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COMPONENT RESPONSE TO RANDOM VIBRATORY MOTION OF THE CARRIER VEHICLE

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SECTION 1. INTRODUCTION

In this treatment of component response to local random vibratory motion of the carrier vehicle, the component plus supporting structure is modeled as the system shown in Figure 1. The component model is allowed two degrees-of-freedom, one translational and one rotational, and is excited by a random translatory motion of the base whose acceleration power spectral density (PSD), herein denoted by $G_{\ddot{u}}(f)$, is presumed known. Prescription of the base acceleration PSD is done in the manner indicated by the inset in Figure 2 which admits the analytical representation appearing beneath the diagram of Figure 3.

Since the base motion is prescribed only to the extent that its acceleration PSD is given, the "time response," i.e., a time history of the system configuration coordinates and their first and second time derivatives, is out of the question. The word "response" is here to be interpreted as "mean square response," that implying the mean squares of the system coordinates and their time derivatives pertinent to the frequency interval over which $G_{\ddot{u}}(f)$ is specified.

SECTION 2. FUNDAMENTAL RELATIONS

Whether interest lies in "time response" or "mean square response," the source of certain fundamental relations, necessary to computation, is the system of differential equations descriptive of the motion. Treating the component model as a perfectly rigid body, invoking Newton and the principle of angular momentum, and making the usual small angle approximations, the equations of motion may be written as equations (1) and (2).

$$m\ddot{x} = -mg - K_1 (x - \delta_{ST,1} - L_1 \theta - u) - C_1 (\dot{x} - D_1 \dot{\theta} - \dot{u})$$

$$- K_2 (x - \delta_{ST,2} + L_2 \theta - u) - C_2 (\dot{x} + D_2 \dot{\theta} - \dot{u}) \tag{1}$$

$$I\ddot{\theta} = K_1 L_1 (x - \delta_{ST,1} - L_1 \theta - u) + C_1 D_1 (\dot{x} - D_1 \dot{\theta} - \dot{u})$$

$$- K_2 L_2 (x - \delta_{ST,2} + L_2 \theta - u) - C_2 D_2 (\dot{x} + D_2 \dot{\theta} - \dot{u}) \quad . \tag{2}$$

Recognizing the simplifications possible via the relations (3), the conditions for static equilibrium,

$$K_1 \delta_{ST,1} + K_2 \delta_{ST,2} = mg \quad , \quad K_1 L_1 \delta_{ST,1} = K_2 L_2 \delta_{ST,2} \quad , \tag{3}$$

one can write the matrix equivalent of equations (1) and (2) as

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{x} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & C_2D_2 - C_1D_1 \\ C_2D_2 - C_1D_1 & C_1D_1^2 + C_2D_2^2 \end{bmatrix} \begin{bmatrix} \dot{x} \\ \dot{\theta} \end{bmatrix}$$

$$+ \begin{bmatrix} K_1 + K_2 & K_2L_2 - K_1L_1 \\ K_2L_2 - K_1L_1 & K_1L_1^2 + K_2L_2^2 \end{bmatrix} \begin{bmatrix} x \\ \theta \end{bmatrix} = \begin{bmatrix} C_1 + C_2 & K_1 + K_2 \\ C_2D_2 - C_1D_1 & K_2L_2 - K_1L_1 \end{bmatrix} \begin{bmatrix} \dot{u} \\ u \end{bmatrix} . \tag{4}$$

If the (y,θ)-description of system configuration is preferred to the (x,θ)-description, then one has only to make the substitution $x = y+u$ in equation (4) to get

$$\begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \ddot{y} \\ \ddot{\theta} \end{bmatrix} + \begin{bmatrix} C_1 + C_2 & C_2D_2 - C_1D_1 \\ C_2D_2 - C_1D_1 & C_1D_1^2 + C_2D_2^2 \end{bmatrix} \begin{bmatrix} \dot{y} \\ \dot{\theta} \end{bmatrix}$$

$$+ \begin{bmatrix} K_1 + K_2 & K_2L_2 - K_1L_1 \\ K_2L_2 - K_1L_1 & K_1L_1^2 + K_2L_2^2 \end{bmatrix} \begin{bmatrix} y \\ \theta \end{bmatrix} = \begin{bmatrix} -m\ddot{u} \\ 0 \end{bmatrix} . \tag{5}$$

The transfer functions $T_{\xi/u}(s)$, $\xi = x,y,\theta$, essential to computation of the mean squares of the system output variables, may be found by applying the Laplace transformation to equations (4) and (5), assuming zero initial conditions, and solving for the transform ratios $\mathcal{L}(\xi)/\mathcal{L}(u)$, $\xi = x,y,\theta$. (Obviously, it is not necessary to apply the transformation to both equations (4) and (5) since one can choose to work with either equation (4) or (5), then, having found either $\mathcal{L}(x)/\mathcal{L}(u)$ or $\mathcal{L}(y)/\mathcal{L}(u)$, find the other via the relation $x = y+u$.) As should be expected, the expression for $T_{\theta/u}(s)$ as determined by equation (4) is equivalent to that determined by equation (5), Thus,

$$T_{x/u}(s) = \left\{ a_{22}(s)\ [a_{11}(s) - ms^2] - a_{12}^2(s) \right\} / A(s) \tag{6}$$

$$T_{y/u}(s) = - ms^2\ a_{22}(s)\ /\ A(s) \tag{7}$$

$$T_{\theta/u}(s) = ms^2\ a_{12}(s)\ /\ A(s) \tag{8}$$

where

$$\begin{aligned} a_{11}(s) &= ms^2 + (C_1 + C_2)\ s + K_1 + K_2 \\ a_{12}(s) &= (C_2D_2 - C_1D_1)\ s + K_2L_2 - K_1L_1 \\ a_{22}(s) &= Is^2 + (C_1D_1{}^2 + C_2D_2{}^2)\ s + K_1L_1{}^2 + K_2L_2{}^2 \\ A(s) &= a_{11}(s)\ a_{22}(s) - a_{12}^2(s) \quad . \end{aligned} \tag{9}$$

Having found $T_{\xi/u}(s)$, $\xi = x, y, \theta$, it is an easy matter to find an expression for the PSD of ξ, $G_\xi(f)$, by appealing to the well known general relation (valid for linear systems and accepted here without dispute)

$$G_\zeta(f) = |T_{\zeta/\eta}\ (2\ \pi\ jf)|^2\ G_\eta(f) \quad , \tag{10}$$

the symbol ζ denoting an output quantity of a system with input η. Obviously,

$$G_\xi(f) = |T_{\xi/u}\ (2\ \pi\ jf)|^2\ G_u(f) \quad , \quad \xi = x, y, \theta \quad . \tag{11}$$

But, since it is $G_{\ddot{u}}(f)$ that is prescribed, not $G_u(f)$, equation (11) will not completely define $G_\xi(f)$ until an expression for $G_u(f)$ is found. To that end one can write

$$T_{u/\ddot{u}}(s) = \mathcal{L}(u)/\mathcal{L}(\ddot{u}) = \frac{\mathcal{L}(u)}{S^2\,\mathcal{L}(u)} = \frac{1}{s^2}$$

which, in conjunction with equation (10), yields

$$G_u(f) = \left| \frac{1}{(2\pi j f)^2} \right|^2 \tilde{G}_{\ddot{u}}(f) = (2\pi f)^{-4}\, \tilde{G}_{\ddot{u}}(f) \quad . \tag{12}$$

It is important to point out that in equation (12) the dimension of $G_u(f)$ is supposed in.^{2}/Hz, thereby requiring that $\tilde{G}_{\ddot{u}}(f)$ have the dimension (in./sec^2)^2/Hz, the use of the tilde (~) serving to distinguish between $\tilde{G}_{\ddot{u}}(f)$ and $G_{\ddot{u}}(f)$, which has the dimension g^2/Hz. In terms of $G_{\ddot{u}}(f)$

$$G_u(f) = \gamma\, f^{-4}\, G_{\ddot{u}}(f) \tag{13}$$

where the numerical value of γ is given by

$$\gamma = (386.08858)^2/(2\pi)^4 \quad ,$$

it being assumed that the local acceleration due to gravity is 386.08858 (in./sec^2).

In a manner similar to that of arriving at equation (13), one can argue that

$$G_{\dot{u}}(f) = \gamma'\, f^{-2}\, G_{\ddot{u}}(f) \quad . \tag{14}$$

The dimension of $G_{\dot{u}}(f)$ is (in./sec)^2/HZ and γ' is given by

$$\gamma' = (386.08858/2\pi)^2 \quad .$$

Between $G_u(f)$ and $G_{\dot{u}}(f)$ is the obvious relation

$$G_u(f) = (2\,\pi\, f)^{-2}\, G_{\dot{u}}(f) \quad . \tag{15}$$

Among other obvious relations are the following:

$$\frac{\mathcal{L}(\ddot{\xi})}{\mathcal{L}(\ddot{u})} = \frac{\mathcal{L}(\dot{\xi})}{\mathcal{L}(\dot{u})} = \frac{\mathcal{L}(\xi)}{\mathcal{L}(u)} \quad , \qquad \xi = x, y, \theta$$

$$T_{\ddot{\xi}/\ddot{u}}(s) = T_{\dot{\xi}/\dot{u}}(s) = T_{\xi/u}(s) \quad , \qquad \xi = x, y, \theta$$

$$G_{\ddot{\xi}}(f) = |T_{\xi/u}(2\pi j f)|^2\, G_{\ddot{u}}(f) \quad , \qquad \xi = x, y \tag{16}$$

$$G_{\ddot{\theta}}(f) = (386.08858)^2 \; |T_{\theta/u}(2\pi jf)|^2 \; G_{\ddot{u}}(f) \tag{17}$$

$$G_{\dot{\xi}}(f) = |T_{\xi/u}(2\pi jf)|^2 \; G_{\dot{u}}(f) \quad , \quad \xi = x,y,\theta \tag{18}$$

$$G_{\xi}(f) = |T_{\xi/u}(2\pi jf)|^2 \; G_{u}(f) \quad , \quad \xi = x,y,\theta \quad . \tag{19}$$

The numerical factor was introduced in equation (17) because, as mentioned before, the dimension of $G_{\ddot{u}}(f)$ is g^2/HZ.

The most efficient sequence of instructions to be executed in computing the mean squares and root mean squares of both input and output variables is the following:

1. Assign a value to f and compute $G_{\ddot{u}}(f)$ in accordance with the expressions (defining the curve fit) appearing beneath the hypothetical plot of Figure 3.
2. Compute and store $G_{\dot{u}}(f)$ in accordance with equation (14).
3. Compute and store $G_{u}(f)$ in accordance with equation (15).
4. Compute and store $G_{\ddot{\xi}}(f)$ in accordance with equations (16) and (17), $\xi = x,y,\theta$.
5. Compute and store $G_{\dot{\xi}}(f)$ in accordance with equation (18), $\xi = x,y,\theta$.
6. Compute and store $G_{\xi}(f)$ in accordance with equation (19), $\xi = x,y,\theta$.
7. Increase f by Δf.
8. Repeat 1 through 7 until the frequency interval over which $G_{\ddot{u}}(f)$ is prescribed has been covered. (In this paragraph f_1 and f_N will denote the left and right extremes of that interval.)
9. Via some numerical integration scheme, compute the mean square of ξ, denoted by $\overline{\xi^2}(f_1,f_N)$ pertinent to the interval (f_1,f_N) in accordance with

$$\overline{\xi^2}(f_1,f_N) = \int_{f_1}^{f_N} G_{\xi}(f)\, df \quad , \quad \xi = \ddot{u},\dot{u},u,\ddot{x},\dot{x},x,\ddot{\theta},\dot{\theta},\theta,\ddot{y},\dot{y},y \quad . \tag{20}$$

10. Extract the square root of $\overline{\xi^2}(f_1,f_N)$ to get the root mean square (RMS) of ξ.

$$\xi_{RMS}(f_1,f_N) = \left\{ \overline{\xi^2}(f_1,f_N) \right\}^{1/2} \quad , \quad \xi = \ddot{u},\dot{u},u,\ddot{x},\dot{x},x,\ddot{y},\dot{y},y,\ddot{\theta},\dot{\theta},\theta \quad .$$

In numerically evaluating the integral in equation (20), the author has found that the simple trapezoidal rule gives satisfactory results, provided a wise choice of Δf is made; but, at this writing, can offer no failure proof method for selecting the "optimum" value of Δf in a given case. Usually, one relies on experience[1] in deciding the value to be assigned to Δf.

To find the mean squares of $\ddot{u}$, $\dot{u}$, and u it is not necessary to resort to any numerical integration scheme since closed expressions are available for their computation. From Reference 1

$$\overline{\ddot{u}^2}(f_1,f_N) = \sum_{\substack{i \\ (b_i \neq -1)}} \frac{1}{1+b_i} \{f_{EX,i+1}\, G_{\ddot{u}}(f_{EX,i+1}) - f_{EX,i}\, G_{\ddot{u}}(f_{EX,i})\}$$

$$+ \sum_{\substack{i \\ (b_i = -1)}} c_i \ln\left(\frac{f_{EX,i+1}}{f_{EX,i}}\right) \quad , \quad (1 \le i \le NSEG) \qquad (21)$$

$$\overline{\dot{u}^2}(f_1,f_N) = \gamma' \sum_{\substack{i \\ (b_i \neq 1)}} \frac{1}{b_i - 1} \left\{f^{-1}_{EX,i+1}\, G_{\ddot{u}}(f_{EX,i+1}) - f^{-1}_{EX,i}\, G_{\ddot{u}}(f_{EX,i})\right\}$$

$$+ \gamma' \sum_{\substack{i \\ (b_i = 1)}} c_i \ln\left(\frac{f_{EX,i+1}}{f_{EX,i}}\right) \quad , \quad (1 \le i \le NSEG) \quad , \qquad (22)$$

$$\overline{u^2}(f_1,f_N) = \gamma \sum_{\substack{i \\ (b_i \neq 3)}} \frac{1}{b_i - 3} \left\{f^{-3}_{EX,i+1}\, G_{\ddot{u}}(f_{EX,i+1}) - f^{-3}_{EX,i}\, G_{\ddot{u}}(f_{EX,i})\right\}$$

$$+ \gamma \sum_{\substack{i \\ (b_i = 3)}} c_i \ln\left(\frac{f_{EX,i+1}}{f_{EX,i}}\right) \quad , \quad (1 \le i \le NSEG) \quad . \qquad (23)$$

---

1. A visual examination of the plot of $G_\xi(f)$ could be of some use in deciding whether to pronounce a specific value of Δf as satisfactory or unsatisfactory.

In equations (21), (22), and (23), the symbols $f_{EX,i}$ and $f_{EX,i+1}$ denote, respectively, the abscissa of the left extremity and right extremity of the ith straight line segment in the log-log plot of $G_{\ddot{u}}(f)$, there being NSEG such segments (see Figure 3), and $f_1 \equiv f_{EX,1}$, $f_N \equiv f_{EX,NSEG+1}$. Notice that equations (22) and (23) have meaning only if $f_N > f_1 > 0$, and further, that when $G_{\ddot{u}}(f) = W(g^2/HZ)$ = a constant for $f_1 \le f \le f_N$, equations (21), (22), and (23) become equations (21)', (22)', and (23)', respectively.

$$\overline{\ddot{u}^2}(f_1,f_N) = W(f_N - f_1) \tag{21'}$$

$$\overline{\dot{u}^2}(f_1,f_N) = \gamma' \, W(f_1^{-1} - f_N^{-1}) \tag{22'}$$

$$\overline{u^2}(f_1,f_N) = \frac{\gamma W}{3} \, (f_1^{-3} - f_N^{-3}) \tag{23'}$$

While dwelling on "closed expressions," mention should be made of the existence of closed expressions for $\ddot{x}$, $\ddot{\theta}$, $\dot{\theta}$, θ, $\dot{y}$, and y in the very special case wherein $G_{\ddot{u}}(f)$ is constant[2] for $0 \le f < \infty$. In this case, it is not difficult, with the aid of the table of integrals in Reference 2 (see also References 3 and 4, both of which cite Reference 2), to show that the mean square of ξ, pertinent to the semi-infinite frequency interval $(0,\infty)$, is given in closed form by

$$\begin{aligned}\overline{\xi^2}(0,\infty) = \frac{\gamma^* W}{4} \Big\{ & (B_0^2/A_0)\,(A_2A_3 - A_1A_4) + A_3\,(B_1^2 - 2\,B_0B_2) \\ & + A_1\,(B_2^2 - 2\,B_1B_3) + (B_3^2/A_4)\,(A_1A_2 - A_0A_3) \Big\}_{(\xi)} \Big/ \Big\{ -A_0A_3^2 \\ & + A_1\,(A_2A_3 - A_1A_4) \Big\} \quad , \quad \xi = \ddot{x}, \ddot{\theta}, \dot{\theta}, \theta, \dot{y}, y \quad ,\end{aligned} \tag{24}$$

where W denotes the constant value of $G_{\ddot{u}}(f)$ and the numerical factor γ^* depends upon which of the variables ξ represents, that is,

$$\gamma^* = \begin{cases} 1.0 \text{ if } \xi = \ddot{x} \\ (386.08858)^2 \quad \text{if} \quad \xi = \ddot{\theta}, \dot{\theta}, \theta, \dot{y}, y \end{cases} .$$

2. In the jargon of vibration engineers the base acceleration in this case is termed "white noise."

The subscript ξ on the right brace in the numerator of equation (24) serves to indicate that the B_K $(K = 0,1,2,3)$ are pertinent to the particular ξ being dealt with. The A_K $(K = 0,1,2,3,4)$ are the same for all ξ, A_K being the coefficient of S^K in the system characteristic polynomial, $A(S)$, defined by the last of equations (9). On performing the indicated multiplications in equation (9) and collecting terms one will find

$$A_0 = K_1K_2 (L_1 + L_2)^2$$

$$A_1 = (C_1 + C_2) (K_1L_1^2 + K_2L_2^2) + (K_1 + K_2) (C_1D_1^2 + C_2D_2^2)$$
$$- 2 (C_2D_2 - C_1D_1) (K_2L_2 - K_1L_1)$$

$$A_2 = m (K_1L_1^2 + K_2L_2^2) + I (K_1 + K_2) + C_1C_2 (D_1 + D_2)^2$$

$$A_3 = m (C_1D_1^2 + C_2D_2^2) + I (C_1 + C_2)$$

$$A_4 = mI \quad .$$

Pertinent to $\ddot{x}$, the B_K, $K = 0,1,2,3$, are

$$B_0 = A_0$$

$$B_1 = A_1$$

$$B_2 = A_2 - m (K_1L_1^2 + K_2L_2^2)$$

$$B_3 = I (C_1 + C_2)$$

Pertinent to $\ddot{\theta}$, the B_K, $K = 0,1,2,3$, are

$$B_0 = 0$$

$$B_1 = 0$$

$$B_2 = m\ (K_2L_2 - K_1L_1)$$

$$B_3 = m\ (C_2D_2 - C_1D_1)$$

Pertinent to $\dot{\theta}$, the B_K, $K = 0,1,2,3$, are

$$B_0 = 0$$

$$B_1 = m\ (K_2L_2 - K_1L_1)$$

$$B_2 = 0$$

$$B_3 = 0$$

Pertinent to θ, the B_K, $K = 0,1,2,3$, are

$$B_0 = m\ (K_2L_2 - K_1L_1)$$

$$B_1 = B_2 = B_3 = 0 \quad .$$

Pertinent to $\dot{y}$, the B_K, $K = 0,1,2,3$, are

$$B_0 = 0$$

$$B_1 = m\ (K_1L_1^2 + K_2L_2^2)$$

$$B_2 = m\ (C_1D_1^2 + C_2D_2^2)$$

$$B_3 = mI \quad .$$

Pertinent to y, the B_K, $K = 0,1,2,3$, are

$$B_0 = m\ (K_1L_1^2 + K_2L_2^2)$$

$$B_1 = m\,(C_1D_1^2 + C_2D_2^2)$$

$$B_2 = mI$$

$$B_3 = 0\quad .$$

The structure of the transfer functions relevant to $\dot{x}$, x, and $\ddot{y}$ is such as to preclude use of the referenced list of integrals to find the mean squares of $\dot{x}$, x, and $\ddot{y}$.

Equations (1) through (23), plus attendant relations (Appendix A), constitute the basis for program TRROBM (a mnemonic for "Translational and Rotational Response to Base Motion") which has been operational since 1983. A recent revision of the 1983 version was made so that the program output would include items of importance to the author in dealing with a related assignment. Before further comment regarding the related assignment is made, the author would like to call attention to Table 1 which shows the remarkably close approximations, afforded by equation (20), to the mean squares $\overline{\xi^2}(0,\infty)$, $\xi = \ddot{x},\ddot{\theta},\dot{\theta},\theta,\dot{y},y$, whose exact values are determined by equation (24). Below the table are the specifications defining the case which was processed by program TRROBM to get the entries in the third column. The coding of program TRROBM requires that the input include the items appearing in the left hand column of Table 2. Consequently, when certain of the system parameters are "indirectly" specified, as in the manner beneath Table 1, one must resort to some preliminary computation in accordance with the equations of APPENDIX B to determine the numerical values of C_1, C_2, D_1, D_2, K_1, K_2, L_1, L_2, and I.

Not shown in the list of input items in Table 2 are other input items which are "implied" by the presence of $G_{\ddot{u}}(f)$ in that list and by the expressions for the curve fit parameters under the diagram of Figure 3. These items include NSEG, $f_{EX,i}$ ($i = 1,\ldots,$NSEG+1), NCORN, GCORN, and ΔDB(i,i+1). $i = 1,\ldots,$NSEG, all essential in the computation of $G_{\ddot{u}}(f)$ for a given value of f. In program TRROBM the $f_{EX,i}$ and ΔDB(i, i+1) are embedded in the one-dimensional arrays identified by the FORTRAN symbols FEX and DELDB, respectively. By mere inspection of the PSD specification, one has immediately the input data designated NSEG, FEX, DELDB, NCORN, and GCORN. Pertinent to the sample PSD of Figure 2 these items are

NSEG = 7

$$\mathrm{FEX} \equiv \begin{bmatrix} f_{EX,1} \\ f_{EX,2} \\ f_{EX,3} \\ f_{EX,4} \\ f_{EX,5} \\ f_{EX,6} \\ f_{EX,7} \\ f_{EX,8} \end{bmatrix} = \begin{bmatrix} 20. \\ 30. \\ 120. \\ 210. \\ 400. \\ 480. \\ 900. \\ 2000. \end{bmatrix} \text{ (HZ)}, \quad \mathrm{DELDB} = \begin{bmatrix} +6. \\ 0 \\ +6. \\ 0 \\ +9. \\ 0. \\ -12. \end{bmatrix} \text{ (DB/OCTAVE)}$$

NCORN = 1 , GCORN = 0.15 (g^2/HZ)

The choice of the combination NCORN = 1, GCORN = 0.15 was but one of several available. The admissible combinations of NCORN and GCORN in this case are shown in the following table.

NCORN	1	2	3	4	5	6	7	8
GCORN	0.15	0.32	0.32	1.0	1.0	1.7	1.7	0.075

It is evident from Table 2 that the entries in the second column of Table 1 are not to be found among the items output by program TRROBM. Instead, they are the output of a smaller program, an auxiliary to TRROBM (aptly named program AUXRBM), which was coded only recently, in July 1986. It was long after the author had developed program TRROBM and two similar programs[3] that he learned, through browsing the literature (References 2, 3, and 4 in particular), of the existence of the table of integrals which served as a guide in writing equation (24) upon which program AUXRBM is based. Neither was he aware, until he surveyed the literature, that much of the work he had done in developing the two programs described in the footnote had already been done years ago.

3. Program RESPBM (response to base motion) treats the single d.o.f. mass-spring-damper system excited by the random vibratory motion of the base whose acceleration PSD is prescribed as in this paper. Program RESBM2 deals with the randomly base driven 2 mass-2 d.o.f. system, the two d.o.f.'s being translational.

Program AUXRBM, a sample output of which is given in Appendix C, provided the numerical data necessary to the construction of the families[4] of curves in Figures 4 through 11. The data could have been generated by program TRROBM but at a greater cost of computer time, not to mention the slight inaccuracies in the data due to the necessity of restricting the mean square computation to a finite frequency interval whose left extremity must be positive. The use of the word "inaccuracies" tends to unjustly discredit program TRROBM. In defense of TRROBM the author should point out that even in those cases wherein $G_{\ddot{u}}(f)$ is constant, which is the only kind of case to which AUXRBM is applicable, there is hardly a discernible difference between the plots[5] of RMS's made from the output of TRROBM and those made from the output of AUXRBM (after the numerical values have been rounded to at most three significant digits and plotting is done using the same scales for both sets of output). The author has made this assertion on the assumption that, in processing a case by TRROBM, a wise choice of Δf is made, and further, that a sufficiently wide[6] frequency interval is used in the mean square computation. As support of his assertion, the author invites the reader to compare the values of $\ddot{\theta}_{RMS}$, $\dot{\theta}_{RMS}$, θ_{RMS}, y_{RMS}, $\dot{y}_{RMS}$, and $\ddot{x}_{RMS}$ found among the items of the sample TRROBM output in Appendix D with the appropriate encircled values or inset tabular entries of Figures 6 through 11.

The source of the RMS's in Figures 12, 13, and 14 was program TRROBM. In each of these figures the prevailing conditions are the same as those pertinent to the encircled points of Figure 6. The previously cited tables of integrals, and hence, program AUXRBM, were of no utility in the computation of $\overline{x^2}$, $\overline{\dot{x}^2}$, and $\overline{\dot{y}^2}$ because, as mentioned in a previous paragraph, the rational functions

$$T_{x/\ddot{u}}(s) = \frac{1}{s^2} T_{x/u}(s) \quad ,$$

$$T_{\dot{x}/\ddot{u}}(s) = \frac{1}{s} T_{x/u}(s) \quad ,$$

4. For the purpose of comparing the behavior of the plotted function, as depicted by the solid curves, with its behavior under slightly different conditions, some of the figures have either an inset table of values or encircled values of the function corresponding to the changes in system parameters.

5. Plots of RMS's versus r_f (holding r_L constant).

6. Confining the mean square computation to the interval (1., $2f_{2c}$) results in excellent approximations.

and

$$T_{\ddot{y}/\ddot{u}}(s) = T_{y/u}(s)$$

do not have the requisite structure. The first thing one will notice about these figures is that no attempt has been made to draw a "best fit" curve through any of the several sets of points, the reason being an insufficient number of points to accurately determine the behavior of the variable plotted.

All of the programs described in this paper are coded in FORTRAN V for a punched card machine (one of the UNIVAC series in particular). However, one with the expertise can translate the FORTRAN language into that of another computer. A fellow employee[7] here at MSFC has, in fact, already effected the translation of program RESPBM, described in one of the footnotes, into TEKTRONIX language (models 4051, 4052, and 4054).

When the author was approached by his supervisor with questions about the mean values of system[8] kinetic energy, potential energy and energy dissipated, his first thought was of the system coordinate velocities whose mean squares are not a part of the output[9] of the 1983 version of TRROBM. It was the need for the mean squares of the coordinate velocities, as well as $\overline{y_P^2}$ (whose need will become apparent later), that prompted the 1986 revision of the 1983 version of TRROBM. In response to the questions asked, the author has developed the following expressions for the mean values of the kinetic energy (KE) and potential energy (PE), the symbol E in $E(\xi)$ being the familiar expectation operator or mean value operator.

$$E(K.E.) = \frac{1}{2} (m\, \dot{x}^2_{RMS} + I\, \dot{\theta}^2_{RMS})$$

$$E(P.E.) = \frac{m\, \omega^2_{nx}}{2}\, y^2_{RMS} + \frac{I\, \omega^2_{n\theta}}{2}\, \theta^2_{RMS} + \frac{1}{2\, L_p} (K_2L_2 - K_1L_1)\, (y^2_{P,RMS} - y^2_{RMS}$$

$$- L_P^2\, \theta^2_{RMS}) + \frac{1}{2} (K_1\, \delta^2_{ST,1} + K_2\, \delta^2_{ST,2}) \quad .$$

---

7. Pat Lewallen, ED24.
8. See Figure 1.
9. When the work which culminated in the 1983 version of TRROBM was done, interest was primarily in accelerations and displacements.

The definitions of ω^2_{nx}, $\omega^2_{n\theta}$, y_P, and L_P are given elsewhere but are repeated here.

$$\omega^2_{nx} = (K_1 + K_2) \;/\; m \quad ,$$

$$\omega^2_{n\theta} = (K_1L_1{}^2 + K_1L_2{}^2) \;/\; I \quad ,$$

y_P = displacement of arbitrary point P (not the CM) relative to the base

L_P = lateral distance of point P from the CM, positive or negative according as point P is right or left of the cm .

The expression for E(PE) was derived on the assumption that the mean value of u, the base displacement, is zero, and that the zero level for gravitational potential energy is the static equilibrium level of the CM. Assuming further that $D_1 = D_2 = D$ and $C_1 = C_2 = C$, it is not difficult to show that the mean value of the rate at which energy is dissipated through viscous damping is $2C(\dot{y}^2_{RMS} + D^2\ \dot{\theta}^2_{RMS})$. Development of the expressions for the mean values in this paragraph was the "related assignment" alluded to earlier.

Attention is now called to Figure 4 wherein the symbol $\{\overline{\ddot{x}^2(0,\infty)}\}_{\theta\equiv 0}$ denotes the mean square of $\ddot{x}$ (for $0 \leq f < \infty$) when component rotation has been suppressed entirely by enforcing the relations $K_1L_1 - K_2L_2 = 0$ and $C_1D_1 - C_2D_2 = 0$ so that θ is identically zero (provided θ and $\dot{\theta}$ are initially zero). A cursory examination of this family of curves reveals that suppressing rotation merely serves to increase the mean square of $\ddot{x}$ (otherwise, the plotted mean square ratios would be greater than one).

It is evident in Figure 5 that for some combinations of r_f and r_L, which admit rotation, the mean square of y is larger than it is when there is no rotation while for other combinations it is smaller.

Figures 6 through 11 show whether the imposition of the conditions $\{D_1 = D_2, D_i \neq L_i, i = 1,2\}$ instead of $\{D_i = L_i, i = 1,2\}$, other conditions being the same, results in an increase or decrease in the RMS of the response variable in question. These figures, when complemented by the output of TRROBM plotted in Figures 12, 13, and 14, provide the RMS's of the system coordinates and their first two time derivatives for both the (x, θ) and (y, θ) descriptions of system configuration. This collection of figures does not represent an exhaustive parameter study, but is exemplary of parameter studies made possible by program TRROBM (with or without the support of its auxiliary AUXRBM, which is of limited application).

At least one paragraph should be devoted to stability, if only to go so far as to write the conditions (on the system parameters) whose satisfaction guarantees system stability. Such conditions are indirectly realized by conditions on the coefficients of the system characteristic polynomial, $A(S) = \sum_{i=0}^{4} A_i S^i$ [see equations (9)], those conditions being available via the Routh-Hurwitz criterion. The Routh-Hurwitz array pertinent to A(S) is the following:

ROW			
0	A_4	A_2	A_0
1	A_3	A_1	
2	$\frac{A_3A_2 - A_4A_1}{A_3}$	A_0	
3	$\frac{A_1(A_2A_3 - A_1A_4) - A_0A_3^2}{A_3A_2 - A_4A_1}$		
4	A_0		

By the expressions defining them, A_4, A_3, and A_0 are intrinsically positive. Hence, by the Routh-Hurwitz criterion for stability, system stability is assured if the other two elements in the first column of the array are positive, or, equivalently, if both of the inequalities

$$A_3A_2 > A_4A_1 \quad , \quad A_1(A_2A_3 - A_1A_4) > A_0A_3^2 \quad ,$$

are satisfied. The problem of assessing the "degree" of stability will not be addressed in this paper. The technique for handling the situation wherein a left column element is zero will be found in the literature.

The author has given some thought to models other than that of Figure 1, those of Figure 15 in particular, which, in certain cases, could be more "credible" or "plausible" models. Though there is slight difference between the "appearances" of the models in Figures 1 and 15-a, that difference due, obviously to the elastically supported dampers in Figure 15-a, there is a marked difference between the respective mathematical descriptions of model motion. While the differential equations governing the motion of the model in Figure 1 are of second order, those determining the motion of that in Figure 15-a are of third order.

Pertinent to Figure 15-a, the author has derived the following equations.

$$
\begin{aligned}
m(\ddot{y}+\ddot{u}) = &-(K_1 + K_2)\, y + (K_1L_1 - K_2L_2)\,\theta - C_1\,(\dot{y} - D_1\dot{\theta}) - C_2\,(\dot{y} + D_2\dot{\theta}) \\
&+ \frac{C_1}{\widetilde{K}_1(D_1+D_2)} \Big\{ I\,\dddot{\theta} - m\,D_2\,(\dddot{y} + \dddot{u}) + [K_2L_2 - K_1L_1 - D_2\,(K_1 + K_2)]\,\dot{y} \\
&\qquad + [K_1L_1^2 + K_2L_2^2 + D_2\,(K_1L_1 - K_2L_2)]\,\dot{\theta} \Big\} \\
&+ \frac{C_2}{\widetilde{K}_2(D_1+D_2)} \Big\{ - I\,\dddot{\theta} - m\,D_1\,(\dddot{y} + \dddot{u}) - [K_2L_2 - K_1L_1 + D_1\,(K_1 + K_2)]\,\dot{y} \\
&\qquad - [K_1L_1^2 + K_2L_2^2 - D_1\,(K_1L_1 - K_2L_2)]\,\dot{\theta} \Big\}
\end{aligned}
\tag{25}
$$

$$
\begin{aligned}
I\,\ddot{\theta} = &(K_1L_1 - K_2L_2)\, y - (K_1L_1^2 + K_2L_2^2)\,\theta + C_1D_1\,(\dot{y} - D_1\dot{\theta}) - C_2D_2\,(\dot{y} + D_2\dot{\theta}) \\
&- \frac{C_1D_1}{\widetilde{K}_1(D_1+D_2)} \Big\{ I\,\dddot{\theta} - m\,D_2\,(\dddot{y} + \dddot{u}) + [K_2L_2 - K_1L_1 - D_2\,(K_1 + K_2)]\,\dot{y} \\
&\qquad + [K_1L_1^2 + K_2L_2^2 + D_2\,(K_1L_1 - K_2L_2)]\,\dot{\theta} \Big\} \\
&+ \frac{C_2D_2}{\widetilde{K}_2(D_1+D_2)} \Big\{ - I\,\dddot{\theta} - m\,D_1\,(\dddot{y} + \dddot{u}) - [K_2L_2 - K_1L_1 + D_1\,(K_1 + K_2)]\,\dot{y} \\
&\qquad - [K_1L_1^2 + K_2L_2^2 - D_1\,(K_1L_1 - K_2L_2)]\,\dot{\theta} \Big\}\,.
\end{aligned}
\tag{26}
$$

Notice that if one makes the substitution $x = y + u$ and allows $\tilde{K}_1$ and $\tilde{K}_2$ to become infinite, equations (25) and (26) become, in view of relations (3), the equations of motion of the model in Figure 1 pertinent to the (x, θ) description of system configuration, the equivalent of the matrix equation (4); or, if one merely permits $\tilde{K}_1$ and $\tilde{K}_2$ to approach infinity, equations (25) and (26) become the equivalent of equation (5).

Considerable simplification of equations (25) and (26) is realized in the special case wherein $K_1 = K_2 = K$, $C_1 = C_2 = C$, $\tilde{K}_1 = \tilde{K}_2 = \tilde{K}$. In that case they read as equations (27) and (28).

$$\frac{mC}{\tilde{K}} \dddot{y} + m \ddot{y} + 2C \left(1 + \frac{K}{\tilde{K}}\right) \dot{y} + 2 K y + C \left[D_2 - D_1 + \frac{K}{\tilde{K}} (L_2 - L_1)\right] \dot{\theta}$$

$$+ K (L_2 - L_1) \theta = \frac{-mC}{\tilde{K}} \dddot{u} - m \ddot{u} \tag{27}$$

$$\frac{IC}{\tilde{K}} \dddot{\theta} + I \ddot{\theta} + C \left[D_1^2 + D_2^2 + \frac{K}{\tilde{K}} (L_1^2 + L_2^2)\right] \dot{\theta} + K (L_1^2 + L_2^2) \theta$$

$$+ C \left[D_2 - D_1 + \frac{K}{\tilde{K}} (L_2 - L_1)\right] \dot{y} + K (L_2 - L_1) y = 0 \quad . \tag{28}$$

Having written the equations of motion, the next step toward a mean square computation is the deduction of the relevant transfer functions. Pertinent to the system comprised of equations (27) and (28), it is easily deduced that

$$\tilde{T}_{y/u}(s) = \sum_{i=0}^{6} \tilde{\beta}_i s^i \Big/ \sum_{i=0}^{6} \tilde{A}_i s^i$$

$$\tilde{T}_{\theta/u}(s) = \sum_{i=0}^{4} \tilde{\gamma}_i s^i \Big/ \sum_{i=0}^{6} \tilde{A}_i s^i \quad . \tag{29}$$

where

$$\tilde{A}_0 = K^2 \, (L_1 + L_2)^2$$

$$\tilde{A}_1 = 2 \, K \, C \left[D_1{}^2 + D_2{}^2 - (D_2 - D_1) \, (L_2 - L_1) + L_1{}^2 + L_2{}^2 + \frac{K}{\tilde{K}} \, (L_1 + L_2)^2 \right]$$

$$\tilde{A}_2 = m \, K \, (L_1{}^2 + L_2{}^2) + 2 \, K \, I + 2 \, C^2 \left(1 + \frac{K}{\tilde{K}} \right) \left[D_1{}^2 + D_2{}^2 + \frac{K}{\tilde{K}} \, (L_1{}^2 + L_2{}^2) \right]$$

$$- C^2 \left[D_2 - D_1 + \frac{K}{\tilde{K}} \, (L_2 - L_1) \right]^2 \tag{30}$$

$$A_3 = \frac{m \, K \, C}{\tilde{K}} \, (L_1{}^2 + L_2{}^2) + \frac{2 \, K \, C \, I}{\tilde{K}} + m \, C \left[D_1{}^2 + D_2{}^2 + \frac{K}{\tilde{K}} \, (L_1{}^2 + L_2{}^2) \right]$$

$$+ 2 \, I \, C \left(1 + \frac{K}{\tilde{K}} \right)$$

$$\tilde{A}_4 = \frac{m \, C^2}{\tilde{K}} \left[D_1{}^2 + D_2{}^2 + \frac{K}{\tilde{K}} \, (L_1{}^2 + L_2{}^2) \right] + m \, I + \frac{2 \, I \, C^2}{\tilde{K}} \left(1 + \frac{K}{\tilde{K}} \right)$$

$$\tilde{A}_5 = 2 \, m \, I \, C \, / \, \tilde{K}$$

$$\tilde{A}_6 = m \, I \, C^2 \, / \, \tilde{K}^2$$

$$\tilde{\beta}_0 = \tilde{\beta}_1 = 0$$

$$\tilde{\beta}_2 = - \, m \, K \, (L_1{}^2 + L_2{}^2)$$

$$\tilde{\beta}_3 = \frac{-2 \, m \, C \, K}{\tilde{K}} \, (L_1{}^2 + L_2{}^2) - m \, C \, (D_1{}^2 + D_2{}^2)$$

$$\tilde{\beta}_4 = \frac{-m \, C^2}{\tilde{K}} \left[D_1{}^2 + D_2{}^2 + \frac{K}{\tilde{K}} \, (L_1{}^2 + L_2{}^2) \right] - m \, I \tag{31}$$

$$\tilde{\beta}_5 = - \, 2 \, m \, C \, I \, / \, \tilde{K}$$

$$\tilde{\beta}_6 = -m \, I \, C^2 \, / \, \tilde{K}^2$$

$$\tilde{\gamma}_0 = \tilde{\gamma}_1 = 0$$

$$\tilde{\gamma}_2 = m\ K\ (L_2 - L_1)$$

$$\tilde{\gamma}_3 = \frac{2\ m\ C\ K}{\tilde{K}}\ (L_2 - L_1) + m\ C\ (D_2 - D_1) \tag{32}$$

$$\tilde{\gamma}_4 = \frac{m\ C^2}{\tilde{K}} \left[D_2 - D_1 + \frac{K}{\tilde{K}}\ (L_2 - L_1) \right] .$$

Imposition of the additional conditions $D_1 = L_1$ and $D_2 = L_2$, as in Figure 15-b, results in a simplification of equations (27) through (32). Further simplification is possible by setting $\tilde{K} = K$.

Upon examining the structure of the transfer functions in equation (29), along with that of the five Laplace transform ratios

$$\frac{\mathcal{L}(y)}{\mathcal{L}(\ddot{u})} = \frac{1}{S^2}\ \tilde{T}_{y/u}\ (S)$$

$$\frac{\mathcal{L}(\dot{y})}{\mathcal{L}(\ddot{u})} = \frac{1}{S}\ \tilde{T}_{y/u}\ (S)$$

$$\frac{\mathcal{L}(\theta)}{\mathcal{L}(\ddot{u})} = \frac{1}{S^2}\ \tilde{T}_{\theta/u}\ (S)$$

$$\frac{\mathcal{L}(\dot{\theta})}{\mathcal{L}(\ddot{u})} = \frac{1}{S}\ \tilde{T}_{\theta/u}\ (S)$$

$$\frac{\mathcal{L}(\ddot{\theta})}{\mathcal{L}(\ddot{u})} = \tilde{T}_{\theta/u}\ (S) \quad ,$$

it is evident, in light of the table of integrals previously cited, that the mean squares $\overline{\xi^2}$, $\xi = y, \dot{y}, \theta, \dot{\theta}, \ddot{\theta}$, are expressible in closed form when $G_{\ddot{u}}(f)$ is constant for $0 \leq f < \infty$. In the notation of this paper, the closed expression for $\overline{\xi^2}$ is

$$\overline{\xi^2} = \frac{\gamma^* W}{4\,\Delta_6}\ \{\tilde{\breve{B}}_5^{\,2}\, n_0 + n_1\,(\breve{B}_4^{\,2} - 2\,\breve{B}_3\tilde{B}_5) + n_2\,(\tilde{\breve{B}}_3^{\,2} - 2\,\tilde{\breve{B}}_2\breve{B}_4 + 2\,\tilde{B}_1\tilde{\breve{B}}_5)$$

$$+\, n_3\,(\tilde{B}_2^{\,2} - 2\,\tilde{\breve{B}}_1\tilde{B}_3 + 2\,\tilde{\breve{B}}_0\tilde{B}_4) + n_4\,(\breve{B}_1^{\,2} - 2\,\tilde{B}_0\breve{B}_2) + n_5\,\tilde{B}_o^{\,2}\}_{(\xi)} \quad , \qquad (33)$$

$$\xi = y, \dot{y}, \theta, \dot{\theta}, \ddot{\theta}$$

the significance of the factors γ^* and W being the same as in equation (24). The constant W, incidentally, is herein assumed to have the dimension g^2/HZ. The subscript ξ on the right brace in equation (33), as one should expect, indicates that the $\tilde{\breve{B}}_i$, i = 0, 1, ..., 5, are associated with the ξ in question. The $\tilde{\breve{A}}_i$, i = 0, 1, ..., 6, are given by equation (30) while Δ_6 and the n_i, i = 0, 1, ..., 5, are defined by

$$n_0 = \frac{1}{\tilde{A}_6}\,(\tilde{A}_4\, n_1 - \tilde{A}_2\, n_2 + \tilde{A}_0\, n_3)$$

$$n_1 = \tilde{A}_0\,(\tilde{A}_3^{\,2} - \tilde{A}_1\tilde{A}_5) + \tilde{A}_1\,(\tilde{A}_1\tilde{A}_4 - \tilde{A}_2\tilde{A}_3)$$

$$n_2 = \tilde{A}_0\tilde{A}_3\tilde{A}_5 + \tilde{A}_1\,(\tilde{A}_1\tilde{A}_6 - \tilde{A}_2\tilde{A}_5)$$

$$n_3 = \tilde{A}_5\,(\tilde{A}_0\tilde{A}_5 - \tilde{A}_1\tilde{A}_4) + \tilde{A}_1\tilde{A}_3\tilde{A}_6$$

$$n_4 = \frac{1}{\tilde{A}_0}\,(\tilde{A}_2\, n_3 - \tilde{A}_4\, n_2 + \tilde{A}_6\, n_1)$$

$$n_5 = \frac{1}{\tilde{A}_0}\,(\tilde{A}_2\, n_4 - \tilde{A}_4\, n_3 + \tilde{A}_6\, n_2)$$

$$\Delta_6 = \tilde{A}_0\,(\tilde{A}_1\, n_5 - \tilde{A}_3\, n_4 + \tilde{A}_5\, n_3) \quad .$$

Pertinent to y the $\tilde{B}_i$, i = 0, 1, ..., 5, are given by

$$\tilde{B}_i = \tilde{\beta}_{i+2} \quad , \quad i = 0,1,2,3,4 \quad \text{[the } \tilde{\beta}_i,\ i = 0,1,\ldots,6, \text{ are given by equations (31)]}$$

$$\tilde{B}_5 = 0 \quad .$$

Pertinent to $\dot{y}$,

$$\tilde{B}_i = \tilde{\beta}_{i+1} \quad , \quad i = 0,1,\ldots,5 \quad .$$

Pertinent to $\ddot{\theta}$,

$$\tilde{B}_i = \tilde{\gamma}_i \quad , \; i = 0,1,2,3,4 \quad [\text{the } \tilde{\gamma}_i,\ i = 0,1,\ldots,4, \text{ are given by equations (32)}]$$

$$\tilde{B}_5 = 0 \quad .$$

Pertinent to $\dot{\theta}$,

$$\tilde{B}_i = \tilde{\gamma}_{i+1} \quad , \quad i = 0,1,2,3$$

$$\tilde{B}_4 = \tilde{B}_5 = 0 \quad .$$

Pertinent to θ,

$$\tilde{B}_i = \tilde{\gamma}_{i+2} \quad , \quad i = 0,1,2$$

$$\tilde{B}_3 = \tilde{B}_4 = \tilde{B}_5 = 0 \quad .$$

To date, no attempt has been made to code a program based on equations (25) and (26) or any of their simplified forms. Programming, on the part of the author, has been pursued only so far as programs TRROBM and AUXRBM, which mark the culmination of the author's effort in this area.

TABLE 1

ξ	$\overline{\xi^2}\ (0,\infty)$	$\overline{\xi^2}\ (1.,\ 2f_{2c})$	Percent Error
$\ddot{x}$	$10^3\ (0.78567859)\ (g^2)$	$10^3\ (0.78498616)\ (g^2)$	0.088
$\ddot{\theta}$	$10^4\ (0.20118432)\ (\text{rad./sec}^2)^2$	$10^4\ (0.20111040)\ (\text{rad./sec}^2)^2$	0.037
$\dot{\theta}$	$10^{-1}\ (0.50960584)\ (\text{rad./sec})^2$	$10^{-1}\ (0.51124543)\ (\text{rad./sec})^2$	0.32
θ	$10^{-4}\ (0.12217110)\ (\text{rad.})^2$	$10^{-4}\ (0.12244709)\ (\text{rad.})^2$	0.23
$\dot{y}$	$10^3\ (0.29642678)\ (\text{in./sec})^2$	$10^3\ (0.29412964)\ (\text{in./sec})^2$	0.77
y	$10^{-3}\ (0.75068497)\ (\text{in.})^2$	$10^{-3}\ (0.75001788)\ (\text{in.})^2$	0.089
	VIA EQ. (24)	VIA EQ. (20)	

SPECIFICATIONS:

$G_{\ddot{u}}(f) = 0.1\ (g^2/\text{HZ})\ ,\ 1 \leq f \leq 200.0$

$K_1/K_2 = 1.,\ C_1/C_2 = 1.,\ D_1/D_2 = 1.,\ \zeta_x = \zeta_\theta = 0.01,\ f_{nx} = 100.\ (\text{HZ}),$

$r_f = 0.1,\ r_L = 2/3$

$m = 1.0\ (\frac{\text{lb.*sec}^2}{\text{in.}})\quad ,\quad \rho = 5.\ (\text{in.})$

VALUES OF I, K_1, K_2, C_1, C_2, D_1, D_2, L_1, L_2 (ENFORCED BY THE SPECIFICATIONS):

$I = 25.\ (\text{lb.*sec}^2\text{*in.})$

$K_1 = K_2 = 10^6\ (0.19739209)\ (\text{lb/in.})$

$C_1 = C_2 = 10\ (0.62831853)\ (\text{lb/(in./sec)})$

$D_1 = D_2 = 10\ (0.15811388)\ (\text{in.})$

$L_1 = 0.39223227\quad ,\quad L_2 = 0.58834841\ (\text{in.})$

TABLE 2

PROGRAM TRROBM

Program Input	Program Output*	
$G_{\ddot{u}}(f)$	$G_{\xi}(f)$	
Δf	$\overline{\xi^2}\ (f_1,f_N)$	$\xi = \ddot{u},\dot{u},u,\ddot{x},\dot{x},x,\ddot{\theta},\dot{\theta},\theta,\ddot{y},\dot{y},y,\ddot{y}_P,\dot{y}_P,y_P$
(f_1,f_N)	$\xi_{rms}\ (f_1,f_N)$	
C_1	f_{ic} , $i = 1,2$	
C_2	Modal Column Corresponding to f_{ic} , $i = 1,2$	
D_1	f_{nx}	
D_2	$f_{n\theta}$	
K_1	$\lvert T_{\xi/u}(2\pi jf)\rvert^2$, $\xi = x,\theta,y,y_P$	
K_2		
L_1		
L_2		
L_P		
m		
I		

* A print of all tabulated functions of frequency is optional

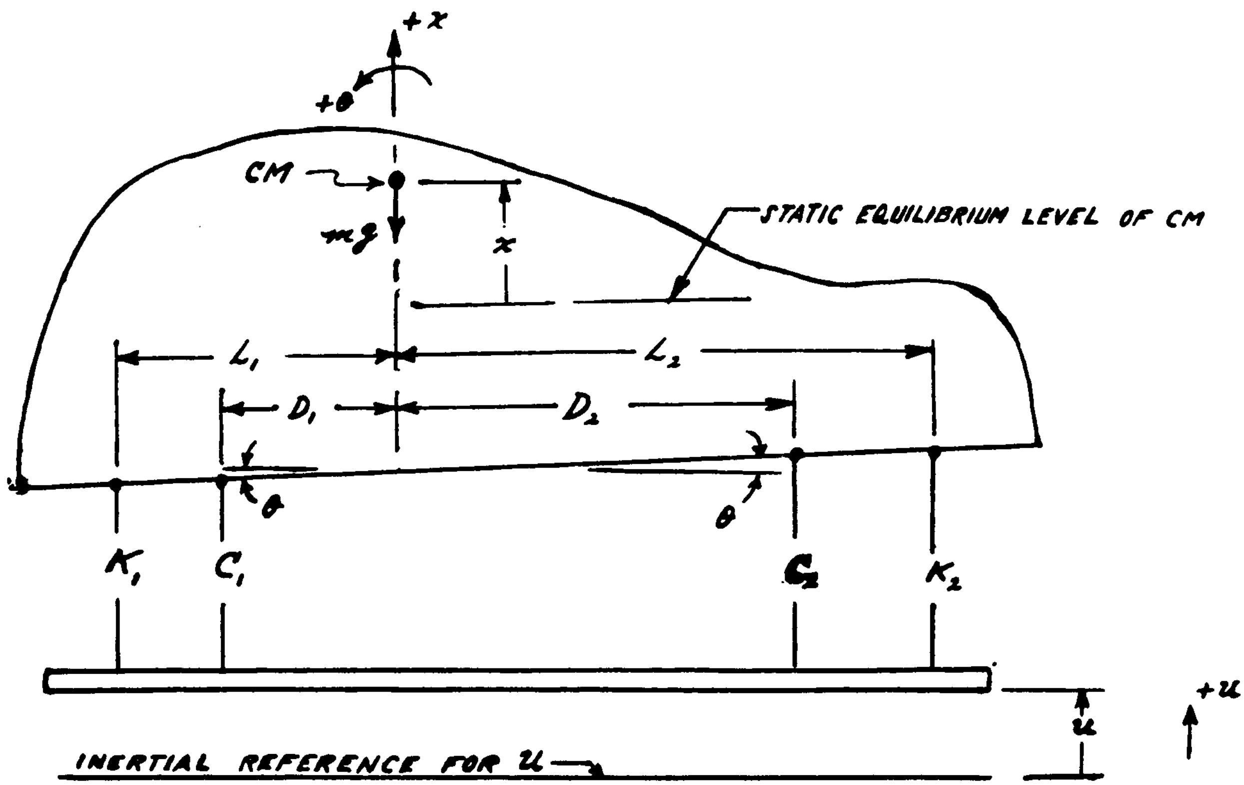

+x
+θ
CM
mg
x
STATIC EQUILIBRIUM LEVEL OF CM
L1
L2
D1
D2
θ
θ
K1
C1
C2
K2
+u
u
INERTIAL REFERENCE FOR u

FIGURE 1

ORIGINAL PAGE IS
OF POOR QUALITY

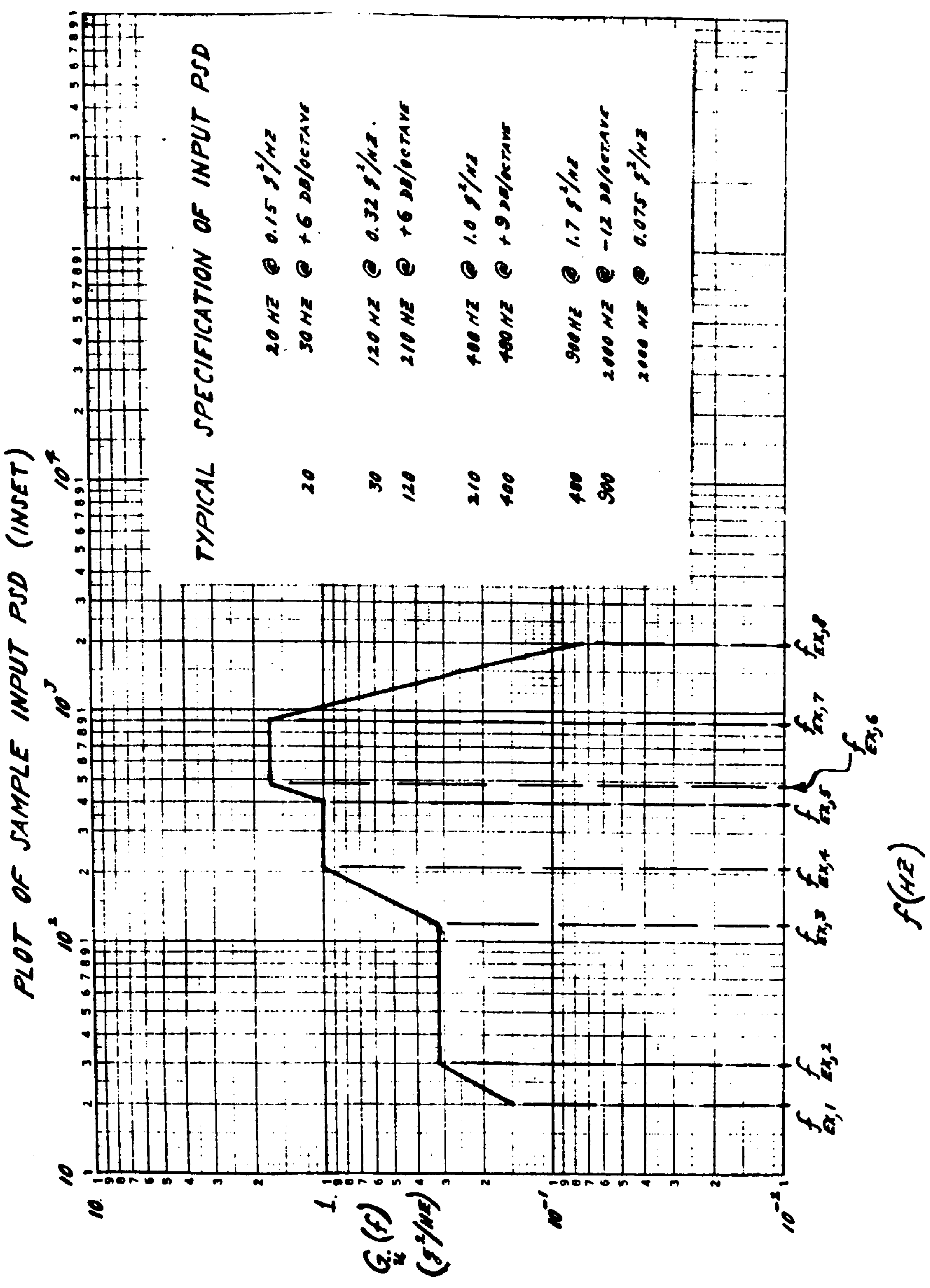
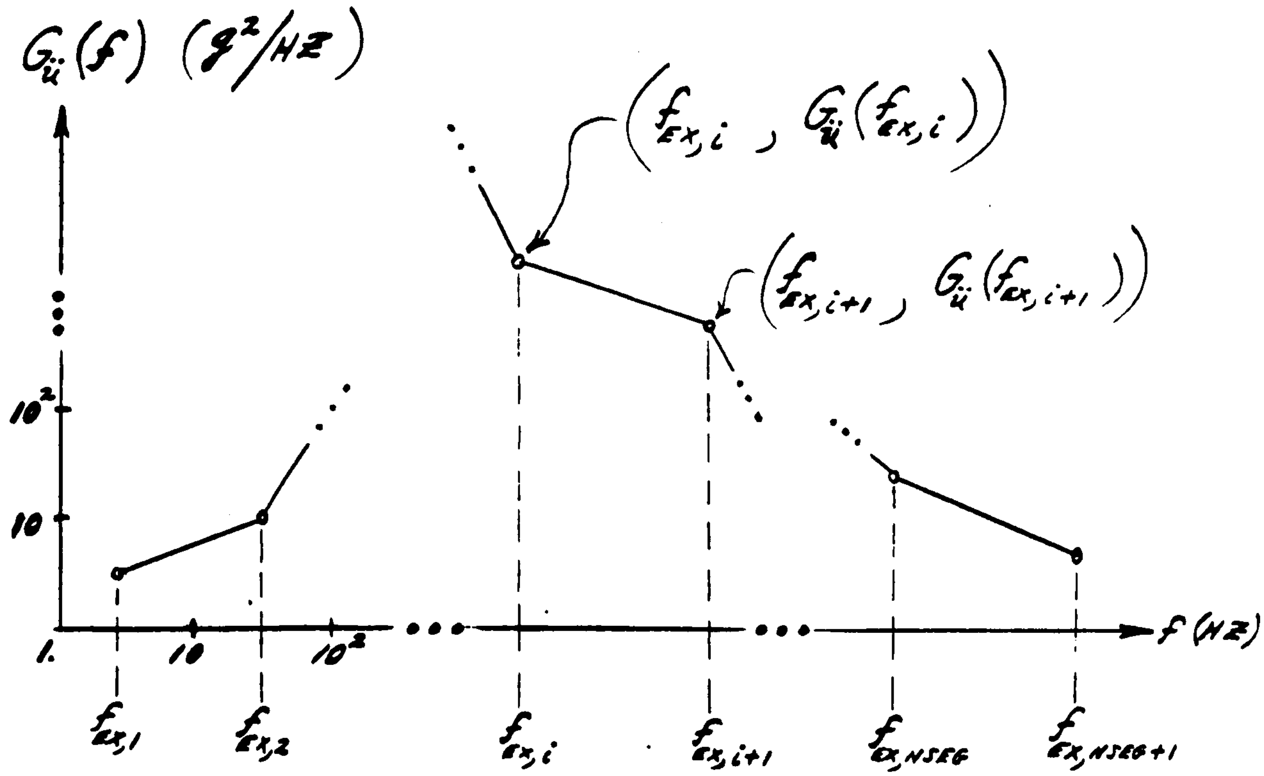


FIGURE 2

PLOT OF HYPOTHETICAL INPUT ACCELERATION PSD ON LOG-LOG PAPER



$$G_{\ddot{u}}(f) = \alpha_i\, f^{b_i}\ ,\quad f_{EX,i} \le f \le f_{EX,i+1}\ ,\quad i = 1, \cdots, NSEG$$

$$b_i = \frac{\log_{10}\left\{\dfrac{G_{\ddot{u}}(f_{EX,i+1})}{G_{\ddot{u}}(f_{EX,i})}\right\}}{\log_{10}\left\{\dfrac{f_{EX,i+1}}{f_{EX,i}}\right\}} = \frac{\Delta DB(i, i+1)}{10 \log_{10} 2}$$

$$\alpha_i = G_{\ddot{u}}(f_{EX,i}) \Big/ f_{EX,i}^{\,b_i} = G_{\ddot{u}}(f_{EX,i+1}) \Big/ f_{EX,i+1}^{\,b_i}$$

FIGURE 3

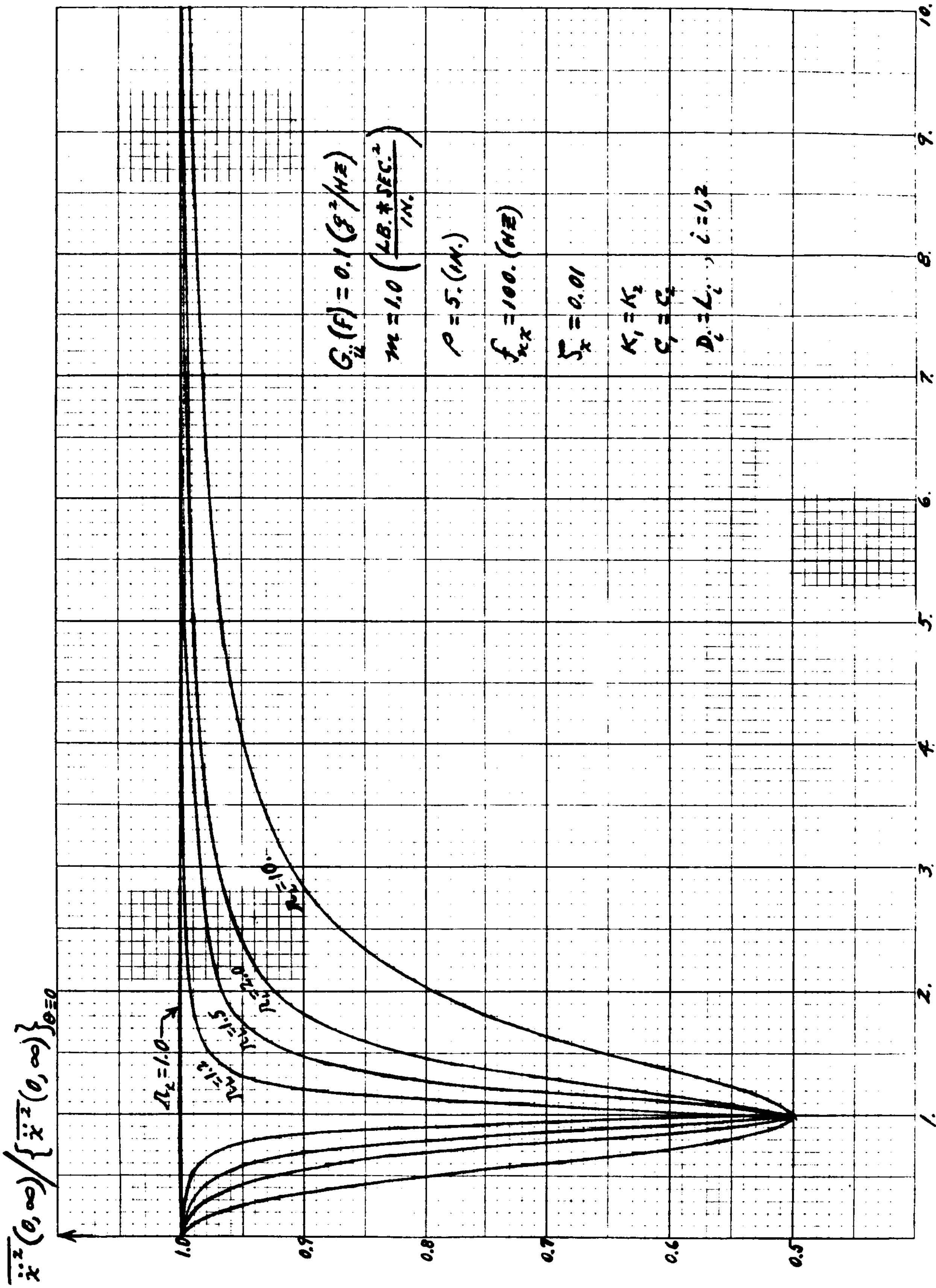

Gẍ(F) = 0.1 (g²/HZ)
m = 1.0 (LB.*SEC.²/IN.)
ρ = 5.(IN.)
f = 100.(HZ)
ζ = 0.01
K1 = K2
C1 = C2
Di = Li , i = 1,2
1.0
0.9
0.8
0.7
0.6
0.5
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.

FIGURE 4

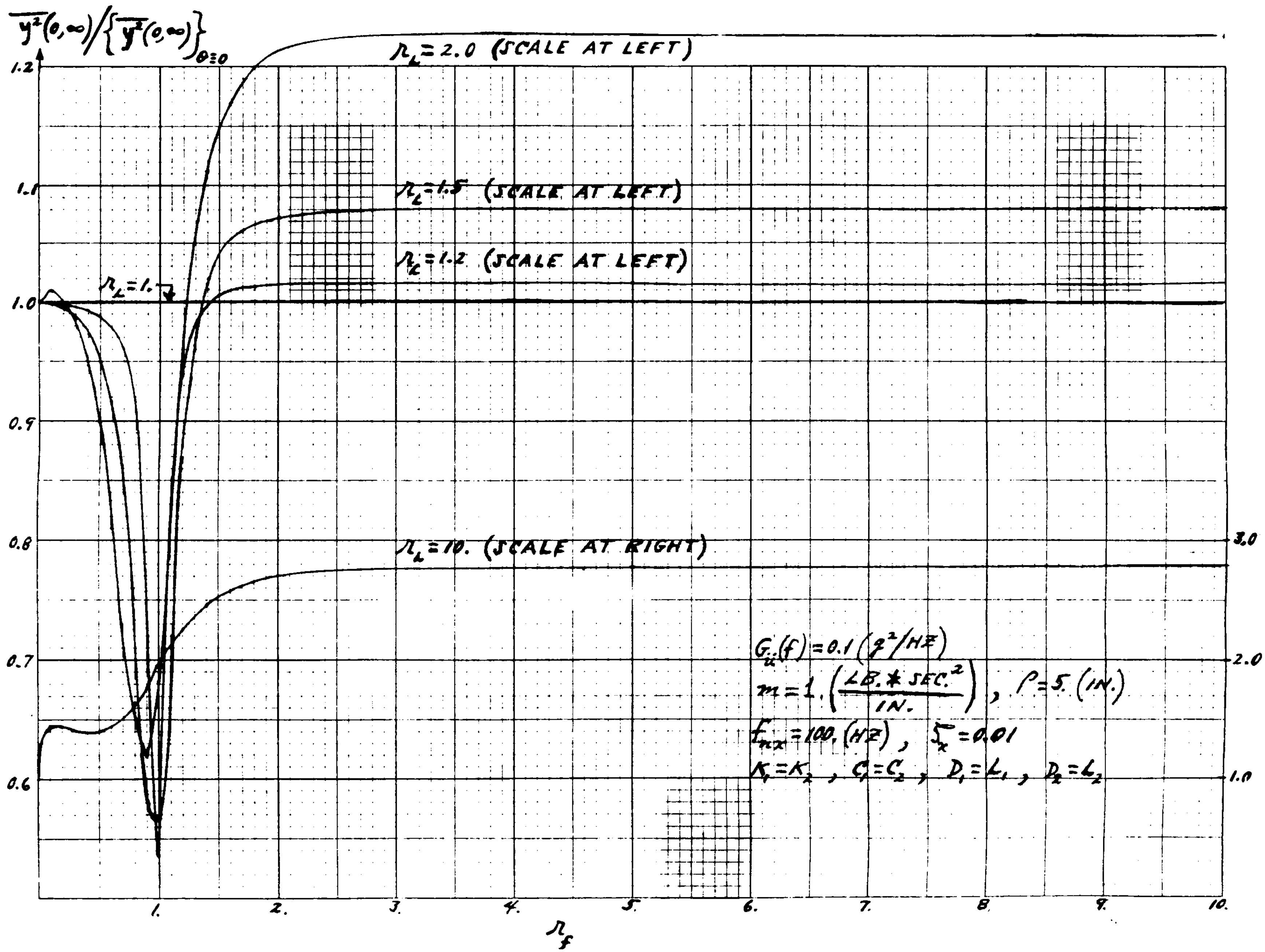

$\overline{y^2}(0,\infty) / \{\overline{y^2}(0,\infty)\}_{\theta=0}$
1.2
1.1
1.0
0.9
0.8
0.7
0.6
$r_L = 2.0$ (SCALE AT LEFT)
$r_L = 1.5$ (SCALE AT LEFT)
$r_L = 1.2$ (SCALE AT LEFT)
$r_L = 1.$
$r_L = 10.$ (SCALE AT RIGHT)
3.0
2.0
1.0
$G_{\ddot{u}}(f) = 0.1\ (g^2/HZ)$
$m = 1.\left(\frac{LB. * SEC.^2}{IN.}\right)$, $\rho = 5.\ (IN.)$
$f_{nx} = 100.\ (HZ)$, $\zeta_x = 0.01$
$K_1 = K_2$, $C_1 = C_2$, $D_1 = L_1$, $D_2 = L_2$
1.
2.
3.
4.
5.
6.
7.
8.
9.
10.
r_f

FIGURE 5

ORIGINAL PAGE IS
OF POOR QUALITY

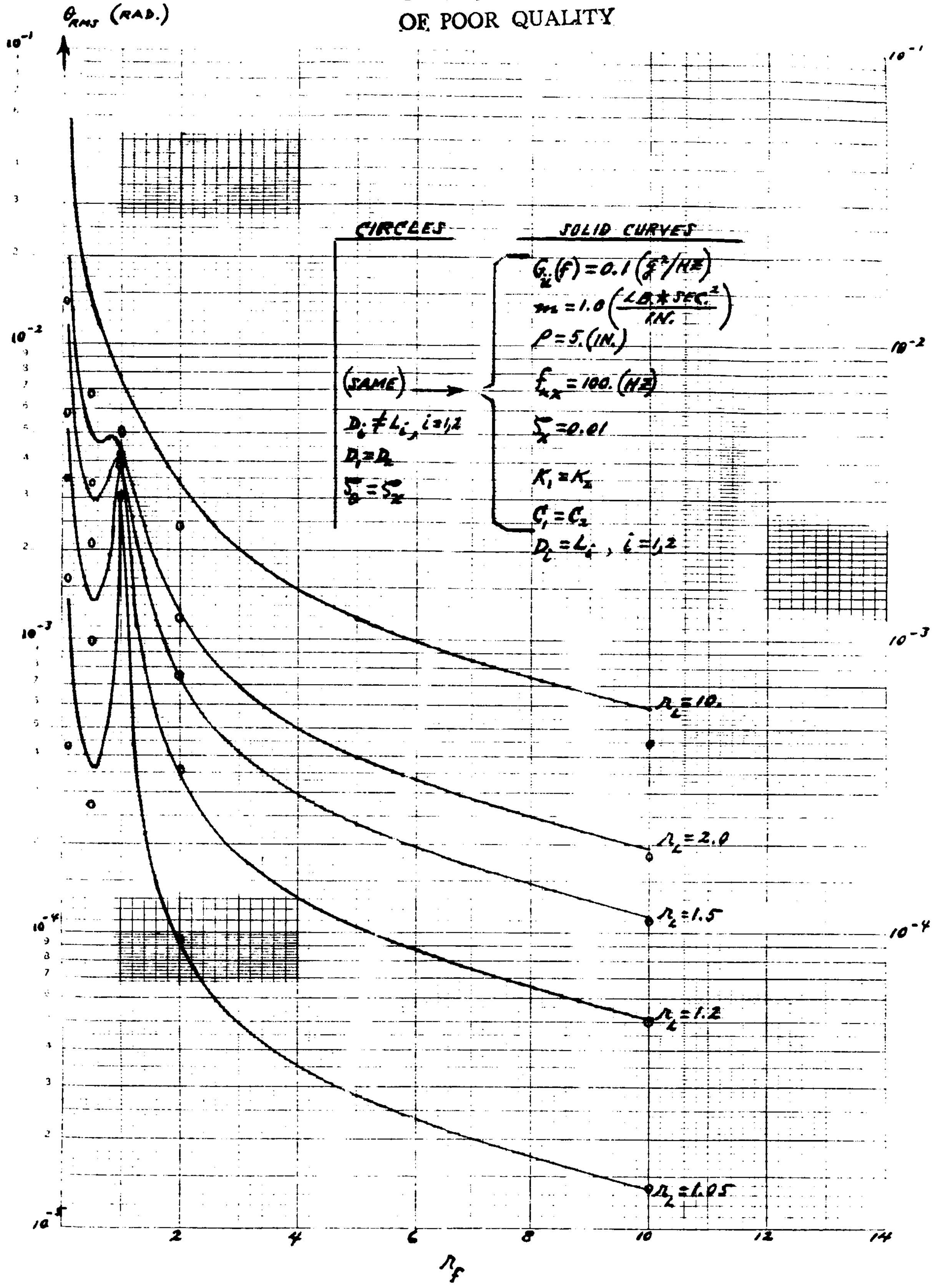
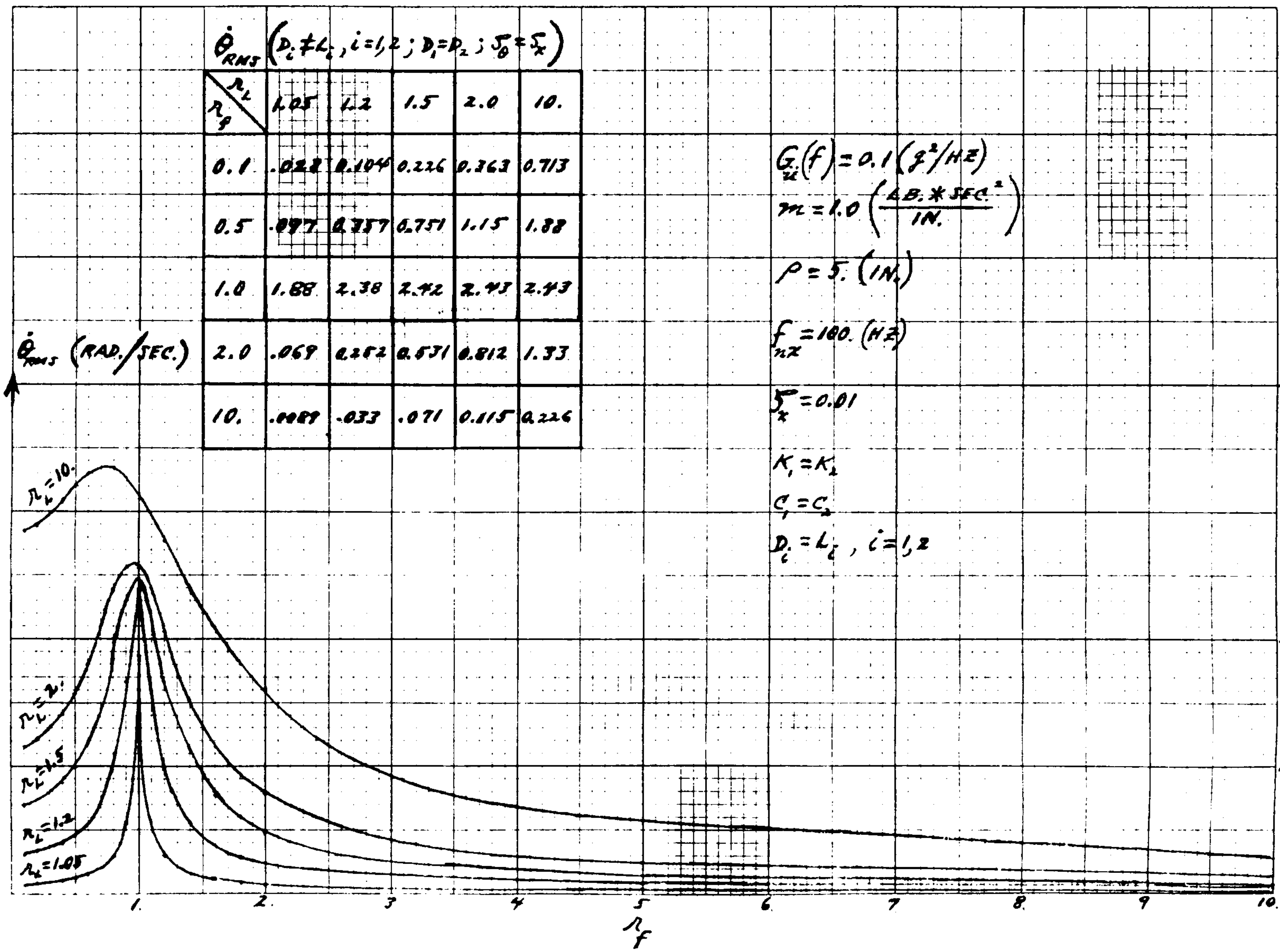


FIGURE 6



$\dot{\theta}_{RMS}$ $(D_i \neq L_i\ ,\ i=1,2\ ;\ D_1 = D_2\ ;\ \zeta_\theta = \zeta_x)$

R_f \ R_L	1.05	1.2	1.5	2.0	10.
0.1	.028	0.104	0.226	0.363	0.713
0.5	.097	0.357	0.751	1.15	1.88
1.0	1.88	2.38	2.42	2.43	2.43
2.0	.069	0.252	0.531	0.812	1.33
10.	.0089	.033	.071	0.115	0.226

FIGURE 7

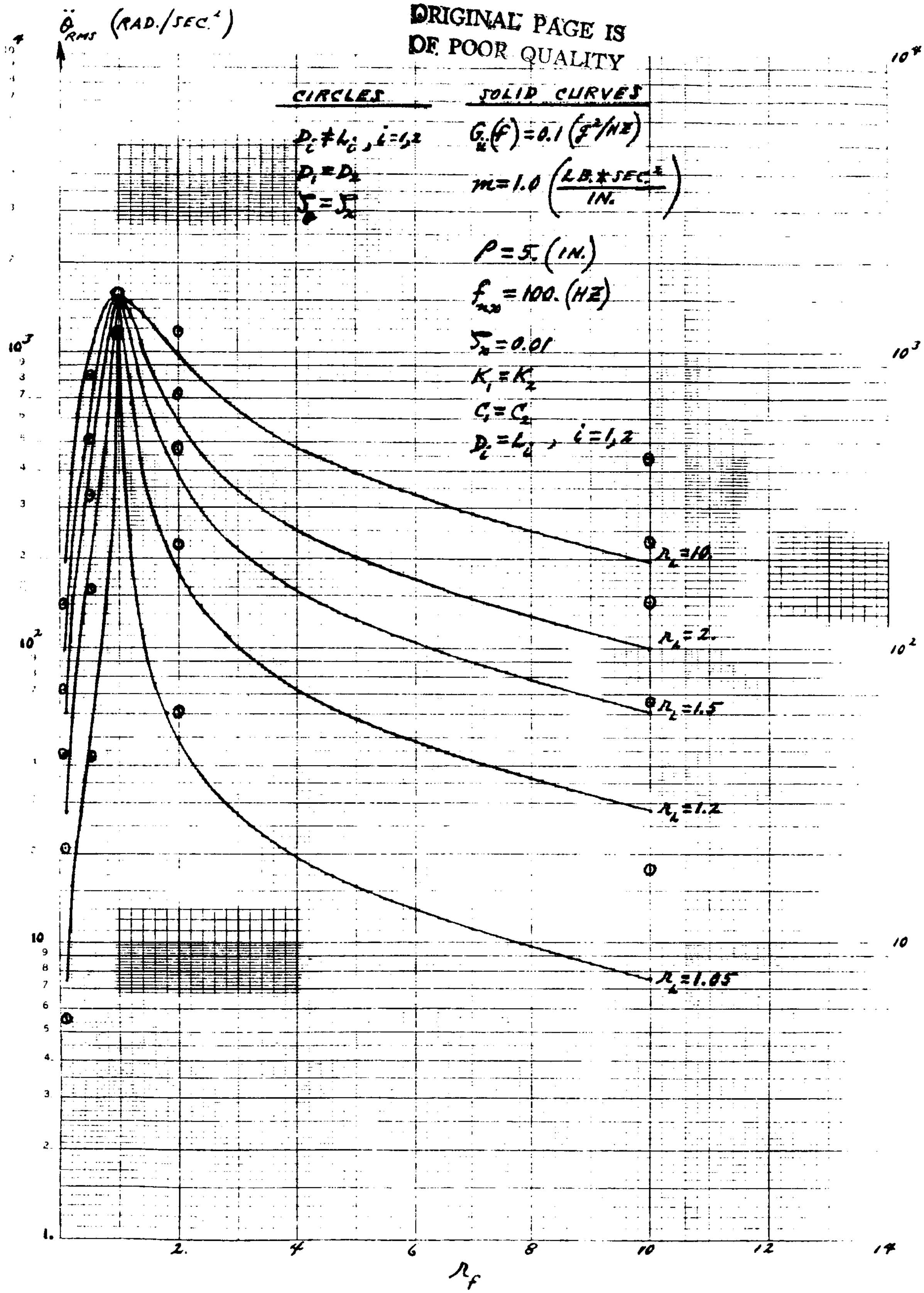
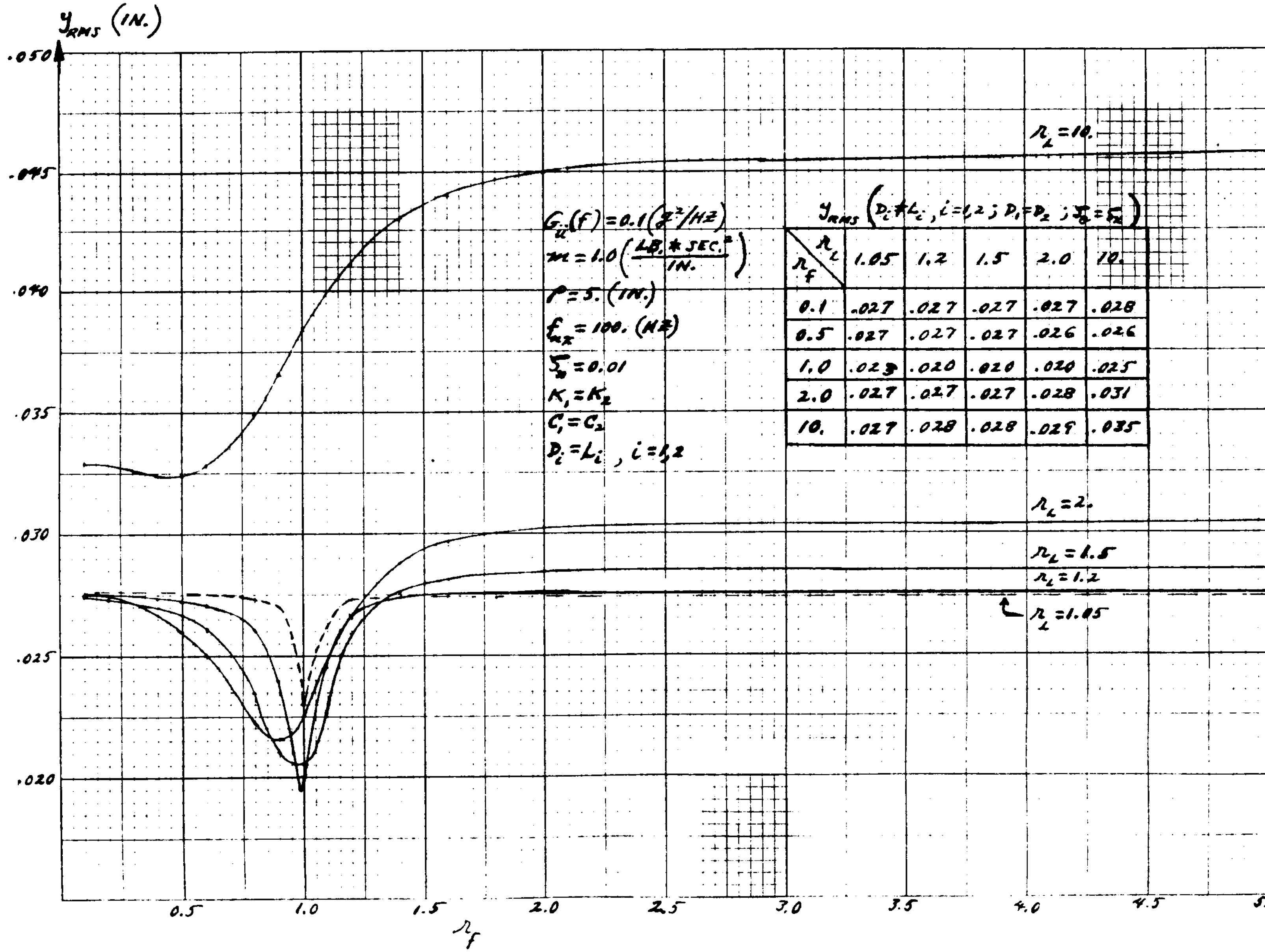
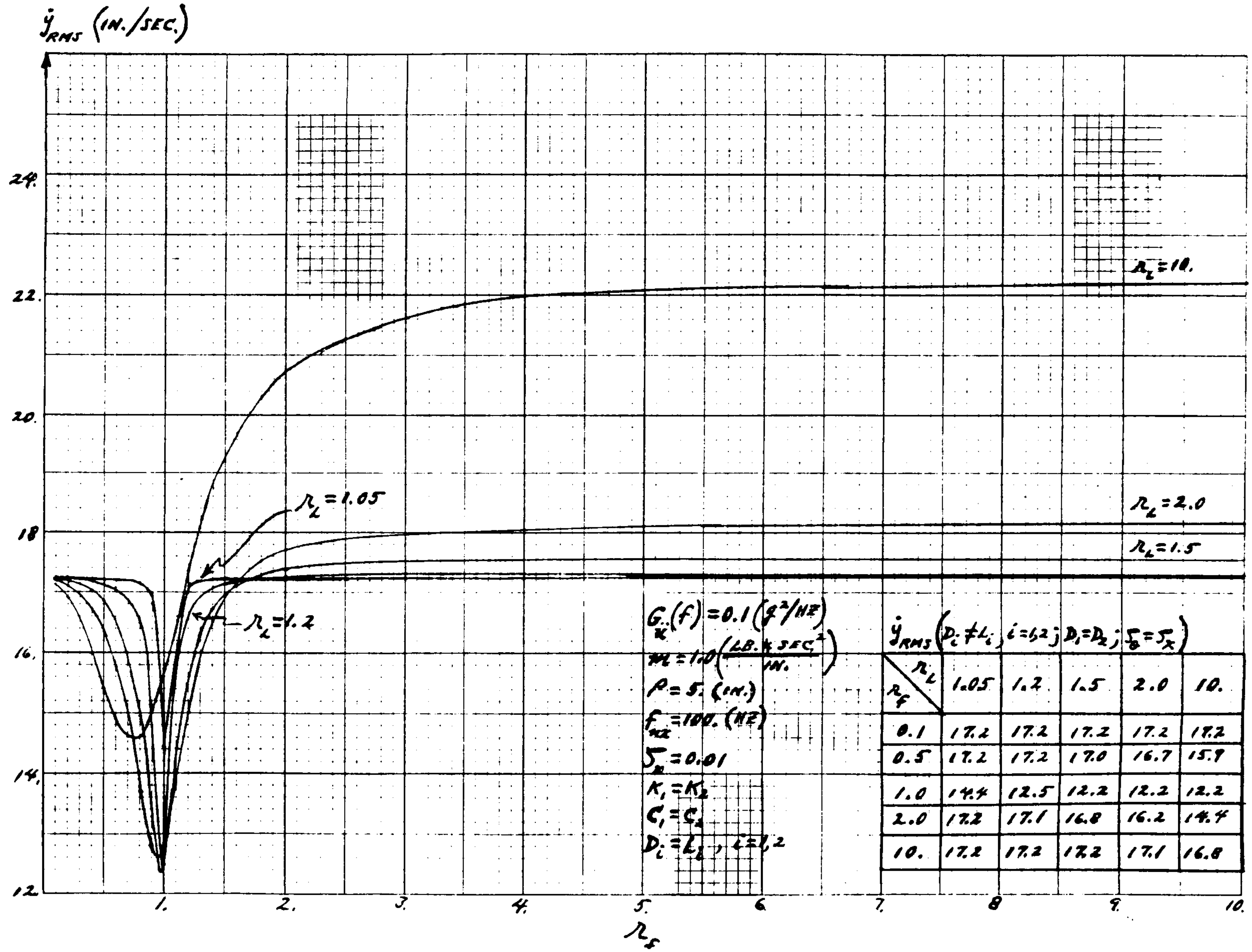

$\ddot{\theta}_{RMS}$ (RAD./SEC.^{2})
CIRCLES
$D_i \neq L_i$, $i=1,2$
$D_1 = D_2$
$\zeta_1 = \zeta_2$
SOLID CURVES
$G_{\ddot{u}}(f) = 0.1\ (g^2/HZ)$
$m = 1.0\ \left(\frac{LB.*SEC^2}{IN.}\right)$
$\rho = 5.\ (IN.)$
$f_{n_{20}} = 100.\ (HZ)$
$\zeta_{20} = 0.01$
$K_1 = K_2$
$C_1 = C_2$
$D_i = L_i$, $i=1,2$
$\lambda_L = 10.$
$\lambda_L = 2.$
$\lambda_L = 1.5$
$\lambda_L = 1.2$
$\lambda_L = 1.05$
10^4
10^3
10^2
10
1.
2.
4
6
8
10
12
14
λ_f

FIGURE 8



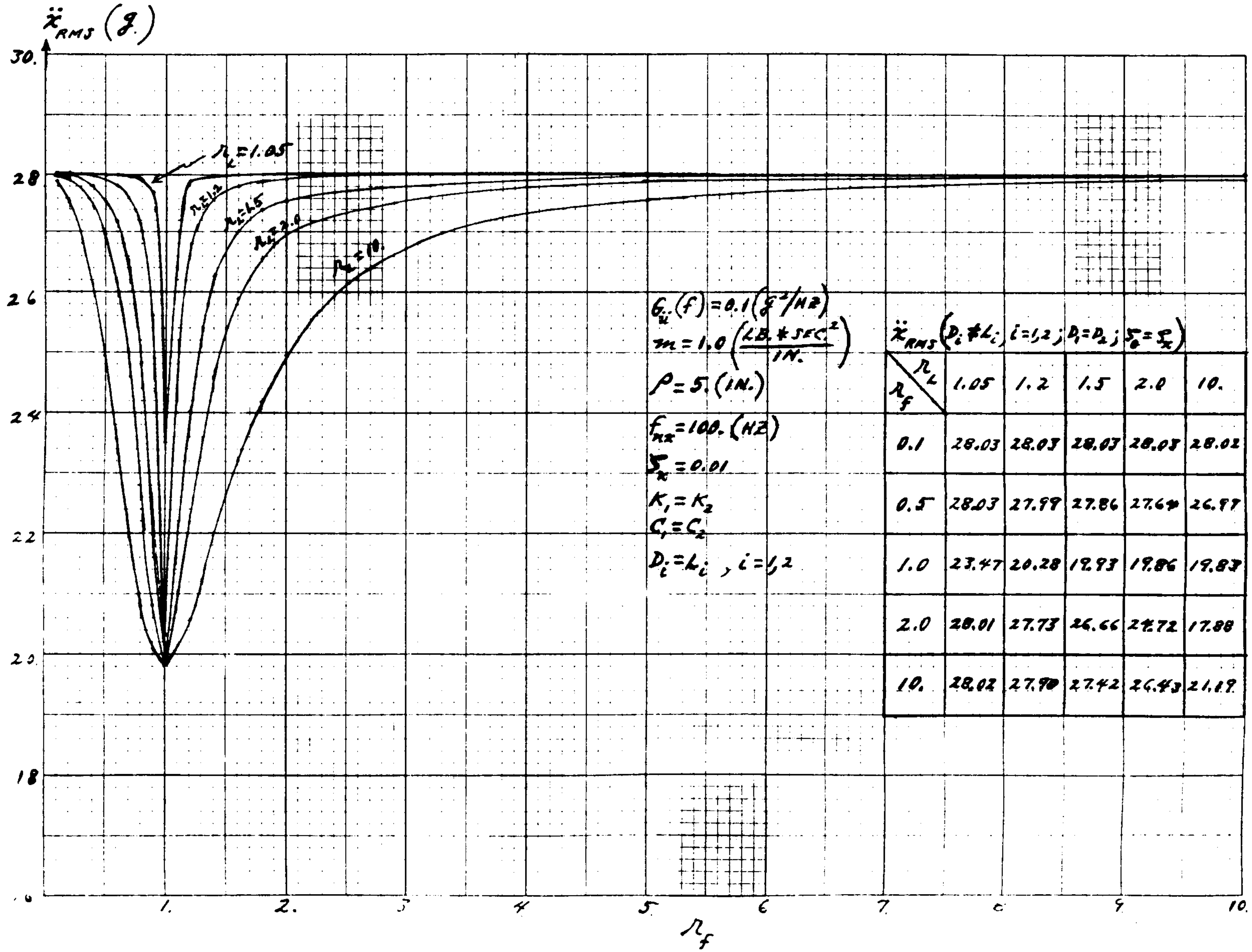
r_f \ r_L	1.05	1.2	1.5	2.0	10.
0.1	.027	.027	.027	.027	.028
0.5	.027	.027	.027	.026	.026
1.0	.023	.020	.020	.020	.025
2.0	.027	.027	.027	.028	.031
10.	.027	.028	.028	.029	.035

FIGURE 9



$\dot{y}_{RMS}$ (IN./SEC.)
24.
22.
20.
18
16.
14.
12
1.
2.
3.
4.
5.
6
7.
8
9.
10.
λ_f
$\lambda_L = 10.$
$\lambda_L = 2.0$
$\lambda_L = 1.5$
$\lambda_L = 1.05$
$\lambda_L = 1.2$
$G_{\ddot{x}}(f) = 0.1\ (g^2/HZ)$
$M = 1.0\ \left(\frac{LB.\cdot SEC^2}{IN.}\right)$
$\rho = 5.\ (IN.)$
$f_{nz} = 100.\ (HZ)$
$\zeta_n = 0.01$
$K_1 = K_2$
$C_1 = C_2$
$D_i = L_i\ ,\ i = 1,2$
$\dot{y}_{RMS}\ (D_i \neq L_i\ ,\ i = 1,2\ ;\ D_1 = D_2\ ;\ \zeta_\theta = \zeta_x)$
λ_f \ λ_L | 1.05 | 1.2 | 1.5 | 2.0 | 10.
0.1 | 17.2 | 17.2 | 17.2 | 17.2 | 17.2
0.5 | 17.2 | 17.2 | 17.0 | 16.7 | 15.9
1.0 | 14.4 | 12.5 | 12.2 | 12.2 | 12.2
2.0 | 17.2 | 17.1 | 16.8 | 16.2 | 14.4
10. | 17.2 | 17.2 | 17.2 | 17.1 | 16.8

FIGURE 10



$\ddot{x}_{RMS}$ $(D_i \neq L_i;\ i=1,2;\ D_1 = D_2;\ \zeta_\theta = \zeta_x)$

Ω_f \ Ω_L	1.05	1.2	1.5	2.0	10.
0.1	28.03	28.03	28.03	28.03	28.02
0.5	28.03	27.99	27.86	27.64	26.97
1.0	23.47	20.28	19.93	19.86	19.83
2.0	28.01	27.73	26.66	24.72	17.88
10.	28.02	27.90	27.42	26.43	21.19

FIGURE 11

ORIGINAL PAGE IS
OF POOR QUALITY

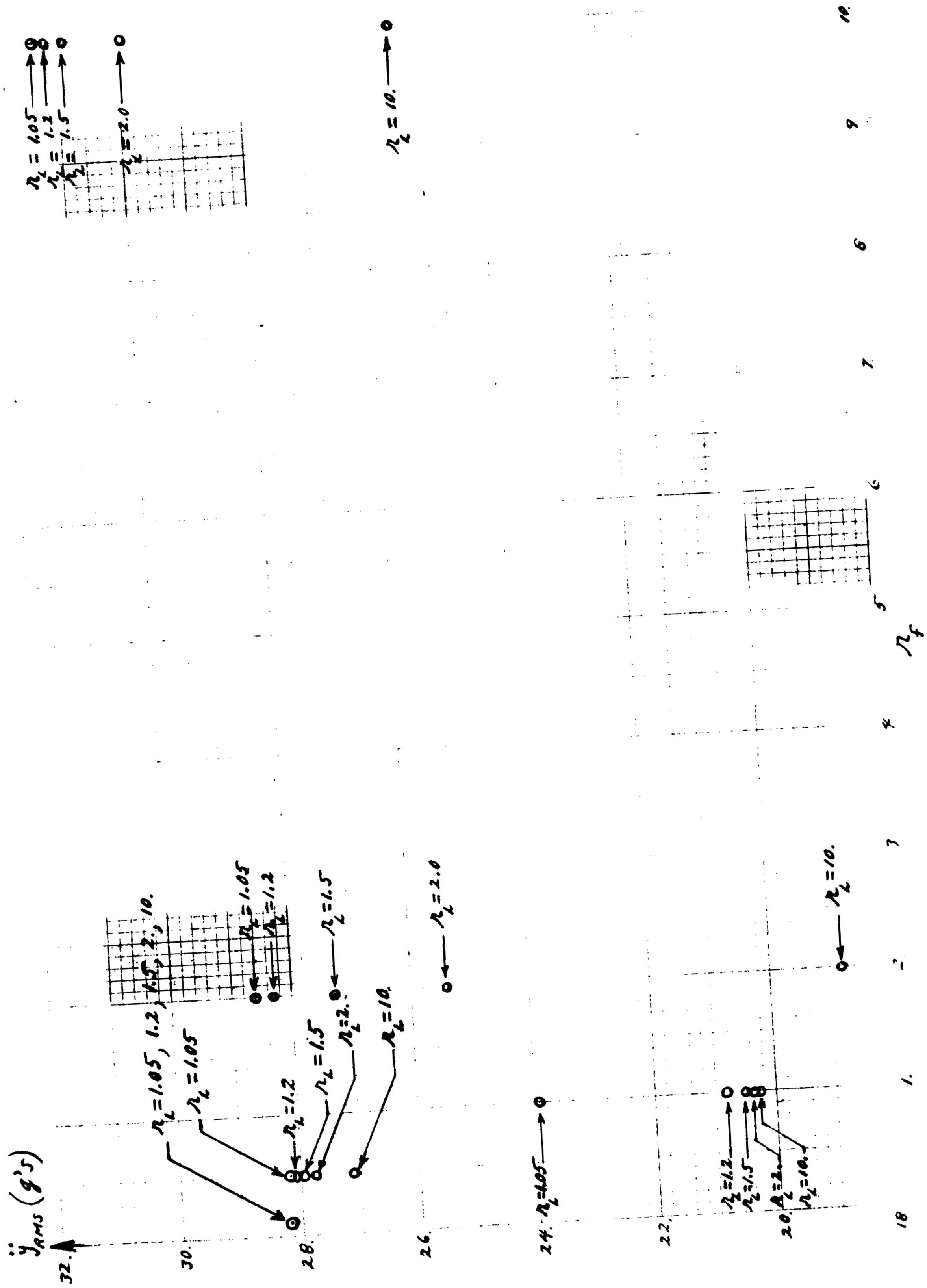


FIGURE 12

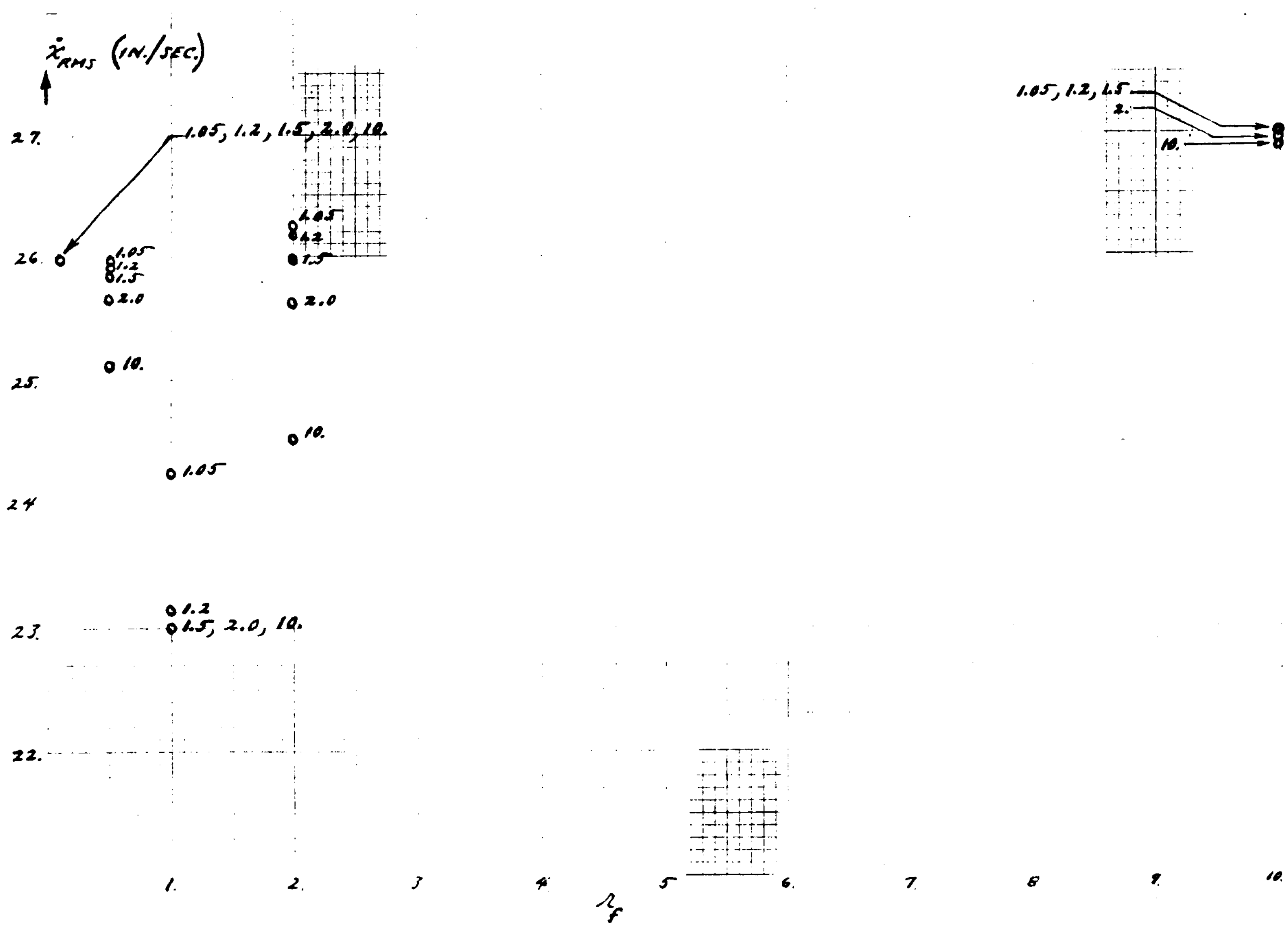

$\dot{x}_{RMS}$ (IN./SEC.)
27.
26.
25.
24
23.
22.
1.05, 1.2, 1.5, 2.0, 10.
1.05
1.2
1.5
2.0
10.
1.05
1.2, 1.5, 2.0, 10.
1.05, 1.2, 1.5
2.
10.
1.
2.
3
4
5
6.
7.
8
9.
10.
λ_f

FIGURE 13

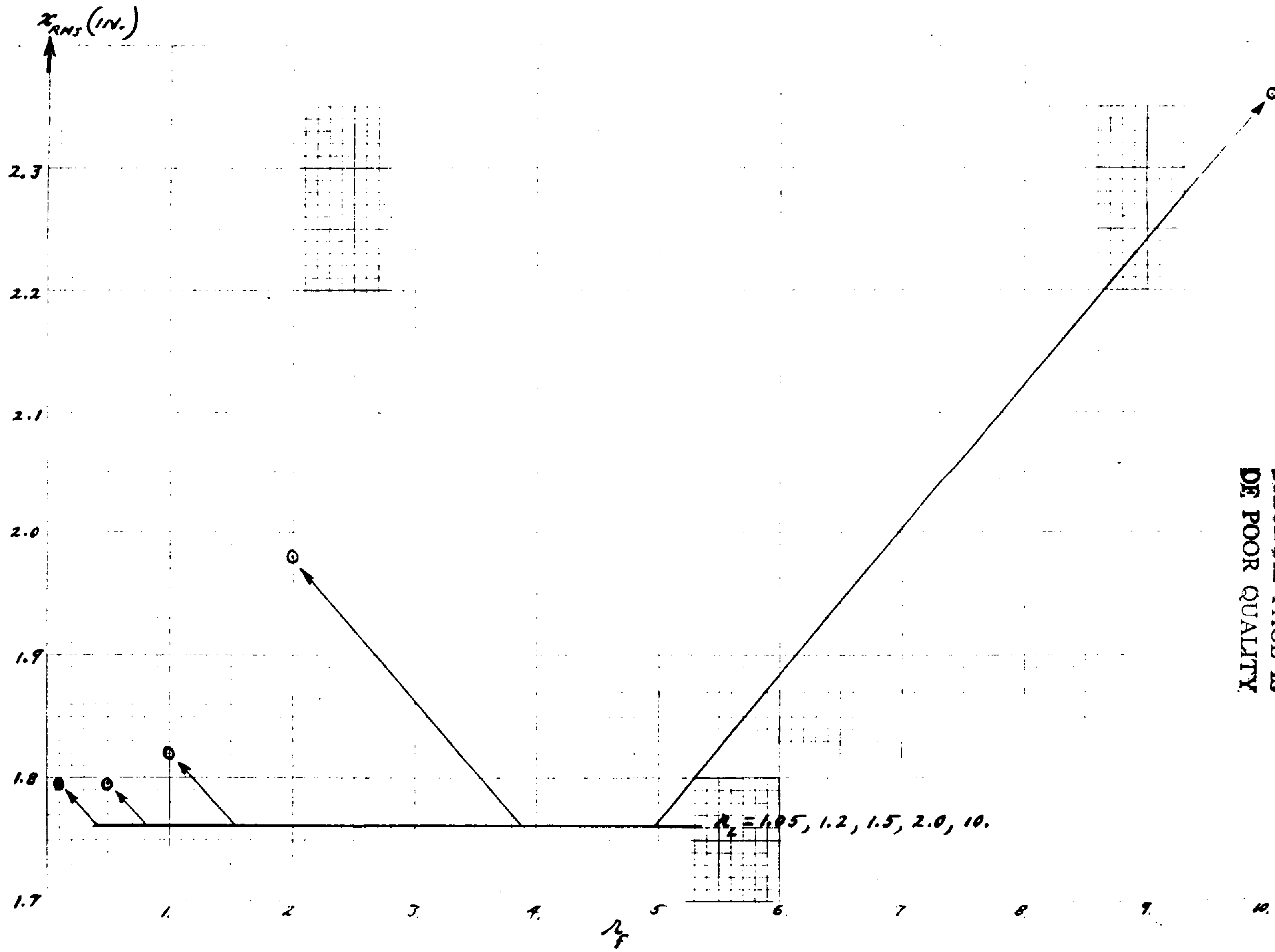
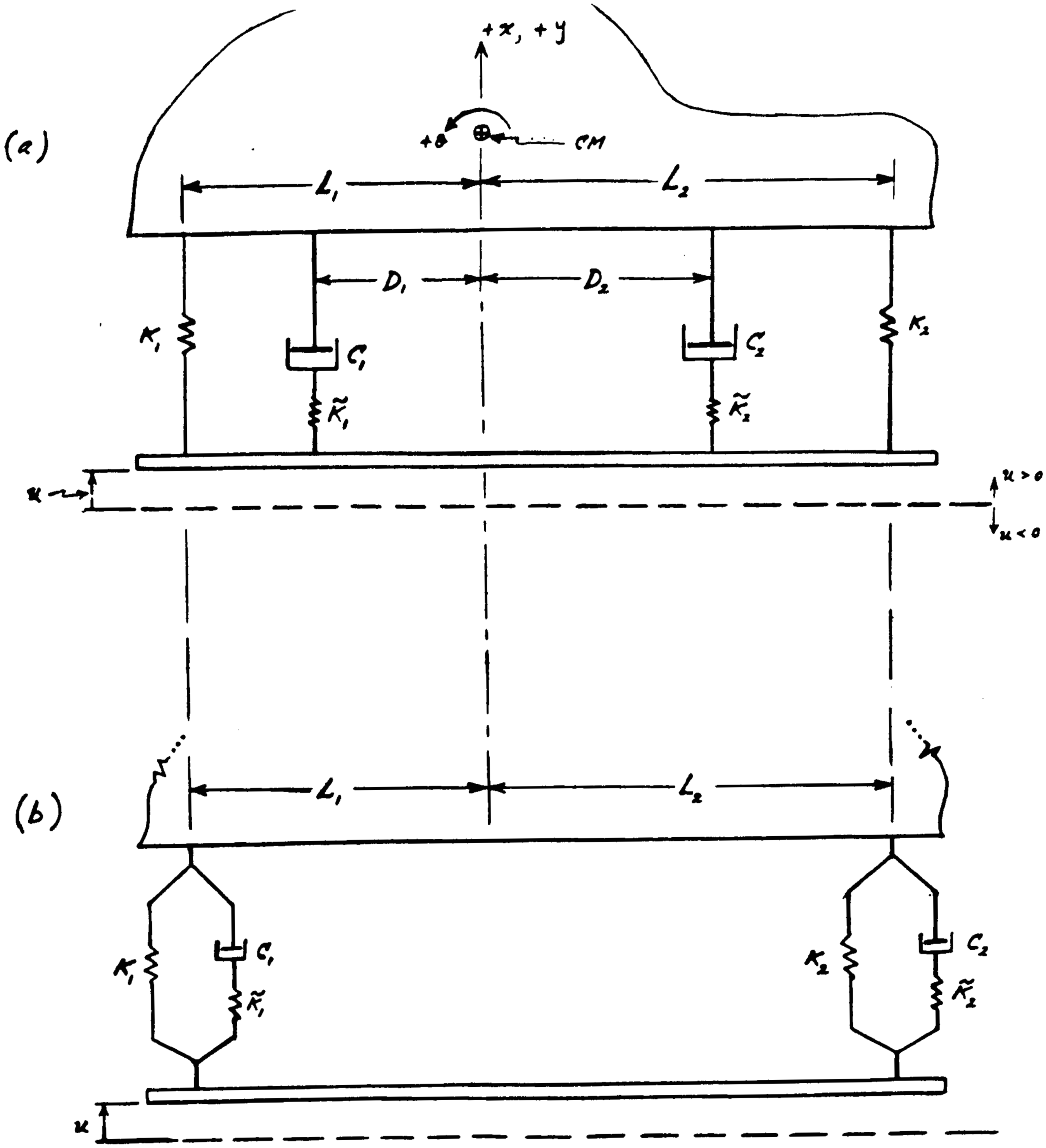


FIGURE 14



x = ABSOLUTE DISPLACEMENT OF CM FROM STATIC EQUILIBRIUM POSITION

$y = x - u$ = DISPLACEMENT OF CM RELATIVE TO CARRIER VEHICLE

FIGURE 15

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3. Seifert, W. W., and Steeg, C. W.: Control Systems Engineering. McGraw-Hill, 1960.

4. Grandall, S. H., and Mark, W. D.: Random Vibration in Mechanical Systems. Academic Press, 1963.

5. Tse, F. S.; Morse, I. E.; and Hinkle, R. T.: Mechanical Vibrations. Allyn and Bacon, 1963.

DEFINITIONS

C_i	Damping coefficient [lb/(in./sec)] of viscous damper i, i = 1,2
D_i	Lateral distance (in.) between CM and point at which damper i is attached to model, i = 1,2 ($D_i \geq 0$)
r_D	D_1/D_2
f	Frequency (HZ), $f_1 \leq f \leq f_N$
f_{ic}	Frequency (HZ) of undamped coupled natural mode i, i = 1,2
f_{nx}	Frequency (HZ) of undamped uncoupled translational mode
$f_{n\theta}$	Frequency (HZ) of undamped uncoupled rotational mode
g	Acceleration due to gravity (in./sec^2)
$G_{\ddot{u}}(f)$	PSD or $\ddot{u}$ (g^2/HZ)
$G_{\dot{u}}(f)$	PSD of $\dot{u}$ [(in./sec)^2/HZ]
$G_u(f)$	PSD of u (in.^{2}/HZ)
$G_{\ddot{x}}(f)$	PSD of $\ddot{x}$ (g^2/HZ)
$G_{\dot{x}}(f)$	PSD of $\dot{x}$ [(in./sec)^2/HZ]
$G_x(f)$	PSD of x (in.^{2}/HZ)
$G_{\ddot{\theta}}(f)$	PSD of $\ddot{\theta}$ [(rad./sec^2)^2/HZ]
$G_{\dot{\theta}}(f)$	PSD of $\dot{\theta}$ [(rad./sec)^2/HZ]
$G_\theta(f)$	PSD of θ (rad.^{2}/HZ)
$G_{\ddot{y}}(f)$	PSD of $\ddot{y}$ (g^2/HZ)
$G_{\dot{y}}(f)$	PSD of $\dot{y}$ [(in./sec)^2/HZ]
$G_y(f)$	PSD of y (in.^{2}/HZ)
$G_{\ddot{y}_p}(f)$	PSD of $\ddot{y}_p$ (g^2/HZ)

$G_{\dot{y}_P}(f)$	PSD of $\dot{y}_P$ [(in./sec)^2/HZ]
$G_{y_P}(f)$	PSD of y_P (in.^{2}/HZ)
θ	Angular displacement (rad.) of model from static equilibrium orientation
$\dot{\theta}$	Angular velocity (rad./sec) of model
$\ddot{\theta}$	Angular acceleration (rad./sec^2) of model
$\overline{\theta^2}(f_1,f_N)$	Mean square value (rad.^{2}) of θ in the interval (f_1,f_N)
$\theta_{rms}(f_1,f_N)$	Root mean square (rad.) of θ on the interval (f_1,f_N)
$\overline{\dot{\theta}^2}(f_1,f_N)$	Mean square value, (rad./sec)^2, of $\dot{\theta}$ on the interval (f_1,f_N)
$\dot{\theta}_{rms}(f_1,f_N)$	Root mean square (rad./sec) of $\dot{\theta}$ on the interval (f_1,f_N)
$\overline{\ddot{\theta}^2}(f_1,f_N)$	Mean square value [(rad./sec^2)^2] of $\ddot{\theta}$ on the interval (f_1,f_N)
$\ddot{\theta}_{rms}(f_1,f_N)$	Root mean square (rad./sec^2) of $\ddot{\theta}$ on the interval (f_1,f_N)
y	Displacement (in.) of model CM relative to the base
$\dot{y}$	Velocity (in./sec) of model CM relative to the base
$\ddot{y}$	Acceleration (in./sec^2) of model CM relative to the base
$\overline{y^2}(f_1,f_N)$	Mean square value (in.^{2}) of y on the interval (f_1,f_N)
$y_{rms}(f_1,f_N)$	Root mean square (in.) of y on the interval (f_1,f_N)
$\overline{\dot{y}^2}(f_1,f_N)$	Mean square value, (in./sec)^2, of $\dot{y}$ on the interval (f_1,f_N)
$\dot{y}_{rms}(f_1,f_N)$	Root mean square (in./sec) of $\dot{y}$ on the interval (f_1,f_N)
$\overline{\ddot{y}^2}(f_1,f_N)$	Mean square value (g^2) of $\ddot{y}$ on interval (f_1,f_N)
$\ddot{y}_{rms}(f_1,f_N)$	Root mean square (g) of $\ddot{y}$ on the interval (f_1,f_N)
y_P	Displacement (in.) of point P relative to the base
$\dot{y}_P$	Velocity (in./sec) of point P relative to the base

$\ddot{y}_P$	Acceleration (in./sec^2) of point P relative to the base
$\overline{y_P^2}(f_1, f_N)$	Mean square value (in.^{2}) of y_P on the interval (f_1, f_N)
$y_{P,rms}(f_1, f_N)$	Root mean square (in.) of y_P on the interval (f_1, f_N)
$\overline{\dot{y}_P^2}(f_1, f_N)$	Mean square value, (in./sec)^2, of $\dot{y}_P$ on the interval (f_1, f_N)
$\dot{y}_{P,rms}(f_1, f_N)$	Root mean square (in./sec) of $\dot{y}_P$ on the interval (f_1, f_N)
$\overline{\ddot{y}_P^2}(f_1, f_N)$	Mean square value (g^2) of $\ddot{y}_P$ on the interval (f_1, f_N)
$\ddot{y}_{P,rms}(f_1, f_N)$	Root mean square (g) of $\ddot{y}_P$ on the interval (f_1, f_N)
$\delta_{ST,i}$	Static deflection (in.) of spring i, i = 1,2 (considered positive despite my sign convention)
ω_{ic}	$2\pi f_{ic}$ (rad./sec), i = 1,2
Δf	Both the print step (HZ) and the frequency increment (HZ) used in the numerical evaluation of the definite integrals defining the mean squares
u	Displacement (in.) of the base from its static equilibrium position
$\dot{u}$	Base velocity (in./sec)
$\ddot{u}$	Base acceleration (in./sec^2)
$\overline{u^2}(f_1, f_N)$	Mean square value (in.^{2}) of u on the interval (f_1, f_N)
$u_{rms}(f_1, f_N)$	Root mean square value (in.) of u on the interval (f_1, f_N)
$\overline{\dot{u}^2}(f_1, f_N)$	Mean square value (in./sec)^2 of $\dot{u}$ on the interval (f_1, f_N)
$\dot{u}_{rms}(f_1, f_N)$	Root mean square value (in./sec) of $\dot{u}$ on the interval (f_1, f_N)
$\overline{\ddot{u}^2}(f_1, f_N)$	Mean square value (g^2) of $\ddot{u}$ on the interval (f_1, f_N)
$\ddot{u}_{rms}(f_1, f_N)$	Root mean square value (g) of $\ddot{u}$ on the interval (f_1, f_N)
x	Displacement (in.) of model CM from its static equilibrium position (x is an absolute displacement)
$\dot{x}$	Absolute velocity (in./sec) of model CM

$\ddot{x}$	Absolute acceleration (in./sec^2) of model CM
$\overline{x^2}(f_1,f_N)$	Mean square value (in.^{2}) of x on the interval (f_1,f_N)
$x_{rms}(f_1,f_N)$	Root mean square (in.) of x on the interval (f_1,f_N)
$\overline{\dot{x}^2}(f_1,f_N)$	Mean square value (in./sec)^2 of $\dot{x}$ on the interval (f_1,f_N)
$\dot{x}_{rms}(f_1,f_N)$	Root mean square value (in./sec) of $\dot{x}$ on the interval (f_1,f_N)
$\overline{\ddot{x}^2}(f_1,f_N)$	Mean square value (g^2) of $\ddot{x}$ on the interval (f_1,f_N)
$\ddot{x}_{rms}(f_1,f_N)$	Root mean square (g) of $\ddot{x}$ on the interval (f_1,f_N)
s	The complex variable of the laplace transformation (rad./sec)
$\lvert T_{\xi/u}(2\pi fj)\rvert^2$	Square of the magnitude of the frequency response function between ξ and u (dimensionless), $\xi = x,y,y_P$ ($j = \sqrt{-1.}$)
$\lvert T_{\theta/u}(2\pi fj)\rvert^2$	Square of the magnitude of the frequency response function between θ and u (rad./in.)^2
I	Moment of inertia about an axis through the CM and perpendicular to the plane of motion (lb*sec^2*in.)
K_i	Stiffness (lb/in.) of linear spring i, i = 1,2
$\tilde{K}_i$	Stiffness (lb/in.) of elastic support of damper i in alternate model (Fig. 15), i = 1,2
L_i	Lateral distance (in.) between model CM and the point at which spring i is attached to the model, i = 1,2
r_L	L_1/L_2
m	Mass of component [lb/(in./sec^2)]
ρ	Radius of gyration (in.)
L_P	Lateral distance (in.) between model CM and point P ($L_P > 0$ or $L_P < 0$ according as point P is right or left of the CM)
r_f	$f_{n\theta}/f_{nx}$
ζ_x	Fraction of critical damping associated with translation
ζ_θ	Fraction of critical damping associated with rotation

b_i, c_i	Parameters appearing in the analytical representation of $G_{\ddot{u}}(f)$ on the interval $(f_{EX,i}, f_{EX,i+1})$, $i = 1,\ldots,NSEG$
NSEG	Number of straight line segments in the log-log plot of the prescribed base acceleration PSD
$f_{EX,i}$	Abscissa (HZ) of the i'th "corner" point in the log-log plot of $G_{\ddot{u}}(f)$, $i = 1,\ldots NSEG+1$
NCORN	A positive integer specifying that "corner" point of the straight line segment representation of the input base acceleration PSD on log-log graph paper at which the value of the input base acceleration PSD is given
GCORN	The value (g^2/HZ) of the input base acceleration PSD at the "corner" point specified by the integer NCORN
$\Delta DB(i,i+1)$	Rate of change, in decibels/octave, of the input base acceleration PSD as the frequency f varies from $f_{EX,i}$ to $f_{EX,i+1}$, $i = 1,\ldots NSEG$

APPENDIX A

SUBSIDIARY RELATIONS

$$|T_{x/u}(2\pi jf)|^2 = \frac{R_x^{\ 2} + \mathscr{I}_x^{\ 2}}{\{Re(D)\}^2 + \{\mathscr{I}m(D)\}^2}$$

$$R_x = K_1K_2\ (L_1 + L_2)^2 - [I(K_1 + K_2) + C_1C_2\ (D_1 + D_2)^2]\ (2\pi f)^2$$

$$\mathscr{I}_x = -\ I\ (C_1 + C_2)\ (2\pi f)^3 + (2\pi f)\ [(C_1 + C_2)\ (K_1\ L_1^{\ 2} + K_2\ L_2^{\ 2})$$

$$+ (K_1 + K_2)\ (C_1D_1^{\ 2} + C_2D_2^{\ 2}) + 2\ (K_1L_1 - K_2L_2)\ (C_2D_2 - C_1D_1)]$$

$$|T_{\theta/u}(2\pi jf)|^2 = \frac{R_\theta^{\ 2} + \mathscr{I}_\theta^{\ 2}}{\{Re(D)\}^2 + \{\mathscr{I}m(D)\}^2}$$

$$R_\theta = -\ m\ (K_2L_2 - K_1L_1)\ (2\pi f)^2$$

$$\mathscr{I}_\theta = -\ m\ (C_2D_2 - C_1D_1)\ (2\pi f)^3$$

$$Re(D) = m\ I\ (2\pi f)^4 - [m\ (K_1L_1^{\ 2} + K_2L_2^{\ 2}) + I\ (K_1 + K_2) + C_1C_2\ (D_1 + D_2)^2]\ (2\pi f)$$

$$+ K_1K_2\ (L_1 + L_2)^2$$

$$\mathscr{I}m(D) = [(C_1 + C_2)\ (K_1L_1^{\ 2} + K_2L_2^{\ 2}) + (K_1 + K_2)\ (C_1D_1^{\ 2} + C_2D_2^{\ 2})$$

$$+ 2\ (K_1L_1 - K_2L_2)\ (C_2D_2 - C_1D_1)]\ (2\pi f) - [m\ (C_1D_1^{\ 2} + C_2D_2^{\ 2})$$

$$+ I\ (C_1 + C_2)]\ (2\pi f)^3$$

$$\omega_{1C}^2 = \frac{1}{2} \left\{ \frac{1}{I} (K_1L_1^{\ 2} + K_2L_2^{\ 2}) + \frac{1}{m} (k_1 + K_2) \right.$$

$$\left. - \sqrt{\left[\frac{1}{I} (K_1L_1^{\ 2} + K_2L_2^{\ 2}) + \frac{1}{m} (K_1 + K_2)\right]^2 - \frac{4\ K_1K_2\ (L_1 + L_2)^2}{m\ I}} \right\}$$

$$\omega_{2C}^2 = \frac{1}{2} \left\{ \frac{1}{I} (K_1L_1^{\ 2} + K_2L_2^{\ 2}) + \frac{1}{m} (K_1 + K_2) \right.$$

$$\left. + \sqrt{\left[\frac{1}{I} (K_1L_1^{\ 2} + K_2L_2^{2}) + \frac{1}{m} (K_1 + K_2)\right]^2 - \frac{4\ K_1K_2\ (L_1 + L_2)^2}{mI}} \right\}$$

$$f_{iC} = \omega_{iC}/2\pi \quad , \quad i = 1,2$$

$$\omega_{nx}^2 = \frac{K_1 + K_2}{m}$$

$$\omega_{n\theta}^2 = \frac{K_1L_1^{\ 2} + K_2L_2^{\ 2}}{I}$$

$$f_{nx} = \omega_{nx}/2\pi$$

$$f_{n\theta} = \omega_{n\theta}/2\pi$$

FOUR ADMISSIBLE EXPRESSIONS FOR THE
UNDAMPED MODAL MATRIX (NON-NORMALIZED)
($K_1L_1 \neq K_2L_2$ PRESUMED)

$$\begin{bmatrix} \omega_{1C}^2 - \dfrac{K_1L_1^2 + K_2L_2^2}{I} & \omega_{2C}^2 - \dfrac{K_1L_1^2 + K_2L_2^2}{I} \\[2ex] \dfrac{K_2L_2 - K_1L_1}{I} & \dfrac{K_2L_2 - K_1L_1}{I} \end{bmatrix}$$

$$\begin{bmatrix} \omega_{1C}^2 - \dfrac{K_1L_1^2 + K_2L_2^2}{I} & \dfrac{K_2L_2 - K_1L_1}{m} \\[2ex] \dfrac{K_2L_2 - K_1L_1}{I} & \omega_{2C}^2 - \dfrac{K_1 + K_2}{m} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{K_2L_2 - K_1L_1}{m} & \dfrac{K_2L_2 - K_1L_1}{m} \\[2ex] \omega_{1C}^2 - \dfrac{K_1 + K_2}{m} & \omega_{2C}^2 - \dfrac{K_1 + K_2}{m} \end{bmatrix}$$

$$\begin{bmatrix} \dfrac{K_2L_2 - K_1L_1}{m} & \omega_{2C}^2 - \dfrac{K_1L_1^2 + K_2L_2^2}{I} \\[2ex] \omega_{1C}^2 - \dfrac{K_1 + K_2}{m} & \dfrac{K_2L_2 - K_1L_1}{I} \end{bmatrix}$$

$$|T_{y/u}(2\pi jf)|^2 = \frac{\{R_x - Re(D)\}^2 + \{\mathcal{J}_x - \mathcal{J}m(D)\}^2}{\{Re(D)\}^2 + \{\mathcal{J}m(D)\}^2}$$

$$|T_{y_{P/u}}(2\pi jf)|^2 = \frac{\{R_x + L_P R_\theta - Re(D)\}^2 + \{\mathcal{J}_x + L_P \mathcal{J}_\theta - \mathcal{J}m(D)\}^2}{\{Re(D)\}^2 + \{\mathcal{J}m(D)\}^2}$$

APPENDIX B

ON THE CONSEQUENCES OF CERTAIN SPECIFICATIONS

When the coupling coefficients vanish simultaneously, that is, when $K_1L_1 - K_2L_2 = 0$ and $C_1D_1 - C_2D_2 = 0$, the equations of motion, (1) and (2), assume their uncoupled forms (B-1) and (B-2).

$$m\ddot{x} + (C_1 + C_2)\,\dot{x} + (K_1 + K_2)\,x = (C_1 + C_2)\,\dot{u} + (K_1 + K_2)\,u \tag{B-1}$$

$$I\ddot{\theta} + (C_1D_1^2 + C_2D_2^2)\,\dot{\theta} + (K_1L_1^2 + K_2L_2^2)\theta = 0 \tag{B-2}$$

Upon inspecting equations (B-1) and (B-2), it is easily seen that the uncoupled undamped natural frequencies and damping ratios satisfy

$$\omega_{nx}^2 = \frac{K_1 + K_2}{m} \quad , \quad 2\,\zeta_x\,\omega_{nx} = \frac{C_1 + C_2}{m} \quad ,$$

$$\omega_{n\theta}^2 = \frac{1}{I}\,(K_1L_1^2 + K_2L_2^2) \quad , \quad 2\,\zeta_\theta\,\omega_{n\theta} = \frac{1}{I}\,(C_1D_1^2 + C_2D_2^2) \quad .$$

If one imposes the conditions

$$K_1 = K_2 \quad , \quad C_1 = C_2 \quad , \quad D_1 = D_2 \quad , \quad \zeta_x = \zeta_\theta \quad ,$$

and assigns values to m, ρ, ω_{nx}, ζ_x, r_f, r_L, where

$$r_f = \omega_{n\theta}/\omega_{nx} = f_{n\theta}/f_{nx} \qquad \text{and} \qquad r_L = L_1/L_2$$

then some simple algebraic manipulation (bearing in mind the definition $\rho = \sqrt{I/m}$) will show that the numerical values of D_i, C_i, L_i, and K_i, $i = 1,2$, are determined by

$$D_i = \rho\,\sqrt{r_f} \quad , \quad i = 1,2$$

$$C_i = m\,\zeta_x\,\omega_{nx} \quad , \quad i = 1,2$$

$$K_i = m\,\omega_{nx}^2/2 \quad , \qquad i = 1,2$$

$$L_2 = \rho\, r_f \left(\frac{2}{r_L^2 + 1}\right)^{1/2}$$

$$L_1 = r_L\, L_2 \quad .$$

If the condition $D_1 = D_2$ is replaced by $D_i = L_i$, $i = 1,2$, the numerical values of $G_{\ddot{u}}(f)$, m, ρ, ω_{nx}, ζ_x, r_f, and r_L being prescribed as before, the expressions for I, K_1, K_2, C_1, C_2, L_1, and L_2 are the same as before, but the equality of D_i and L_i, $i = 1,2$, requires that $\zeta_\theta = r_f\, \zeta_x$. In this case one will find

$$T_{x/u}(s) = \sum_{K=0}^{3} B_K\, s^K \Big/ \sum_{K=0}^{4} A_K\, s^K$$

$$A_0 = \frac{\omega_{nx}^4\, r_f^2\, (1 + r_L)^2}{2\,(1 + r_L^2)} \quad , \qquad A_1 = \frac{2\,\zeta_x\, \omega_{nx}^3\, r_f^2\, (1 + r_L)^2}{1 + r_L^2}$$

$$A_2 = \omega_{nx}^2 \left\{ 1 + r_f^2 + \frac{2\,\zeta_x^2\, r_f^2\, (1 + r_L)^2}{1 + r_L^2} \right\} \quad , \qquad A_3 = 2\,\zeta_x\, \omega_{nx}(1 + r_f^2) \quad ,$$

$$A_4 = 1 \quad .$$

$$B_0 = A_0 \quad , \quad B_1 = A_1 \quad , \quad B_2 = A_2 - \omega_{nx}^2\, r_f^2 \quad , \quad B_3 = 2\,\zeta_x\, \omega_{nx}$$

$$T_{\theta/u}(s) = \sum_{K=0}^{3} \gamma_K\, s^K \Big/ \sum_{K=0}^{4} A_K\, s^K$$

$$\gamma_0 = \gamma_1 = 0 \quad , \qquad \gamma_2 = \frac{r_f\, \omega_{nx}^2}{\rho} \left[\frac{1 - r_L}{\sqrt{2\,(1 + r_L^2)}} \right] \quad ,$$

$$\gamma_3 = \frac{2\, r_f\, \zeta_x\, \omega_{nx}}{\rho} \left[\frac{1 - r_L}{\sqrt{2\,(1 + r_L^2)}} \right]$$

$$T_{y/u}(s) = \sum_{K=0}^{4} \beta_K\, s^K \Big/ \sum_{K=0}^{4} A_K\, s^K$$

$$\beta_0 = \beta_1 = 0 \quad , \quad \beta_2 = -\, r_f^2\, \omega_{nx}^2 \quad , \quad \beta_3 = -2\, r_f^2\, \zeta_x\, \omega_{nx} \quad , \quad \beta_4 = -1 \quad .$$

If neither the condition $D_1 = D_2$ nor the condition $D_i = L_i$, $i = 1,2$, is imposed, while enforcing the conditions $K_1 = K_2$, $C_1 = C_2$, and prescribing the numerical values of $G_{\ddot{u}}(f)$, m, ρ, ω_{nx}, ζ_x, ζ_θ, r_f, r_L, and $r_D = D_1/D_2$, there is still no change in the expressions for I, K_1, K_2, C_1, and C_2, but D_1 and D_2 will be determined by

$$D_2 = \rho \left\{ \frac{2\, \zeta_\theta\, r_f}{\zeta_x\, (1 + r_D^2)} \right\}^{1/2} \quad , \quad D_1 = r_D\, D_2 \quad .$$

In this special case, the transfer functions relevant to x, θ, and y are

$$T_{x/u}(s) = \sum_{K=0}^{3} B_K'\, s^K \Big/ \sum_{K=0}^{4} A_K'\, s^K$$

$A_K' = A_K$ of the preceding paragraph for $K = 0,3,4$

$$A_1 = 2\, \zeta_x\, \omega_{nx}^3\, r_f \left\{ r_f + \frac{\zeta_\theta}{\zeta_x} - (1 - r_D)\,(1 - r_L) \left[\frac{\zeta_\theta\, r_f}{\zeta_x\, (1 + r_D^2)\,(1 + r_L^2)} \right]^{1/2} \right\}$$

$$A_2 = \omega_{nx}^2 \left[1 + r_f^2 + \frac{2\, r_f\, \zeta_x\, \zeta_\theta\, (1 + r_D)^2}{1 + r_D^2} \right]$$

$B_K' = B_K$ of the preceding paragraph for $K = 0,3$

$$B_1' = A_1' \quad , \quad B_2' = A_2' - \omega_{nx}^2 \, r_f^2$$

$$T_{\theta/u}(s) = \sum_{K=0}^{3} \gamma_K' \, s^K \Big/ \sum_{K=0}^{4} A_K' \, s^K$$

$\gamma_K' = \gamma_K$ of the preceding paragraph for $K = 0,1,2$

$$\gamma_3' = \frac{\omega_{nx}}{\rho} (1 - r_D) \left(\frac{2 \, \zeta_\theta \, \zeta_x \, r_f}{1 + r_D^2} \right)^{1/2}$$

$$T_{y/u}(s) = \sum_{k=0}^{4} \beta_K' \, s^K \Big/ \sum_{K=0}^{4} A_K' \, s^K$$

$\beta_K' = \beta_K$ of the preceding paragraph for $K = 0,1,2,4$

$$\beta_3' = - 2 \, \zeta_\theta \, r_f \, \omega_{nx} \quad .$$

It is worthy of note, insofar as economy of computer time is concerned, that the moduli of the transfer functions of this and the preceding paragraph are unchanged (for a specific value of the complex variable s) if both r_L and r_D are replaced by their reciprocals. The same can be said of the transfer functions defined by equations (6), (7), and (8) when their numerator and denominator coefficients are such as to meet specifications similar to those beneath Table 1.

APPENDIX C

SAMPLE OUTPUT OF PROGRAM AUXRBM

On the following pages of this Appendix are two sets of mean squares and RMS's of $\ddot{x}$, $\ddot{\theta}$, $\dot{\theta}$, θ, $\dot{y}$, and y as found by program AUXRBM in two computer runs, there being seven cases processed in each run. In both runs r_L assumes the values 2/3, 1.0, 1.05, 1.2, 1.5, 2.0, and 10, in turn, and

$$r_f = 0.1 \quad , \quad f_{nx} = 100.\ (\mathrm{HZ}) \quad , \quad \zeta_x = 0.01 \quad ,$$

$$K_1 = K_2 \quad , \quad C_1 = C_2 \quad , \quad \rho = 5.\ (\mathrm{in.}) \quad , \quad m = 1.0\ (\mathrm{lb{*}sec^2/in.}) \quad ,$$

$$G_{\ddot{u}}(f) = 0.1\ (g^2/\mathrm{HZ}) \quad \text{for} \quad 0 \leq f < \infty \quad .$$

In one run the conditions $D_1 = D_2$ and $D_i \neq L_i$ $(i = 1,2)$ apply, while in the other $D_i = L_i$ $(i = 1,2)$.

Observe that in each run the output mean squares in case 5 duplicate those of case 1, that being due to the fact that the values of r_L and r_D in case 5 are the reciprocals of those in case 1. See Appendix B (last two sentences).

Notice also that the numerical results in case 2 of each run verify that θ is identically zero (assuming zero initial conditions) when the relations $C_1D_1 = C_2D_2$ and $K_1L_1 = K_2L_2$ hold.

The following table will serve to define the FORTRAN symbols appearing on the AUXRBM printout.

Symbol	$\ddot{x}$	$\ddot{\theta}$	$\dot{\theta}$	θ	$\dot{y}$	y	r_L	r_f	K_1	K_2	L_1	L_2	D_1	D_2	C_1	C_2	RMS
FORTRAN Equivalent	XDD	TDD	TD	T	YD	Y	R	RF	K1	K2	L1	L1	D1	D2	C1	C2	RMS

CASE 1 $(D_1 = D_2 \ , D_i \neq L_i \ ; i = 1, 2)$

K1 =	.19739209+06	L1 =	.39223227+00	K2 =	.19739209+06	L2 =	.58834841+00	R =	.66666667+00
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78567858+03	RMS OF XDD =	.28029959+02
MEAN SQUARE OF TDD =	.20118432+04	RMS OF TDD =	.44853575+02
MEAN SQUARE OF TD =	.50960585-01	RMS OF TD =	.22574451+00
MEAN SQUARE OF T =	.12217110-04	RMS OF T =	.34952983-02
MEAN SQUARE OF YD =	.29642678+03	RMS OF YD =	.17217049+02
MEAN SQUARE OF Y =	.75068497-03	RMS OF Y =	.27398631-01

CASE 2 $(D_1 = D_2,\ D_i \neq L_i,\ i = 1,2)$

K1 =	.19739209+06	L1 =	.50000000+00	K2 =	.19739209+06	L2 =	.50000000+00	R =	.10000000+01
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78571233+03	RMS OF XDD =	.28030560+02
MEAN SQUARE OF TDD =	.00000000	RMS OF TDD =	.00000000
MEAN SQUARE OF TD =	.00000000	RMS OF TD =	.00000000
MEAN SQUARE OF T =	.00000000	RMS OF T =	.00000000
MEAN SQUARE OF YD =	.29655418+03	RMS OF YD =	.17220749+02
MEAN SQUARE OF Y =	.75118053-03	RMS OF Y =	.27407673-01

CASE 3 $(D_1 = D_2\ ,\ D_i \neq L_i\ ,\ i = 1,2)$

K1 =	.19739209+06	L1 =	.51204284+00	K2 =	.19739209+06	L2 =	.48765985+00	R =	.10500000+01
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78571179+03	RMS OF XDD =	.28030551+02
MEAN SQUARE OF TDD =	.31244010+02	RMS OF TDD =	.55896342+01
MEAN SQUARE OF TD =	.79142003-03	RMS OF TD =	.28132189-01
MEAN SQUARE OF T =	.18254337-06	RMS OF T =	.42725094-03
MEAN SQUARE OF YD =	.29655220+03	RMS OF YD =	.17220691+02
MEAN SQUARE OF Y =	.75117103-03	RMS OF Y =	.27407499-01

CASE 4 $(D_1 = D_2 \;,\; D_i \neq L_i \;,\; i = 1,2)$

K1 =	.19739209+06	L1 =	.54321448+00	K2 =	.19739209+06	L2 =	.45267873+00	R =	.12000000+01
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78570509+03	RMS OF XDD =	.28030432+02
MEAN SQUARE OF TDD =	.43035373+03	RMS OF TDD =	.20744969+02
MEAN SQUARE OF TD =	.10900987-01	RMS OF TD =	.10440779+00
MEAN SQUARE OF T =	.25336174-05	RMS OF T =	.15917341-02
MEAN SQUARE OF YD =	.29652693+03	RMS OF YD =	.17219957+02
MEAN SQUARE OF Y =	.75105458-03	RMS OF Y =	.27405375-01

CASE 5 $(D_1 = D_2\ ,\ D_i \neq L_i\ ,\ i = 1,2)$

K1 =	.19739209+06	L1 =	.58834841+00	K2 =	.19739209+06	L2 =	.39223227+00	R =	.15000000+01
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78567858+03	RMS OF XDD =	.28029959+02
MEAN SQUARE OF TDD =	.20118432+04	RMS OF TDD =	.44853575+02
MEAN SQUARE OF TD =	.50960585-01	RMS OF TD =	.22574451+00
MEAN SQUARE OF T =	.12217110-04	RMS OF T =	.34952983-02
MEAN SQUARE OF YD =	.29642678+03	RMS OF YD =	.17217049+02
MEAN SQUARE OF Y =	.75068497-03	RMS OF Y =	.27398631-01

CASE 6 $(D_1 = D_2 \, , \, D_i \neq L_i \, , \, i = 1,2)$

K1 =	.19739209+06	L1 =	.63245553+00	K2 =	.19739209+06	L2 =	.31622777+00	R =	.20000000+01
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78562525+03	RMS OF XDD =	.28029007+02
MEAN SQUARE OF TDD =	.51915380+04	RMS OF TDD =	.72052328+02
MEAN SQUARE OF TD =	.13150320+00	RMS OF TD =	.36263370+00
MEAN SQUARE OF T =	.33681742-04	RMS OF T =	.58035973-02
MEAN SQUARE OF YD =	.29622542+03	RMS OF YD =	.17211200+02
MEAN SQUARE OF Y =	.75044067-03	RMS OF Y =	.27394172-01

CASE 7 $(D_1 = D_2 ,\ D_i \neq L_i ,\ i = 1,2)$

K1 =	.19739209+06	L1 =	.70359755+00	K2 =	.19739209+06	L2 =	.70359755-01	R =	.10000000+02
C1 =	.62831853+01	D1 =	.15811388+01	C2 =	.62831853+01	D2 =	.15811388+01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78537553+03	RMS OF XDD =	.28024552+02
MEAN SQUARE OF TDD =	.20080513+05	RMS OF TDD =	.14170573+03
MEAN SQUARE OF TD =	.50864536+00	RMS OF TD =	.71319377+00
MEAN SQUARE OF T =	.19574100-03	RMS OF T =	.13990747-01
MEAN SQUARE OF YD =	.29528257+03	RMS OF YD =	.17183788+02
MEAN SQUARE OF Y =	.76468440-03	RMS OF Y =	.27652927-01

$D_i = L_i \;, \; i = 1,2$

CASE 1

K1 =	.19739209+06	L1 =	.39223227+00	K2 =	.19739209+06	L2 =	.58834841+00	R =	.66666667+00
C1 =	.62831853+01	D1 =	.39223227+00	C2 =	.62831853+01	D2 =	.58834841+00	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78509668+03	RMS OF XDD =	.28019577+02
MEAN SQUARE OF TDD =	.36703982+04	RMS OF TDD =	.60583811+02
MEAN SQUARE OF TD =	.48819175+00	RMS OF TD =	.69870719+00
MEAN SQUARE OF T =	.12744467-03	RMS OF T =	.11289140-01
MEAN SQUARE OF YD =	.29621145+03	RMS OF YD =	.17210795+02
MEAN SQUARE OF Y =	.75125880-03	RMS OF Y =	.27409101-01

$D_i = L_i\ ,\ i = 1,2$

CASE 2

K1 =	.19[illegible]209+06	L1 =	.50000000+00	K2 =	.19739209+06	L2 =	.50000000+00	R =	.10000000+01
C1 =	.62[illegible]1853+01	D1 =	.50000000+00	C2 =	.62831853+01	D2 =	.50000000+00	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78571230+03	RMS OF XDD =	.28030560+02
MEAN SQUARE OF TDD =	.00000000	RMS OF TDD =	.00000000
MEAN SQUARE OF TD =	.00000000	RMS OF TD =	.00000000
MEAN SQUARE OF T =	.00000000	RMS OF T =	.00000000
MEAN SQUARE OF YD =	.29655419+03	RMS OF YD =	.17220749+02
MEAN SQUARE OF Y =	.75118053-03	RMS OF Y =	.27407673-01

$D_i = L_i , \; i = 1,2$

CASE 3

K1 =	.19739209+06	L1 =	.51204284+00	K2 =	.19739209+06	L2 =	.48765985+00	R =	.10500000+01
C1 =	.62831853+01	D1 =	.51204284+00	C2 =	.62831853+01	D2 =	.48765985+00	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78570277+03	RMS OF XDD =	.28030390+02
MEAN SQUARE OF TDD =	.56823895+02	RMS OF TDD =	.75381626+01
MEAN SQUARE OF TD =	.72716537-02	RMS OF TD =	.85273992-01
MEAN SQUARE OF T =	.18249908-05	RMS OF T =	.13509222-02
MEAN SQUARE OF YD =	.29654880+03	RMS OF YD =	.17220592+02
MEAN SQUARE OF Y =	.75116265-03	RMS OF Y =	.27407347-01

$D_i = L_i,\ i = 1, 2$

CASE 4

K1 =	.19739209+06	L1 =	.54321448+00	K2 =	.19739209+06	L2 =	.45267873+00	R =	.12000000+01
C1 =	.62831853+01	D1 =	.54321448+00	C2 =	.62831853+01	D2 =	.45267873+00	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78558094+03	RMS OF XDD =	.28028217+02
MEAN SQUARE OF TDD =	.78318156+03	RMS OF TDD =	.27985381+02
MEAN SQUARE OF TD =	.10099057+00	RMS OF TD =	.31779014+00
MEAN SQUARE OF T =	.25544096-04	RMS OF T =	.50541167-02
MEAN SQUARE OF YD =	.29648025+03	RMS OF YD =	.17218602+02
MEAN SQUARE OF Y =	.75098402-03	RMS OF Y =	.27404088-01

$D_i = L_i \,, \; i = 1,2$

CASE 5

K1 =	.19739209+06	L1 =	.58834841+00	K2 =	.19739209+06	L2 =	.39223227+00	R =	.15000000+01
C1 =	.62831853+01	D1 =	.58834841+00	C2 =	.62831853+01	D2 =	.39223227+00	RF =	.10000000+00

MEAN SQUARE OF XPD =	.78509668+03	RMS OF XDD =	.28019577+02
MEAN SQUARE OF TPD =	.36703982+04	RMS OF TDD =	.60583811+02
MEAN SQUARE OF TP =	.48819175+00	RMS OF TD =	.69870719+00
MEAN SQUARE OF T =	.12744467-03	RMS OF T =	.11289140-01
MEAN SQUARE OF YP =	.29621145+03	RMS OF YD =	.17210795+02
MEAN SQUARE OF Y =	.75125880-03	RMS OF Y =	.27409101-01

$D_i = L_i \;,\; i = 1,2$

CASE 6

K1 =	.19739209+06	L1 =	.63245553+00	K2 =	.19739209+06	L2 =	.31622777+00	R =	.20000000+01
C1 =	.62831853+01	D1 =	.63245553+00	C2 =	.62831853+01	D2 =	.31622777+00	RF =	.10000000+00

MEAN SQUARE OF XDD =	.78411581+03	RMS OF XDD =	.28002068+02
MEAN SQUARE OF TDD =	.95191688+04	RMS OF TDD =	.97566228+02
MEAN SQUARE OF TD =	.13526967+01	RMS OF TD =	.11630549+01
MEAN SQUARE OF T =	.37773693-03	RMS OF T =	.19435456-01
MEAN SQUARE OF YD =	.29568699+03	RMS OF YD =	.17195552+02
MEAN SQUARE OF Y =	.75775851-03	RMS OF Y =	.27527414-01

$D_i = L_i \, , \; i = 1,2$

CASE 7

K1 =	.19739209+06	L1 =	.70359755+00	K2 =	.19739209+06	L2 =	.70359755-01	R =	.10000000+02
C1 =	.62831853+01	D1 =	.70359755+00	C2 =	.62831853+01	D2 =	.70359755-01	RF =	.10000000+00

MEAN SQUARE OF XDD =	.77938792+03	RMS OF XDD =	.27917520+02
MEAN SQUARE OF TDD =	.37709679+05	RMS OF TDD =	.19418980+03
MEAN SQUARE OF TD =	.80512397+01	RMS OF TD =	.28374707+01
MEAN SQUARE OF T =	.33982930-02	RMS OF T =	.58294880-01
MEAN SQUARE OF YD =	.29379198+03	RMS OF YD =	.17140361+02
MEAN SQUARE OF Y =	.10839108-02	RMS OF Y =	.32922801-01

APPENDIX D

SAMPLE OUTPUT OF PROGRAM TRROBM

The specifications $G_{\ddot{u}}(f) = 0.1\ (g^2/HZ)$, $m = 1.0\ (lb*sec^2/in.)$, $\rho = 5.\ (in.)$, $f_{nx} = 100.\ (HZ)$, $\zeta_x = \zeta_\theta = 0.01$, $K_1 = K_2$, $C_1 = C_2$, $D_1 = D_2$, $D_i \neq L_i\ (i = 1,2)$, $r_f = 2.0$, $1. \leq f \leq 400.$, $\Delta f = 0.5$, and

$$r_L = \begin{cases} 1.05, & \text{(Case 1)} \\ 1.2\ , & \text{(Case 2)} \\ 1.5\ , & \text{(Case 3)} \\ 2.0\ , & \text{(Case 4)} \\ 10.\ , & \text{(Case 5)} \end{cases}$$

led to the numerical values of I, K_1, K_2, L_1, L_2, C_1, C_2, D_1, D_2, and other items (with the exception of L_p) essential to the mean square computation, shown on the input print which precedes the output print of program TRROBM. In each of the five cases processed by TRROBM, the frequency interval (1., 400.) HZ was slightly less in length than the recommended interval (1., $2f_{2c}$) HZ, but the approximations to the mean squares and RMS's were surprisingly good. The reader should compare θ_{RMS}, $\dot{\theta}_{RMS}$, $\ddot{\theta}_{RMS}$, y_{RMS}, $\dot{y}_{RMS}$, and $\ddot{x}_{RMS}$ found in the output print* with the encircled values and inset tabular values, corresponding to $r_f = 2.0$, in Figures 6 through 11.

The reader is due an explanation of the items appearing on the last page of the output print for each case. As it pertains to matrices, the word adjoint has its usual meaning, that is, the adjoint of a matrix is the transpose of the associated matrix of cofactors. The 2 x 2 matrix identified as "ADJOINT CORRESPONDING TO OMEGA1C" is merely the adjoint of the characteristic matrix, to be defined subsequently, when the elements of the characteristic matrix are evaluated at $\omega = \omega_{1C}$. A similar statement applies to the matrix identified as "ADJOINT CORRESPONDING TO OMEGA2C" (with ω_{1C} replaced by ω_{2C}). The characteristic matrix, here denoted by $Ch(H,\omega^2)$, is given by

* The author has exercised the option to avoid printing all tabulated functions of frequency.

$$Ch(H,\omega^2) = \omega^2 \begin{bmatrix} 1. & 0 \\ 0 & 1. \end{bmatrix} - H$$

where H, known as the dynamic matrix, is defined by

$$H = M^{-1} K \quad ,$$

the matrices M and K being, respectively, the system mass matrix and stiffness matrix, that is [see equation (4) or (5)],

$$M = \begin{bmatrix} m & 0 \\ 0 & I \end{bmatrix} \quad , \quad K = \begin{bmatrix} K_1 + K_2 & K_2L_2 - K_1L_1 \\ K_2L_2 - K_1L_1 & K_1L_1{}^2 + K_2L_2{}^2 \end{bmatrix} \quad .$$

The equation formed by setting the determinant of $Ch(H,\omega^2)$ to zero is a quadratic in ω^2 whose roots, ω_{1C}^2 and ω_{2C}^2, are the characteristic values (or eigenvalues of H) and also the squares of the undamped coupled natural frequencies. As the characteristic vector (or eigenvector) of H corresponding to ω_{iC}^2, one may choose any non-zero scalar multiple of either column of the adjoint of the matrix $Ch(H,\omega_{iC}^2)$, $i = 1,2$. Program TRROBM selects the second column of the adjoint and the reciprocal of the 2,2 element as the scalar multiplier to get the vectors identified as "normalized" characteristic vectors on the output print.

The vectors of the preceding paragraph could also be called modal columns of the "undamped" modal matrix. Notice that the modal columns have been "normalized" in a certain fashion, the "fashion" indicated. The author has not declared that such a normalization renders the modal matrix normalized with respect to the mass matrix. If it is desired that the modal matrix be normalized with respect to the mass matrix, one should select one of the four admissible expressions for the undamped modal matrix*, here denoted by

$$\Phi = \begin{bmatrix} \phi_{11} & \phi_{12} \\ \phi_{21} & \phi_{22} \end{bmatrix} \quad ,$$

*See Appendix A.

and post multiply it by the diagonal matrix

$$\eta = \begin{bmatrix} \eta_1 & 0 \\ 0 & \eta_2 \end{bmatrix}$$

where η_1 and η_2 are computed by

$$\eta_1 = (m \; \phi_{11}^2 + I \; \phi_{21}^2)^{-1/2} \quad , \quad \eta_2 = (m \; \phi_{12}^2 + I \; \phi_{22}^2)^{-1/2} \quad .$$

The expressions for η_1 and η_2 were found by simply demanding that the matrix $\Psi = \Phi \, \eta$ be such as to satisfy

$$\Psi^T \; M \; \Psi = \begin{bmatrix} 1. & 0 \\ 0 & 1. \end{bmatrix} \quad .$$

```
MASS =    .10000000+01     MOMENT OF INERTIA =    .25000000+02     DELTA F =    .50000000+00

K1 =   .19739209+06     L1 =    .10240857+02     K2 =    .19739209+06     L2 =    .97531970+01     LP=    .10000000+02  CASE   1

K1 =   .19739209+06     L1 =    .10864290+02     K2 =    .19739209+06     L2 =    .90535746+01     LP=    .10000000+02  CASE   2

K1 =   .19739209+06     L1 =    .11766968+02     K2 =    .19739209+06     L2 =    .78446454+01     LP=    .10000000+02  CASE   3

K1 =   .19739209+06     L1 =    .12649111+02     K2 =    .19739209+06     L2 =    .63245553+01     LP=    .10000000+02  CASE   4

K1 =   .19739209+06     L1 =    .14071951+02     K2 =    .19739209+06     L2 =    .14071951+01     LP=    .50000000+01  CASE   5

C1 =   .62831853+01     D1 =    .70710678+01     C2 =    .62831853+01     D2 =    .70710678+01

NSEG = 1

FEX
  .10000000+01    .40000000+03

NCORN = 1        GCORN =    .10000000+00

DELDB
  .00000000

GUDDEX
  .10000000+00    .10000000+00

SEG
 798

N =  799
```

```
MEAN SQUARE OF INPUT ACCELERATION (EXACT) =    .39900000+02  (G**2)

RMS OF INPUT ACCELERATION (EXACT) =    .63166447+01 (G'S)

MEAN SQUARE OF INPUT VELOCITY (EXACT) =    .37664055+03 ((IN/SEC)**2)

RMS OF INPUT VELOCITY (EXACT) =    .19407229+02 (IN/SEC)

MEAN SQUARE OF INPUT DISPLACEMENT (EXACT) =    .31881091+01 (IN**2)

RMS OF INPUT DISPLACEMENT (EXACT) =    .17855277+01 (IN.)

THE EXACT MEAN SQUARES AND RMS'S ABOVE ARE PERTINENT TO THE ENTIRE INTERVAL (FEX(1),FEX(NSEG+1))
```

U DENOTES THE ABSOLUTE DISPLACEMENT OF THE BASE (INCHES)

UD DENOTES THE ABSOLUTE VELOCITY OF THE BASE (INCHES/SEC)

UDD DENOTES THE ABSOLUTE ACCELERATION OF THE BASE (G'S)

X DENOTES THE ABSOLUTE TRANSLATIONAL DISPLACEMENT OF THE CM (INCHES)

XD DENOTES THE ABSOLUTE TRANSLATIONAL VELOCITY OF THE CM (INCHES/SEC)

XDD DENOTES THE ABSOLUTE TRANSLATIONAL ACCELERATION OF THE CM (G'S)

T DENOTES THE ANGULAR DISPLACEMENT OF THE MASS (RADIANS)

TD DENOTES THE ANGULAR VELOCITY OF THE MASS (RAD/SEC)

TDD DENOTES THE ANGULAR ACCELERATION OF THE MASS (RAD/SEC**2)

Y DENOTES THE RELATIVE DISPLACEMENT OF THE CM FROM THE BASE (INCHES)

YD DENOTES THE VELOCITY OF THE CM RELATIVE TO THE BASE (INCHES/SEC)

YDD DENOTES THE ACCELERATION OF THE CM RELATIVE TO THE BASE (G'S)

YP DENOTES THE RELATIVE DISPLACEMENT OF POINT P FROM THE BASE (INCHES)

YPD DENOTES THE VELOCITY OF POINT P RELATIVE TO THE BASE (INCHES/SEC)

YPDD DENOTES THE ACCELERATION OF POINT P RELATIVE TO THE BASE (G'S)

(ABS(HX&U))**2 DENOTES THE SQUARE OF THE ABSOLUTE VALUE OF THE FREQUENCY RESPONSE FUNCTION BETWEEN X AND U

(ABS(HT&U))**2 DENOTES THE SQUARE OF THE ABSOLUTE VALUE OF THE FREQUENCY RESPONSE FUNCTION BETWEEN T AND U

(ABS(HY&U))**2 DENOTES THE SQUARE OF THE ABSOLUTE VALUE OF THE FREQUENCY RESPONSE FUNCTION BETWEEN Y AND U

(ABS(HYP&U))**2 DENOTES THE SQUARE OF THE ABSOLUTE VALUE OF THE FREQUENCY RESPONSE FUNCTION BETWEEN YP AND U

CASE 1 FNTHETA/FNX = .20000000+01 L1/L2 = .10500000+01

THE MEAN SQUARES BELOW WERE COMPUTED VIA THE TRAPEZOIDAL RULE APPLIED TO THE DEFINITE INTEGRALS DEFINING THEM

MEAN SQUARE OF X =	.39105849+01	RMS OF X =	.19775199+01
MEAN SQUARE OF XD =	.68888029+03	RMS OF XD =	.26246529+02
MEAN SQUARE OF XDD =	.78431451+03	RMS OF XDD =	.28005616+02
MEAN SQUARE OF T =	.89213796-08	RMS OF T =	.94453055-04
MEAN SQUARE OF TD =	.46921707-02	RMS OF TD =	.68499422-01
MEAN SQUARE OF TDD =	.37037480+04	RMS OF TDD =	.60858426+02
MEAN SQUARE OF Y =	.75103380-03	RMS OF Y =	.27404996-01
MEAN SQUARE OF YD =	.29533451+03	RMS OF YD =	.17185299+02
MEAN SQUARE OF YDD =	.81867986+03	RMS OF YDD =	.28612582+02
MEAN SQUARE OF YP =	.80073075-03	RMS OF YP =	.28297186-01
MEAN SQUARE OF YPD =	.31505280+03	RMS OF YPD =	.17749727+02
MEAN SQUARE OF YPDD =	.87259097+03	RMS OF YPDD =	.29539651+02

[illegible]

$\Delta f = 0.5$

$L_p = 10.$

UNDAMPED COUPLED NATURAL FREQUENCIES : FC1 = .99960366+02 (HZ) FC2 = .20001981+03 (HZ)

UNDAMPED UNCOUPLED NATURAL FREQUENCIES : FNX = .10000000+03 (HZ) FNTHETA = .20000000+03 (HZ)

```
ADJOINT CORRESPONDING TO OMEGA1C
 -.11846654+07   -.96260234+05
 -.38504094+04   -.31287891+03

ADJOINT CORRESPONDING TO OMEGA2C
  .31287500+03   -.96260234+05
 -.38504094+04    .11846654+07

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA1C
   .30765971+03   (INCHES)

   .10000000+01   (RADIANS)

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA2C
  -.81255207-01   (INCHES)

   .10000000+01   (RADIANS)
```

1

CASE 2 FNTHETA/FNX = .20000000+01 L1/L2 = .12000000+01

THE MEAN SQUARES BELOW WERE COMPUTED VIA THE TRAPEZOIDAL RULE APPLIED TO THE DEFINITE INTEGRALS DEFINING THEM

MEAN SQUARE OF X =	.39106003+01	RMS OF X =	.19775238+01
MEAN SQUARE OF XD =	.68592783+03	RMS OF XD =	.26190224+02
MEAN SQUARE OF XDD =	.76867935+03	RMS OF XDD =	.27725067+02
MEAN SQUARE OF T =	.12208796-06	RMS OF T =	.34941087-03
MEAN SQUARE OF TD =	.63723363-01	RMS OF TD =	.25243487+00
MEAN SQUARE OF TDD =	.50299617+05	RMS OF TDD =	.22427576+03
MEAN SQUARE OF Y =	.75113441-03	RMS OF Y =	.27406832-01
MEAN SQUARE OF YD =	.29238262+03	RMS OF YD =	.17099200+02
MEAN SQUARE OF YDD =	.80304079+03	RMS OF YDD =	.28337974+02
MEAN SQUARE OF YP =	.94390161-03	RMS OF YP =	.30722982-01
MEAN SQUARE OF YPD =	.36916010+03	RMS OF YPD =	.19213539+02
MEAN SQUARE OF YPDD =	.10220771+04	RMS OF YPDD =	.31969940+02

UNDAMPED COUPLED NATURAL FREQUENCIES : FC1 = .99454039+02 (HZ) FC2 = .20027205+03 (HZ)

UNDAMPED UNCOUPLED NATURAL FREQUENCIES : FNX = .10000000+03 (HZ) FNTHETA = .20000000+03 (HZ)

$1 \leq f \leq 400.$

$\Delta f = 0.5$

$L_p = 10.$

```
ADJOINT CORRESPONDING TO OMEGA1C
 -.11886515+07   -.35742091+06
 -.14296836+05   -.42989766+04

ADJOINT CORRESPONDING TO OMEGA2C
  .42989531+04   -.35742091+06
 -.14296836+05    .11886515+07

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA1C
   .83140929+02   (INCHES)


   .10000000+01   (RADIANS)

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA2C
  -.30069444+00   (INCHES)


   .10000000+01   (RADIANS)
```

CASE 2

CASE 3 FNTHETA/FNX = .19999999+01 L1/L2 = .15000000+01

THE MEAN SQUARES BELOW WERE COMPUTED VIA THE TRAPEZOIDAL RULE APPLIED TO THE DEFINITE INTEGRALS DEFINING THEM

MEAN SQUARE OF X =	.39106656+01	RMS OF X =	.19775403+01
MEAN SQUARE OF XD =	.67500014+03	RMS OF XD =	.25980765+02
MEAN SQUARE OF XDD =	.71081042+03	RMS OF XDD =	.26661028+02
MEAN SQUARE OF T =	.55771350-06	RMS OF T =	.74680218-03
MEAN SQUARE OF TD =	.28221526+00	RMS OF TD =	.53123936+00
MEAN SQUARE OF TDD =	.22276100+06	RMS OF TDD =	.47197564+03
MEAN SQUARE OF Y =	.75318346-03	RMS OF Y =	.27444188-01
MEAN SQUARE OF YD =	.28145707+03	RMS OF YD =	.16776682+02
MEAN SQUARE OF YDD =	.74515552+03	RMS OF YDD =	.27297537+02
MEAN SQUARE OF YP =	.11954587-02	RMS OF YP =	.34575406-01
MEAN SQUARE OF YPD =	.45362490+03	RMS OF YPD =	.21298472+02
MEAN SQUARE OF YPDD =	.12506108+04	RMS OF YPDD =	.35363976+02

$1. \leq f \leq 400.$

$\Delta f = 0.5$

$L_p = 10.$

UNDAMPED COUPLED NATURAL FREQUENCIES : FC1 = .97445668+02 (HZ) FC2 = .20125690+03 (HZ)

UNDAMPED UNCOUPLED NATURAL FREQUENCIES : FNX = .10000000+03 (HZ) FNTHETA = .20000000+03 (HZ)

```
ADJOINT CORRESPONDING TO OMEGA1C
 -.12042631+07   -.77423547+06
 -.30969419+05   -.19910625+05

ADJOINT CORRESPONDING TO OMEGA2C
  .19910609+05   -.77423547+06
 -.30969419+05    .12042631+07

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA1C
   .38885543+02  (INCHES)

   .10000000+01  (RADIANS)

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA2C
  -.64291222+00  (INCHES)

   .10000000+01  (RADIANS)
```

CASE 3

CASE 4   FNTHETA/FNX = .20000000+01   L1/L2 = .20000001+01

THE MEAN SQUARES BELOW WERE COMPUTED VIA THE TRAPEZOIDAL RULE APPLIED TO THE DEFINITE INTEGRALS DEFINING THEM

MEAN SQUARE OF X =	.39108186+01	RMS OF X =	.19775790+01
MEAN SQUARE OF XD =	.65617628+03	RMS OF XD =	.25615938+02
MEAN SQUARE OF XDD =	.61113393+03	RMS OF XDD =	.24721123+02
MEAN SQUARE OF T =	.13903946-05	RMS OF T =	.11791499-02
MEAN SQUARE OF TD =	.65854909+00	RMS OF TD =	.81151037+00
MEAN SQUARE OF TDD =	.51979502+06	RMS OF TDD =	.72096811+03
MEAN SQUARE OF Y =	.76494496-03	RMS OF Y =	.27657638-01
MEAN SQUARE OF YD =	.26263816+03	RMS OF YD =	.16206115+02
MEAN SQUARE OF YDD =	.64544617+03	RMS OF YDD =	.25405632+02
MEAN SQUARE OF YP =	.15195937-02	RMS OF YP =	.38981966-01
MEAN SQUARE OF YPD =	.53681444+03	RMS OF YPD =	.23169256+02
MEAN SQUARE OF YPDD =	.14422373+04	RMS OF YPDD =	.37976799+02

$1 \leq f \leq 4..$

$\Delta f = 0.5$

(exceptionally good approximations)

$k_p = 10.$

UNDAMPED COUPLED NATURAL FREQUENCIES :   FC1 = .93387255+02 (HZ)   FC2 = .20317190+03 (HZ)

UNDAMPED UNCOUPLED NATURAL FREQUENCIES :   FNX = .10000000+03 (HZ)   FNTHETA = .20000000+03 (HZ)

```
ADJOINT CORRESPONDING TO OMEGA1C
 -.12348384+07   -.12484173+07
 -.49936691+05   -.50485824+05

ADJOINT CORRESPONDING TO OMEGA2C
  .50485797+05   -.12484173+07
 -.49936691+05    .12348384+07

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA1C
   .24728076+02   (INCHES)

   .10000000+01   (RADIANS)

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA2C
  -.10109965+01   (INCHES)

   .10000000+01   (RADIANS)
```

CASE 4

CASE 5 FNTHETA/FNX = .20000000+01 L1/L2 = .99999999+01

THE MEAN SQUARES BELOW WERE COMPUTED VIA THE TRAPEZOIDAL RULE APPLIED TO THE DEFINITE INTEGRALS DEFINING THEM

MEAN SQUARE OF X =	.39121378+01	RMS OF X =	.19779125+01
MEAN SQUARE OF XD =	.60111256+03	RMS OF XD =	.24517599+02
MEAN SQUARE OF XDD =	.31969037+03	RMS OF XDD =	.17879887+02
MEAN SQUARE OF T =	.55790598-05	RMS OF T =	.23620031-02
MEAN SQUARE OF TD =	.17589059+01	RMS OF TD =	.13262375+01
MEAN SQUARE OF TDD =	.13880725+07	RMS OF TDD =	.11781649+04
MEAN SQUARE OF Y =	.97470391-03	RMS OF Y =	.31220248-01
MEAN SQUARE OF YD =	.20761468+03	RMS OF YD =	.14408840+02
MEAN SQUARE OF YDD =	.35384100+03	RMS OF YDD =	.18810662+02
MEAN SQUARE OF YP =	.18193216-02	RMS OF YP =	.42653507-01
MEAN SQUARE OF YPD =	.39053956+03	RMS OF YPD =	.19762074+02
MEAN SQUARE OF YPDD =	.66649741+03	RMS OF YPDD =	.25816611+02

$1 \le f \le 400$

$\Delta f = 0.5$

$L_p = 5.$

UNDAMPED COUPLED NATURAL FREQUENCIES : FC1 = .73270006+02 (HZ) FC2 = .21126170+03 (HZ)

UNDAMPED UNCOUPLED NATURAL FREQUENCIES : FNX = .10000000+03 (HZ) FNTHETA = .20000000+03 (HZ)

```
ADJOINT CORRESPONDING TO OMEGA1C
 -.13671971+07   -.24999227+07
 -.99996906+05   -.18284454+06

ADJOINT CORRESPONDING TO OMEGA2C
  .18284453+06   -.24999227+07
 -.99996906+05    .13671970+07

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA1C
   .13672394+02   (INCHES)

   .10000000+01   (RADIANS)

''NORMALIZED'' CHARACTERISTIC VECTOR CORRESPONDING TO OMEGA2C
  -.18285021+01   (INCHES)

   .10000000+01   (RADIANS)
```

CASE 5

APPROVAL

COMPONENT RESPONSE TO RANDOM VIBRATORY MOTION OF THE CARRIER VEHICLE

By L. P. Tuell

The information in this report has been reviewed for technical content. Review of any information concerning Department of Defense or nuclear energy activities or programs has been made by the MSFC Security Classification Officer. This report, in its entirety, has been determined to be unclassified.

G. F. McDONOUGH
Director, Structures and Dynamics Laboratory

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16. ABSTRACT

Two physical models of component plus supporting substructure are considered. Each model consists of a rigid body attached to a moving base by means of linear springs and viscous dampers. The second model differs from the first in that its dampers are elastically supported. The first model receives the more extensive treatment. Base motion, assumed a random translational motion parallel to a fixed axis, is prescribed only to the extent that the power spectral density (PSD) of its acceleration is given; and, as given, its plot on log-log graph paper is a series of straight line segments, each segment having an extremity in common with the adjacent segment. Closed expressions are given for the mean squares of base acceleration, base velocity, and base displacement. The component is restricted to planar motion and allowed two degrees of freedom, one translational and one rotational. Integral expressions are given for the mean squares of component response variables, the transfer functions essential to mean square computation being available via the equations of motion. Closed expressions are given for mean squares of certain of the response variables for the case wherein the base acceleration PSD is constant. A very brief paragraph is given to stability of motion.

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