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DYNAMICS DURING THRUST MANEUVRES OF FLEXIBLE SPINNING SATELLITES WITH AXIAL AND RADIAL BOOMS

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ABSTRACT

Analyses are given for the dynamic response to operational maneuvers for spinning symmetric spacecraft with radial and axial booms. The research was performed as part of the prelaunch dynamic analysis of the ISEE-3 spacecraft placed in a halo orbit around an Earth-Sun libration point, and later renamed ICE when it was directed to fly-by comet Giacobini-Zinner. The results presented use simple spacecraft models with the aim of developing understanding, and they frequently gave predictions that were good and easily obtained when the results from using a general purpose multi-body dynamics program were very time consuming to obtain. The operations encountered during the spacecraft history that are analyzed here are, deployment of radial booms, spin-up after partial deployment, station keeping, and trajectory changes. The latter two can involve both axial thrusting and pulsed radial thrusting once per revolution.

Keywords: Flexible spacecraft, attitude dynamics, deployment, spin-up, vibration modes during thrusting.

1. INTRODUCTION

This paper addresses a series of problems associated with the dynamic response of flexible spinning spacecraft during thrust maneuvers. It is a companion work to Reference 1, which determined spacecraft vibration modes and frequencies, and dynamic response during spin-up, of the IMP-J (Interplanetary Monitoring Platform) spin stabilized spacecraft with radial wire antennas. The work reported here was done as part of the pre-launch dynamic analysis of the ISEE (International Sun-Earth Explorer) series of spacecraft (whose name had just been changed from IME at the time of the study). This series represented a follow-on program to the IMP series, employing very similar spacecraft configurations. Particular attention is given the ISEE-3 spin stabilized spacecraft with four long radial wire antennas, and shorter spin axis booms.

The spacecraft was subjected to various thrust maneuvers while the radial and axial booms were deployed, and this paper aims at developing simple models to predict the associated vibrational response of the flexible vehicle. The maneuvers considered include the following. After orbit injection the radial wire antennas are deployed, and the effect of the deployment rate history on spin rate and spin rate vibations is studied. In order to maintain at least a minimum spin rate, deployment is periodically interupted for a spin-up maneuver. After the radial and axial wires and booms are deployed, orbit maintenance maneuvers are required to maintain the satellite in its unstable halo orbit about the libration point. Because of reliability considerations it is undesirable to retract and later extend booms and wires for such maneuvers. In addition to the orbit maintenance maneuvers, there are orbit change maneuvers to go from the libration point orbit to a large Earth orbit, and eventually to the comet fly-by after various moon fly-bys for gravity assist. The satellite has the capability of maneuvering in the plane of the spin using pulsed radial thrusting that is turned on for a portion of each spin cycle. Maneuvering along the spin axis can be accomplished with spin axis thrusters, or by performing an attitude change and using the radial thruster.

The aim of the analysis here is to use simple models to obtain an understanding of the spacecraft vibrational response to these maneuvers. The prelaunch dynamic analysis also included many computer simulations using large general purpose multibody dynamics programs. Often, such simulations were quite time consuming, while the equations derived here predicted the results accurately and quickly in terms that were easily understood and gave more insight.

2. SPIN-UP AND DEPLOYMENT MANEUVERS

The spacecraft is modeled as a cylindrical rigid hub with radial and axial booms represented as rigid spherical pendulums, with a linear spring constant restoring torque at their pivot points. The spring constant is chosen to match the first bending mode frequency of the boom as a cantilever beam. This simple model makes many of the problems tractable. In one case it was possible to obtain results for the first vibrational mode starting with partial differential equation models, and this resulted in the same ordinary differential equation as the above simple model, except that the gain coefficient in the driving term was altered from 1.5 to 1.566. This small difference gives confidence in the other results obtained from this simple model.

Reference 1 develops the fundamental spacecraft mode shapes for the satellite before the spin axis booms

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are deployed. Both deployment and spin-up directly excite only the α mode shown in Fig. 1, in which all radial booms simultaneously lag or lead, while the hub rotates in the opposite direction. Only this mode will be considered here.

2.1 Derivation of differential equations

This section derives differential equations in sufficient generality that they can subsequently be used for both spin-up and for deployment. In the former case, there is a torque applied to the spacecraft hub, and in the latter there is kinetic energy associated with the radial motion of the booms. Figure 2 gives the nomenclature used, where x,y are inertial, θ is the angle of rotation of the hub and q is the radial position of a generic volume element on a boom of lenght $\ell(t)$. The portion of each boom remaining in the hub at any time is modeled as a point mass at the radius r of the hub. The total kinetic energy is the sum of: 1) The kinetic energy of the hub $1/2 I_{H0}\dot{\theta}^2$ and that of the coiled booms. 2) The kinetic energy of the tip masses of mass m_t. 3)The kinetic energy of the booms. The potential energy of all radial booms together is $1/2\, k\alpha^2$. Let ρ be the density per unit length of the booms and let their total length be ℓ_T. Then by Lagranges equations one obtains the α equation

$$
\begin{aligned}
I &= I_H + M_1 r^2 + 2M_2 r\ell + M_3\ell^2 \\
M_1 &= 4(m_t + \rho\ell) \\
M_2 &= 4(m_t + \tfrac{1}{2}\rho\ell) \qquad\qquad (1)\\
M_3 &= 4(m_t + \tfrac{1}{3}\rho\ell)
\end{aligned}
$$

$$
\begin{aligned}
&[M_3\ell + M_2 r\cos\alpha]\ell\ddot{\theta} + M_3\ell^2\ddot{\alpha} + [M_2 r\ell\sin\alpha]\dot{\theta}^2 \\
&\qquad + 2M_2\ell\dot{\ell}(\dot{\alpha}+\dot{\theta}) + k\alpha = 0 \qquad (2)
\end{aligned}
$$

The θ equation has a first integral, Eq. 3, when no torque is applied to the hub. C is a constant of integration, here evaluated for zero initial conditions. The Lagrange equation for θ is Eq. 4 below.

$$
\begin{aligned}
&[I - 2M_2 r\ell(1-\cos\alpha)]\dot{\theta} + [M_3\ell + M_2 r\cos\alpha]\ell\dot{\alpha} \qquad (3)\\
&\qquad + M_1 r\dot{\ell}\sin\alpha = C \\
&C = I(0)\omega_0 = (I_{H0} + 4\rho\ell_T r^2 + 4m_t r^2)\omega_0
\end{aligned}
$$

$$
\begin{aligned}
&[I - 2M_2 r\ell(1-\cos\alpha)]\ddot{\theta} + [M_3\ell + M_2 r\cos\alpha]\ell\ddot{\alpha} \\
&+[2M_1 r\dot{\ell}\cos\alpha]\dot{\alpha} + [\dot{I} - 2M_1 r\dot{\ell}(1-\cos\alpha) - 2M_2 r\ell\sin\alpha\,\dot{\alpha}]\dot{\theta} \\
&+ [M_1 r\ddot{\ell} + 4\rho r\dot{\ell}^2]\sin\alpha \\
&+ [M_1 r\dot{\ell}\cos\alpha - (M_2 r\ell\sin\alpha)\dot{\alpha} + 2M_2\ell\dot{\ell}]\dot{\alpha} = Q \qquad (4)
\end{aligned}
$$

$$
\dot{I} = 2[M_1 r + M_2\ell]\dot{\ell}
$$

Here Q is the spin-up torque, and ω_0 the initial spin rate.

2.2 Radial boom dynamics during deployment

We are interested in isolating a differential equation for radial boom deformation α alone, by eliminating θ dependence in Eq. 2. We assume the boom deformations are small so that linearization in α and $\dot{\alpha}$ about zero is appropriate. There is no need to pick a nominal θ history to linearize about. Decoupling of the linearized equations is complicated by the presence of time dependent coefficients, so that methods such as Laplace transforms cannot be used. Making use of all three equations, Eqs. 2-4, one can obtain

$$
\begin{aligned}
&I[IM_3\ell^2 - M_{23}^2]\ddot{\alpha} + [2I^2 M_2\ell - IM_{23}(3M_1 r + 4M_2\ell) \\
&+ 2(M_1 r + M_2\ell)M_{23}^2]\dot{\ell}\dot{\alpha} + [I^2 k + M_2 r\ell C^2 \\
&-(2IM_1M_2 r\ell + 4IM_{23}\rho r - 2(M_1 r + M_2\ell)M_1M_{23}r)\dot{\ell}^2 \\
&- IM_1M_{23}r\ddot{\ell}]\alpha \\
&= -2[IM_2\ell - (M_1 r + M_2\ell)M_{23}]\dot{\ell}C \qquad (5)
\end{aligned}
$$

$$
M_{23} = M_2 r\ell + M_3\ell^2
$$

This equation is too complex for analytic solution. If one considers a constant deployment rate, then the coefficients become polynomials in t, but the degree of the polynomials is high. For example, the coefficient of $\ddot{\alpha}$ contains t^9.

If the deployment rate is slow one could conceive of a quasi-static steady state associated with the particular solution. Then the approximate particular solution is given by dividing the right hand side of Eq. 5 by the coefficient of α on the left hand side. This solution often gives rather good results. It can be simplified by making further assumptions in order to obtain the offset angle during spin-up that was derived by approximate means in Ref. 2: $\alpha_p \cong -2\dot{\ell}/(r\omega_0)$.

2.3 Dynamic response of radial booms during spin-up

To obtain a differential equation for the radial boom deflection α during a constant torque Q spin-up, linearize Eqs. 2 and 4 for α about zero, and for $\dot{\theta}$ about the nominal spin rate that would exist if the spacecraft were rigid. Decoupling the equations, introducing a new independent variable $\tau = 1 + \epsilon t$ and denoting differentiation with respect to τ by a prime results in the following equation for α

$$
\begin{aligned}
\alpha'' + (\bar{k} + \tau^2)\Omega_\alpha^2\alpha &= -K \qquad (6)\\
\epsilon &= Q/(I\omega_0) \\
\bar{k} &= k/(M_2 r\ell\omega_0^2) \\
\Omega_\alpha^2 &= (IM_2 r\ell\omega_0^2)/[\epsilon^2(IM_3\ell^2 - M_{23}^2)] \\
K &= QM_{23}/[\epsilon(IM_3\ell^2 - M_{23}^2)]
\end{aligned}
$$

The quantity ϵ is small for the spacecraft considered, since the spin-up torque Q is small. Ω_α is the radian natural frequency of the α spacecraft mode when spinning at ω_0.

In the companion paper, Ref. 1, Eq. 6 was solved analytically for the case of wire booms with k=0. The contribution here is to study the effect of a nonzero boom stiffness on the solution to Eq. 6. In Ref. 1 a transformation was found to convert the equation to a Bessel equation, for which the complementary function is in terms of Bessel functions of order ±1/4. The particular solution for a constant torque Q was found to be expressible in terms of a Struve function of order -1/4. An alternative expression for the particular solution was given as an infinite series in Bessel functions. Simpler approximate solutions were also obtained, by the WKB method (Ref. 3) for the complementary function, and in the form of an asymptotic expansion for the particular solution.

It is found here that the introduction of the boom stiffness term $\bar{k}$ precludes obtaining analytic soltions in terms of Bessel and Struve functions. However, the WKB method and the asymptotic expansion method can still be applied, although the results are more complicated.

2.3.1 The WKB approximation to the complementary function

Generating the WKB approximation as in Refs. 1,3, one obtains the complementary function, Eq. 7

$$\alpha_c \approx \frac{A}{(\bar{k}+\tau^2)^{1/4}} \cos\left\{\tfrac{1}{2}\Omega_\alpha\left[\tau(\bar{k}+\tau^2)^{1/2} + \bar{k}\log\left(\tau+(\bar{k}+\tau^2)^{1/2}\right)\right] + \delta\right\} \qquad (7)$$

$$(\bar{k}+\tau^2)\Omega_\alpha^2 \gg \frac{1}{2}\frac{1}{(\bar{k}+\tau^2)} - \frac{5}{4}\frac{\tau^2}{(\bar{k}+\tau^2)^2} \qquad (8)$$

which is a good approximation whenever Eq. 8 is satisfied which it is in this case because of the ϵ^2 divisor in Ω_α^2. A and δ are arbitrary constants. It is of interest to determine whether this approximate solution is sufficiently accurate that *there is no large accumulation of phase error after* many oscillations. Let $C_\pm$ be the arbitrary consttants associated with the alternative exponential form of Eq. 7. The error in the approximation can be characterized by these constants becoming functions of time τ. An approximation to the total change $\Delta C_\pm$ in $C_\pm$ going from $\tau=1$ to ∞ gives (Refs. 1,3)

$$|\Delta C_\pm / C_\pm| \leq 7/(8\Omega_\alpha) \qquad (9)$$

The bound could be made much tighter, but the presence of ϵ as a divisor in Ω_α nevertheless makes this bound small. We conclude that the approximate solution Eq. 7 is valid for the whole range of interest from $\tau=1$ to ∞.

2.3.2 Asymptotic expansion solution of the forced response. The asymptotic expansion solution obtained when $\bar{k}$ = 0 was in a simple form of a divergent *series in inverse powers of* τ. Because of its simple form it was easy to obtain the solution to any prescribed power, and to obtain error bounds as a function of the number of terms chosen. It was found that use of only two terms in the expansion gave results with very little error, and the third term is down by a factor of 10^{-8} compared to the first. Of course, if too many terms are taken the error will start to grow again, since the expansion is divergent.

When stiffness $\bar{k}$ is present it is no longer a simple matter to obtain the expansion in general form for any order, but the same technique nevertheless applies to develop the expansion term by term, and to bound the error at each stage.

The expansion is obtained by first assuming that the time varying coefficient is sufficiently slowly varying that it can be considered constant for the purpose of generating a particular solution. This solution is plugged into the left hand side of Eq. 6. The difference between the result and the right hand side is the forcing function for a differential equation whose solution is the error in the previous solution, and whose differential operator is the same as in Eq. 6. This equation is solved by the same method, and the procedure is repeated indefinitely. When three terms are developed by this method one obtains

$$\alpha_p \sim -\frac{K}{\Omega_\alpha^2}\left[\frac{1}{\bar{k}+\tau^2}\right] + \left(\frac{2K}{\Omega_\alpha^4}\right)\left[\frac{1}{(\bar{k}+\tau^2)^3} - \frac{4\tau^2}{(\bar{k}+\tau^2)^4}\right] + \left(\frac{2K}{\Omega_\alpha^6}\right)\left[\frac{2}{(\bar{k}+\tau^2)^5} + \frac{208\tau^2}{(\bar{k}+\tau^2)^6} - \frac{320\tau^4}{(\bar{k}+\tau^2)^7}\right] \qquad (10)$$

A bound on the error, obtained starting from the variation of parameters solution, and written in terms of the forcing function for the next error equation, $F(\tau)$, is

$$|\Delta\alpha| \leq \frac{1}{[(\bar{k}+\tau^2)\Omega_\alpha^2]^{1/4}} \int_1^\tau \frac{1}{2[(\bar{k}+\xi^2)\Omega_\alpha^2]^{1/4}} |F(\xi)| d\xi \qquad (11)$$

The general solution of Eq. 6 obtained as the sum of Eqs. 7 and 10, applied to the case k=0, and using only two terms in the expansion, predicted well: the initial amplitude of the oscillations resulting from applying the step change in torque to the hub, the decay in amplitude and the increase in frequency as the spacecraft spins-up. The nonoscillatory part of the solution coming from the particularsolution predicted the initially large angles which converge to zero as τ gets large due to growth of centrifugal force on the booms. These predictions are simple to obtain from the equations here, *but the multibody dynamics program simulation* of 11 minutes of spin-up, required 30 minutes of computatuion time on an IBM 360-91.

3. AXIAL THRUSTING MANEUVERS

During station keeping maneuvers it would be natural to require velocity and position corrections both in the plane of the spin and along the spin axis. This section is devoted to studying the effect of axial thrusting on the boom dynamics, both for the radial booms and for the axial booms. Such thrusting can be destabilizing to the spin axis booms, and the interplay and trade-offs are established here between such factors as: axial boom stiffness, axial boom length, axial acceleration magnitude, spacecraft spin rate, displacement of the boom axis from the spin axis, and misalignment of the boom direction from the spin axis direction. The spacecraft is assumed to be in a steady spin, and in the case of spin axis boom dynamics, the spacecraft is assumed to undergo a prescribed acceleration along the spin axis, with the boom motion having no influence on the hub motion.

3.1 Response of radial booms to axial thrusting

Axial thrusting directly excites the β spacecraft mode in Fig. 1. Use of Lagranges equations gives the boom deflection β in terms of arbitrary constants C_1 and C_2 as

$$\begin{aligned} \beta &= \beta_p + C_1\cos\Omega_\beta t + C_2\sin\Omega_\beta t \\ \beta_p &= -(M_2 \ell Q)/(k+M_{23}\omega_0^2) \\ \Omega_\beta &= \{[k+M_{23}\omega_0^2]/[(M_3-M_2^2/M)\ell^2]\}^{1/2} \end{aligned} \qquad (12)$$

3.2 Axial boom dynamics with boom misalignment and offset from spin axis

Figure 3 indicates nomenclature needed for the analysis of the axial boom response to axial thrust. The coordinates $\hat{I}$, $\hat{J}$, $\hat{K}$ are inertial. The position of the center of the top surface of the satellite hub is given as $\frac{1}{2}ct^2\hat{K}$ so that the axial acceleration of the hub is prescribed as c. The base of the axial boom is displaced a distance r_0 from the spin axis, and hub fixed coordinates are centered at the base of the boom with $\hat{I}'$ along the direction of the displacement r_0. The angular orientation of the boom is prescribed by angles Θ_1 in the $\hat{i}', \hat{k}'$ plane and angle Θ_2. If the boom unstressed direction is misaligned and not parallel to the spin axis, the misalignment is assumed to be small enough for linearization to apply, and is denoted by Θ_{10}, Θ_{20}.

As before, due to space limitations, derivations must be deleted. Developing Lagranges equations for $\delta\Theta_1$, $\delta\Theta_2$ and linearizing in these quantities as well as in Θ_{10}, Θ_{20} produces

$$\delta\ddot{\Theta} + G\delta\dot{\Theta} + K\delta\Theta = \omega_0^2\omega_3^2[1\ \ 0]^T + \omega_0^2\omega_2^2\Theta_0$$

$$\delta\Theta = [\delta\Theta_1\ \ \delta\Theta_2]^T \quad ; \quad \Theta_0 = [\Theta_{10}\ \ \Theta_{20}]^T \qquad (13)$$

$$\Theta_i = \Theta_{i0} + \delta\Theta_i \qquad i = 1,2$$

$$G = \begin{bmatrix} 0 & -2\omega_0 \\ 2\omega_0 & 0 \end{bmatrix} \quad ; \quad K = \begin{bmatrix} K_1 & K_2 \\ K_2 & K_1 \end{bmatrix}$$

$$K_1 = \omega_0^2(\omega_1^2 - \omega_2^2 + \omega_3^2\Theta_{10})$$

$$K_2 = \omega_0^2\omega_3^2\Theta_{20}$$

$$\omega_1^2 = \Omega^2/\omega_0^2$$

$$\omega_2^2 = 1 + 3c/(2\ell\omega_0^2)$$

$$\omega_3^2 = (3/2)(r_0/\ell)$$

$$\Omega^2 = k/(\tfrac{1}{3}m_B\ell^2)$$

Here Ω is the radian natural frequency of the first mode of the boom vibration as a cantilever beam, and m_B is the boom mass. Consider one of these equations explicitly

$$\underset{(1)}{\delta\ddot{\Theta}_1 - 2\omega_0\delta\dot{\Theta}_2} + [\underset{(2)}{\Omega^2} - \underset{(3)}{\omega_0^2} - \underset{(4)}{\tfrac{3c}{2\ell}} + \underset{(5)}{\tfrac{3}{2}\omega_0^2\left(\tfrac{r_0}{\ell}\right)\Theta_{10}}]\delta\Theta_1 \qquad (14)$$

$$+ \underset{(6)}{\tfrac{3}{2}\omega_0^2\left(\tfrac{r_0}{\ell}\right)\Theta_{20}}\delta\Theta_2 = \underset{(7)}{\tfrac{3}{2}\omega_0^2\left(\tfrac{r_0}{\ell}\right)} + \underset{(8)}{\left(\omega_0^2 + \tfrac{3c}{2\ell}\right)\Theta_{10}}$$

Term (1) represents gyroscopic coupling. The bracket multiplying $\delta\Theta_1$ would represent the square of the vibration frequency of the boom, if there were *no coupling with the* $\delta\Theta_2$ *equation, but is still* related to the natural frequency here. This bracket contains the square of the boom's natural frequency, term (2), which is decreased by the satellite spin rate, term (3), and by the axial acceleration of the spacecraft, term (4). Term (5) can either decrease or increase the value in the bracket depending on the sign of the boom misalignment Θ_{10}. Term (6) represents coupling in the stiffness matrix due to misalignment. The offset of the base of the boom from the spin axis produces a forcing function to the differential equation, term (7), as does the boom misalignment Θ_{10}, term (8).

The general solution for Θ_1, Θ_2 is

$$\Theta(t) = \Theta_0 + \delta\Theta_p + C_1\sin w_1t + C_2\cos w_1t + C_3\sin w_2t + C_4\cos w_2t \qquad (15)$$

$$\delta\Theta_{p1} = \frac{(\omega_1^2 - \omega_2^2 + \omega_3^2\Theta_{10})(\omega_3^2 + \omega_2^2\Theta_{10}) - \omega_2^2\omega_3^2\Theta_{20}^2}{(\omega_1^2 - \omega_2^2 + \omega_3^2\Theta_{10})^2 - (\omega_3^2\Theta_{20})^2}$$

$$\delta\Theta_{p2} = \left[\frac{\omega_3^4 + \omega_2^2(\omega_1^2 - \omega_2^2)}{(\omega_1^2 - \omega_2^2 + \omega_3^2\Theta_{10})^2 - (\omega_3^2\Theta_{20})^2}\right]\Theta_{20}$$

$$w_{1,2} = \{(2 + \omega_1^2 - \omega_2^2 + \omega_3^2\Theta_{10}) \pm [4(1 + \omega_1^2 - \omega_2^2 + \omega_3^2\Theta_{10}) + \omega_3^4\Theta_{20}^2]^{1/2}\}^{1/2}$$

$$C_j = [C_{j1},\ C_{j2}]^T \quad ; \quad j = 1,2,3,4$$

$$C_{11} = (2\omega_0 w_1 C_{22} + K_2C_{12})/(K_1 - w_1^2)$$

$$C_{21} = (K_2C_{22} - 2\omega_0 w_1 C_{12})/(K_1 - w_1^2)$$

$$C_{31} = (2\omega_0 w_2 C_{42} + K_2C_{32})/(K_1 - w_2^2)$$

$$C_{41} = (K_2C_{42} - 2\omega_0 w_2 C_{32})/(K_1 - w_2^2)$$

$$C_1 + w_2C_3 = 0$$

$$C_2 + C_4 = -\delta\Theta_p + \delta\Theta_p\big|_{c=0}$$

The last two equations evaluate the arbitrary constants C_{12}, C_{22}, $C32$, C_{42} for the case of zero initial conditions. $\delta\Theta_p$ represents the linearized expression for the altered equilibrium position of the boom due to spin and acceleration. Certain *special cases are of interest:*

$$w_{1,2} = \left|\omega_0 \pm \sqrt{\Omega^2 - \tfrac{3c}{2\ell}}\right| \quad ; \quad \Theta_0 = 0 \text{ or } r_0 = 0$$

$$\delta\Theta_p = \left[-\tfrac{3}{2}\omega_0^2\tfrac{r_0}{\ell}/\left(\Omega^2 - \omega_0^2 - \tfrac{3}{2}\tfrac{c}{\ell}\right),\ 0\right]^T \quad ; \quad \Theta_0 = 0$$

$$\delta\Theta_p = \left(\omega_0^2 + \tfrac{3c}{2\ell}\right)\Theta_0/\left(\Omega^2 - \omega_0^2 - \tfrac{3}{2}\tfrac{c}{\ell}\right) \quad ; \quad r_0 = 0$$

Ignoring stability due to gyroscopic coupling, which is of no practical significance since it disappears in the presence of energy dissipation in the beam, stability of the differential equation occurs if and only if the stiffness matrix K is positive definite. This requires $K_1>0$ and hence the square bracket in Eq. 14 must be positive. For any given boom stiffness k and boom lenght ℓ, there exists an upper limit on the allowable spacecraft spin rate ω_0 combined with the allowable axial acceleration, expressed by this condition. The same effect *is shown by the particular solution* $\delta\Theta_p$.

4. PULSED RADIAL THRUSTING

Because the spacecraft is spin stabilized, stationkeeping and trajectory modifications requiring corrections in the plane of the spin are accomplished by turning on a radial thruster for a fraction of each revolution of the hub. Here we study the dynamic response of the spin axis booms to this repetitive transverse excitation at their bases. As in the previous section we assume that the hub motion is unaffected by the boom vibrations. An offset r_0 of the boom from the spin axis is included again, but misalignment is ignored, i. e. $\Theta_0 = 0$.

Figure 4 describes the nomenclature for the problem. Axes $\hat{i}_1$, $\hat{j}_1$, $\hat{k}_1$ are inertial coordinates, axes $\hat{i}_2$, $\hat{j}_2$, $\hat{k}_2$ are centered on the top of the spacecraft with $\hat{k}_2$ along the spin axis and $\hat{i}_2$ along the thrust direction, and axes $\hat{i}_3$, $\hat{j}_3$, $\hat{k}_3$ are rotated with respect to the hub coordinates by angle γ about the spin axis and translated along the resulting $\hat{i}$ direction by r_0 so that the origin of the coordinates is at the base of the boom. The path of the origin O_2 of the hub coordinates executes a prescribed zig-zag trajectory in the $\hat{i}_1$, $\hat{j}_1$ plane as a result of the pulsed radial thrusting.

Developing Lagranges equations for Θ_1 and Θ_2, linearizing, and defining $\Theta = \Theta_1 + i\Theta_2$, $i = \sqrt{-1}$, results in the differential equation

$$\ddot{\Theta} + 2\omega_0 i\dot{\Theta} + (\Omega^2 - \omega_0^2)\Theta = -\tfrac{3}{2}\tfrac{1}{\ell}(A_1 + iA_2) + \tfrac{3}{2}\tfrac{\omega_0^2 r_0}{\ell} \qquad (16)$$

$$A_1 = (F/m)\cos\gamma$$
$$A_2 = (F/m)\sin\gamma$$
$$F = \begin{cases} 0 & \text{for } t \in [nT, nT+\tau] \,, \quad n = 0, 1, 2, \ldots \\ F_{max} & \text{for } t \in [nT+\tau, (n+1)T] \end{cases}$$

Here A_1, A_2 are the components of the inertial acceleration of the origin O_2 in 3 axes, F is the thrust magnitude, m is the total spacecraft mass, and T is the period of revolution of the hub. F is pulsed as indicated.

The solution of the homogeneous equation is

$$\theta_H = C_H \exp(i\Omega_1 t) + D_H \exp(i\Omega_2 t) \tag{17}$$
$$\Omega_1 = \Omega - \omega_0$$
$$\Omega_2 = -\Omega - \omega_0$$

The process of finding a particular solution is somewhat complicated. The forcing function is piecewise constant, and within each thrust-on or thrust-off interval a constant particular solution would apply. But using such a combined solution is simplistic in the sense that it does not correspond to a solution of the differential equation for any initial conditions. The particular solution for any chosen initial conditions would normally require solving the differential equation on the first interval from 0 to τ, evaluating the arbitrary constants, and then finding $\theta(\tau)$, $\dot{\theta}(\tau)$. These would be used as initial conditions for the arbitrary constants in the solution from τ to T using the constant particular solution. The process would have to be repeated indefinitely to obtain the solution for all t.

However, the forcing function is a periodic function, which suggests that there should be a periodic particular solution for properly chosen initial conditions. By finding these initial conditions one only needs to find the particular solution over one period, and then it is known for all t. Hence, we look for a solution in the following form where the arbitrary constants are determined by the given boundary conditions:

$$\theta(t) = C_A \exp(i\Omega_1 t) + D_A \exp(i\Omega_2 t) + K_3 \qquad 0 \le t < \tau$$
$$\theta(t) = C_B \exp(i\Omega_1 t) + D_B \exp(i\Omega_2 t) - (K_1 + iK_2) + K_3 \qquad \tau \le t \le T$$
$$K_j = (3/2)\, A_j / [\ell(\Omega^2 - \omega_0^2)] \qquad j = 1, 2$$
$$K_3 = (3/2)\, \omega_0^2 r_0 / [\ell(\Omega^2 - \omega_0^2)]$$
$$\theta(\tau^-) = \theta(\tau^+)$$
$$\dot{\theta}(\tau^-) = \dot{\theta}(\tau^+)$$
$$\theta(0) = \theta(T)$$
$$\dot{\theta}(0) = \dot{\theta}(T) \tag{18}$$

The general solution for the boom response to axial thrusting, after employing the particular solution resulting from the above and after evaluating the arbitrary constants in terms of $\theta(0)$, $\dot{\theta}(0)$, is

$$\theta(t) = \theta_p(t) + C_H e^{i\Omega_1 t} + D_H e^{i\Omega_2 t} \tag{19}$$

$$\theta_p(t) = \begin{cases} \theta_A(t) & \text{for } nT \le t < nT+\tau \qquad n = 0, 1, 2, \ldots \\ \theta_A(t) + \dfrac{K_1 + iK_2}{\left[1 - \frac{\Omega_1}{\Omega_2}\right]} \left\{ e^{i\Omega_1(t-\tau-nT)} - \dfrac{\Omega_1}{\Omega_2} e^{i\Omega_2(t-\tau-nT)} \right\} \\ \qquad \text{for } nT+\tau \le t < (n+1)T \end{cases}$$

$$\theta_A(t) = C_A e^{i\Omega_1(t-nT)} + D_A e^{i\Omega_2(t-nT)} + K_3$$

$$C_A = -\frac{K_1 + iK_2}{\left[1 - \frac{\Omega_1}{\Omega_2}\right]} \left\{ e^{-i\Omega_1\tau} - e^{-i\Omega_1(T+\tau)/2} \frac{\sin(\frac{1}{2}\Omega_1\tau)}{\sin(\frac{1}{2}\Omega_1 T)} \right\}$$

$$D_A = \frac{\Omega_1}{\Omega_2} \frac{(K_1 + iK_2)}{\left[1 - \frac{\Omega_1}{\Omega_2}\right]} \left\{ e^{-i\Omega_2\tau} - e^{-i\Omega_2(T+\tau)/2} \frac{\sin(\frac{1}{2}\Omega_2\tau)}{\sin(\frac{1}{2}\Omega_2 T)} \right\}$$

$$C_H = -C_A - \frac{i\dot{\theta}(0) + \Omega_2\theta(0) - \Omega_2 K_3}{\Omega_1 - \Omega_2}$$

$$D_H = -D_A + \frac{i\dot{\theta}(0) + \Omega_1\theta(0) - \Omega_1 K_3}{\Omega_1 - \Omega_2}$$

If pulsed radial thrusting is maintained long enough that the steady state oscillations are reached, then the periodic particular solution represents the boom response. The complete solution is needed to evaluate the initial transient response, and to determine the maximum deflections encountered in a maneuver.

Figure 5 presents numerical results using these equations. The thrust level produced 0.4 ft/sec acceleration (corresponding to 10 lbs thrust), and thrust was kept on for 90° of rotation. The boom lenght was ℓ = 23 ft with a natural frequency of Ω = 2.756 rad/sec (corresponding to a 1-1/8 inch STEM boom). The boom offset r_0 was chosen as 0.1 ft, which was considered large but possible, spacecraft spin was ω_0 = 20 rpm = 2.09 rad/sec, and the thrust angle γ = 0. The top two graphs in the figure give the response of $\theta_1(t)$ and $\theta_2(t)$ for zero initial conditions. The second two graphs give the periodic particular solution for θ_1 and θ_2, while the final graph gives the associated thrust-on intervals. Although the vertical axes are labeled as angles, the units on the axes are converted to tip deflection in feet. These results agree well with an independent analysis using a nonlinear multibody dynamics program.

5. REFERENCES

1. Longman R W & Fedor J V 1976, Dynamics of flexible spinning satellites with radial wire antennas, *Acta Astronautica* vol 3, 17-37.

2. EFME wire antenna, Mid-program progress report 746-403-PM-2, May 26, 1971, Fairchild Space and Electronics Division, Germantown, MD, NASA contract NAS5-11822 (derivation from UK galactic noise experimenters handwritten paper: Dipole antenna deployment).

3. Mathews J & Walker R L 1965, *Mathematical methods of physics* W A Benjamin, Inc., New York, 29-32.

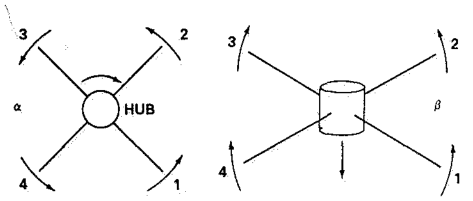

Figure 1. Spacecraft modes directly excited by operational maneuvers.

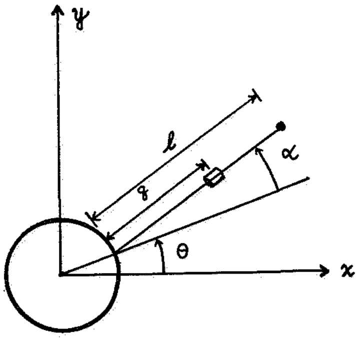

Figure 2. Symbols used for deployment and spin-up derivations.

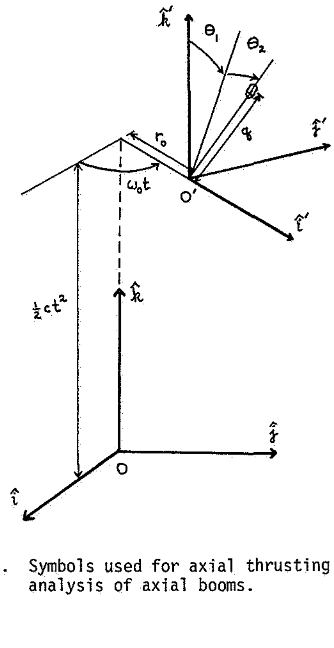

Figure 3. Symbols used for axial thrusting analysis of axial booms.

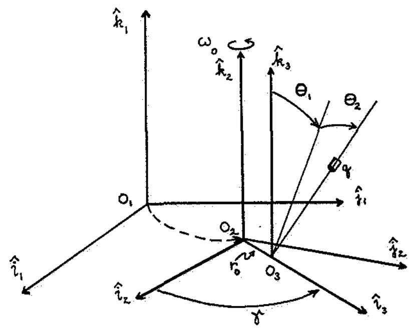

Figure 4. Symbols used for pulsed radial thrusting analysis.

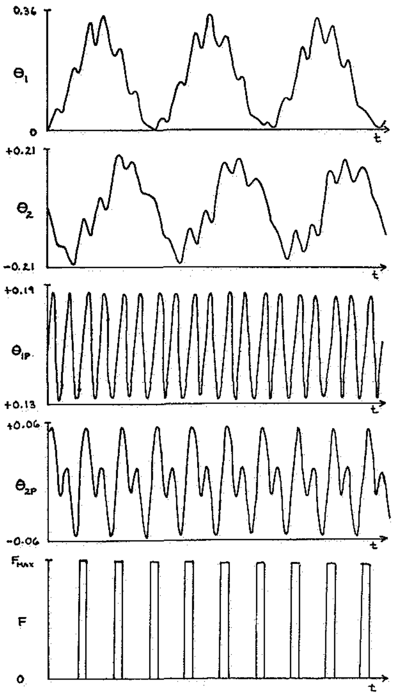

Figure 5. Axial boom response to pulsed radial thrusting.