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NUMERICAL SOLUTIONS OF THE COMPLETE

NAVIER-STOKES EQUATIONS

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During this period, most of the issues pertaining to the Hyperbolic Navier-Stokes equations have been sorted out and an Abstract (copies enclosed) was submitted to the CFD meeting in Honolulu.

Work is continuing on incorporating the viscous terms into the reacting flow code. When this is completed and checked, we intend to incorporate a chemistry model based on the complete H_2-O_2 reaction. Moreover, all transport properties will be based on the 12-6 potential.

BOUNDARY CONDITIONS FOR THE NAVIER-STOKES EQUATIONS*

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ABSTRACT

Using ideas from kinetic theory, the Navier-Stokes equations are modified in such a way that they can be cast as a set of first order hyperbolic equations. This is achieved by incorporating time dependent terms into the definition of the stress tensor and the heat flux vectors. The boundary conditions are then determined from the theory of characteristics. Because the resulting equations reduce to the traditional Navier-Stokes equations when the steady state is reached, the present approach provides a straightforward scheme for the determination of inflow and outflow boundary conditions. The method is validated by comparing its predictions with known exact solutions of the steady Navier-Stokes equations.

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Introduction and Approach

Solutions of the Navier-Stokes equations have not matured to the point where one can routinely simulate transition, separation and turbulence. One of the major obstacles that prevents us from achieving these goals is our inability to formulate accurate inflow and outflow boundary conditions.[1] Attempts at formulating such conditions centered around the assumption that, in the far field, the governing equations reduce to a simple form. Such forms may include a wave equation, a Poisson's equation or, some other simple equation. Asymptotic solutions of these equations are then constructed and used to provide boundary conditions for the Navier-Stokes equations at the computational boundaries.[2-8]

Because viscous effects are important in the vicinity of bodies and in their wakes, it is not clear that the Euler equations (or a simpler subset) should govern the far field behavior especially in the wake region. Because of this and the desire to provide a simple approach to this complicated problem, attention is focused in this work on modifying the physics of the problem. The modification is such that, if one uses time marching methods to obtain a steady state solution, the net result of the approach is to provide a different path of convergence to the steady state.

Using index notation, the conservation of mass, momentum and energy equations can be written as

$$\frac{\partial \rho}{\partial t} + \frac{\partial}{\partial x_i}\left(\rho u_i\right) = 0 \tag{1}$$

$$\frac{\partial}{\partial t}\left(\rho u_i\right) + \frac{\partial}{\partial x_j}\left(\rho u_i u_j + p\delta_{ij} + \sigma_{ij}\right) = 0 \tag{2}$$

$$\frac{\partial}{\partial t}(\rho E) + \frac{\partial}{\partial x_j}\left(\rho u_j H + u_i \sigma_{ij} + q_i\right) = 0 \tag{3}$$

where ρ is the density, u_i is the velocity component in the direction of x_i, E and H are the total energy and enthalpy per unit mass, p is the pressure, σ_{ij} is the stress tensor, q_i is the heat flux vector and δ_{ij} is the Kronecker delta. Expressions for σ_{ij} and q_i are needed to close the above system. In the traditional Navier-Stokes equations, the stress tensor and the heat flux vector have the form

$$\sigma_{ij} = -2\mu\left\{\frac{\partial u_i}{\partial x_j}\right\} \tag{4}$$

$$q_i = -\lambda\ \partial T/\partial x_i \tag{5}$$

where

$$\left\{A_{ij}\right\} = \frac{1}{2}\left(A_{ij} + A_{ji}\right) + \left(\frac{K}{2\mu} - \frac{1}{3}\right)\delta_{ij}A_{mm} \tag{6}$$

with μ, λ and K being the coefficients of shear viscosity, heat conductivity and bulk viscosity. The first order system of equations (1) - (5) cannot be hyperbolic because of the absence of time derivatives in equations (4) and (5). A necessary (but not sufficient) modification that may make the system hyperbolic is the addition of time dependent terms to equations (4) and (5), i.e., change them to

$$\alpha \quad \frac{\partial \sigma_{ij}}{\partial t} + 2\left\{\frac{\partial u_i}{\partial x_i}\right\} + \frac{\sigma_{ij}}{\mu} = 0 \tag{7}$$

$$\beta \quad \frac{\partial q_i}{\partial t} + \left(\frac{\partial T}{\partial x_i} + \frac{q_i}{\lambda}\right) = 0 \tag{8}$$

where α and β are unknown parameters. It is seen from equations (7) and (8) that when the steady state is reached one will recover equations (4) and (5).

The α and β have to be chosen in such a way that the resulting system is hyperbolic. Rather than make aribtrary choices, attention was focused on the results of kinetic theory. As is well known, the stress tensor and the heat flux vector can be determined from the solution of the Baltzmann equation.[9] The traditional Hilbert or Chapman-Enskog methods yield equations similar to equations (4) and (5). On the other hand, the thirteen moment method of Grad[10] yields equations similar to equations (7) and (8). By comparing equations (7) and (8) with the results of the thirteen moment method (equations (7.5-7) and (7.5-8) of Ref. 9), it is seen that α and β should be chosen as

$$\alpha = 1/p \qquad , \qquad \beta = 1/c_p p \tag{9}$$

where c_p is the specific heat at constant pressure.

Using the above choices, the governing system of equations, (1)-(3), (7)-(8) can be written as

$$\frac{\partial U}{\partial t} + A \frac{\partial U}{\partial x} + B \frac{\partial U}{\partial y} + C \frac{\partial U}{\partial z} = F(U) \tag{10}$$

where

$$U = \left\{\rho,\ \rho u_i,\ \rho E,\ \sigma_{ij},\ q_i\right\}^t \tag{11}$$

with A, B and C being appropriate matrices. In the most general case, some of the eigenvalues of A, B and C cannot be determined in closed form. It was necessary to use numerical computations to show that all of the eigenvalues were real. Thus, the resulting system is hyperbolic.

The presence of source terms in the resulting formation renders the scheme somewhat stiff. To overcome this problem a semi-implicit treatment, similar to that of Ref. 11, was employed. A finite volume, cell-centered, formulation and a four-step Runge-Kutta time stepping scheme, similar to that of Ref. 12, is used to obtain the solution. Local time stepping and residual smoothing[13] were employed to accelerate convergence to the steady state.

Boundary Conditions

In order to demonstrate the validity of this approach, it is necessary to compare its predictions with some of the well-known exact solutions. As an illustration, the flow past a semi-infinite plate (Blasius problem) is considered. Because the normal stresses and the axial heat flux are very small, a "thin layer" approximation in which σ_{xx}, σ_{yy} and q_x were assumed negligible was developed. For this case, it was possible to derive analytical expressions for the eigenvalues of the matrices A and B.

It may be recalled that the eigenvalues of A for the two-dimensional Euler equations are u, u, u±a with v replacing u for the eigenvalues of B. In the present case, where the thin layer approximation is used, the eigenvalues of A are

$$u,\ u \pm a,\ \left[u \pm \left(u^2 + \frac{4a^2}{\gamma}\right)^{\frac{1}{2}}\right] / 2 \tag{11}$$

where a is the speed of sound. The matrix A can be written as

$$A = P \Lambda P^{-1} \tag{12}$$

where Λ is a diagonal matrix whose elements are the eigenvalues of A, P is a matrix whose columns are the eigenvectors of A and P^{-1} is the inverse of P. The characteristic variables are given by

$$W = P^{-1} U \tag{13}$$

or

$$W_1 = - \frac{(\gamma - 1)\ \rho uv\ \sigma_{xy}}{p^2} + \frac{S - S_\infty}{c_v} - \frac{(\gamma - 1)(\sigma_{xy})^2}{p^2}$$

$$W_2,\ W_3 = u \pm \frac{2a}{\gamma-1}$$

$$W_4, W_5 = \frac{1}{2}\rho v \left[u \pm \left(u^2 + \frac{4a^2}{\gamma}\right)^{\frac{1}{2}}\right] + \delta_{xy} \tag{14}$$

where S is the entropy.

At boundaries normal to the x-axis, equation (10) may be approximated by

$$\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x} = F(U) \tag{15}$$

Multiplying equation (15) by P^{-1}, using eq. (13), and assuming that P^{-1} is aprpoximately constant, one obtains

$$P^{-1}\left(\frac{\partial U}{\partial t} + A\frac{\partial U}{\partial x}\right) = \frac{\partial W}{\partial t} + \Lambda\frac{\partial W}{\partial x} = P^{-1}F \tag{16}$$

If a steady state solution is desired, the boundary conditions are given by

$$\Lambda\frac{\partial W}{\partial x} = P^{-1}F \tag{17}$$

Upwind differencing is used to express the derivatives at the boundary.

Results and Discussion

Results for a flow past a flat plate at a Mach number of .3 and a Reynolds number of 5,000 are presented. Figure 1 shows a comparison of the dimensionless velocity in the axial direction, as a function of the Blasius coordinate with the Blasius solution. Similarly, Figure 2 compares the calculated C_f, the skin friction coefficient, as a function of Re_x with the result obtained from the Blasius solution. Good agreement is indicated in both cases. Figure 3 is taken from Ref. 14 and is intended to compare our results with those of calculations based on the traditional Navier-Stokes equations. Again, good agreement is indicated.

As a further validation of this approach we hope to include in the final paper the calculation of subsonic separated flow past a cylinder.[15] The wake of the cylinder is dominated by a Karman-Vortex street. Because of lack of adequate boundary conditions, the problem proved to be difficult to simulate. The present approach, with boundary conditions provided by equation (16), and similar relations, is highly suited for the solution of this problem.

In conclusion, we have developed a rather straightforward and general approach for determining inflow and outflow boundary conditions for the Navier-Stokes equations. It is hoped that this approach will make it possible to tackle more difficlut problems involving transition, separation and turbulance.

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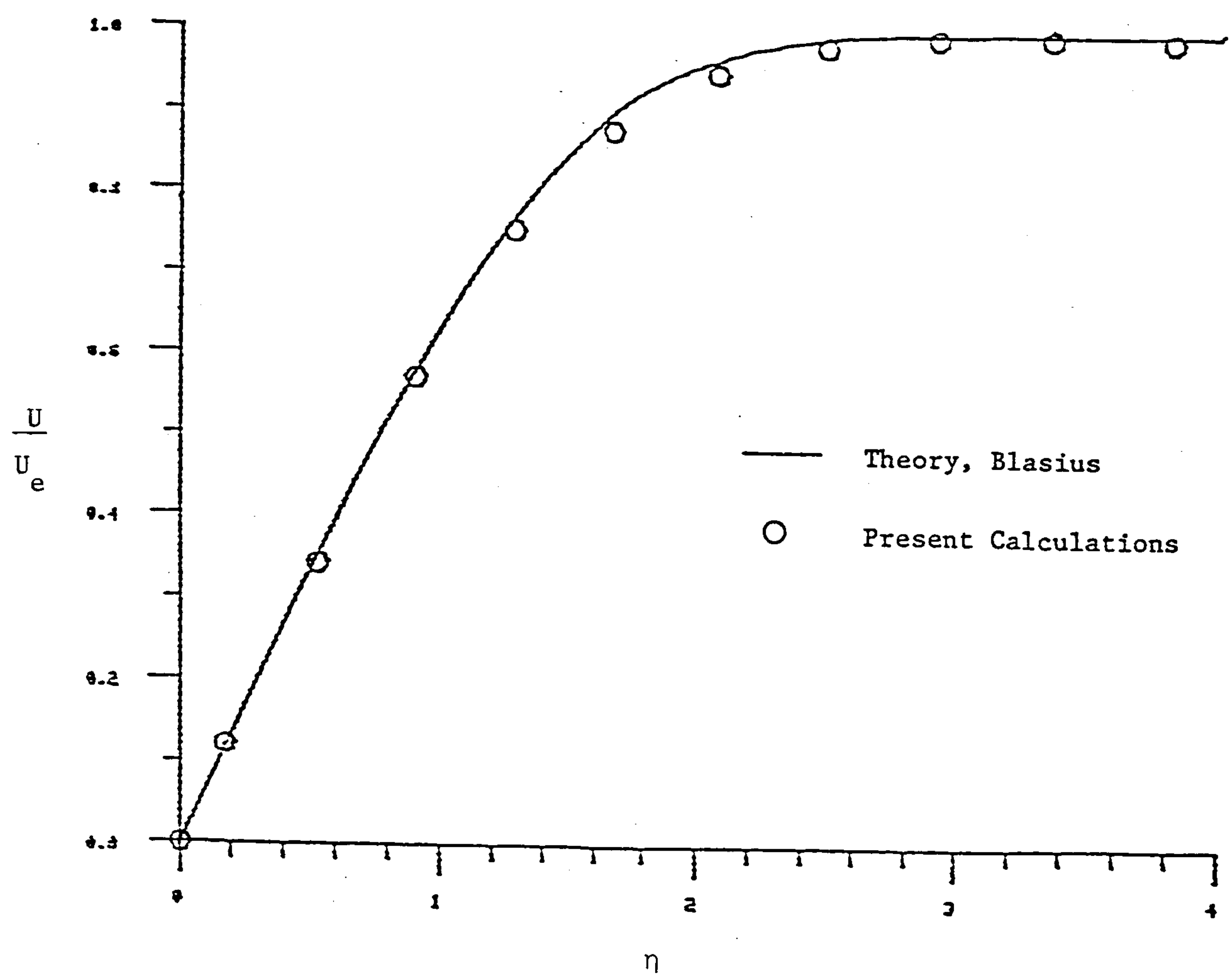

Figure 1. Comparison of the dimensionless velocity as a function of the Blasius coordinate.

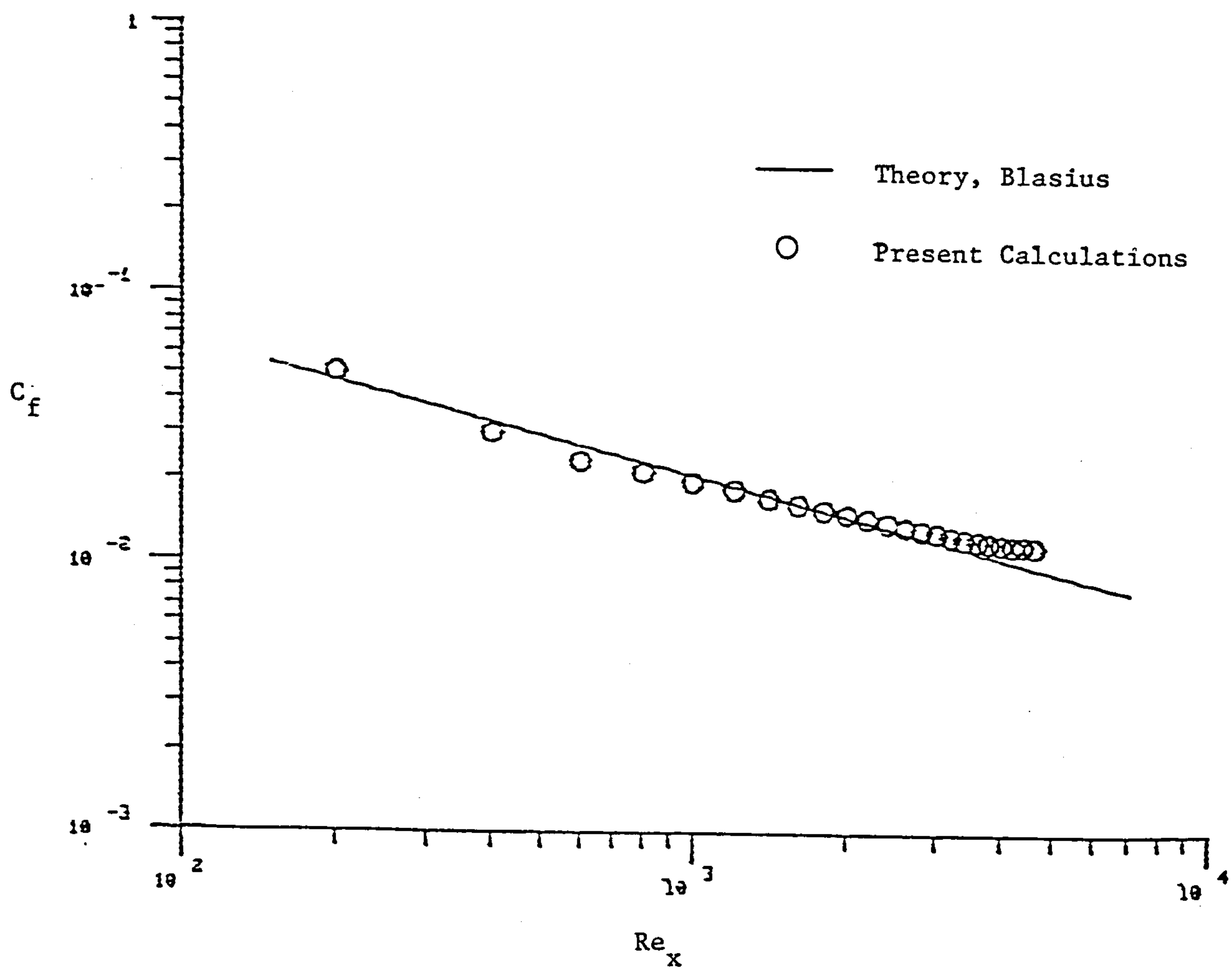

Figure 2. Comparison of skin-firction distributions for laminar flat plate flow based on the Hyperbolic Navier-Stokes equations.

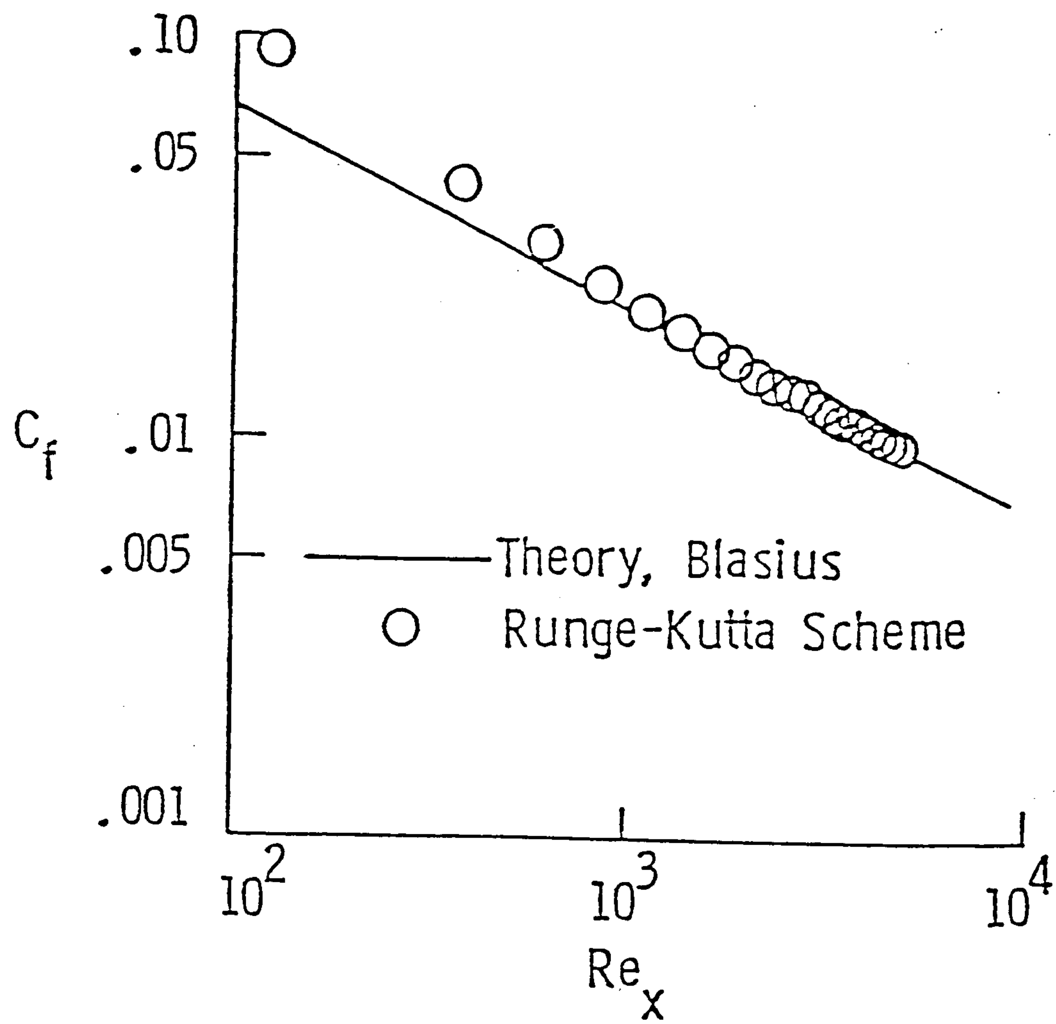

Figure 3. Comparison of skin-friction distributions for laminar flat plate flow based on the traditional Navier-Stokes equations.