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Minimum Time Attitude Slewing Maneuvers of a Rigid Spacecraft*

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ABSTRACT

The problems of large-angle attitude maneuvers of a spacecraft have gained much consideration in recent years [1-8]. In these papers, the configurations of the spacecraft considered are: (1) completely rigid, (2) a combination of rigid and flexible parts, or (3) gyrostat-type systems. The performance indices usually include minimum torque integration, power criterion, and frequency-shaped cost functionals, etc. Also some of these papers used feedback control techniques. In this paper, we try to concentrate on the minimum time slewing problem of a rigid spacecraft, i.e., minimizing the total slewing time period

$$t_f = \int_0^{t_f} dt \qquad (1)$$

The control torques have their upper and lower limits, respectively

$$u_{j_{min}} \leq u_j \leq u_{j_{max}} \qquad j=1,2,\ldots n \qquad (2)$$

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In Ref. [2], the author studied the rapid torque-limited slewing of SCOLE about a single axis (x-axis) about which the spacecraft has a small moment of inertia. The control torque about this axis is of a bang-bang type or a bang-pause-bang type. The author computed the slewing motion on the simplified model of the rigidized SCOLE[1], then worked on the practical rigidized model (with nonzero products of inertia); hence, this leads to a large error of the attitude after the slewing. Also it seems that no details were given for the controls about the other two axes (y,z).

In the present paper, we apply optimal control theory (Maximum Principle) to the slewing motion of a general rigid spacecraft (include the rigidized SCOLE, without simplification). The slewing motion need not be restricted to a single-axis slewing. The attitude error after the slewing can be made as small as required. Control torques about all three axes are computed.

Generally speaking, minimization of t_f under the constraints (2) will result in a so-called two point boundary value problem in which several controls (at least one) will reach their bounds during the slewing time, t_f. Particularly, if one considers the case of minimum time rotation of a rigid spacecraft about one of its principal axes of inertia, the control torque about this axis is of a bang-bang type, while other torques remain zero. On the other hand, if the slewing motion is not about a single principal axis, none of the controls remain zero, but we can reason that at least one of the control inputs reaches its bounds (saturation level) during the period, t_f, in order to minimize it.

To handle the problem in which some controls reach their bounds and others do not, we introduce an additional cost function

$$J = \frac{1}{2} \int_{0}^{t_f} u^T R u dt \qquad (3)$$

where u is the control vector, T denotes the transpose of a vector, and R is a proper weighting function matrix. From Refs. [3] and [8] we can see that, if we use only (3) as a criterion, and the t_f is long enough, the control torques are approximately linear functions of time, and the controls will not reach their saturation levels. But if we shorten t_f for our purpose, some of the controls must reach their bounds in order to contribute more effort to the slewing. By continuing the shortening of t_f, we can get a particular value, t_f^*, during which at least one of the controls remains as bang-bang with one switching point, while others are generally not of the bang-bang type. This value, t_f^*, is called the minimum time which is required.

The equations for the system are composed of the Euler dynamical equations in the spacecraft body axes and the quaternion kinematical equation. By introducing the costates for the quaternion and the angular velocity, we can form the Hamiltonian of the system and obtain the optimal controls (with upper and lower bounds). The necessary conditions for the optimal control leads to the two point boundary value problem. The associated boundary conditions are:

$q(0)$, $\omega(0)$, $q(t_f)$. $\omega(t_f)$ known

$p(0)$, $r(0)$, $p(t_f)$, $r(t_f)$ unknown

The time t_f is also to be determined.

We choose the following procedure to solve our problem: first, select a fixed value of $t_f^{(0)}$, solve the combinational dynamical equations for the angular velocities, quaterion components and costates using a quasilinearization algorithm; if one of the controls is of a bang-bang type, then stop the computation. If not, then shorten the t_f by a reasonable amount and start the quasilinearization algorithm again.

Before starting the quasilinearization algorithm, we need to guess the initial unknown boundary values of the costates $p(0)$, $r(0)$. Since the quaternion is subject to a constraint (i.e. its norm equals to unity), this leads to the norm of the associated costates being equal to a constant (but not equal to unity). This extra constant is usually treated as an unknown and is determined by iteration [3]. This results in more computational effort. However, through several steps of reasoning, we can show that this unknown constant can be easily selected without changes in the optimal controls [8]. Therefore, this property simplifies the problem and saves computational time.

By means of Euler's eigenaxis rotation theorem, from the known attitudes at the initial and final time, $q(0)$ and $q(t_f)$, we can find a unit vector (eigenaxis) $\bar{\varepsilon}$ which is fixed in both the body axes and inertial coordinate system, and a rotation angle, θ^*. Then the attitude changes from $q(0)$ to $q(t_f)$ can be realized by rotating the spacecraft about the axis, $\bar{\varepsilon}$, through the angle, θ^*. By properly choosing $\theta(t)$ as a function of time, which satisfies the boundary conditions $\theta(0)=0$, $\theta(t_f)=\theta^*$, we obtain a nominal trajectory of the attitude slewing motion, which will be used as the starting values of the quasilinearization algorithm.

The starting value $t_f^{(0)}$ needs to be made as close to the minimum time, t_f^*, as possible. This can be done by using similiar techniques. Suppose the slewing motion is a rotation about the vector, $\bar{\varepsilon}$, through an angle, $\theta(t)$. Then, we get three similiar equations for $\theta(t)$:

$$a_i \ddot{\theta} = b_i \dot{\theta}^2 + c_i \tau_i \qquad i=1,2,3 \qquad (4)$$

$$\theta(0) = 0, \quad \theta(t_f) = \theta^*$$

where a_i, b_i, c_i are constants, τ_i is the control about the body axis

$$|\tau_i| \leq 1 \qquad i=1,2,3 \qquad (5)$$

For each i we have a minimum time control problem; we then get three minimum time values, t_{fi}^*, $i=1,2,3$; then, we can select the longest of them as our initial guess for t_f.

Finally, we apply these methods to the SCOLE slewing motion[1]. Fig. 1 shows the configuration of the SCOLE. Due to the configuration of the SCOLE, the inertia matrix is not diagonal. The control variables include three control moments on the Shuttle and two control forces on the reflector. We have the following numerical results:

(a) Assuming control torques only on the Shuttle and a diagonal inertia matrix, slewing the SCOLE about one of three principal axes through $\theta^* = 20$ deg, from rest to rest. The result is exactly the same as described above, i.e., the control torque about the axis is of a bang-bang type, while the other control components remain zero.

(b) Using the same conditions in case (a), except that using the inertia matrix of the SCOLE (non-diagonal), the nominal slewing motion is about each of the three spacecraft axes, respectively. Fig. 2 and Fig. 3 give the control torques of these slewing motions.

(c) Following the case (b), add forces on the reflector, Fig. 4 and Fig. 5 show these results.

Conclusions

(1). There is a good agreement between the initial guessed value of t_f and the value of t_f to which the algorithm converges in the case (b).

(2). The guessed value of costates: p(0), r(0) are sufficient to arrive at values of these parameters supplied by the algorithm.

(3). The methods used in this paper are easily implemented for practical control sources, which may have more constraints. For example, in the neighborhood of the switching point of the bang-bang control, we can replace the jump by a linear function of time.

(4). The control profiles obtained in this paper give us a good reference for future use. For example, an extension to the minimum time slewing motion of the SCOLE model containing both rigid and flexible components is planned.

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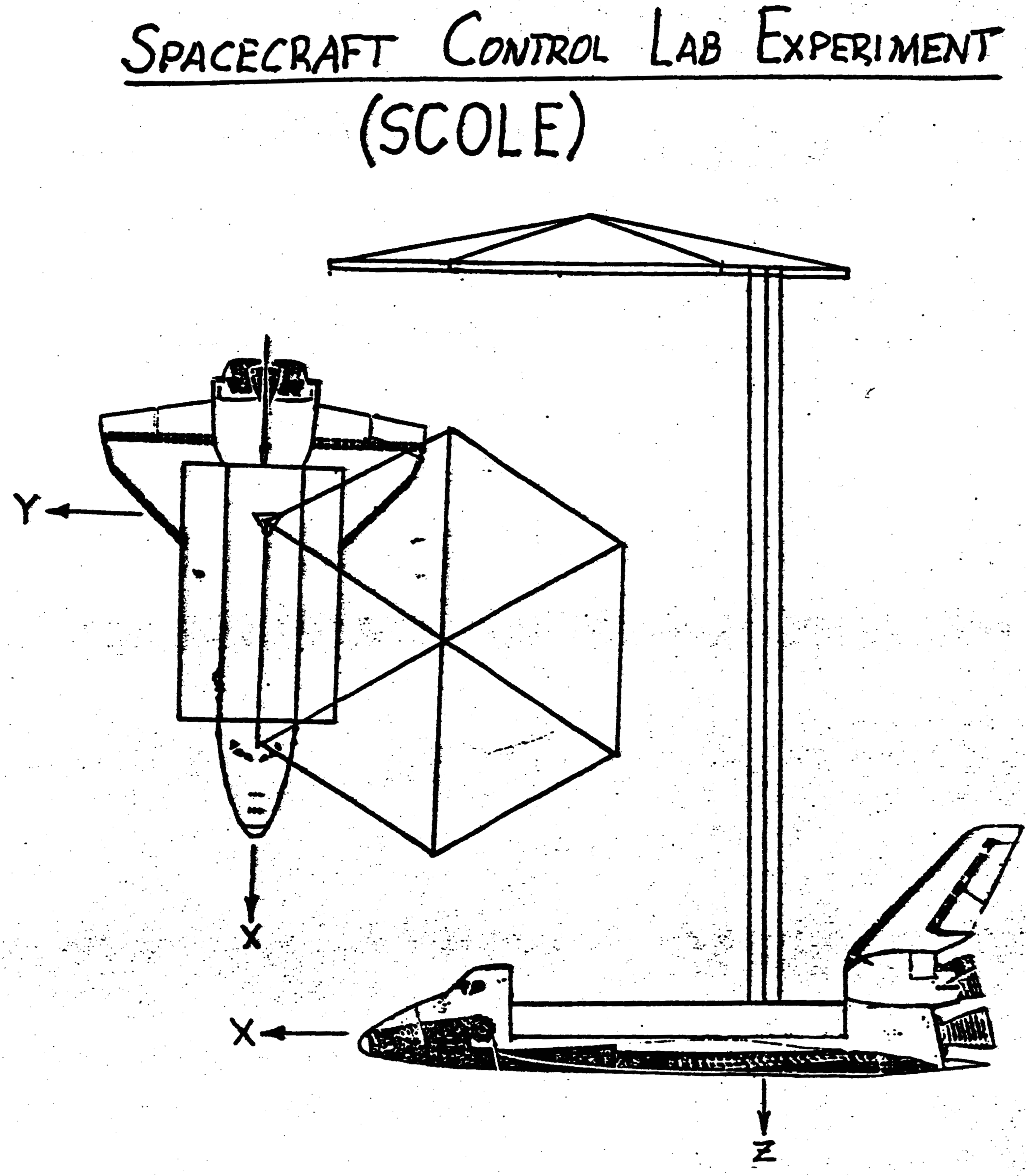
Figure 1. Drawing of the Shuttle/Antenna Configuration.



CONTROL TORQUES U (X-AXIS SLEWING)

(NO FORCES F)

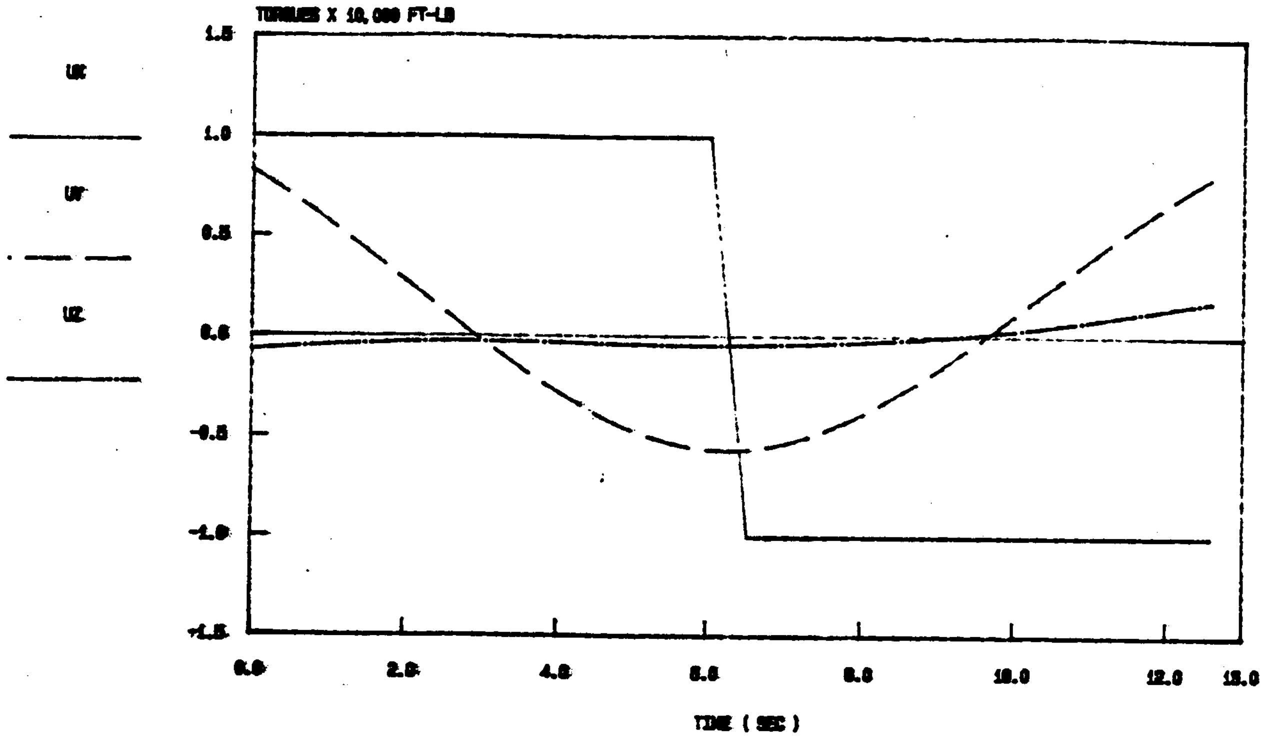


Fig 2

CONTROL TORQUES U (Z-AXIS SLEWING)

(NO FORCES F)

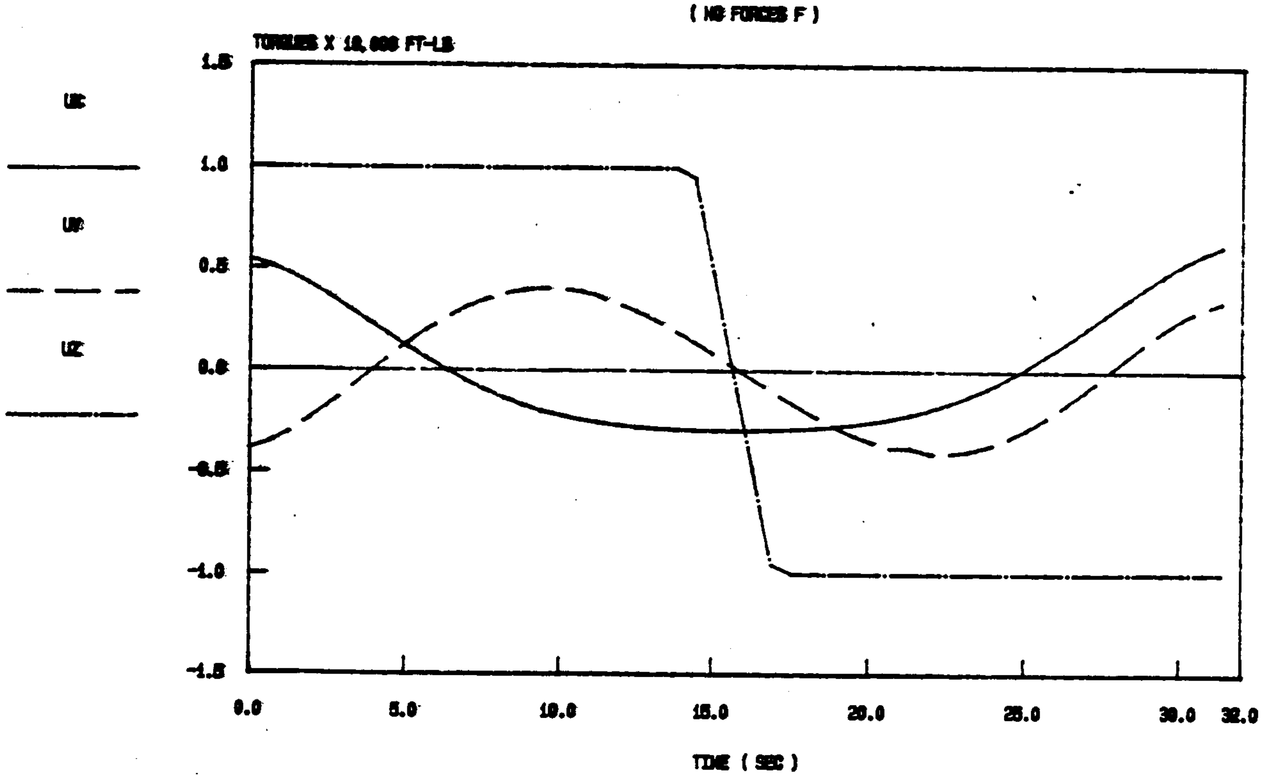
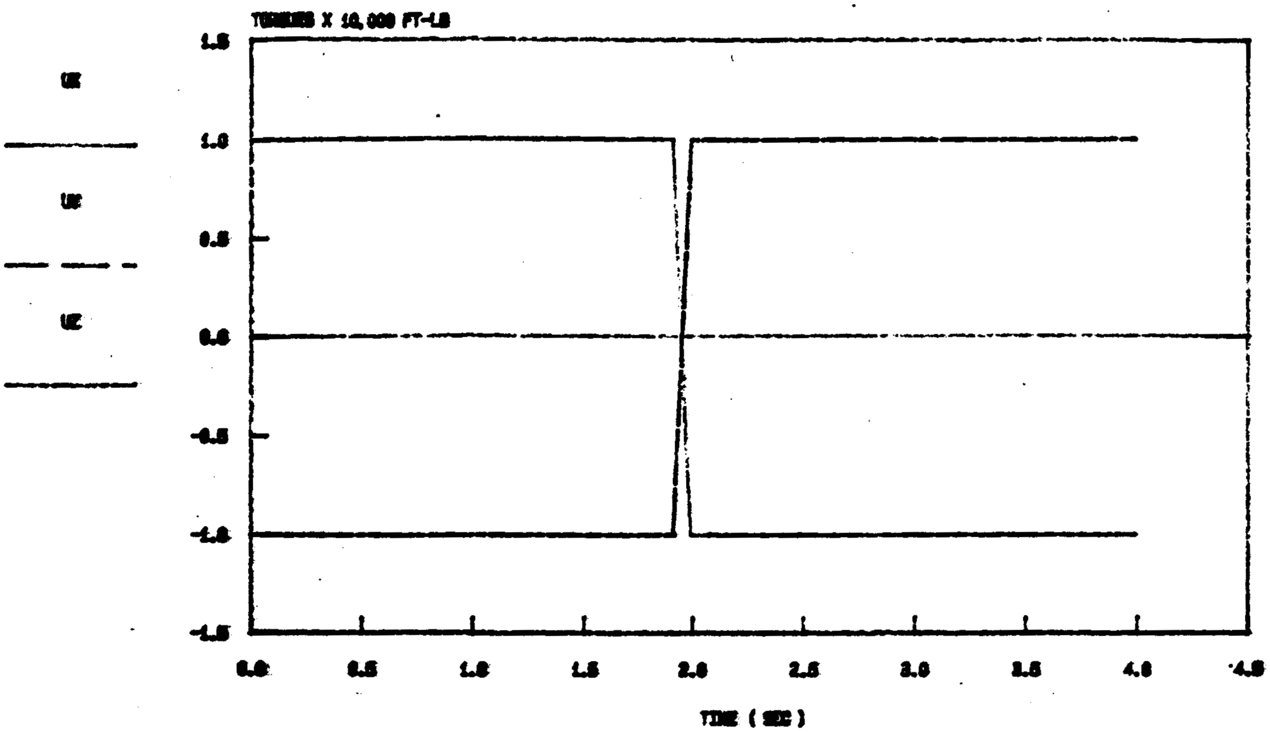


Fig. 3

CONTROL TORQUES U (X-AXIS SLEWING)



CONTROL FORCES F (X-AXIS SLEWING)

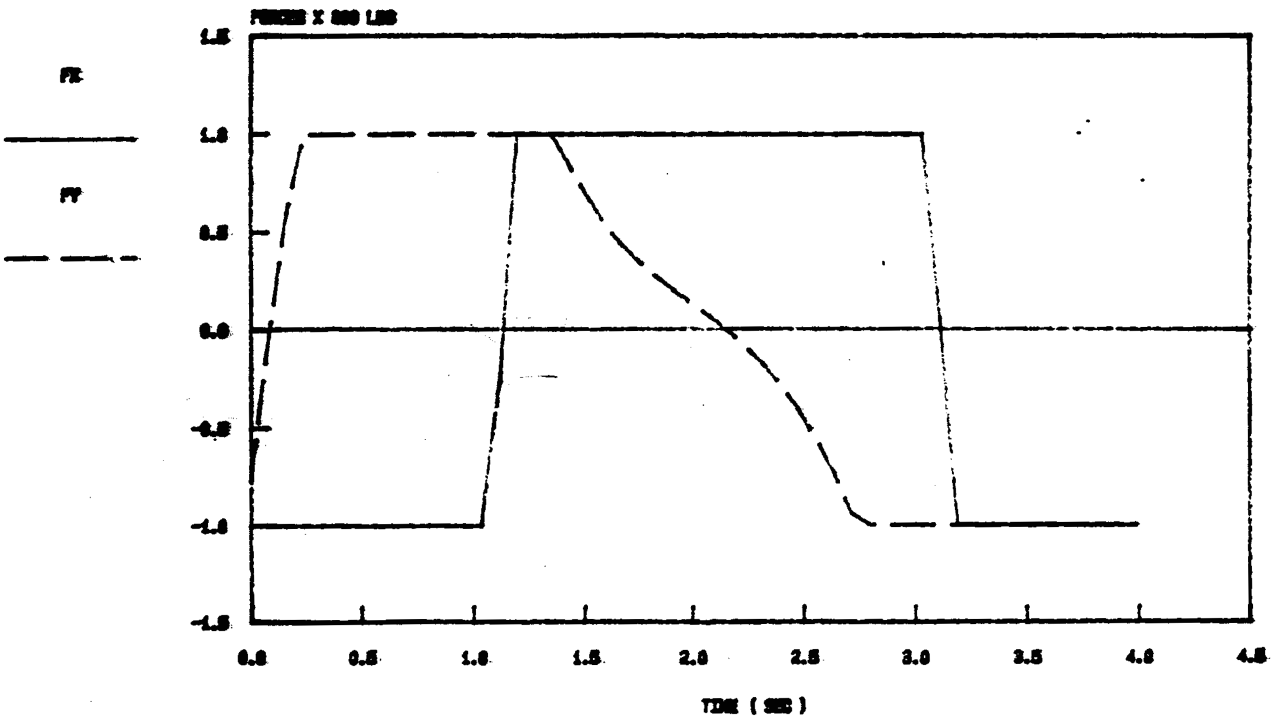


Fig. 4

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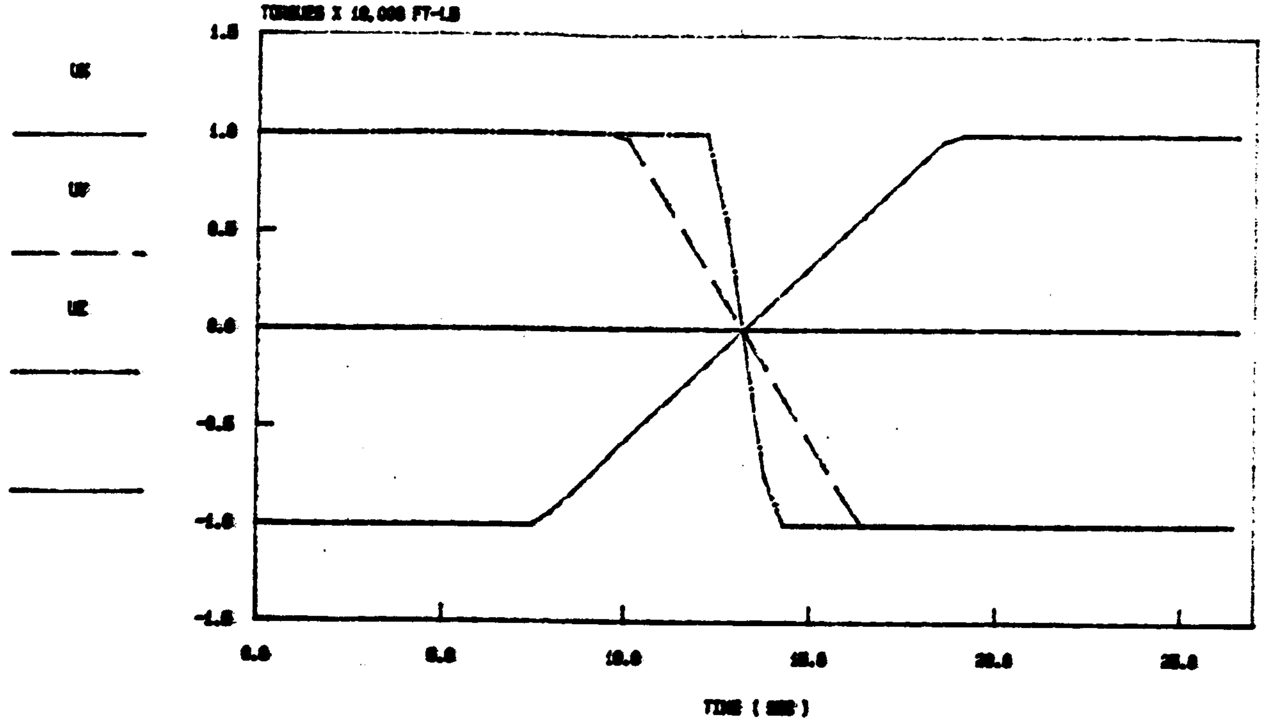
CONTROL TORQUES U (Z-AXIS SLEWING)



CONTROL FORCES F (Z-AXIS SLEWING)

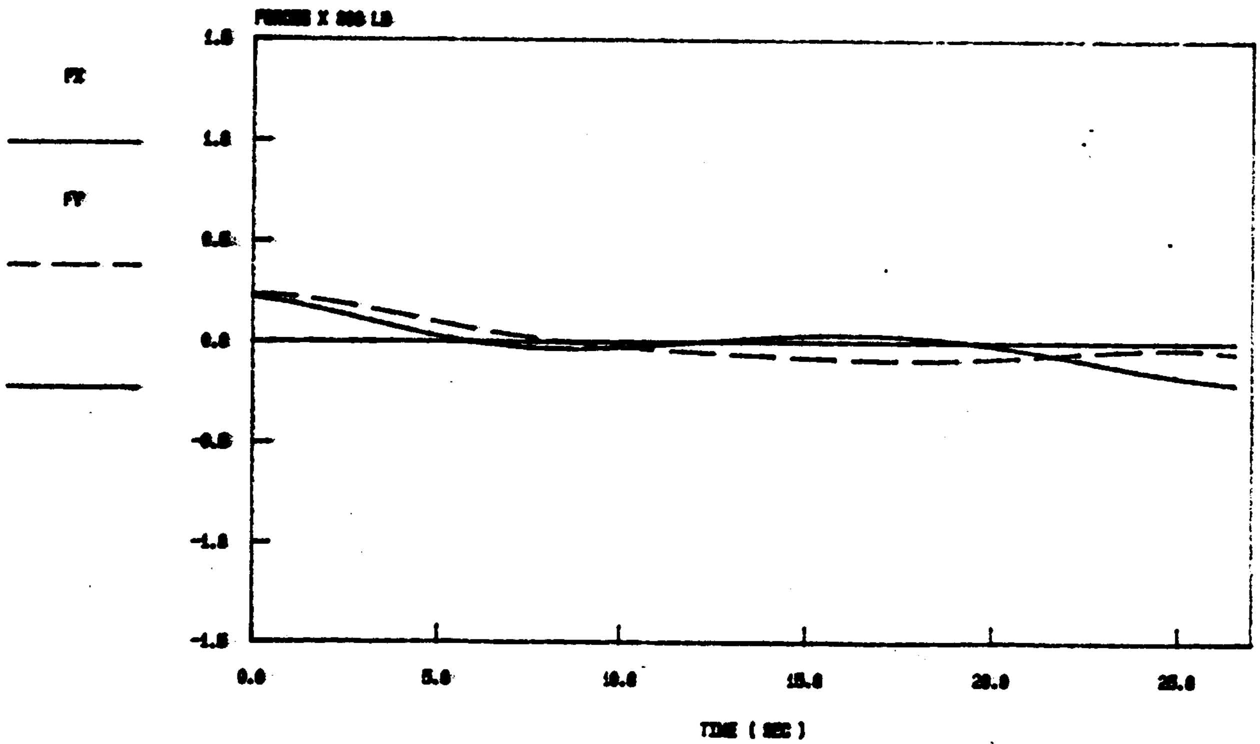


Fig. 5