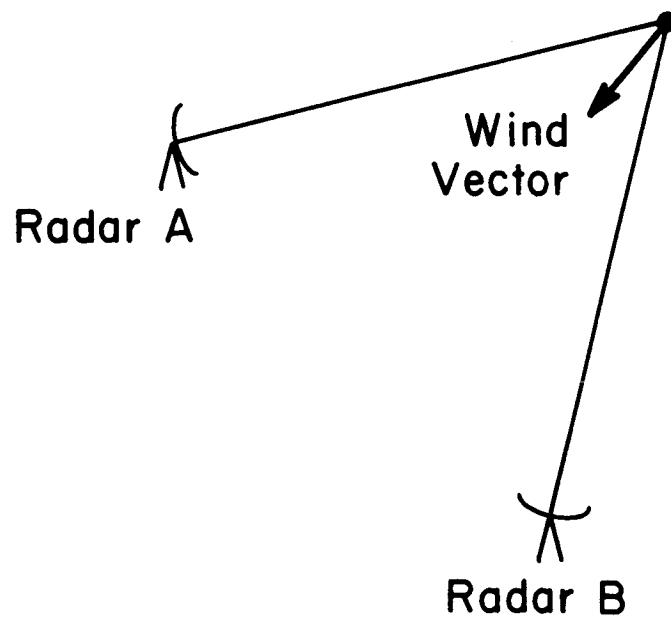
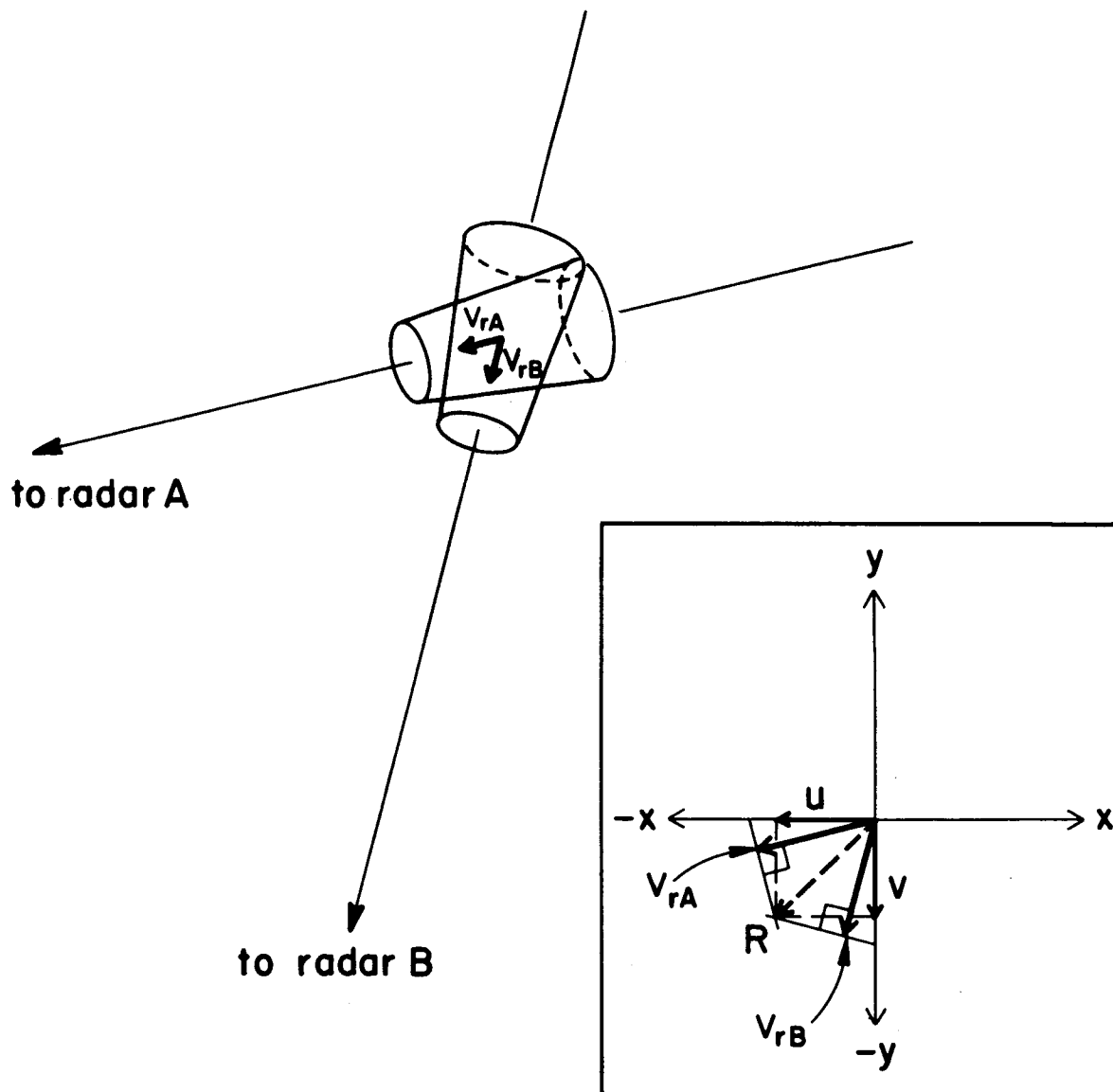Figure 1. Two Doppler radars sampling a common point in space.

Figure 2.   The resolution of V_{rA} and V_{rB} into orothogonal
components.   Inset shows graphical resolution of two
non-orthogonal into two orthogonal components using direction
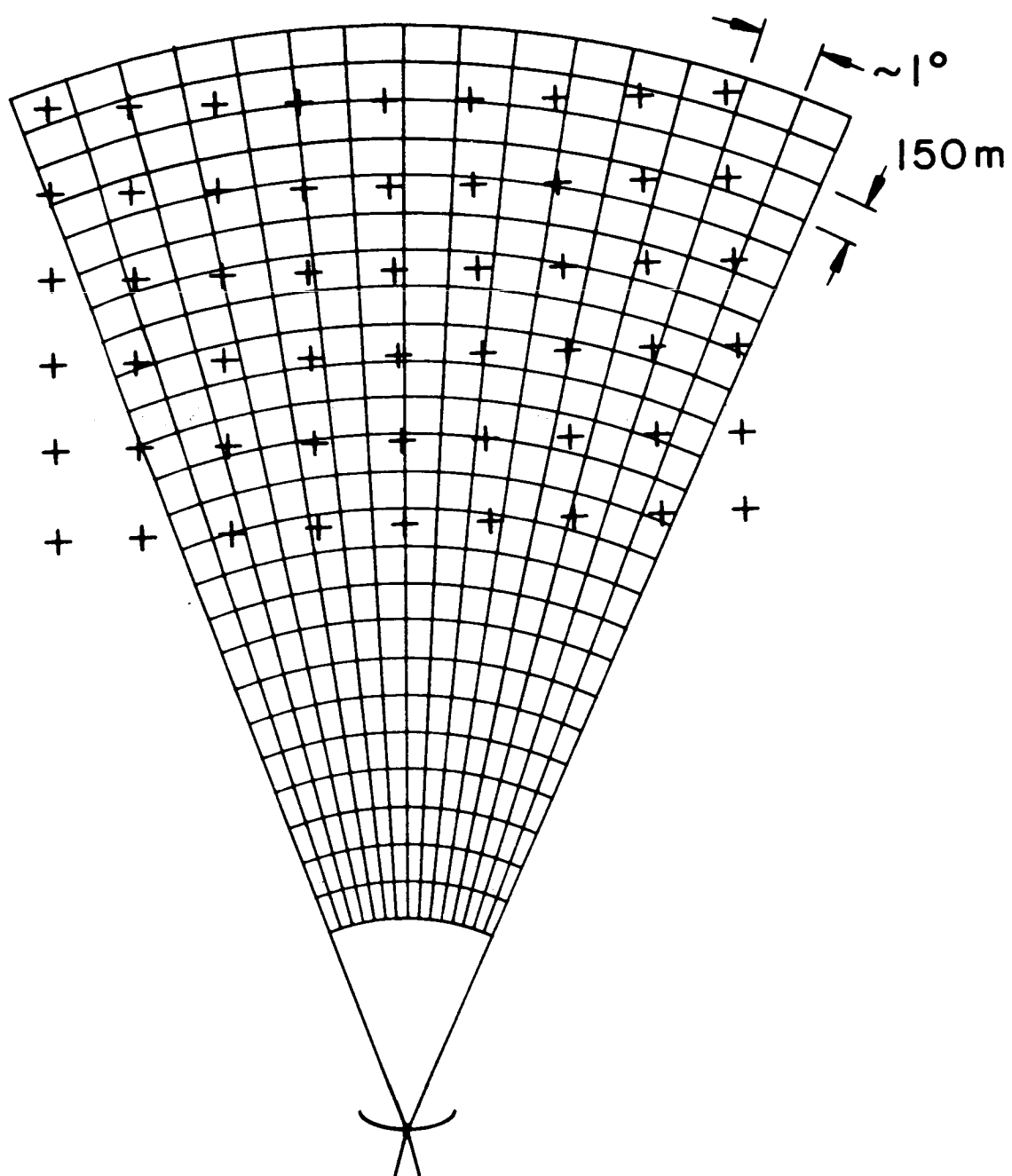cosines.

Figure 3. Two-dimensional schematic view (not to scale) of Doppler radar data overlaid by an orthogonal Cartesian grid.
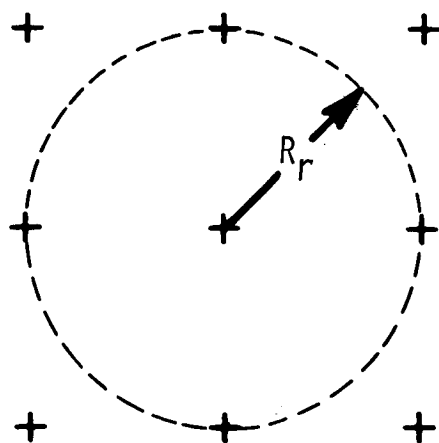
Figure 4. A close-up of the Cartesian grid point that is used in a
Cressman objective analysis scheme.

where g_i is the weight of the i th datum, d_i is the distance from the grid point to the i th datum, and R_r is the radius of influence which, for our case, is equal to the grid spacing.

An objectively analyzed grid point value is defined as

$$G = \frac{\Sigma V_r(i) g_i}{\Sigma g_i},$$

(2)

where G is the objectively analyzed grid point value, $V_r(i)$ is the value of the i th datum, and g_i is the weight assigned to the i th datum. In reality, radar data are three-dimensional, not two-dimensional, and an influence volume, rather than an influence circle, is used. The influence volume is spheroidal in shape, since the Cartesian grids we use may not always have the same vertical and horizontal spacing.

This objective analysis is performed on the radial velocity and reflectivity data gathered by each radar. Thus, at the end of the objective analysis step we will have fields of radial velocity and reflectivity from each radar all on a common grid. Both the radial velocity and reflectivity are used in the next step: three-dimensional wind field synthesis.

THREE-DIMENSIONAL WIND FIELD SYNTHESIS

We want to synthesize the horizontal wind components u and v, as well as the vertical wind component, from only two knowns (radial velocity from each of two radars). It would seem our system of equations is seriously under-determined, but this apparent dilemma is solved using the equation of continuity.

In its simplest terms, the equation of continuity states that whatever goes into a volume must come out of it somewhere else, thus conserving the mass within the volume. The volume may not accrue a mass excess or suffer a mass deficit. Figure 5 shows the concept schematically. For this example, assume that the bottom of the box is a solid boundary, like the ground. Since air cannot go into or come out of the ground, whatever enters the top of the box must exit out the sides (divergence). Conversely, if air enters the sides of the box (convergence), air exits through the top.

The radial velocity measured from any radar, i, is given by

$$V_i = \frac{1}{R_i} (ux_i + vy_i + Wz_i)$$

(3)

where i is the radar index, x_i, y_i, and z_i are the Cartesian distances from the pulse volume to the i th radar, W is the vertical motion of the raindrops measured by the radar, and R_i is the slant range from radar i to the pulse volume, defined as

$$R_i = x_i^2 + y_i^2 + z_i^2 .$$
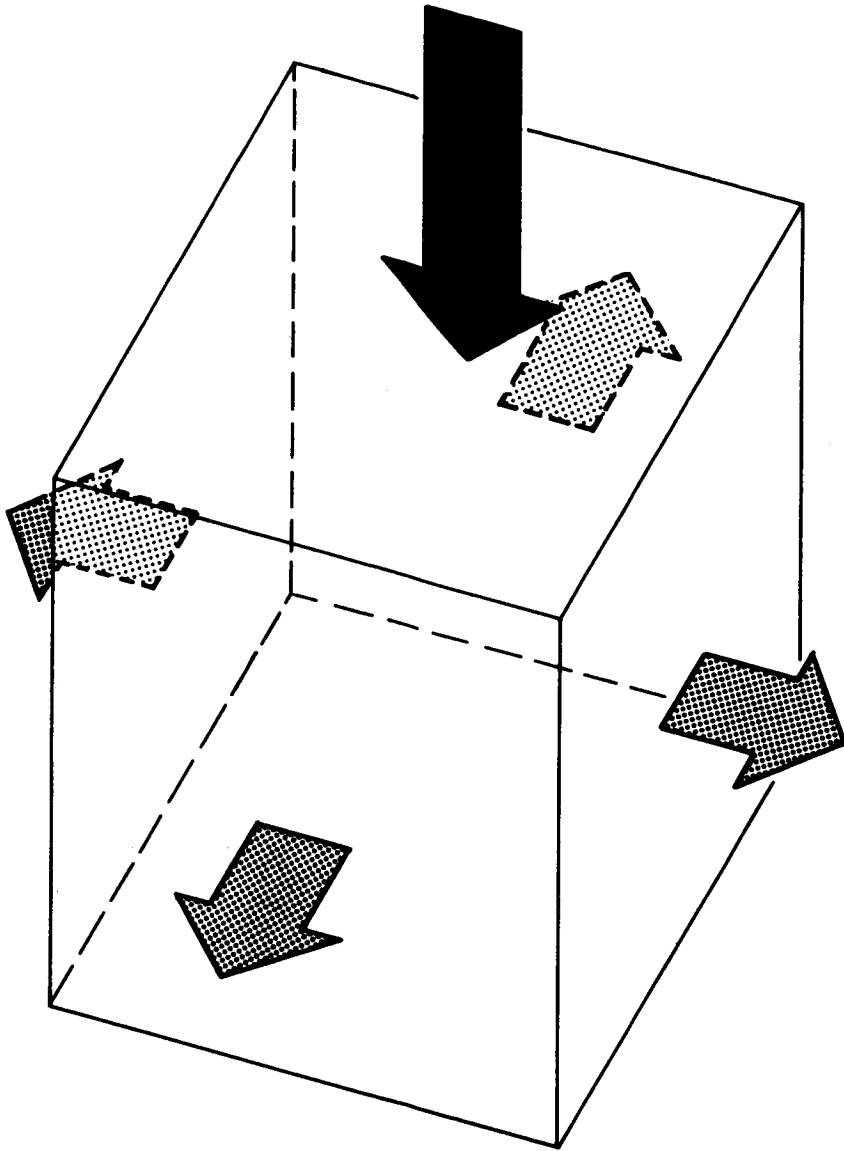
Mass In = Mass Out

Figure 5. The concept of continuity as it applies to JAWS analysis.

A radar actually measures the motion of raindrops, but what we really want is the air motion. Studies have shown that raindrops are remarkably good tracers of horizontal air motions, but they tend to fall at speeds that are on the order of the vertical airspeed; therefore, they make poor vertical air motion tracers. Thus, vertical fall speed somehow must be accounted for since we really want the motion of the air, not of the raindrops.

Recall that reflectivity can be used to estimate the size of the raindrops. By knowing their size, we can estimate quite well how fast they are falling through the air. With this estimation, we can correct the radial velocity from each radar to make a better estimate of the actual air motion, uncontaminated by the fallspeed of the raindrops.

So W in Equation (3) can now be broken into two parts: w, the actual vertical air motion and V_t, the terminal fallspeed of the raindrops. The equation describing terminal fallspeed has the following form:

$$V_t = -3.8 \left(\frac{\rho_{sfc}}{\rho(z)}\right)^{0.4} Z_e^{0.0174} \tag{4}$$

where $\rho(z)$ is density at some height z, ρ_{sfc} is density at the surface, and Z_e is the equivalent radar reflectivity. Finally, the form of the continuity equation used for JAWS is the anelastic (or compressible) continuity equation, given by

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z} = -\frac{w}{\rho}\frac{\partial \rho}{\partial z}. \tag{5}$$

We now have four equations, (3)-(5), (since (3) counts as two equations), and four unknowns, u, v, w, and V_t.

Given the equation for V_1 and V_2 (the radial velocity from each radar), the equations for u and v become

$$u = A + B(w + V_t), \tag{6}$$

$$v = C + D(w + V_t), \tag{7}$$

where

$$A = \frac{R_1 V_1 y_2 - R_2 V_2 y_1}{x_1 y_2 - x_2 y_1}, \tag{8}$$

$$B = \frac{-z_1 y_2 - z_2 y_1}{x_1 y_2 - x_2 y_1}, \tag{9}$$

$$C = \frac{-R_1 V_1 x_2 + R_2 V_2 x_1}{x_1 y_2 - x_2 y_1}, \tag{10}$$

and $D = \dfrac{z_1 x_2 - z_2 x_1}{x_1 y_2 - x_2 y_1}$. (11)

We use Equation (4) to eliminate V_t from Equations (6) and (7), and then integrate Equation (5) to yield

$$w_k = w_{sfc} \frac{\rho_{sfc}}{\rho_k} - \frac{1}{\rho_k} \int_{sfc}^{z_k} \rho(z)(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y})dz , \qquad (12)$$

where subscript sfc indicates a boundary (surface value), subscript k indicates the grid level for which the computation is being carried out, ρ is density, and $\rho(z)$ is the density at any height z.

In Equations (6) and (7), u, v, and w are all functions of each other and so are solved using an iterative predictor-corrector process. Starting at the bottom of the data (the surface), we assume w = 0. Based on this assumption, we solve for u and v. Then w at the next level up is specified using w at the previous level as a first guess. Since we've specified w at this k = 2 level, we can compute u and v as we did at the surface. We now have u's and v's at two levels, so we can use them to compute a divergence over the depth from the surface to the second grid level. We then integrate this divergence over this depth using a numerical approximation of Equation (12) and come up with a new w at the second level. This new w is used to compute a new u and v at the second level, which is, in turn, used to compute a new divergence, and so on until divergence, and so w, converges on some constant value. Then the process repeats between the second and third levels, then the third and fourth levels, etc., until the top of the data is reached. The iterative process described above usually converges after about five to seven iterations for each level.

ERRORS

Comparing the numbers derived from these equations with the real world, the horizontal wind components are good to about 2 kts and the vertical wind component is good to about 6 kts, depending upon how far above the ground the measurement is made. Errors result for several reasons. The radar itself can measure velocity inside a gate or pulse volume to within 0.02 kts, but not everything in a pulse volume is moving in unison. The radar calculates a number that represents only the ensemble average motion of all the raindrops in the gate. Therefore, a radial velocity estimate within each individual pulse volume is good to roughly 0.5 kts.

We have to interpolate radial velocities from radar (spherical) space to a regular, common Cartesian grid. This is the most costly step in terms of accuracy. The interpolation process along with previously mentioned effects yields gridded radial velocities good to a little less than 2 kts.

With these errors, we synthesize u and v. Each of the radar beams is not always perpendicular; in fact, that is an extremely rare event since it occurs at only two points. Also, the radial velocity from each radar is not measured

at the same point in space at the same time. Finally, it takes a finite amount
of time (about 2 minutes) to gather all the necessary radial velocities from
each radar over the region of interest. This can be likened to "moving the
camera" while taking a photograph; it tends to degrade the quality of the pic-
ture. Yet, given all this, u and v are still good to just a little over 2 kts.

In computing w, we have additional problems. First, we compute derivatives
using a three- or five-point numerical finite difference which has well-known
error properties. Next, we integrate these derivative estimates vertically
upward from a known boundary condition using density weighting. Because of the
density weighting, any errors in w made at low levels will be amplified as we
integrate upward because an error in w really translates into an error in mass
flux. The mass flux error actually remains constant as we go upward, but because
density decreases upward, a larger w is required to maintain the same erroneous
mass flux.

In all cases, we assume that w at the ground is zero, which in and of
itself is a very good assumption. But, in fact, the lowest data level gathered
by a Doppler radar is not at a height of zero meters; it is usually at least
a few meters above the surface and can be tens of meters above the surface.
Obviously, w is not zero a few meters above the surface, and this is a source
of bias error in w. Orography can also play a role in degrading w. For example,
if horizontal windspeeds at the surface are 40 kts and the terrain slopes at
an angle of two degrees (a 3.5% grade), w at the surface will be 1.3 kts. This
will substantially bias w at the higher levels.

Because w is a derived quantity, it is the least accurate. At the top of
the data (roughly 3000 feet AGL), w is good to only about 6 kts. However, near
the ground, where approaches and takeoffs are simulated, w is very nearly as
good as u and v.

The following table summarizes the expected errors in JAWS data:

PARAMETER IN ERROR	MAGNITUDE	SOURCE(S)
fundamental radial velocity (pulse volume)	$\sim$0.5 kts	mainly turbulence
gridded radial velocity	<2 kts	interpolation
u and v	$\sim$2 kts	all data not simultaneous in time and space
w	$\sim$2-6 kts (height dependent)	1) truncation errors 2) improper boundary conditions

JAWS has independently verified these u, v, and w velocity estimates with airborne, vertically pointing Doppler radar and in situ, instrumented aircraft measurements; thus their accuracy, at least in cases we have been able to compare, is within the limits given above [1].

WHY JAWS DATA ARE SIGNIFICANT

A comparison of flight data recorder (FDR) reconstructions with JAWS multiple Doppler data reveals advantages unique to the FDR reconstructions:
1) FDR data come from wind shear that presumably caused the crash; and
2) FDR resolution is, in a sense, better than Doppler data since FDR's collect data at a frequency of 1 Hz, which at an approach speed of 150 kts, corresponds to a spatial resolution of 75 m compared to 150 m for Doppler data.

However, on the multiple Doppler data side, advantages include: 1) actual winds are measured; 2) few assumptions are required to obtain all three wind components; and 3) multiple Doppler radar analyses are fully three-dimensional.

The only obvious disadvantage to a multiple Doppler radar analysis is the best fundamental resolution of the instrument, which is 150 m compared to FDR resolution which is roughly 75 m. However, FDR disadvantages are somewhat more serious. The older FDR's, from which most accident reconstructions come and which make up the vast majority of FDR's flying today, are not very accurate. FDR's do not measure the actual winds; the winds must be derived through a complicated process involving many, often crude, assumptions. Finally, and most importantly, FDR's provide only one-dimensional data--a noodle along the aircraft's final path. The real world that we fly in is fully three-dimensional; for realistic simulations and control analysis, the input winds must also be three-dimensional. Multiple radar analyses provide a sufficiently better product for aviation uses than previous FDR reconstructions that a sizable implementation and utilization effort is warranted.

The following table will help in summarizing the pros and cons of FDR reconstruction vs. multiple Doppler analyses.

FDR ADVANTAGES	DOPPLER DISADVANTAGES
Data came from wind shear that presumably caused the crash.	
In a sense, better one-dimensional (along track) resolution (75 m).................	Data only every 150 m (492 ft)

DOPPLER ADVANTAGES	FDR DISADVANTAGES
	Old FDR's not very accurate
actual winds are measured.......................	Actual winds not measured-- must be derived
few assumptions required to get all three components............	Derivation of winds requires too many, often crude, assumptions
fully three-dimensional........................	only one-dimensional.

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REFERENCE

1. Rodi, A. R., K. L. Elmore, and W. P. Mahoney: Aircraft and Doppler air motion comparisons in a JAWS microburst. AMS Preprint, 21st Radar Conference on Meteorology, Edmonton, Alberta, Canada, 19-23 Sept. 1983.