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COMPUTATIONAL ASTROPHYSICS

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Introduction

Astronomy is an area of applied physics in which unusually beautiful objects challenge our imaginations to explain observed phenomena in terms of known laws of physics. It is a field that has stimulated the development of lots of physical laws and lots of mathematical and computational methods. Many statistical and numerical methods may be traced to 19th century efforts to determine orbits of solar system objects such as comets and asteroids. Numerical calculations were to be done by hand, with pencil and paper. Methods were needed that would guarantee a reliable orbit with minimal effort, based on imperfect observations that were not evenly spaced in time. In another area, some well-known and powerful stability techniques were developed to study the classical 3-body problem by mathematicians like Poincaré. The stability of the solar system is a problem that continues to challenge mathematicians. The story of Newton, apples, and the moon's orbit is well known to all of you.

Outstanding scientists were drawn to astronomical problems by their charm and beauty, not to deny practical usefulness, in an earlier era. The opening paragraphs of Maxwell's Adams Prize Essay (1859) on the structure and stability of Saturn's rings eloquently appeal to the charm of trying to understand a natural phenomenon for which no practical use could be imagined. He did that in an era when astronomy was the quintessence of "practical" science for navigation at sea. Techniques worked out to attack astronomical problems are useful in many other contexts, and they promise to remain useful for years to come.

Charm and beauty characterize many problems of present-day astronomy as well. As in earlier eras, astronomical problems drive us to consider phenomena in extraordinary physical conditions. Physical conditions in astronomical objects take on far more extreme values than can be mimicked in any terrestrial laboratory. Densities, for example, range from less than one atom per cubic centimeter ($\rho \sim 2 \times 10^{-24}$gm/cm^3) in interstellar space to nuclear densities ($\rho \sim 10^{15} - 10^{16}$gm/cm^3) in neutron stars, rising a bit higher if the star is on the verge of collapsing to a black hole. Temperatures range from around $100°K$ in molecular clouds to 10^{10} $°K$ at the centers of pre-supernova stars. Magnetic fields can be found that range from a microgauss in interstellar space to 10^{12} gauss at the surface of a neutron star. Magnetic fields are important because the interstellar medium tends to be a highly ionized plasma, thus to have high conductivity.

Boundary conditions are much cleaner in astronomical problems than in laboratory experiments as well. Astrophysical plasmas (e.g., the solar wind, solar atmosphere, interstellar medium) are not contaminated by boundaries as are laboratory plasmas. Problems in which laboratory boundary conditions differ from real-world situations are familiar to people working on aircraft design as well.

Astronomers have been in supercomputing for the past century and a half as well, if we define supercomputing to mean numerical computation on an unprecedented scale for its time. Imagine the numerical work in computing ephemerides for navigational purposes in the 19th and first half of the 20th centuries. An astronomical work still stands as a landmark from the days when it was first done with punched-card calculators: the "Coordinates of the 5 Outer Planets 1653-2060," by Eckert, Brouwer, and Clemence (1951). A sign of the times: five years ago, amateur astronomers took it as a challenge to repeat that calculation on PC's like the TRS-80. Fortunately, they could refer to the original work for methods and for numerical values that could be used to check their calculations.

Three features drive present research efforts that impel astronomers to seek the computational power of supercomputers.

(1) A complex interplay of many physical processes. Interactions are almost always nonlinear. General physical situations must be taken into account to fit observations.
(2) Higher quality observational material discloses details of flow and emission patterns that require complex models. Several factors contribute. (a) Improved (multi-element, low-noise) detectors, (b) Extension to new wavelength bands (radio, infrared, ultra-violet, x-ray, gamma rays), with (c) high resolution imaging and (d) spectroscopic resolution as well. These features come from excellent new instruments such as the VLA and VLBI in radio astronomy, from KAO and IRAS and the future SOFIA and SIRTF in infrared astronomy, from IUE and the Hubble Space Telescope in visible and ultra-violet wavelengths, and a variety of satellites for x- and gamma rays. European and Japanese satellites are coming on line as well, and we saw cooperative programs with them and with the Soviets in the recent Halley missions. Scientific use of all these superb instruments requires source modeling in at least as great detail as the instruments can yield.
(3) Reduction of raw observational data to a form comprehensible by people. Instruments like the VLA (a 27-element interferometer or synthetic aperture) produce what is essentially a Fourier transform of the source brightness distribution (in two spatial dimensions for each spectral passband for possibly as many as 256 spectral bands), corrupted by sky noise, by receiver and antenna noise, by a "dirty" raw synthesized beam, and messed up by the atmosphere. Other instruments produce data encoded differently, but the data reduction problem is much the same. High data rates exacerbate the problem. These newer instruments produce a lot of data in a short time, then they switch to a new source and produce a lot more data.

We see again the pattern that techniques developed for astronomical use have uses in other areas: Modern oil field exploration uses techniques roughly equivalent to turning the VLA upside down so its antennas detect signals coming from inside the earth. Some of the astronomical imaging methods such as CLEAN, self-calibration, hybrid mapping, etc., are finding use there. Maximum entropy restoration methods are being developed actively in both camps, as they are elsewhere.

Image restoration in radio astronomy is truly impressive. Within the past couple of years, a map of the radio source Cygnus-A has been produced with 1000 or more pixels along an edge, and with a dynamic range (ratio of brightest spot to faintest detail recorded) of about 10,000 to 1. This is in the presence of sidelobes that can range to 10% for a "dirty beam." It is difficult to look at that map without a tingle of admiration and excitement.

Current Computations

Computation has had an impact on nearly every area of astrophysics. Many of the problems of astrophysical interest have their origin in trying to understand the structure and aging (evolution) of stars. Particularly interesting computational problems arise with studies of the final death-throes of a massive star–a supernova outburst. A core like a neutron star (density around 10^{15}gm/cm^3, temperature about 100 Million degrees K, size about 10 miles across, about 1 solar mass or 10^{33} gm) is thought to pass a stability limit in its equation of state that leads to a collapse. A shock is sent outward into the surrounding envelope. Time scales are milliseconds (sound speed is nearly the velocity of light). Nuclear burning in the shock front transmutes most nuclei to the doubly magic Ni^{56} (28 protons and 28 neutrons). So many neutrinos are produced, and they carry such a large fraction of the energy, that neutrino opacity limits energy transport. Things become particularly interesting when the shock encounters the innermost nuclear-burning shell of the pre-supernova star where silicon is being burnt. The upshot is that a burst of energy (10^{54} ergs) is produced that makes a star as bright as an entire galaxy (10^{11} solar luminosity or about 10^{44} ergs/sec) that takes weeks or months to expand and cool, as the outburst gradually fades.

This is but one of the fascinating problems astrophysicists are tackling computationally today. Star formation, stellar evolution, and physics of the interstellar medium are others. Common threads run through the physics of all these situations. That physics includes (1) Radiative transfer (2) Transient fluid flows, and (3) Radiation hydrodynamics. This last term refers to situations in which radiation pressure dominates mass motions. Actual physical situations involve some cases where radiation dominates, others where fluid flows dominate, and transition cases. The fluid flows influence the radiation field and radiative transport as well. Strong shocks frequently occur in examples of astrophysical interest. The state of the art can cope with these cases, and nuclear burning is included in modern codes. Nuclear burning provides an energy source (and a sink in some cases). Nuclear reaction networks bring in several hundred nuclear species with vastly different lifetimes, so the problems are very "stiff." Magnetic fields have so far not been successfully included, so we cannot yet handle hydromagnetic cases with radiation and nuclear burning. But that is coming. It should be here in a couple of years.

Numerical relativity is another area receiving a lot of attention. Relativity here means general relativity–well known for numerical intractability because the nonlinearities affect the geometry of spacetime. Black holes were made to collide a few years ago (lots of gravitational radiation emitted). Star clusters going relativistic are just now being studied–but even that with restrictive symmetries because of numerical and conceptual difficulties. A star cluster goes relativistic when its central density and size become great enough to be near the Schwarzschild radius for a black hole.

We see once more that new methods have to be developed to handle extreme physical conditions that first come to our attention in astrophysical situations. Often these methods can help with problems in other scientific and technical areas, even if conditions are not so extreme. Algorithms must be robust and dependable.

Dynamics of Galaxies

Let me turn to another particularly beautiful area of computational astrophysics, the area of my own research, the dynamics of galaxies. Galaxies are beautiful both in photographs and in VLA maps, and they are dynamical objects. I will do this by describing an investigation that has been occupying our attention for the past couple of years. It is a good example of the method of numerical experimentation.

Experiments in terrestrial laboratories are out of the question for problems in the dynamics of galaxies as well. Distances are so great that there is no hope of going out beyond the object to get a look from the other side. There is no way to kick a galaxy to see if it bounces–to check its stability as you would check the stability (or robustness) of everyday objects in the laboratory. Even if we could, the time scales are so long we could not wait. Dynamical times are typically about 100 million years. The need for experimental checks on analytical theories is as great in galaxy dynamics as in any area of physics. Numerical experiments, based on large scale computations, are the closest thing we have to the laboratory experiments of other parts of physics. They are much closer in style and spirit to laboratory experiments than to conventional (analytic) theory. Numerical experiments are initial value problems. They are defined by the initial conditions. The same rules are used for carrying out all experiments, and those rules contain as much of the essential physics of galaxies as possible. Once an experiment is started, it runs to "completion" with no interference.

Studies in the dynamics of galaxies produce strikingly beautiful objects as well as charming dynamical insights. Galaxy dynamics is challenging because one's guesses are so often wrong. This happens, of course, because self-consistent systems have so many options open to them that they usually select one you hadn't thought about. It is exciting as you begin to see which option the system selected and as you try to figure out why it chose that option.

The investigation reported here started out as a study of the dynamics of dark lanes in the so-called dark-lane elliptical galaxies. This investigation was started by Dr. Althea Wilkinson of the University of Manchester in England, and it was carried out by a team from which Dr. Bruce F. Smith of NASA-Ames was a driving force. Some totally unexpected and surprising phenomena turned up–phenomena that remind one of the strange things observed at the center of our own Galaxy. I want to concentrate on those phenomena.

The experiments started with a disk of particles embedded within an oblate spheroidal galaxy. Both the disk and the oblate spheroid were formed of particles, 400 000 in all. The experiments consist of following the dynamical development of this system for some time, usually several dynamical time periods, by means of a fully self-consistent fully three-dimensional n-body integration. The bulk of the mass is in the oblate spheroid, and it represents the elliptical galaxy. The disk represents the dark lane. It contains little mass (1000 particles), and notwithstanding its dominant optical appearance it has little, if any, dynamical effect on the main galaxy.

The disk was flat at the start. Its normal formed an angle of 45° with the spheroid axis. Its center coincided with the center of the galaxy. Disk particles orbit around the galaxy center so the disk is centrifugally supported against the gravitational forces of the galaxy. Disk particles have different velocities at different distances from the center, so the disk is said to "rotate differentially." It also precesses differentially, and it soon becomes warped, taking on a sequence of beautiful shapes.

We routinely make motion pictures that show the dynamical development of the systems we study. Shapes and motions are so complex that one cannot understand them otherwise. It is safe to say that every significant discovery we have made in 10 years of experimentation has started from some feature first noticed in a motion picture, whether on film or on a graphics display. Good graphics is essential for an experimenter to understand his own results. Of course, motion pictures are great for showing results to others as well, and a motion picture derived from an experiment run on the NAS Cray-2 will be shown.

The unexpected feature showed up as we were studying an earlier version of this system on the then new IRIS graphics displays at Ames. We had zoomed in for a tight view of the center, watching temporal developments. The center was whipping about. We were concerned lest this be a numerical instability. A motion picture showing a close-up view of the disk center was made as one of our checks. That motion picture will be shown. The motions that troubled us are quite small. The centermost particle moves only as far from the center as the radius of a sphere that would contain but 4×10^{-4} of the galaxy's mass. Nonetheless, it is troubling to an experimenter until he can figure out what causes it. If caused by a numerical instability, it could signal a trouble with the entire sequence of experiments we've done over the past 10 years.

Checks whether we were dealing with a numerical instability led us to study $l = 1$ oscillations of galaxy models, to study the stability of the disk with the galaxy immobilized (i.e., the underlying galaxy was no longer self-consistent), to study the disk in a harmonic oscillator potential rather than the galaxy potential, and to carry out a variety of other checks. The $l = 1$ mode had too low a Q to fit the experiment. Experiments with the galaxy immobilized or with a harmonic potential showed that self-gravitation of the disk was not responsible. Next, we conducted experiments on the galaxy alone, with the disk removed. We had to invent a way to look for motions of the central parts in order to do this. The method used was to locate the extremum of the gravitational potential. We found that the center, as defined by the potential, moved about the center as defined by the mass (the centroid). The most dense part of the galaxy, indicated by the potential center, need not remain fixed, even though the equations of motion require that the mass centroid remain fixed.

Fairly early in this sequence of checks, we plotted the trajectory of the normal to a little patch at the center of the disk. That plot is shown in Figure 1. It looks like the plots of combined precession and nutation in rigid body motion in your elementary mechanics text. Indeed, that is just what it shows–combined precession and nutation. It is not as clean as the motion shown in the mechanics text, but it is the trajectory of a genuine mechanical motion. It is not the trajectory of a numerical instability.

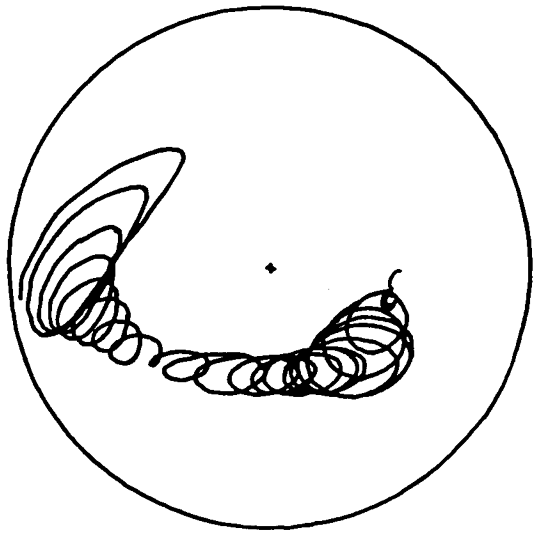

Figure 1. Trajectory of normal to the center of disk as seen on the unit sphere. The trajectory starts at the right side (small amplitude) and winds downward and to the left, ending on the left side with large amplitude.

So it's not a numerical instability. But then, what is it? We finally nailed this down with the experiments on the NAS Cray-2.

The potential near the center of a galaxy is harmonic–it is the potential of a (possibly anisotropic) harmonic oscillator. A particle, initially at rest at the center, feels tugged first in one direction and then in another, by the wanderings of the potential center. It acts like a harmonically bound particle in Brownian motion. That is a well-known problem. We checked whether this might be the character of the motions we were observing by integrating the response of a harmonically bound particle to forces it would feel due to the wandering center, and found those motions to match the wanderings of the centermost particle of our disk fairly well.

Now we come to the nailing. Experiments with different numbers of particles showed that the amplitude by which the potential center wanders about the mass center varied inversely with the square root of the number of particles in the galaxy. Separate checks, with a sequence of initial loads produced by different runs of random numbers produced a scatter of potential center positions about the same as the early stages of an actual integration. That confirms suspicions that $\sqrt{n}$ noise causes the potential center to differ from the mass center. Both checks were run with 25,600, with 100,000, and finally with 400,000 particles. The scatter for the integration increases at later times.

The second part of the check is shown in Figure 2.

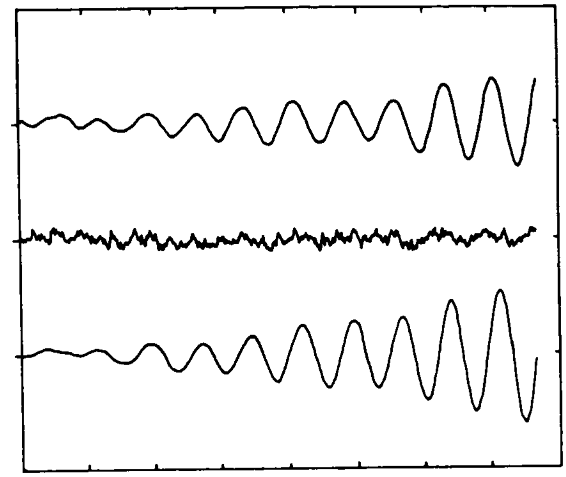

Figure 2. Plots of position *vs.* time. Time in the integration is plotted horizontally, the x-component of position vertically. The motion of the centermost particle is shown in the top trace, the potential center wandering in the middle trace, and the motion of a harmonically bound test particle subject to forces produced by that potential center wandering in the bottom track. All three tracks have the same vertical and horizontal scales.

The actual motion of the centermost particle is shown in the topmost trace, and the potential center wandering in the middle trace. Finally, a separate integration of a harmonically bound test particle under the tuggings of the actual potential center wanderings is shown as the bottom trace. Notice the similarity of the top and bottom tracks–even to shapes of individual wiggles. The amplitudes are a bit different, indicating that something or other about the galaxy affects the centermost particle at late times. Further, we used oscillation frequencies from an earlier experiment for this comparison, rather than re-determining the frequencies from the actual experiment. That accounts for the phase difference in the two tracks at the end of the experiment. The increasing amplitude looks more like a linear growth rate (up to a limiting amplitude) than like the square-root of time amplitude of the classical harmonically bound particle, suggesting that something more is going on. That can be seen in the center track, where one can imagine that the potential center wanderings near the end are beginning to follow the particle. The system is actually unstable, with feedback coming from the potential center's following the driven particles. But that is a physical, not a numerical, instability.

The experiments are vindicated, but our next task is to determine whether this phenomenon is important in real galaxies. As mentioned earlier, the wanderings are very suggestive of things that happen at the center of our Galaxy, so we could be dealing with an important physical effect. But that involves a separate set of arguments, one that necessarily hinges on of astronomical details. We postpone that to a conference on the dynamics of galactic nuclei.

I wish to thank the organizers of the NAS Dedication for the invitation to participate and to show you some of the beautiful results Bruce Smith and I have obtained already with the NAS facilities. You can imagine our hopes and excitement over future prospects. This work has been partially supported by Cooperative Agreement NCC 2-265 between NASA-Ames Research Center and the University of Chicago.