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DEPARTMENT OF MECHANICAL ENGINEERING AND MECHANICS
COLLEGE OF ENGINEERING AND TECHNOLOGY
OLD DOMINION UNIVERSITY
NORFOLK, VIRGINIA 23508

UNSTEADY HYBRID VORTEX TECHNIQUE FOR TRANSONIC VORTEX
FLOWS AND FLUTTER APPLICATIONS

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By

Osama A. Kandil, Principal Investigator

Progress Report
For the period February 16, 1986 to October 15, 1986

Prepared for the
National Aeronautics and Space Administration
Langley Research Center
Hampton, VA 23665

Under
Research Grant NAG-1-648
Dr. Samuel Bland, Technical Monitor
Unsteady Aerodynamics Branch
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Submitted by the
Old Dominion University Research Foundation
P.O. Box 6369
Norfolk, VA 23508

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Unsteady Hybrid Vortex Technique for Transonic Vortex Flows and Flutter Applications

Osama A. Kandil

This report covers the progress of research work performed under this grant in the period of February 16, 1986 to October 15, 1986. During this period of eight months, the following tasks have been accomplished:

1. Integral Equation Method of Full Potential Equations

The integral equation method has been extended to treat transonic flows around airfoils. This is achieved by adding a. volume integral term (corresponding to the full compressibility effect) to the classical surface integral solution. The gradient terms in the volume integral term are calculated by using mixed type differencing which is consistent with the mixed nature of the transonic flow. The solution is obtained through successive iteration cycles for subcritical flows. In addition and for transonic flows, mixed type differencing is used and the Rankine-Hugoniot relations are satisfied across the captured shock. Moreover, shock panels are explicitly introduced and fitted to sharpen the captured shock.

The method is applied to airfoils in subcritical and critical flows and the results are in good agreement with the experimental data and the finite-difference solutions of the full-potential and Euler equations. The method has been applied eariler to transonic vortex-dominated three-dimensional flows using a low-order distribution of vortex panels. Currently the method is extended to solve for unsteady transonic flows around pitching airfoils and to solve for the steady and unsteady transonic flows around three-dimensional wings.

The following papers have been published or submitted for presentations (copies are attached to this report):

a. "Transonic Flow Computation Using the Integral Solution of the Full Potential Equations," Kandil, O. A. and Hong, H., submitted for presentation at the AIAA Fluid and Plasma Dynamics Conference, Honolulu, Hawaii, June 8-10, 1987.

b. "Finite-Volume and Integral-Equation Techniques for Transonic and Supersonic Vortex-Dominated Flows," Kandil, O. A., Chuang, A. and Chu, L-C., ICAS-86-1.5.4, London, England, September 7-12, 1986, pp. 629-640 (Vol. 1).

c. "Transonic Vortex Flow Past Delta Wings-Integral Equation Approach," AIAA Journal, Vol. 24, No. 11, 1986, pp. 1729-1736.

2. <u>Euler Equations Solution Using Central-Differencing Finite Volume Solver with Second and Fourth Order Dissipation Terms</u>

During this period, we have implemented Jameson's Central-Differencing Finite Volume Method, which uses Runge Kutta time stepping with second and fourth order explicit dissipation terms, into two computer codes to solve for vortex dominated flows over a wide range of Mach numbers which includes subsonic, transonic and supersonic regimes.

The first computer code solves for the supersonic vortex-dominated flows which is an exact conical flow for steady inviscid problems. The unsteady Euler equations are used to study conical supersonic vortex-dominated flows around delta wings with sharp and round edges. For sharp edges, separated flow solutions have always been obtained while for round edges, separated and attached flow solutions have been obtained. The solution for round edges is dependent upon the level of numerical dissipations and the grid fineness. We are the first to conclusively show the solution dependence upon the numerical dissipation for round edges. Recently, we have shown two additional parameters upon which the solution depends. These are the total pressure loss due to the entropy production and the accuracy of enforcing the boundary conditions.

The second computer code solves for the three-dimensional flows in subsonic, transonic and supersonic flows. The code is fully vectorized and is very efficient (For 130,000 grid points, it takes 3,000 CRU for 900 iteration cycles until convergence -- average residuals of 10^{-4} - on the VPS 32). The code has been used to solve for transonic and subsonic vortex-dominated flows and for supersonic vortex-dominated flows as well. The results compare well with the experimental results at the chord stations downstream of the 18% chord station with 130,000 grid points. Fine grids (300,000 points) are being tested.

Work is underway to solve for the unsteady pitching and rolling oscillations with time accurate schemes.

The results of this work have been reported in the following papers (copies are attached with this report):

a. "Numerical Dissipation Effects in Finite-Volume Euler Solutions for Conical Vortex-Dominated Flows," Kandil, O. A. and Chuang, A., Springer Verlag, Computational Mechanics, Theory and Applications, Edited by G. Iagawa and S. Atluri, Tokyo, Japan, May 1986, pp. XI57-XI.64 (Vol. II).

b. "Influence of Numerical Dissipation in Computing Supersonic Vortex-Dominated Flows," Kandil, O. A. and Chuang, A., AIAA 86-1073, May 1986. Accepted for publication in AIAA Journal.

c. "State of the Art of Computational Vortex Flows," Kandil, O. A. and Chuang, A., SIAM National Meeting, Boston, Mass., July 1986.

d. "Finite-Volume Euler and Navier-Stokes Solvers for Three-Dimensional and Conical Vortex Flows Over Delta Wings," Kandil, O. A. an Chuang, A., AIAA 87-041, January 1987.

3. <u>Presentations and Briefings:</u>

The P.I. has given three presentations and briefings at the UAB-NASA Langley Research Center:

a. Briefing to Dr. Eligh Turner, Wright Patterson AFB, on the Vortex-Dominated Flow work.

b. Briefing during Division and Directorate Annual Review of the UAB Research work.

c. Presentation to McDonald Douglas visitors to NASA Langley Research Center on the Steady and Unsteady Vortex-Dominated Flow research work, September 17,1986.

4. <u>Organizing and Chairing Sessions on Vortex Dominated Flows</u>

The P.I. has organized and chaired the following sessions in the vortex flow area:

a. "Vortex Flows," AIAA 24th Aerospace Sciences Meeting, January 1986.

b. "Computational Vortex Flows," SIAM National Meeting, July 1986.

c. "Vortex Dominated Flows I & II," AIAA 25th Aerospace Sciences Meeting, January 1987.

Abstract Submittal Form

Print or type all information

I wish to submit an abstract for (conference/meeting): Fluid and Plasma Dynamics Conference

Place: Honolulu, Hawaii   Date: June 8-10, 1987

Session/organizer (this information appears in the call): Dr. Helen Reed, Arizona State

Reminders:

1/ AIAA has first publication rights to all papers presented at its meetings. Manuscripts accepted for the meeting will be evaluated for publication in the appropriate AIAA publication, if the author requests it. If a Progress Series book is planned, papers from relevant sessions will be held for the book.

2/ Authors must inform each organizer if abstract is submitted to more than one session topic organizer.

3/ All program participants and authors will be charged a registration fee (AIAA member rate) at the meeting.

4/ Authors of accepted papers will be expected to submit a photo-ready manuscript 8 weeks before a meeting. Authors desiring journal publication should send 4 copies of their photo-ready manuscript directly to the Editor in Chief of the journal of their choice (see inside front cover of each journal for correct address), along with a covering letter, before the meeting, if they wish to accelerate the review procedure. Government Program Monitors/Security Officers should be alerted as soon as authors receive formal acceptance of their paper.

Paper title (this title will be published in the Program):

Transonic Flow Computation Using the Integral Solution of the Full Potential Equation.

Author/s' name and title, AIAA membership grade, company, full mailing address, telephone number:

1 Author presenting paper
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3

4

Abstract due date: Nov. 17, 1986

Draft of paper included?   ☐ Yes   ☑ No   Extended Abstract is included

Similar results been presented or published elsewhere?   ☐ Yes   ☑ No

Concise statement of problem (its genesis and objective covered):

Currently, the integral solution of the full potential equation for transonic flows is receiving a growing attention and in particular for unsteady flows. The integral solution is computationally efficient as compared to the finite difference solutions of the full potential equation and Euler equations. Far field boundary conditions are automatically satisfied and the computational domain is much smaller than that required for the finite-difference equation.

Scope and methods of approach, with statement of contribution to the state-of-the-art or an application of existing analytical techniques and theories to a problem:

A shock capturing - shock fitting technique has been developed by using the integral solution of the full potential equation. The full compressibility is found from a volume integral term which uses mixed type differencing. Rankine Hugoniot equations are used to calculate the properties across the shock and the shock is sharpened by using shock panels. The technique is applied to unsteady pitching airfoils.

Summary of important conclusions:

The integral solutions have shown good agreement with the experimental data and the finite difference solutions of the FP and Euler equations. It is capable of capturing sharp shocks within a few iteration steps. Addition of shock panels further sharpens the shocks.

Statement of data used to substantiate conclusions, and freehand sketches of major figures to be used (no more than two typed pages):

See the Comparisons in Figures ② - ⑥.

Extended Abstract

Transonic Flow Computation Using the Integral Solution[t] of the Full Potential Equation

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Abstract

The well developed surface panel method is extended by adding a volume integral term allowing the calculation of the full effect of compressibility. The full effect of compressibility is calculated by using mixed type finite difference scheme consistent with the mixed nature of transonic flow. The solution is obtained through successive iteration cycles for subcritical flows and for critical flows the solution is obtained through satisfying the Rankine-Hugoniot relations across the captured shock in addition to the successive iteration cycles. Shock panels are introduced to sharpen the captured shock. The method is applied to airfoils in subcritical and critical flows and the results are in good agreement with the experimental data and finite-difference solutions of the full-potential and Euler equations.

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1. INTRODUCTION AND BACKGROUND

At the present time, computational inviscid flow prediction methods available to the aerodynamic designer can be devided into two types: first, the integral equation methods more commonly known as panel methods or field panel methods of which the first was that developed by Hess and Smith [1] and second, field methods (which encompass Finite-Difference, Finite-Volume and Finite-Element methods) which have been extensively developed for transonic flow following the work of Murman and Cole [2].

A brief review of the state of the art of computational transonic inviscid flows will be presented here.

(a) Field Methods in Steady Inviscid Transonics

The research for inviscid transonics has been made in major three ways: (i) the transonic small-disturbance (TSD) formulation, (ii) full-potential (FP) formulation and (iii) Euler equation formulation which includes the rotationality effect.

The field methods of transonics are the new ones, having essentially started in 1970. Murman and Cole [2] were the first to achieve a stable transonic solution for the two-dimensional TSD equation by using the concept of type-dependent differencing. Soon after, this procedure was extended to three dimensional flows for swept-wing calculations by Ballhaus and Bailey [3] and wing-cylinder calculations by Bailey and Ballhaus [4]. At about this same time, numerical procedures for solving the transonic FP equation were being developed with suitable mapping procedures. Notable contributions are due to Stegar and Lomax [5] and Garabedian and Korn [6], both with nonconservative FP formulations for airfoil configurations. The first three-dimensional calculation (nonconservative) was introduced by Jameson [7] and was used to

solve the transonic flow about wings. Subsequently, a solution procedure for the conservative form of the FP equation was introduced by Jameson [8] and extended to three dimensions by Jameson and Caughey [9, 10].

Much of the recent work in steady transonic potential field methods has been devoted to finding better ways of solving the full potential equation in conservation form and to the development of more computationally efficient relaxation schemes, such as fully implicit approximate factorization (AF) or multigrid. Recently, several methods which use the strong conservative form of the unsteady Euler equation have been developed to solve steady and unsteady transonic flows. A typical method for steady, two-dimensional transonic flow is developed by Jameson [11]. Basic research involving the Euler formulation for steady transonic applications is currently increasing and is expected to continue to do so as the potential algorithms enter a more production-oriented research phase.

(b) Integral Equation Methods in Steady Inviscid Transonics

Although a great deal of progress has been made in solving nonlinear fluid flow problems by finite-difference methods, these methods have not yet proved to be easily adaptable to complex three-dimensional surfaces. The major technical obstacle to computing inviscid transonic flows about complete aircraft is the difficulty in generating suitable grids. More recently, the difficulties and relatively larger computation time associated with field methods prompted a number of workers to reconsider the application of integral equation methods to transonic flows.

Panel methods for subsonic and supersonic aerodynamic applications have been in use in the aerospace community since the 1960s and have become indispensable tools in aerodynamic analysis and design.

In the low-speed regime, methods which are directly or indirectly obtained from the Green's function solution (IEM) have been developed for steady and unsteady 3-D vortex flows. Existing methods of this type are the Nonlinear Discrete-Vortex methods [12-15], the Double-Panel methods [16-20], the Vortex-Panel method [21-22] and the Velocity-Potential-Panel method [23]. Flow compressibility has been accounted for by using the Prandtl-Glauert transformation, based on the free-stream Mach number.

Exact integral equation formulation of the 2-D subsonic full potential problem have been studied by Luu et al. [24] and Stricker [25]. However, these methods are not capable of dealing with flow with shocks.

Relatively little (compared with field methods) attention has been paid, so far, to integral equation methods for transonic flows. certain approximate IEM formulations of the transonic small-perturbation problem were studied already in the pre-computer era, notably by Oswatitsch [26] and Spreiter [27]. Computerized and extended versions of the approximate IEM were developed later by, amongst others, Norstrud [28], Crown [29] and Nixon [30]. It should be remarked that the approximate IEM's mentioned above, are all based on a special, partial-integration form of the integral equation for the transonic small-distrubance problem, which enables easy implementation of approximating assumptions on the decay of the perturbation velocity away from the body. The approximating assumptions on the decay of the perturbation velocity and the shock-fitting character of these methods are not considered to be competitive with FDM's.

Later, Piers and Slooff [31] developed a IEM based on transonic small-distrubance theory. Their method does not contain any approximating assumptions and, through the introduction of artificial viscosity and directional bias, has a shock-capturing capability similar to that of current FDM's.

The development of the numerical methods based on the full potential equation has been made during the last two years. Kandil and Yates [32] developed a IEM for steady transonic flows past delta wing, the results show that the method is promising and efficient. Oskam [33] developed a panel method for the full potential equation applied to multicomponent airfoils. This method is accomplished by adding a field distribution of source singularities to the conventional distribution of singularities over the boundaries of the field. At about the same time, Erickson and Strande [34] used Green's Third Identity as a means to extend panel method to non-linear potential flow, in which the concept of artificial density is employed and optimization technique is used to make sure the total compressibility is conserved. Later on, Sinclair [35] published an exact IEM for 2-D steady transonic flows, which is similar to Erickson's [34] method.

The study of these integral equation methods shows a very good potential for IEM to challenge widely used FD methods and to replace FD methods in some fields of potential transonic flows, but, the development of the IE method is far from complete. Therefore, it is a challenge task to develop transonic integral equation methods.

2. ADVANTAGES OF INTEGRAL EQUATION METHODS

The integral equation method has several advantages over the finite difference method. The IE approach involves evaluation of integrals, which is more accurate and simpler than the FD approach in which the accuracy depends on the grid size. In IE approach, grid refinement and high order source-vorticity and compressibility modeling can be used in order to increase the accuracy. Moreover, the IE method automatically satisfies the far-field boundary conditions as $O(1/r)$ or $O(1/r^2)$ and hence a small limited region

around the source of disturbance is needed. In FD method, grid points are needed over a large region around the source of disturbance and special treatment is required to satisfy the far-field boundary conditions. The IE method is computationally inexpensive, particularly for unsteady flows, and does not suffer from the artificial viscosity effects as compared to FD method for transonic-flow-shock-capturing.

3. FORMULATION

The work presented here is for steady, inviscid transonic flows around two-dimensional configurations.

(a) Differential Governing Equation and B.C.'s

The governing equations of the two-dimensional, steady, inviscid compressible flow are given by

Conservation Of Mass

$$(\rho^* \Phi_x^*)_{x^*} + (\rho^* \Phi_z^*)_{z^*} = 0 \tag{1}$$

Conservation of Energy

$$a_\infty^{*2} + \frac{\gamma-1}{2} V_\infty^{*2} = a^{*2} + \frac{\gamma-1}{2} (\Phi_x^{*2} + \Phi_z^{*2}) \tag{2}$$

Isentropic Gas equation

$$\rho^*/\rho_\infty^* = (a^{*2}/a_\infty^{*2})^{1/\gamma-1} \tag{3}$$

where Φ^* is the total velocity potential, a^* is the speed of sound, ρ^* is the density, γ is the ratio of specific heats, and the subscript ∞ refers to the freestream condition. Combining equations (2) and (3) and using V_∞^*, ρ_∞^* and a length ℓ (ℓ is the chord of airfoil) as the reference parameters, we obtain the dimensionless equation for the density ρ

$$\rho = [1 + \frac{\gamma-1}{2} M_\infty^2 (1 - \Phi_x^2 - \Phi_z^2)]^{\frac{1}{\gamma-1}} \qquad (4)$$

where M_∞ is the freestream Mach number.

The dimensionless form of equation (1) is given by

$$\Phi_{xx} + \Phi_{zz} = G \qquad (5)$$

where

$$G = - \frac{1}{\rho} (\rho_x \Phi_x + \rho_z \Phi_z) \qquad (6)$$

The boundary conditions are given by

(i)   Surface   No-Penetration (kinematic) Condition

$$\bar{V} \cdot \bar{n}_g = 0 \qquad \text{on} \quad g(\bar{r}) = 0 \qquad (7)$$

(ii) Kutta Condition

$$\Delta C_p \Big|_{TE} = 0 \qquad (8)$$

(iii) Infinity Condition

$$\nabla \Phi \to 0 \qquad \text{away from } g \qquad (9)$$

where $\bar{V}$ is total velocity vector, $g(\bar{r}) = 0$ is body (airfoil) surface, C_p is pressure coefficient and TE refers to the trailing edge.

(b)   Integral Equation Solution

The integral equation solution of Eq. (5) for a two-dimensional configuration is given in terms of the velocity field by

$$\bar{V}(x,z) = \bar{e}_\infty + \frac{1}{2\pi} \oint_g q_g(s) \frac{(x-\xi)\bar{i} + (z-\zeta)\bar{k}}{(x-\xi)^2 + (z-\zeta)^2} ds$$

$$+ \frac{1}{2\pi} \oint_g \gamma_g(s) \frac{(z-\zeta)\bar{i} - (x-\xi)\bar{k}}{(x-\xi)^2 + (z-\zeta)^2} ds \qquad (10)$$

$$+ \frac{1}{2\pi} \iint G(\xi, \zeta) \frac{(x-\xi) \, \bar{i} + (z-\zeta) \, \bar{k}}{(x-\xi)^2 + (z-\zeta)^2} d\xi d\zeta$$

$$+ \frac{1}{2\pi} \oint_S q_S (s) \frac{(x-\xi) \, \bar{i} + (z-\zeta) \, \bar{k}}{(x-\xi)^2 + (z-\zeta)^2} ds$$

where S is shock surface, q and γ are surface source and vortex distribution; respectively.

In Eq. (10), first integral is the contribution of the body thickness; second integral is the contribution of lifting or lifting and thickness or thickness only; third integral is the contribution due to the compressibility; while last integral is contribution due to shock.

Not all of the first and second integral terms in Eq. (10) are necessarily included in the calculation of the velocity field. For symmetric flows, either first integral or second integral can be used; while for asymmetric flows, either second integral or both integrals should be used.

Note that the integrand of the volume integral of Eq. (10) decreases rapidly with increasing distance not only because of the factor $1/((x-\xi)^2 + (z-\zeta)^2)$ but also because G diminishes rapidly with increasing distance. Consequently, for computational purposes, the volume integral needs to be addressed only within the immediate vicinity of the body.

The reader should notice that the difference between the present formulation and the formulations given by Sinclair [35] and by Tseng and Morino [36]. The present formulation is based on the velocity field in which the source term G contains first order derivatives of density only, and the normal velocity is discontinuous across the shock. The Sinclair and Tseng and Morino formulations are both based on the velocity potential in which the source term G contains first- and second-order derivatives of the velocity

potential and the velocity potential is continuous across the shock. The present formulation has two advantages over the velocity-potential formulation: (1) only first-order derivatives need to be calculated by finite differencing, and (2) one does not need to calculate derivatives of the velocity potential in order to detect the shock formation since the velocity field is calculated directly in the present formulation.

(c) Shock-capturing and Shock-fitting Technique

Before switching to the next section for numerical procedure it is worth discussing the last integral term of Eq. (10).

For treating transonic flows, we consider two techniques - shock-capturing and shock-fitting techniques. Although these techniques are well known, we examine here their application to the integral equation formulation.

In the shock-fitting technique, the contribution of the shock to the velocity field is represented by an explicit surface - integral term, the last term of Eq. (10), and the shock strength q_s is given by the normal velocity increment across it. Thus

$$q_s = - (V_{1n} - V_{2n}) \qquad (11)$$

But in the shock-capturing technique, the last term of Eq. (10) is not necessarily included, since the volume integral itself implicitly includes the shock surface contribution [32].

Therefore, for shock-capturing technique Eq. (10) becomes,

$$\bar{V}(x,z) \equiv u\,\bar{i} + w\bar{k} = \bar{e}_\infty + \frac{1}{2\pi} \oint_g q_g(s) \frac{(x-\xi)\,\bar{i} + (z-\zeta)\,\bar{k}}{(x-\xi)^2 + (z-\zeta)^2}\, ds$$

$$+ \frac{1}{2\pi} \oint_g \gamma_g(s) \frac{(z-\zeta)\,\bar{i} - (x-\xi)\,\bar{k}}{(x-\xi)^2 + (z-\zeta)^2}\, ds \qquad (12)$$

$$+ \frac{1}{2\pi} \iint G\ (\xi,\zeta)\ \frac{(x-\xi)\ \bar{i} + (z-\zeta)\ \bar{k}}{(x-\xi)^2 + (z-\zeta)^2}\ d\xi d\zeta$$

We will use both shock-capturing and shock-fitting techniques for present work.

4. SOLUTION PROCEDURE

The whole solution procedure will be described by three parts: (1) discretization of equations, (ii) iterative procedure for subcritical flows, and (iii) treatment of critical flows.

(i) Discretization of Equations

Eq. (12) expresses the solution to the potential equation, Eq. (5), as the sum of four terms, the first three terms are the standard panel method terms and the last term is a volume integral in the field described above. The airfoil surface is divided by a number of straight panels with linear source and/or vortex distribution. The field is divided into a number of rectangular meshes except at the airfoil surface where trapezoidal elements are used. The value of G is taken to be constant over each mesh. Figure 1 shows the airfoil panels and field meshes. After discretization of equations, the integral equation solution, Eq. (12), becomes

$$\bar{V}\ (x,z) = \bar{e}_\infty + \frac{1}{2\pi} \sum_{k=1}^{N} \oint q_{gk}\ \frac{(x-\xi)\ \bar{i} + (z-\zeta)\ \bar{k}}{(x-\xi)^2 + (z-\zeta)^2}\ ds_k$$

$$+ \frac{1}{2\pi} \sum_{k=1}^{N} \oint \gamma_{gk}\ \frac{(z-\zeta)\ \bar{i} - (x-\xi)\ \bar{k}}{(x-\xi)^2 + (z-\zeta)^2}\ ds_k \tag{13}$$

$$+ \frac{1}{2\pi} \sum_{i=1}^{IM} \sum_{k=1}^{JM} G_{ij} \iint_{A_{ij}} \frac{(x-\xi) \bar{i} + (z-\zeta) \bar{k}}{(x-\xi)^2 + (z-\zeta)^2} d\xi d\zeta$$

where N is total number of surface panels, IM x JM the total number of field meshes, A_{ij} the area of each field meshes.

(ii) Iterative Procedure for Subcritical Flows

The main difference between standard panel methods and integral equation methods (or field panel methods) is due to the volume integral term in Eq. (13). This term is a non-linear term and therefore unlike the standard panel method, the solution cannot be obtained directly and an iterative procedure is necessary. For simplicity, we discuss the iterative procedure for subcritical flows first.

The iterative cycle is described below.

Step 1

A standard panel method calculation, with G = 0, is employed to get q_g or γ_g.

Step 2

Calculate G for each mesh by using the linear compressibility, $G = M_\infty^2 u_x$, where u_x is calculated from the results of Step 1.

Step 3

Enforce the no-penetration condition and Kutta condition to get new q_g or γ_g. In this step, the contribution of G is included.

Step 4

Calculate G by using non-linear compressibility, Eq. (6). In this step, we first calculate Φ_x and Φ_z by using Eq. (13), compute density (ρ) by Eq. (4) and ρ_x and ρ_z are computed by central differencing for subcritical flows.

Step 5

Enforce no-penetration condition and kutta condition again to solve for new q_g or γ_g .

Step 6

Repeat Steps 4 and 5 until q_g or γ_g and G converge.   In each step, the pressure coefficient is computed by

$$Cp = \frac{2}{\gamma M_\infty^2} \{[1 + \frac{\gamma-1}{2} M_\infty^2 (1-V^2)]^{\frac{\gamma}{\gamma-1}} - 1\} \tag{14}$$

where $V = |\bar{V}|$. Note that when calculating the pressure on the surface, the self-induced velocity must be treated carefully.

(iii) Treatment of Critical Flows

Several key points have been used for treating transonic flows with shocks:

(1) For the critical flow with shocks, a mixed type finite difference must be applied to calculate ρ_x and ρ_z ; consistant with the mixed nuture of transonic flow. The local Mach number is calculated by

$$M = VM_\infty/\rho \frac{\gamma-1}{2} \tag{15}$$

(2) The conditions behind the shock are determined by using the Rankine-Hugoniot relations:

$$V_{2n} = \frac{(\gamma-1) M_{1n}^2 + 2}{(\gamma+1) M_{1n}^2} V_{1n} \tag{16.a}$$

$$V_{2t} = V_{1t} \tag{16.b}$$

$$\rho_2 = \frac{(\gamma+1)\ M_{1n}^2}{(\gamma-1)\ M_{1n}^2 + 2}\ \rho_1 \qquad\qquad (16.c)$$

(3) Shock panels are introduced in order to sharpen the discontinuity in flow properties. The panels are added after several iterations when the location of the shock is fixed. The orientation of the shock panels is determined by Rankine-Hugoniot equation.

$$\beta = \sin^{-1}\ \left[\frac{1.2\ \sin\ (\beta)\ \sin\ (\theta)}{\cos\ (\beta - \theta)} + \frac{1}{M_1^2}\right]^{1/2} \qquad\qquad (17)$$

where β is the shock angle and θ is the relative direction of the flow behind the shock to that ahead of the shock. The strength of shock is give by

$$G\epsilon = -\ (V_{1n} - V_{2n}) = -\ \frac{2\ V_{1n.}}{\gamma + 1}\ (1 - \frac{1}{M_{1n}^2}),\ M_{1n} > 1.0 \qquad\qquad (18)$$

5. NUMERICAL EXAMPLES

A scalar program has been developed to implement the solution procedure of the shock capturing-shock fitting technique. To prove the concept and verify the algorithm, the code is applied to NACA0012 airfoil at different Mach numbers and different angles of attack.

CASE A   Subcritical Flows

Fig. 2 shows a comparison of the present results with experimental data for incompressible flow at $\alpha = 0^0$. A total of 140 panels is used for whole calculations over upper and lower airfoil surfaces. Two different modeling options, source panel modeling and vortex panel modeling, are applied respectively. The comparison shows that the vortex panel modeling is better than the source panel modeling.

Fig. 3a through Fig. 3c are for the case of $\alpha = 0$, $M_\infty = 0.72$. Fig. 3a gives the comparison of the results of the two different modeling methods with Euler's solution, and it is again shown that the vortex panel modeling is better than the source panel modeling. Fig. 3b shows the effect of the size of computational domain, while Fig. 3c shows that the present solution compares well with Euler's solution.

The next case illustrated is a lifting case with $\alpha = 2^0$ and $M_\infty = 0.63$. Fig. 4a shows the effect of the size of the computational domain and Fig. 4b shows the comparison of present solution with finite difference solution of the full potential equation.

CASE B Critical Flows

Fig. 5a shows the present results compared with the experimental data and Garabedian's finite difference solution. The computational domain used is 2c x 1.5 c as shown in Fig. 1. The properties behind the shock are calculated using Rankine-Hugoniot relations as given by Eqs. (16.a) - (16.c). The results show that the present integral solution method is capable of capturing a sharp shock. The location and strength of the shock calculated here are in a very good agreement with the experimental data and the finite difference solution. Fig. 5b shows the effect of shock panels on the shock strength. The comparison shows that the shock panels are effective for sharpening the shock. Fig. 6 shows the lifting case result for $\alpha = 2^0$, $M_\infty = .75$. The comparison shows good agreement.

6. CONCLUDING REMARKS

The standard panel method has been extended to the treatment of transonic flows. The results presented here show that the method is capable of capturing sharp shocks. The location and strength of shocks are in a very good agreement with the experimental data and finite difference solutions. Introduction of shock panels sharpens the shock with relatively coarse grids. Presently, the method is applied to unsteady transonic flow cases where the unsteady density term is treated as another volume integral term. A time-splitting finite difference scheme is used to step the equation in time while the integral solution is used to compute the spatial variation. The full length paper will include transonic flow cases in pitching motion.

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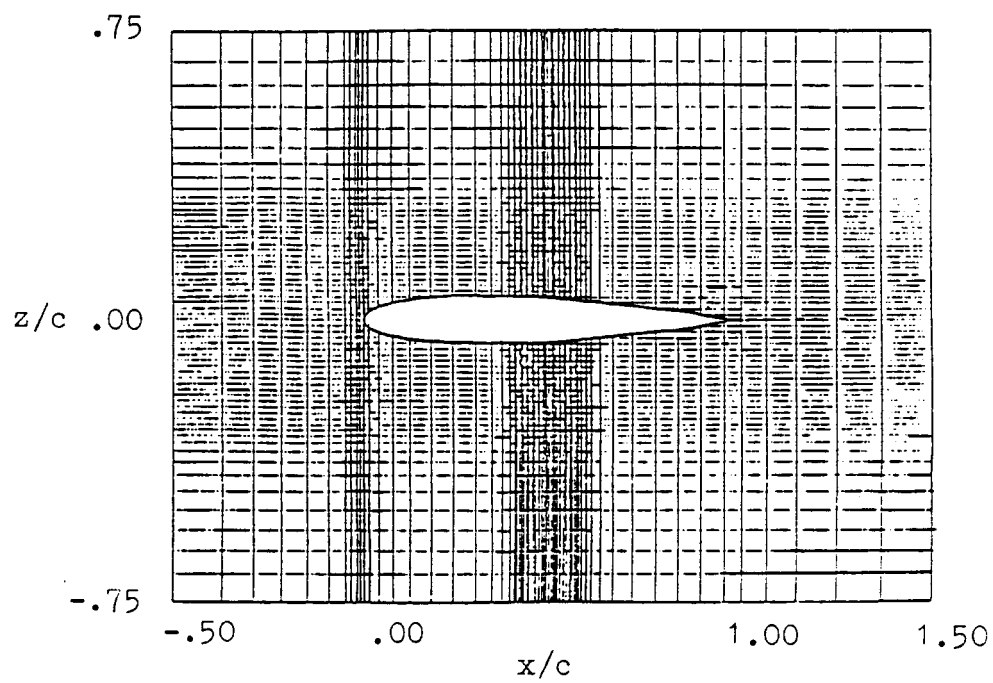
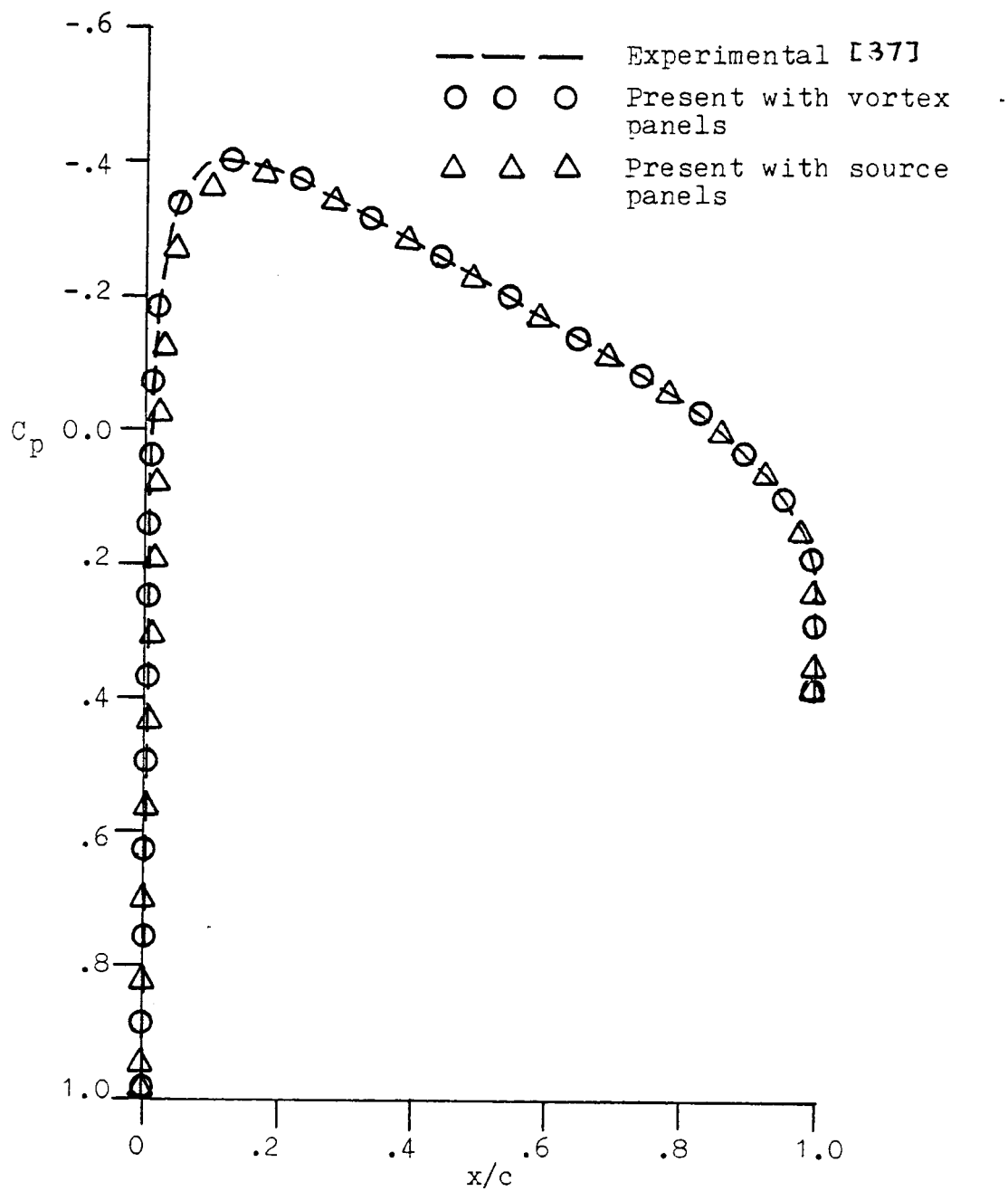Fig. 1   Computational mesh, NACA 0012
aerofoil, 64 x 60 field mesh.

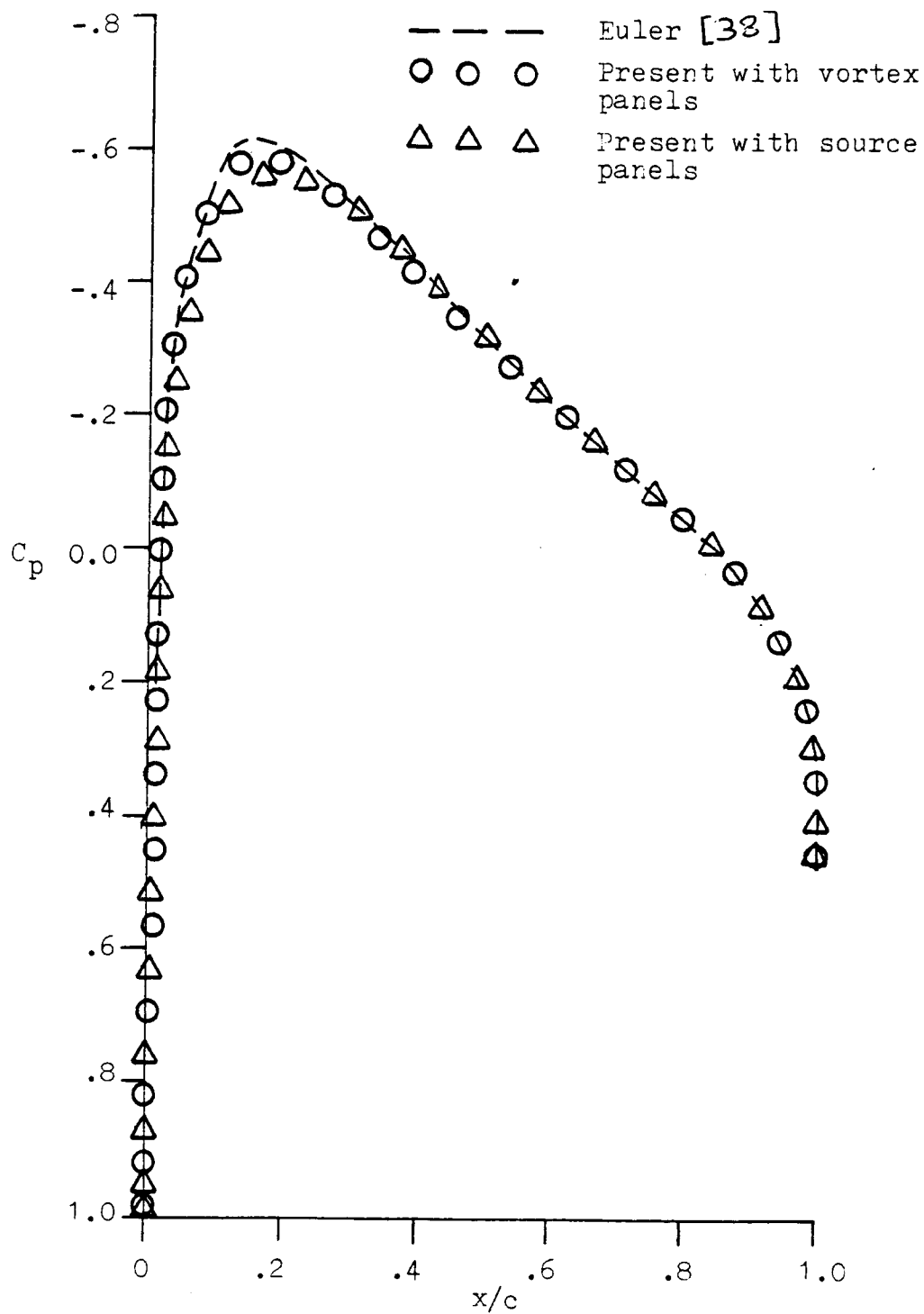Fig. 2   NACA 0012 aerofoil, $M_\infty = 0.0$, $\alpha = 0.0$.

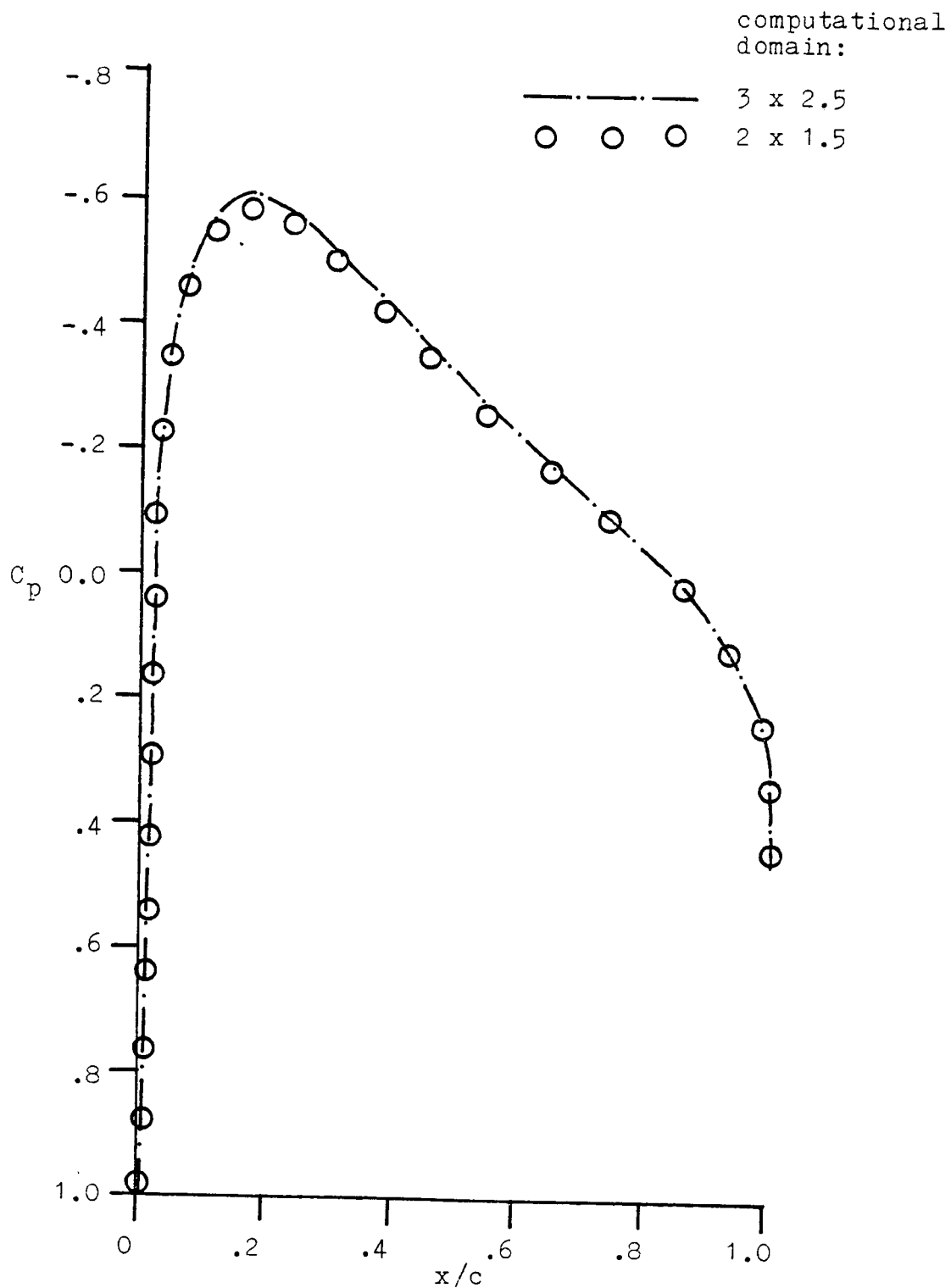Fig. 3a   NACA 0012 aerofoil, M_∞ = .72, α = 0.0, computational domain: 2 x 1.5.

Fig. 3b NACA 0012 aerofoil, M_∞ = .72, α = 0.0, with vortex panels, effect of the size of the computational domain.

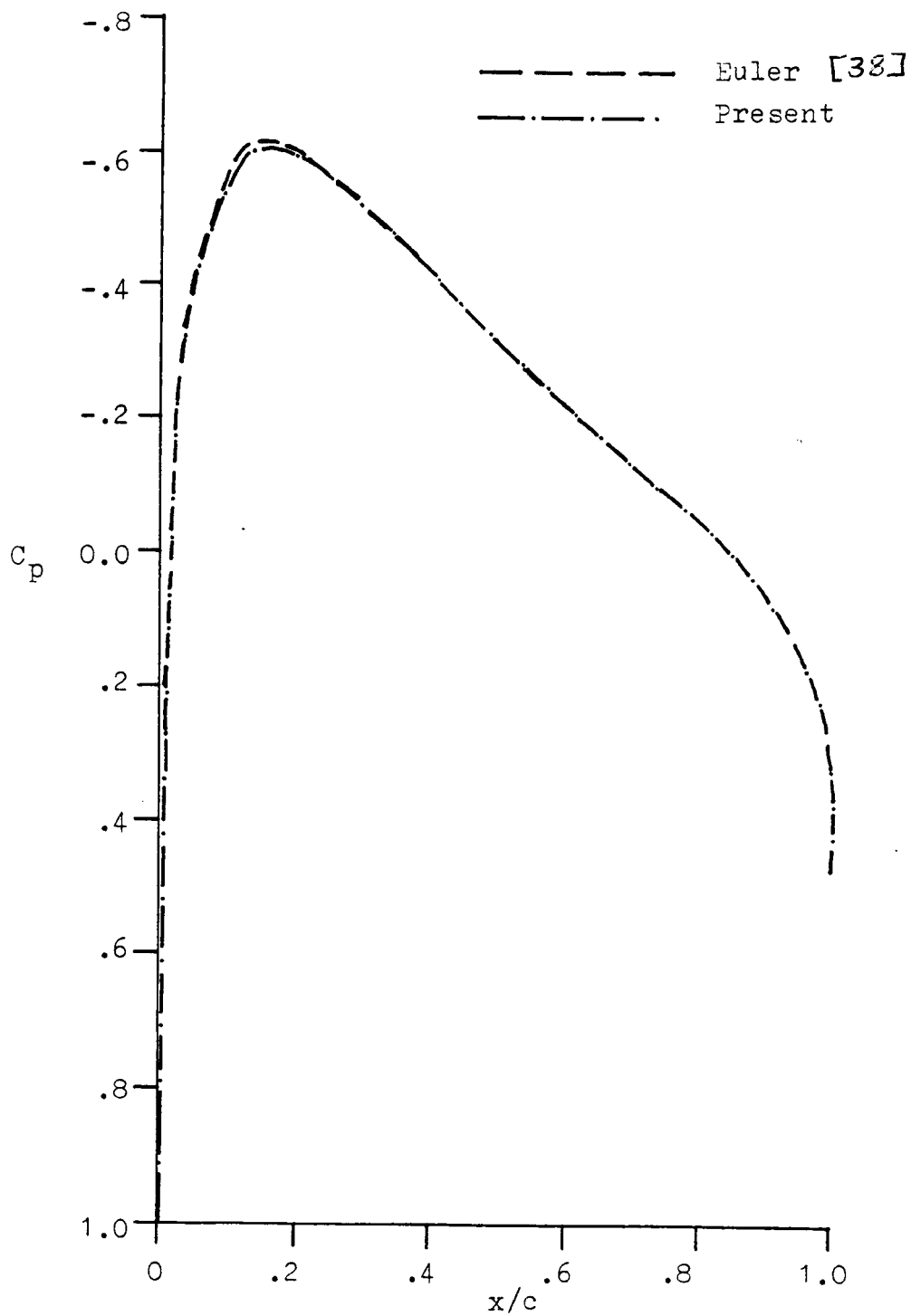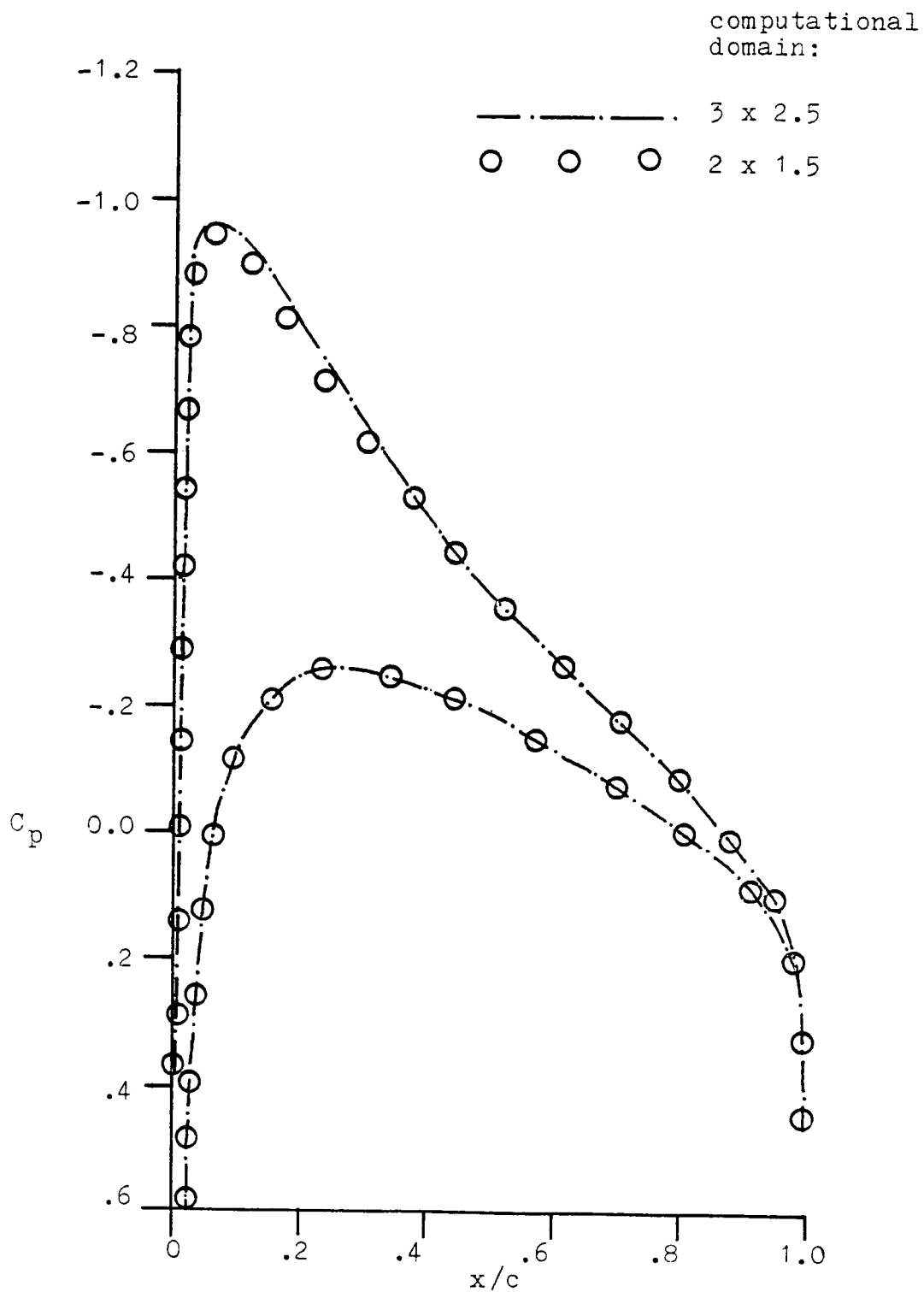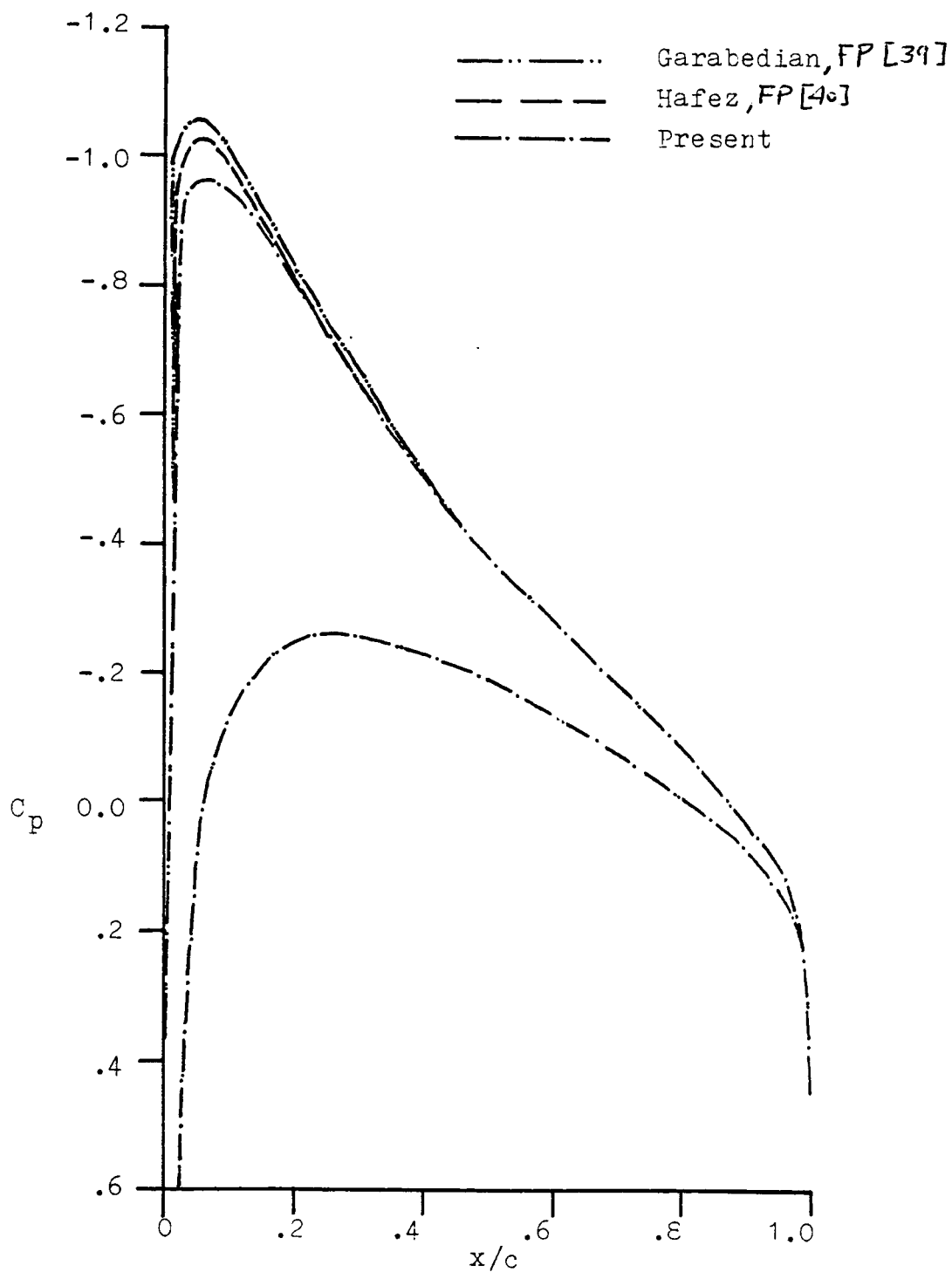Fig. 3c NACA 0012 aerofoil, M_∞ = .72, α = 0.0,
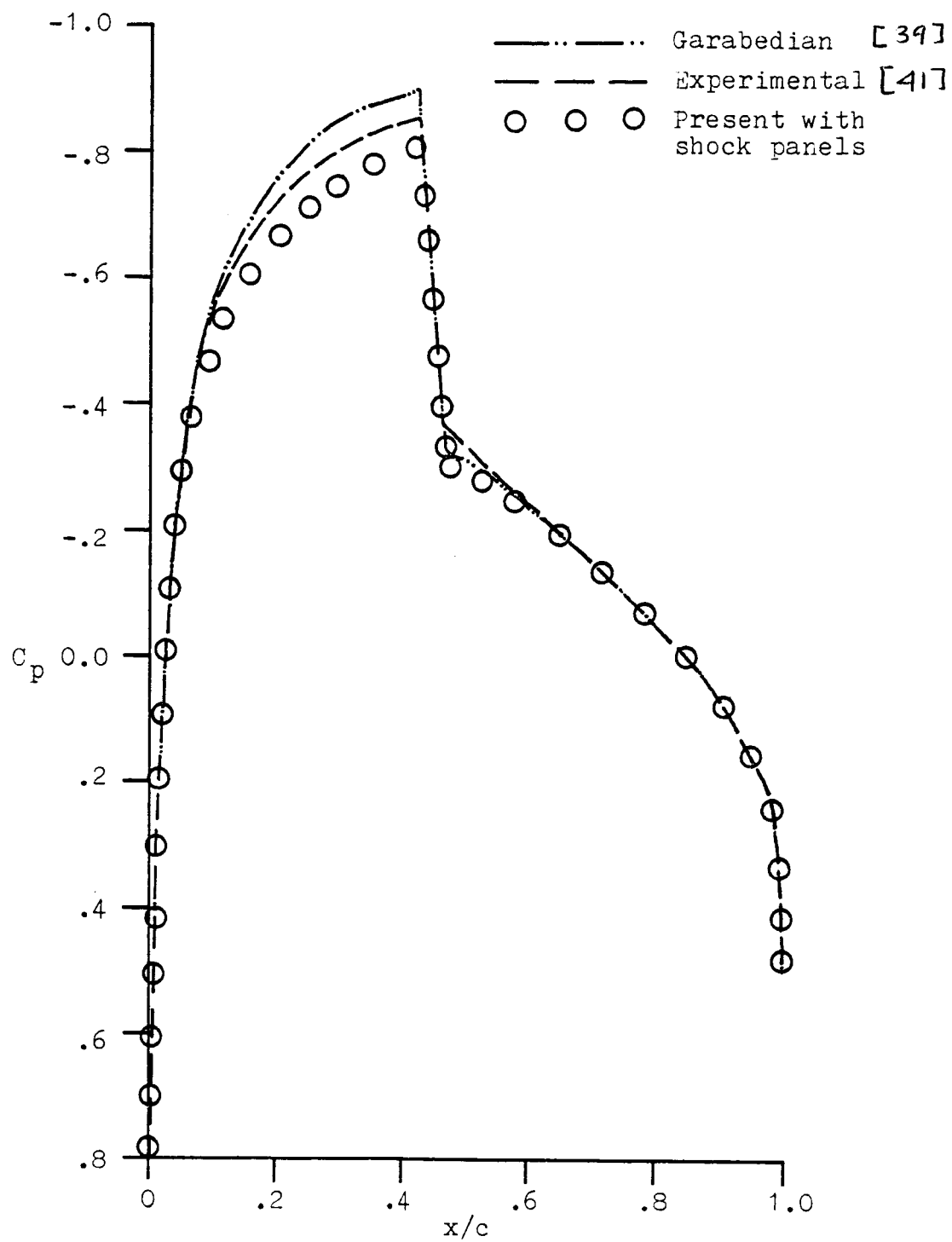   with vortex panels, computational
         domain: 3 x 2.5.

Fig. 4a NACA 0012 aerofoil, $M_\infty = .63$, $\alpha = 2.0$, with vortex panels, effect of the size of the computational domain.

Fig. 4b NACA 0012 aerofoil, M_∞ = .63, α = 2.0,
with vortex panels, computational
domain: 3 x 2.5.

Fig. 5a   NACA 0012 aerofoil, M_∞ = .80,
          α = 0.0, with vortex panels,
          computational domain: 2 x 1.5.
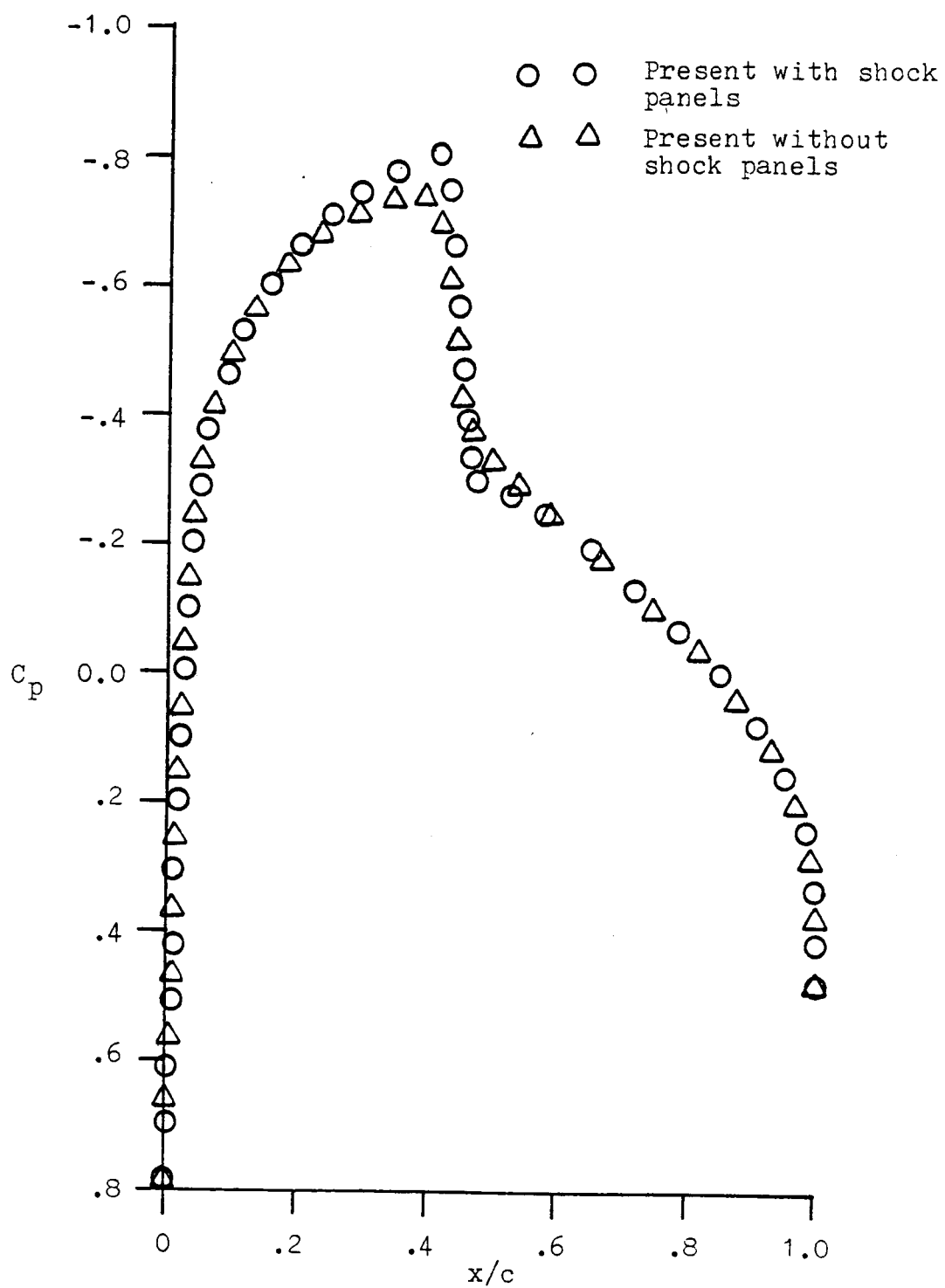
Fig. 5b NACA 0012 aerofoil, M_∞ = .80,
         α = 0.0, with vortex panels,
         computational domain: 2 x 1.5,
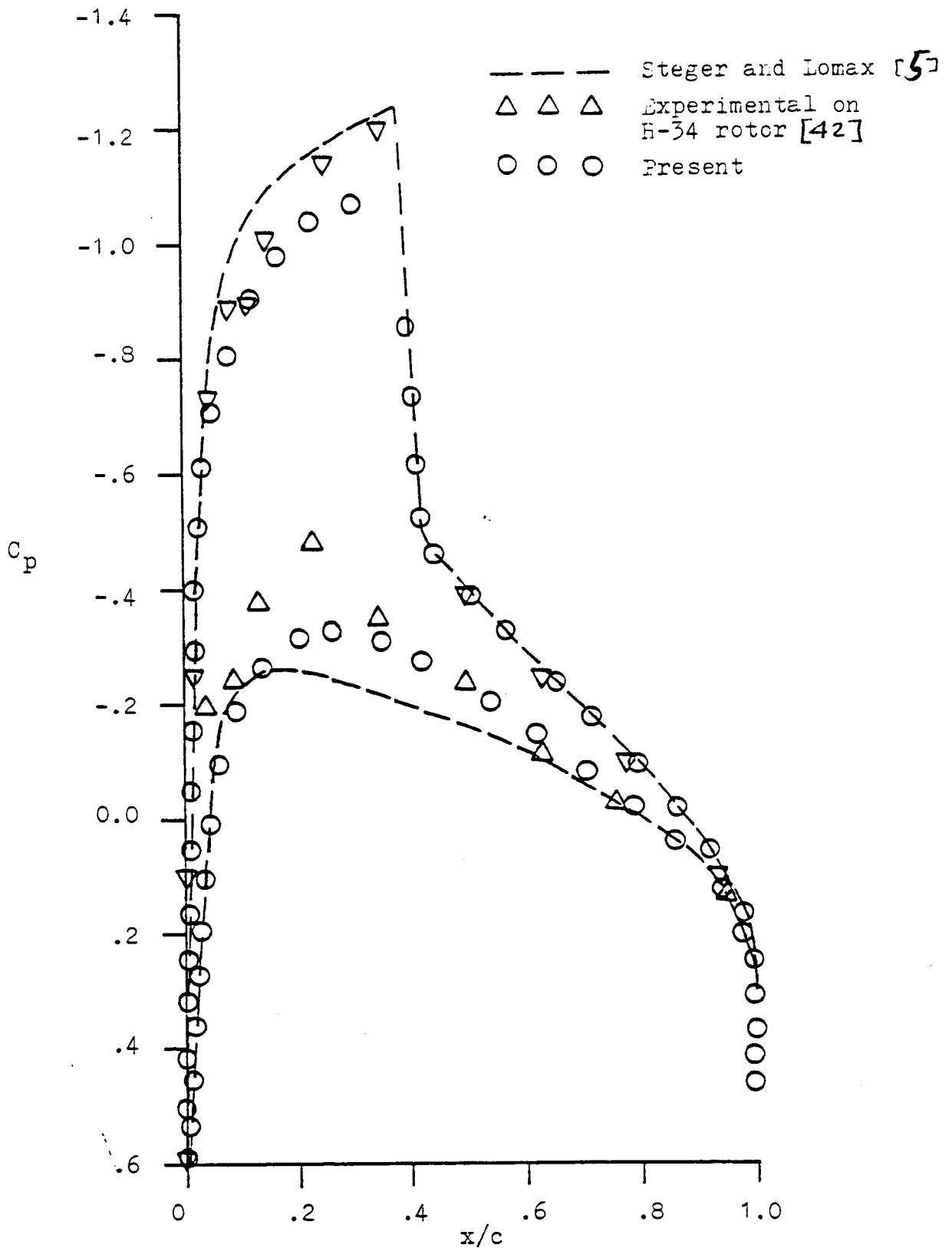   effect of shock panels.

Fig. 6 NACA 0012 aerofoil, M_∞ = .75, α = 2.0,
with vortex panels, the computational
domain: 2 x 1.5.