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Composite Theory Applied to Elastomers

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COMPOSITE THEORY APPLIED TO ELASTOMERS

1. Introduction

Reinforced elastomers form the basis for most of the structural or load carrying applications of rubber products. The reinforcing materials include various synthetic textiles, glass and steel, and are most usually utilized in the form of twisted filamentary assemblages, cords, designed to impart stiffness, strength and to resist fatigue. Due to the geometry of such cords they are almost always used in the form of a parallel array, embedded in rubber, so as to give highly directional properties to the composite cord-rubber lamina.

Computer based structural analysis in the form of finite element codes has been highly successful in refining structural design in both isotropic materials and rigid composites. This has lead the rubber industry to attempt to make use of such techniques in the design of structural cord-rubber composites, among the more prominent being tires of all types where volume production or technical necessity can justify extensive design computation.

While such efforts appear promising, they have not been easy to achieve for several reasons. Among these is a distinct lack of a clearly defined set of material property descriptors suitable for computer analysis. There are substantial differences between conventional steel, aluminum or even rigid composites such as graphite-epoxy, and textile-cord reinforced rubber. These differences are both conceptual and practical. They are discussed in this paper.

Based on the assumption of a linear continuum, it may be shown that ν_{21} and ν_{12} are related so that the measurement of four properties suffices for experimental description.

In many applications the elastic properties are quite close in numerical value for filaments in either tension or compression, and so little or no differentiation is made between them. Similarly, while damping or viscoelastic properties are useful in many structural applications to suppress unwanted vibratory motion, these properties are not usually considered as an integral part of the material property description.

Finally, it should be noted that while the individual filaments usually are themselves orthotropic, with different properties in the filament direction compared to the direction perpendicular to the filament, they are solid monofilaments and they are almost always individually embedded in the matrix. They cannot slip or move relative to one another, and so individually and collectively contribute to the material properties of the composite sheet.

3. Cord Rubber Composites

The comparison of cord-rubber composites with rigid composites is best done using a lamina such as shown conceptually in Fig. 1. Geometrically, the primary differences in the two are in the filamentary reinforcement, where in cord-rubber constructions the filaments are almost always assembled or twisted into strands, and strands into cords, to form large discrete reinforcement elements

2. Rigid Composite Lamina

Most rigid composite structures are composed of filamentary reinforcement, such as glass or graphite, in a matrix proportioned in such a way that the diameter of an individual filament is very much smaller than the thickness of the part in question, and so that a number of filaments occur through the thickness of a section. Reinforcement volume loadings are usually quite high, and in many ways it is reasonable to assume that material stiffness and elastic properties are smeared or averaged over the thickness, giving rise to a composite material property. While mat and roving are widely used for lower strength applications, most structural uses are based on a series of filaments being laid parallel to one another, so as to form a highly anisotropic solid having three principal material directions, these being the direction of the filamentary reinforcement and the two mutually perpendicular directions to the filament, as shown in Fig. 1. Many thin-walled structures are formed by laminating a series of thin sheets of this type, each sheet being considered orthotropic and treated as a two-dimensional load carrying element in the 1-2 plane, those effects in the 3 direction being given a secondary role. Such a sheet is usually described as a linear elastic element, but requires five elastic constants to describe it completely, these being:

E_1 = elastic modulus in 1 direction

E_2 = elastic modulus in 2 direction

G_{12} = shear modulus in 1-2 directions

ν_{12} = Poisson's ratio for stress in the 1 direction

ν_{21} = Poisson's ratio for stress in the 2 direction occupying now a significant fraction of the lamina thickness, as shown in Fig. 2. In some types of reinforcement the cord is impregnated with a dip to promote adhesion, and the dip forms a shell or coating over the cord which tends to cause it to act as a unit. However, in other materials, notably steel, such unitary action is not always evident since the filaments are more nearly individual. Internal filament slip can occur. In any case, the dispersion of reinforcement evident in rigid composites is not present in cord-rubber composites. Rather the reinforcement is agglomerated and the material is more nearly periodic in its properties. The concept of a smeared or averaged set of material properties is still valuable and even necessary at the present level of computational efficiency, but the definition of these properties is more difficult since the reinforcement can be defined clearly only in the direction of the strand or cord. The transverse response of the strand, or its shear response, depend on the material as well as dip and processing variables which are not always well defined. Some variation in averaged material properties is to be expected in cord-rubber composites.

In a rigid composite the reinforcing filament is almost always transversely orthrotropic. In a cord-rubber composite the reinforcement strand is twisted, or the equivalent in steel, so that bending fatigue properties are improved. This twist usually couples tension in the strand to strand twist, so that the transverse orthotropy is lost. This means that the lamina itself is coupled between tension in the cord direction and twist of the 1-2 plane, as shown in Fig. 1. It is common to neglect this twist effect, which is not usually large, and to consider the lamina purely orthotropic in the 1-2 plane. Otherwise it must be described as monoclinic, as has been pointed out [1]*, and would require 13 elastic constants for its definition instead of the five needed otherwise.

One major difference between cord-rubber and rigid composites lies in the ratio of reinforcement modulus to matrix modulus. One measure of this is the ratio of modulus of the lamina of Fig. 1 in the cord direction to the transverse direction, E_1/E_2. Ref. [1] lists values shown in Table 1 below.

Composite System	E_1/E_2
Glass-epoxy	2.9
Graphite-epoxy	40
Nylon-rubber	80
Steel-rubber	850

Table 1

Ratio of Longitudinal to Transverse Moduli

Practically speaking, cord-rubber composites are characterized by very low modulus matrices which provide very little lateral reinforcement against filament buckling. Further, since the internal section of most strands of filaments are not fully penetrated by the dip, they are supported laterally only by adjacent filaments. Due to twist, the initial shape of individual filaments is curved, and the filaments straighten under tension and

*Numbers in parentheses refer to references in the Bibliography.

become stiffer. All of these effects combine to result in the stress-strain curves of lamina in the 1 direction being distinctly non-linear for all commercially important cord-rubber laminates. This is illustrated conceptually in Fig. 3, where it is understood that the transition region may occur at different levels of cord strain E_1 depending on the reinforcing cord material.

Stress-strain response such as illustrated in Fig. 3 has been widely discussed in the literature as bimodular response. Two excellent review articles covering the mathematical description [2] and the mechanics of the phenomenon [3] are available.

Past work on this phenomenon have divided this non-linear response into regions of cord tension and cord compression. Experimental data shows that this is an oversimplification. Figure 4 shows data on rayon from [4], Fig. 5 shows data on nylon from [5], Fig. 6 shows data on steel-rubber from [6], Fig. 7 shows data on polyester-rubber from [7] while Fig. 8 shows data on aramid-rubber, also from [7].

In view of the clearly different behavior exhibited by these various reinforcing materials, possibly for different structural reasons but certainly associated with processing variables, it is proposed that the bimodular simplifcation be adapted in the form illustrated in Fig. 9, taken from Ref. [5]. Here the stress-strain curve in the reinforcement direction is idealized into two linear regions, I and II. Region I is the low cord modulus region, with individual filaments contributing little to lamina stiffness. In region II the filaments are fully effective in enhancing stiffness. The two regions are linearized and their intersection is expressed in terms of cord strain ε_1*, a material property of the lamina.

The elastic constants of the lamina must be measured separately in both regions I and II.

In this descriptive framework, an elastic cord-rubber lamina is now described by eleven elastic constants, these being shown below in Table 2.

Region I	Region I
E_1	E_1
E_2	E_2
G_{12}	G_{12}
ν_{12}	ν_{12}
ν_{21}	ν_{21}
plus ε_1*	

Table 2

Elastic Constants for a Cord-Rubber Lamina

In future notation, the elastic constants will be subscripted with I or II to indicate the region in question.

Some elastic constants have been reported in the literature for both regions I and II, such as in refs. [6] and [8]. As an example, constants are given from Ref. [8] for aramid-rubber and polyester-rubber in Table 3, while the data for nylon is from our current studies. The volume fraction is not quoted.

	Aramid-Rubber		Polyester-Rubber		Nylon-Rubber	
	I	II	I	II	I	II
E_1	12.	3580.	36.9	617.	88.	447.
E_2	12.	9.1	11.	8.	8.4	8.4
G_{12}	3.7	3.7	2.7	2.6	3.5	4.4
ν_{12}	0.205	0.416	0.185	0.475	0.837	0.283

Table 3

Bimodular Elastic Properties

Modulus Values in GPa

In so far as is known, no data has been published on ε_1*. A review of available σ_1 vs ε_1 curves available in the literature and from work done by the writer give values of ε_1* as shown in Table 4. These should be taken as representative only, since they are clearly a function of variables such as cord twist, compound modulus, and construction of the lamina.

Material	ε_1*	Ref.
Nylon 840/2	.015	[5]
Polyester	0	[7]
Fiberglass	~0.005	
Steel	.065	[6]
Aramid	0	[6]

Table 4

Values of ε_1^* for Various Reinforcement Systems

What is clear from Table 4 is that allowance should be made in general for the bimodular transition to occur at a value of $\varepsilon_1{}^*$ which is an inherent material property, just as clearly as the other elastic constants. Computationally, this quantity can be tracked and used as a switch to move from one set of values to another. Such a system probably provides a realistic material framework compatible with present FEM technology, as evidenced in its use in Ref [8].

While the role of the elastic and shear moduli are clear, some discussion is in order concerning the Poisson's ratios ν_{12} and ν_{21}. It is customary in conventional rigid composites to consider the material as an elastic continuum, in which case strain energy considerations lead to the well known reciprocity relation

$$\nu_{12}/E_1 = \nu_{21}/E_2 \qquad (1)$$

With this relation, it is now only necessary to measure one Poisson ratio, the other being obtained from Eq. (1).

There has been some question in the literature on the applicability of Eq. (1) to cord-rubber composites as a general class. This seems to be due to at least two reasons, these being:

(a) Conceptually, a twisted filamentary strand is not a continuum. The filaments may separate under transverse tension, so that load is carried around the strand but not through it. However, the filaments are very effective in carrying load in the longitudinal direction, so that inherently different load carrying mechanisms operate in the two principal directions. The cord rubber composite is not a continuum but a structure, and may not be symmetric for that reason.

(b) Accurate measurements of Poisson's ratio are difficult to make for a variety of reasons which will be dealt with in more detail in a later section. Little published data is available for the two Poisson's ratios for the same material, so that confirmation of reciprocity has yet to be demonstrated convincingly.

For the remainder of this paper it will be accepted that ν_{12} and ν_{21} are elastic constants which will be obtained by independent measurement.

A second major difference between conventional rigid composites and cord-rubber composites lies in the significant energy loss component associated with cyclic stress in cord-rubber laminates. This effect is large enough so that it must be taken into account in describing tests where self-heating can occur. In this paper the notation of Ref. [9] is adapted, so that elastic constants can be expressed as:

$$E^* = E' + iE'' \tag{2}$$

where the real portion E' represents the in-phase elastic modulus obtained by using the slope of the line 0-1 as illustrated in Fig. 10. The imaginary portion E" is obtained from the well known relation

$$E'' = \frac{A}{\pi\, \varepsilon_o^{2}} \tag{3}$$

where A and ε_o are also defined in Fig. 10. The ratio E"/E' is called tan δ and widely used as a measure of material damping. Both E" and tan δ will be used in the following discussion.

Since the elastic constants representing the interaction of stress with strain now appear in the form of both real and imaginary parts, the number of elastic constants needed for description of the lamina of Fig. 1 now is 17, as shown in Table 5.

$E'_{1,I}$	$E''_{1,I}$		$E'_{1,II}$	$E''_{1,II}$
$E'_{2,I}$	$E''_{2,I}$		$E'_{2,II}$	$E''_{2,II}$
$G'_{12,I}$	$G''_{12,I}$		$G'_{12,II}$	$G''_{12,II}$
$\nu_{12,I}$			$\nu_{12,II}$	
$\nu_{21,I}$		$\varepsilon_1{}^*$	$\nu_{21,II}$	

Table 5

Elastic Constants for Cord Rubber Composite

In Table 5, the Poisson's ratios ν_{12} and ν_{21} represent the interaction of two strain terms. On physical principles, and from experimental observation where only in-phase values have been observed, these constants are assigned only real values. Data taken on $\nu_{12,II}$ on 840/2 nylon in rubber is given in Fig. 11 as indicative of observations.

Because of the dependence of elastic properties such as given in Table 5 on cord material, twist, denier and end-count, as well as compound characteristics, only representative values of such constants can be given, and even those have limited validity. For example, Table 6 gives values for a particular nylon/rubber composite under conditions listed there.

$E'_{1,I} = 87.8$	$E''_{1,I} = 18.9$	$E'_{1,II} = 447.3$	$E''_{1,II} = 50.8$
$E'_{2,I} = 8.41$	$E''_{2,I} = 2.19$	$E'_{2,II} = 8.41$	$E''_{2,II} = 2.19$
$G'_{12,I} = 3.54$	$G''_{12,I} = 0.42$	$G'_{12,II} = 4.43$	$G''_{12,II} = 0.42$
$\nu_{12,I} = 0.84$		$\nu_{21,II} = 0.283$	
$\nu_{21,I} = 0.17$		$\nu_{21,II}$ not measured	
		$\varepsilon_1 = 0.015$	

Table 6

Elastic Constants for 840/2 Nylon in NR compound

End count/dm 118, Temperature 22°C, Frequency 1 Hz

Volume Fraction $v_c \simeq 0.17$.   Units: MPa

It is recognized that properties such as quoted in Table 6 are also dependent on both frequency and temperature. Such dependence is a function of the specific material and is difficult to generalize. Studies on selected properties from Table 6 are given in Figs. 12 through 15, but these should be used with caution since there is no evidence that the trends observed have quantitative generality to other constructions or materials. However, the known response of both rubber compounds and most polymeric textile cords supports the generalizations that modulus values increase somewhat with frequency but decrease with increasing temperature.

Insofar as is known, Table 6 represents the only complete set of data on cord-rubber composites expressed in the form of Table 5. However, a number of partial data sets are available in the literature. These have been collected in Appendix 1 for convenient reference.

4. Angle Ply Effects

Most cord-rubber lamina are used in laminates, where a number of plies are bonded together to form a structure, usually forming principal material directions which are not aligned with the cord reinforcement direction. Rigid composite practice involves the use of a compliance or stiffness matrix to represent the plane elastic properties of the lamina. Using the notation of Ref [10] and Fig. 1,

$$\begin{Bmatrix} \sigma_1 \\ \sigma_2 \\ \tau_{12} \end{Bmatrix} = \begin{bmatrix} Q_{11} & Q_{12} & 0 \\ Q_{12} & Q_{22} & 0 \\ 0 & 0 & Q_{66} \end{bmatrix} \begin{Bmatrix} \varepsilon_1 \\ \varepsilon_2 \\ \gamma_{12} \end{Bmatrix} \qquad (4)$$

where the reciprocity relation of Eq (1) has been invoked to obtain the symmetry of the Q matrix, and where all quantities represent the real, or elastic response, so that

$$\begin{aligned} Q_{11} &= E'_1/1-\nu_{12}-\nu_{21} \\ Q_{12} &= \nu_{12}E'_2/(1-\nu_{12}\nu_{21}) = \nu_{21}E'/(1-\nu_{12}\nu_{21}) \\ Q_{22} &= E'_2/(1-\nu_{12}\nu_{21}) \\ Q_{66} &= G'_{12} \end{aligned} \qquad (5)$$

By transformation of the stresses and of the strain measures, it may be shown that in directions different from the 1-2-3 principal directions, the relations of Eq (4) become

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} \\ Q \\ \\ \end{bmatrix} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \gamma_{xy} \end{Bmatrix} \qquad (6)$$

where the Q matrix is now fully populated and is a function of both the Q_{11}, Q_{22}, etc., and the angle θ between the x axis and the 1 direction, as shown in Fig. 16. Q is given in Ref [10].

The effectiveness of this transformation procedure is widely accepted in the analysis of rigid composites. Ref. [11] has documented experimental evidence of its adequacy for cord-rubber composites, where single ply test specimens were studied over a variety of cord angles. Good agreement was obtained by comparing the measured real elastic constants with those calculated from Eq. (6). The reinforcing materials were polyester, steel and aramid, all in rubber compounds. Details of the material properties are given in Appendix 1.

The test techniques used in Ref [11] involved the use of tensile test specimens with the reinforcement at an angle to the tensile direction. Cord compression existed at angles above 60°, and the test data reported represent a mixture of data from what in this paper are denoted as regions I and II. Nevertheless, the agreement with transformation theory for the real part of the elastic constants is surprisingly good, and based on this work its use appears justified for the real portion in the region of cord tension, i.e. region II.

In view of the importance of this experimental evidence, the comparisons of measurements with calculation from Ref. [11] are given in Figs. 17 through 19.

A major question now concerns the application of classical transformation theory to the concept put forth in this paper, namely that there exist two regions of cord-rubber composite behavior, region I and region II, and that inside both regions the modulus values are complex numbers. In order to clarify this question, an extensive series of experiments has been conducted on tubular specimens using a balanced two-ply construction with cord angles $\pm\theta$ with respect to the longitudinal axis of the tube. The reinforcing materials have been nylon, as described in Table 6, polyester, steel and fiberglass. Modulus data was obtained using several test techniques, the most common being cyclic extension of the tubes. In addition, some data was gotten by inflation of the tubes and other data by torsion with and without combinations of tension and inflation. Tube geometry is illustrated in Fig. 20.

The results of these experiments appear to confirm and extend the concept that transformation theory is applicable to the imaginary elastic modulus as well as the real elastic modulus, as might be anticipated from the correspondence principal. Figure 21 shows comparisons of the real modulus of nylon in both Region I and Region II with calculated values over the range of cord angles 0° to 90°. The calculations are based on the formulations of Ref. [12], which for completeness is given in Appendix 2. Similar data is given for the imaginary modulus in Fig. 22. From these comparisons it appears that satisfactory prediction of complex modulus for any angle can be obtained using such transformation equations.

The data of Table 6 shows that the shear modul $G_{12,I}$ and $G_{12,II}$ differ only slightly, probably because they are controlled primarily by the relatively soft matrix which is not affected in moving from Region I to Region II. However, off-angle shear modulus values are sensitive to this bi-modular effect, since there cord stiffness can play an important role, particularly near 45°. Figures 23 and 24 show comparison between measurement and calculation from transformation theory for G' and G". While agreement is not as good here as in the case of the extension modulus, nevertheless the trends are strong enough so as to confirm the adequacy of the transformation equations.

Part of the scatter of data in Figs. 23 and 24 is quite probably due to experimental difficulties in obtaining good measurements of G'_{II} and G''_{II}, since these tests must be performed under conditions of combined tension, internal pressure in the tube, and torsion.

There is considerable data on un-reinforced rubber compounds showing a reduction of modulus values with increasingly large strain. That effect does not appear to be present in the group of cord-reinforced materials studied here, or in Refs. [4] through [7]. One explanation for this is that low cord compliance limits matrix strain values, particularly in multi-ply laminates, so that only limited strains are experienced. In any event, modulus values are observed to be nearly independent of strain level as long as strains are well into Region I or Region II, and will be considered as constants in this paper.

Having available a rational basis for calculation of plane multi-ply laminates stiffness, the complete [A] matrix may be formed in the set of equations relating membrane forces to plane strain, as in Eq. (7).

$$\begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} = [A] \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \tag{7}$$

Non-symmetric laminates and the introduction of bending moments require a more complete set of relations than given in Eq. (7), in the form shown in Eq. (8), as given in Ref. [10].

$$\begin{pmatrix} N \\ M \end{pmatrix} = \begin{bmatrix} A & B \\ B & D \end{bmatrix} \begin{pmatrix} \varepsilon \\ \kappa \end{pmatrix} \tag{8}$$

Depending on the critical parameter ε_1^* at a given location in any lamina, the moduli and $\bar{Q}$ values from Eq. (4) may be assigned to either Region I or Region II. The appropriate summation of those through the thickness of the various lamina in the laminate allows the A, B, and D matrices to be calculated, each lamina contributing both a real and an imaginary portion. In cases where the imaginary component is small, elastic analysis can be carried out using only the real part. An excellent example of this technique is given by Ref. [13] where solutions are obtained for the deflection of bimodular elliptic plates using the concept advanced here of two elastic regions, equivalent to Regions I and II, with $\varepsilon_1^* = 0$.

In those cases where the imaginary component of Q is substantial, or where viscoelastic response is desired, it is necessary to utilize the complete complex form of the A, B and D matrices.

One important effect of a bimodular material model which was brought out in Ref. [8] and Ref. [13], but which should be emphasized here once more, is that with this type of material the matrices [A],[B] and [D] are no longer material invariants for a given laminate. Rather here they depend on the strain state of the cord reinforcement in each individual lamina making up the laminate. Therefore any analysis using Eq. (8) as a basis is often, although not always, a trial and error procedure. This is particularly true if bending is involved, since the [D] matrix will automatically contain components from lamina in Region I and Region II both, unless other factors such as large membrane strains override the bending strain distribution. As an example of this, Ref. [1] gives a listing of reinforcing cord moduli deduced from both tensile and bending tests. The bending moduli listed will contain elements of both the tensile and compressive cord modulus, but the large differences illustrate clearly the major changes occurring quantitatively in the elements of the [D] matrix. This data is reproduced in Table 7.

Cord Type	Construction	Tensile Cord Modulus $E'_{c,II}$	Bending Modulus E'_B,	E'_c/E'_m	Compressive Cord Modulus $E'_{c,I}$
Steel	5 x 0.25	106,000	53,400	2.0	35,700
Steel	4 x 0.21 +1	57,700	13,500	4.2	7,640
Aramid	1500/3	49,600	1,200	40	610
Glass	G75-5/0	59,500	3,300	18	1,700
PET	1300/3	7,700	240	32	120

TABLE 7

Tensile and Bending Moduli of Tire Cords Ref [1]
Compressive Cord Modulus by Calcualtion Units, MPa

An estimate of the compressive cord moduli for the materials given in Table 7 can be obtained by neglecting the shear deformation of the rubber matrix between cord layers and computing an effective cord compressive modulus. The results are also given in Table 7. It should be emphasized that these are only estimates.

One may view the [A],[B] and [D] matrices involving bimodular materials as variables whose values range from one extreme when all lamina are in Region I to another extreme when all lamina are in Region II. As an example of this, one may use properties of a typical Aramid reinforced rubber composite such as given in Appendix I from Ref. [8] to construct three different levels of the [D] matrix. These are given in Table 8 in order to illustrate the extreme variation in numerical values of matrix elements as cord strain moves from Region I to Region II.

Region I: $$[D] = \begin{bmatrix} 12.5 & 2.5 & 0 \\ 2.5 & 12.5 & 0 \\ 0 & 0 & 3.7 \end{bmatrix} \frac{h^3}{12}$$

Intermediate Condition

Pure Bending

$\varepsilon_1{}^* = 0$ $$[D] = \begin{bmatrix} 537 & 3 & 0 \\ 3 & 12.4 & 0 \\ 0 & 0 & 3.7 \end{bmatrix} \frac{h^3}{12}$$

Region II: $$[D] = \begin{bmatrix} 3583 & 6.1 & 0 \\ 6.1 & 12. & 0 \\ 0 & 0 & 3.7 \end{bmatrix} \frac{h^3}{12}$$

Table 8

Variation of [D] Matrix from Region I to Region II

Material: Aramid-Rubber Composite., 0°, thickness h.

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APPENDIX 1

Material Properties in Units of MPa

Reinforcing Material	Measured Constituent Properties	Measured Composite Properties*		Ref.
Rayon	$E_c' = 3410$	$E_1' = 779$		
	$E_m' = 8.8$	$E_2' = 13.9$		[14]
	$v = 0.23$	$G_{12}' = 3.57$		
	$E_c'v = 784$	$\nu_{12} = 0.49$		
Nylon		$E_{1,I}' = 82.6$	$E_{1,II}' = 420.5$	
		$E_{2,I}' = 8.41$	$E_{2,II}' = 8.41$	Table 6
		$G_{12,I}' = 3.24$	$G_{12,II}' = 4.44$	
		$\nu_{12,I} =$	$\nu_{12,II} = 0.283$	
		$\varepsilon_1^* = 0.015$		
Polyester	$E_c' = 4780$	$E_1' = 841$		
	$E_m' = 8.8$	$E_2' = 13.4$		[14]
	$v = 0.17$	$G_{12} = 3.45$		
	$E_c'v = 813$	$\nu_{12} = 0.49$		
	$E_c' = 3970$	$E_1' = 6.7$		
	$E_m' = 5.5$	$E_2' = 10.6$		[11]
	$v = 0.149$	$G_{12}' = 3.13$		
	$E_c'v = 592$	$\nu_{12} = 0.475$		
		$E_{1,I}' = 36.9$	$E_{1,II}' = 617$	
		$E_{2,I}' = 10.6$	$E_{2,II}' = 8.0$	[8]
		$G_{12,I}' = 2.67$	$G_{12,II}' = 2.62$	
		$\nu_{12,I} = 0.185$	$\nu_{12,I} = .475$	
		$\varepsilon_1^* = 0$		

*Region II unless otherwise specified.

Reinforcing Material	Measured Constituent Properties	Measured Composite Properties	Ref.
Aramid	$E_c' = 24800$ $E_m' = 8.0$ $v = 0.14$ $E_c'v = 3472$	$E_1' = 3580$ $E_2' = 12.0$ $G_{12}' = 3.83$ $\nu_{12} = 0.416$	[11]
	$E_c' = 68600$ $E_m' = 7.56$ $v = 0.26$ $E_c'v = 17,840$	$E_1' = 17800$ $E_2' = 13.3$ $G_{12}' = 2.76$ $\nu_{12} = 0.534$	[1]
	$E_c' = 26,100$ $E_m' = 7.93$ $v = 0.2$ $E_cv = 5220$	$E_1' = 5230$ $E_2' = 13.5$ $G_{12}' = 3.45$ $\nu_{12} = 0.49$	[14]
		$E_{1,I}' = 12.0$ $E_{1,II}' = 3580$ $E_{2,I}' = 12.0$ $E_{2,II}' = 9.09$ $G_{12,I}' = 3.7$ $G_{12,II}' = 3.7$ $\nu_{12,I} = 0.205$ $\nu_{12,II} = 0.416$	[8]

Reinforcing Material	Measured Constituent Properties	Measured Composite Properties	Ref.
Steel	$E_c' = 72400$	$E_1' = 14080$	
	$E_m' = 10.0$	$E_2' = 10.$	[11]
	$v = 0.135$	$G_{12}' = 3.13$	
	$E_c'v = 9770$	$\nu_{12} = 0$	
	$E_c' = 103,200$	$E_1' = 26,900$	
	$E_m' = 7.56$	$E_2' = 10.8$	[1]
	$v = 0.267$	$G_{12}' = 2.76$	
	$E_c'v = 28,890$	$\nu_{12} = 0.568$	
	$E_c' = 50,500$	$I_1' = 5720$	
	$E_m' = 15.9$	$E_2' = 19.3$	[14]
	$v = 0.11$	$G_{12}' = 4.92$	
	$E_c'v = 5555$	$\nu_{12} = 0.49$	

APPENDIX 2

For two-dimensional problems, the strain-stress relations, with respect to an arbitrary coordinate system, $\bar{1}$-$\bar{2}$, have the following form in the transformed domain

$$\hat{\bar{\varepsilon}}_{11} = \hat{\bar{a}}_{11}(p)\hat{\bar{\sigma}}_{11}(p) + \hat{\bar{a}}_{12}(p)\hat{\bar{\sigma}}_{22}(p)$$

$$+ \hat{\bar{a}}_{16}(p)\hat{\bar{\sigma}}_{12}(p)$$

$$\hat{\bar{\varepsilon}}_{22} = \hat{\bar{a}}_{21}(p)\hat{\bar{\sigma}}_{11}(p) + \hat{\bar{a}}_{22}(p)\hat{\bar{\sigma}}_{22}(p)$$

$$+ \hat{\bar{a}}_{26}(p)\hat{\bar{\sigma}}_{12}(p)$$

$$\hat{\bar{\varepsilon}}_{12} = \hat{\bar{a}}_{61}(p)\hat{\bar{\sigma}}_{11}(p) + \hat{\bar{a}}_{62}(p)\hat{\bar{\sigma}}_{22}(p)$$

$$+ \hat{\bar{a}}_{66}(p)\hat{\bar{\sigma}}_{12}(p)$$

where the notations with an overhead bar denote quantities in $\bar{1}$-$\bar{2}$ coordinate system. The transformed properties, $\hat{\bar{a}}_{ij}(p)$ are related to engineering complex properties in the $\bar{1}$-$\bar{2}$ coordinate system. The engineering constants in any arbitrary direction, then, can be calculated from $\hat{\bar{a}}ij(p)$, which are given below.

The real and imaginary parts of the complex flexibility moduli in an arbitrary direction can be found from the following relations in terms of the engineering moduli.

$$\alpha = \theta$$

$$\bar{a}'_{11} = +\left(\frac{E'_{11}}{E'^{2}_{11}+E''_{11}}\right)\cos^4\alpha + \left(+\frac{G'_{12}}{G'^{2}_{12}+G''^{2}_{12}} - \frac{2\nu_{12}E'_{11}}{E'^{2}_{11}+E''^{2}_{11}}\right)\cdot$$

$$\sin^2\alpha\cos^2\alpha + \left(\frac{E_{22}}{E'^{2}_{22}+E'^{2}_{22}}\right)\sin^4\alpha$$

$$\bar{a}''_{11} = -\left(\frac{E''_{11}}{E'^{2}_{11}+E''^{2}_{11}}\right)\cos^4\alpha + \left(-\frac{G''_{12}}{G'^{2}_{12}+G''^{2}_{12}} + \frac{2\nu_{12}E''_{11}}{E'^{2}_{11}+E''^{2}_{11}}\right)\cdot$$

$$\sin^2\alpha\cos^2\alpha - \left(\frac{E''_{22}}{E'^{2}_{22}+E''^{2}_{22}}\right)\sin^4\alpha$$

$$\bar{a}'_{22} = \left(\frac{E'_{11}}{E'^2_{11}+E''^2_{11}}\right)\sin^4\alpha + \left(\frac{G'_{12}}{G'^2_{12}+G''^2_{12}} - \frac{2\nu_{12}E'_{11}}{E'^2_{11}+E''^2_{11}}\right)\cdot$$

$$\sin^2\alpha\cos^2\alpha - \left(\frac{E'_{22}}{E'^2_{22}+E''^2_{22}}\right)\cos^4\alpha$$

$$\bar{a}''_{22} = -\left(\frac{E'_{11}}{E'^2_{11}+E''^2_{11}}\right)\sin^4\alpha + \left(-\frac{G''_{12}}{G'^2_{12}+G''^2_{12}} + \frac{2\nu_{12}E''_{11}}{E'^2_{11}+E''^2_{11}}\right)\cdot$$

$$\cos^2\alpha\sin^2\alpha - \left(\frac{E''_{22}}{E'^2_{22}+E''^2_{22}}\right)\cos^4\alpha$$

$$\bar{a}'_{12} = \left[\frac{(1+2\nu_{12})E'_{11}}{E'^2_{11}+E''^2_{11}} + \frac{E'_{22}}{E'^2_{22}+E''^2_{22}} - \frac{G'_{12}}{G'^2_{12}+G''^2_{12}}\right]\cdot$$

$$\sin^2\alpha\cos^2\alpha - \frac{\nu_{12}E'_{11}}{E'^2_{11}+E''^2_{11}}$$

$$\bar{a}''_{12} = -\left[\frac{(1+2\nu_{12})E''_{11}}{E'^2_{11}+E''^2_{11}} + \frac{E''_{22}}{E'^2_{22}+E''^2_{22}} - \frac{G''_{12}}{G'^2_{12}+G''^2_{12}}\right]\cdot$$

$$\sin^2\alpha\cos^2\alpha + \frac{\nu_{12}E''_{11}}{E'^2_{11}+E''^2_{11}}$$

$$\bar{a}'_{66} = 4\left[\frac{(1+2\nu_{12})E'_{11}}{E'^{2}_{11}+E''^{2}_{11}} + \frac{E'_{22}}{E'^{2}_{22}+E''^{2}_{22}} - \frac{G'_{12}}{G'^{2}_{12}+G''^{2}_{12}}\right] \cdot$$

$$\sin^2\alpha\cos^2\alpha \quad + \frac{G'_{12}}{G'^{2}_{12}+G''^{2}_{12}}$$

$$\bar{a}''_{66} = -4\left[\frac{(1+2\nu_{12})E''_{11}}{E'^{2}_{11}+E''^{2}_{11}} + \frac{E''_{22}}{E'^{2}_{22}+E''^{2}_{22}} - \frac{G''_{12}}{G'^{2}_{12}+G''^{2}_{12}}\right] \cdot$$

$$\sin^2\alpha\cos^2\alpha - \frac{G'_{12}}{G'^{2}_{12}+G''^{2}_{12}}$$

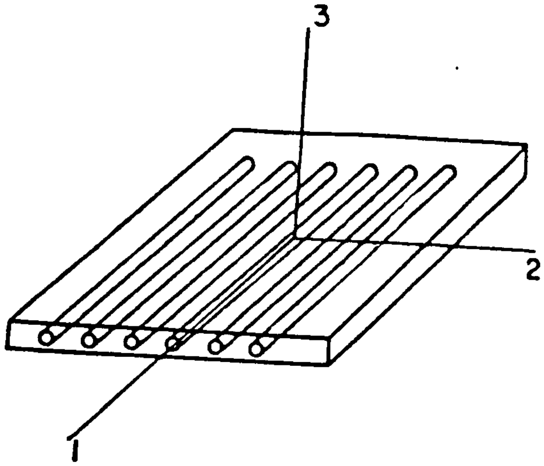
$$a'_{16} = +\left[2\left(\frac{E'_{22}\sin^2\alpha}{E'_{22}+E''^{2}_{22}} - \frac{E'_{11}\cos^2\alpha}{E'^{2}_{22}+E''^{2}_{11}}\right) + \right.$$

$$\left.\left(\frac{G'_{12}}{G''^{2}_{12}+G'^{2}_{12}} - \frac{2\nu_{12}E'_{11}}{E'^{2}_{11}+E''^{2}_{11}}\right) \cdot (\cos^2\alpha-\sin^2\alpha)\right]\sin\alpha\cos\alpha$$

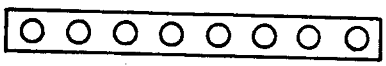
$$\bar{a}''_{16} = -\left[2\left(\frac{E''_{22}\sin^2\alpha}{E'^2_{22}+E''^2_{22}} - \frac{E''_{11}\cos^2\alpha}{E'^2_{11}+E''^2_{11}}\right) + \left(\frac{G''_{12}}{G'^2_{12}+G''^2_{12}} - \frac{2\nu_{12}E''_{11}}{E'^2_{11}+E''^2_{11}}\right) \cdot (\cos^2\alpha-\sin^2\alpha)\right] \sin\alpha\cos\alpha$$

$$\bar{a}'_{26} = \left[2\left(\frac{E'_{22}\cos^2\alpha}{E'^2_{22}+E''^2_{22}} - \frac{E'_{11}\sin^2\alpha}{E'^2_{11}+E''^2_{11}}\right) - \left(\frac{G'_{12}}{G'^2_{12}+G''^2_{12}} - \frac{2\nu_{12}E'_{11}}{E'^2_{11}+E''^2_{11}}\right) (\cos^2\alpha-\sin^2\alpha)\right] \cos\alpha\sin\alpha$$

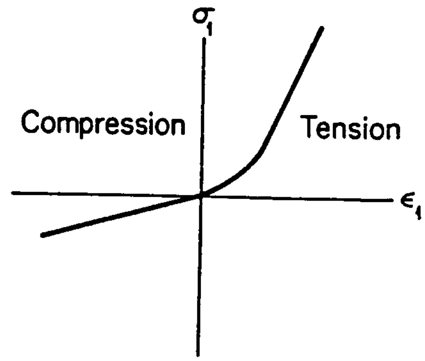
$$\bar{a}''_{26} = -\left[2\left(\frac{{}''_{22}\cos^2\alpha}{E'^2_{22}+E''^2_{22}} - \frac{E''_{11}\sin^2\alpha}{E'^2_{11}+E''^2_{11}}\right) - \left(\frac{G''_{12}}{G'^2_{12}+G''^2_{12}} - \frac{2\ {}_{12}E''_{11}}{E'^2_{11}+E''^2_{11}}\right) (\cos^2\alpha-\sin^2\alpha)\right] \cos\alpha\sin\alpha$$



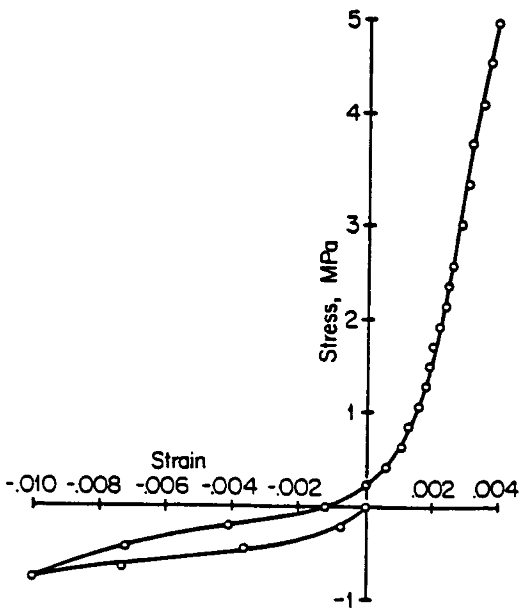
1. Principal material directions for unidirectionally reinforced lamina.



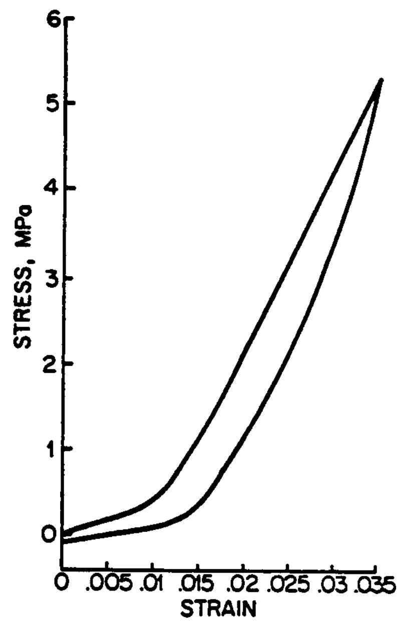
2. End view of cord-rubber composite.



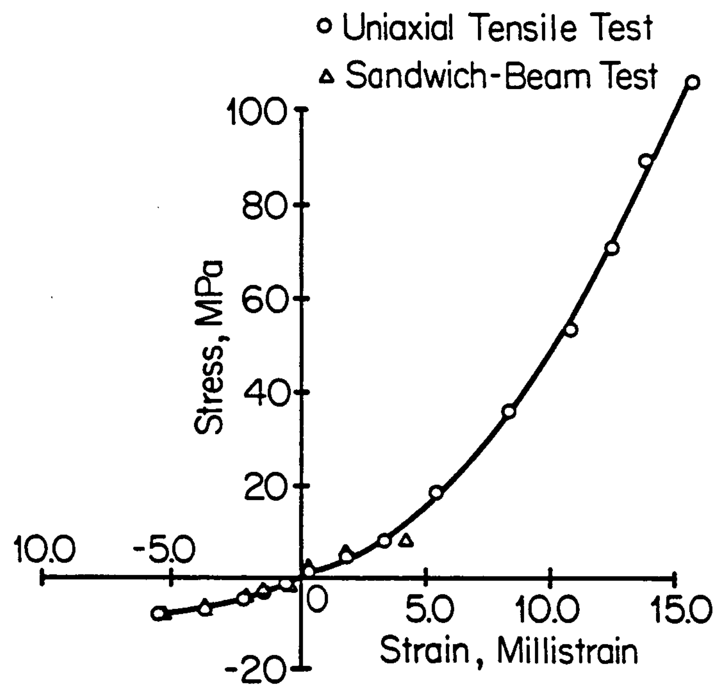
3. Typical bimodular response for unidirectionally reinforced lamina with twisted reinforcement.



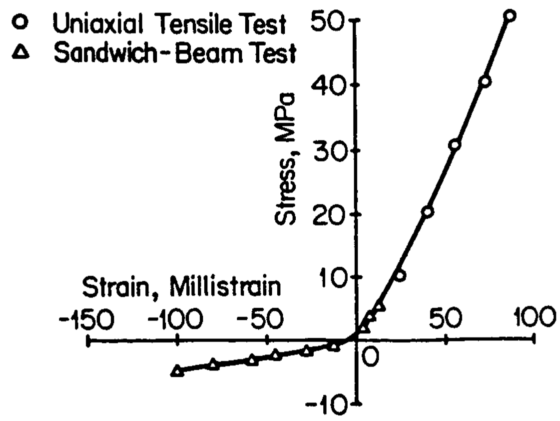
4. Stress-strain curve for rayon-reinforced rubber, 1 direction, Ref. [4].



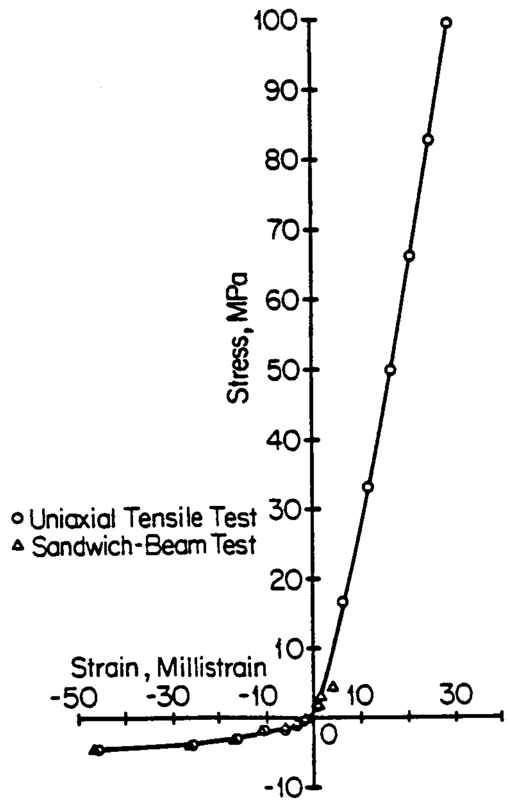
5. Stress-strain curve for nylon reinforced rubber, 1 direction, Ref. [5].



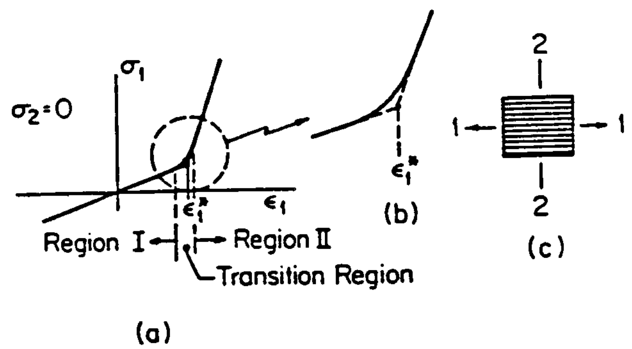
6. Stress-strain curve for steel reinforced rubber, 1 direction, Ref. [6].



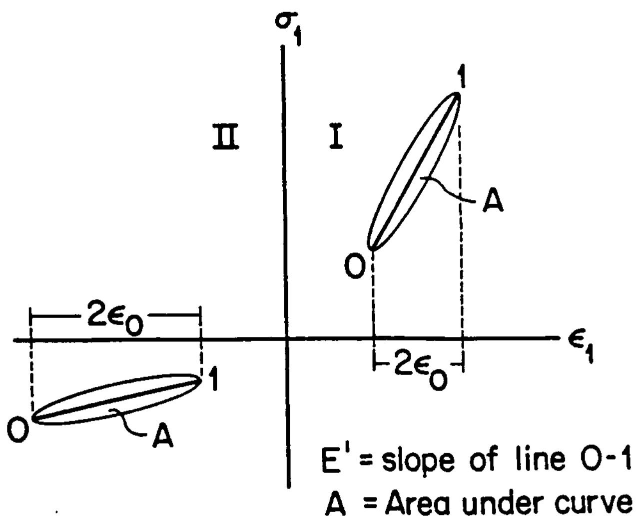
7. Stress-strain curve for polyester reinforced rubber, 1 direction, Ref. [7].



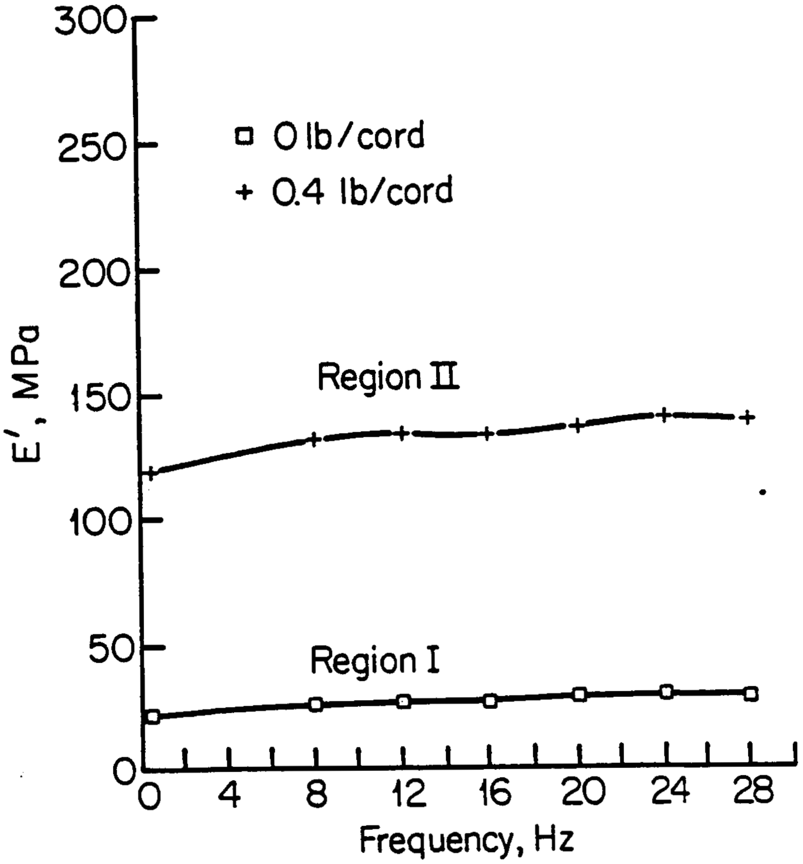
8. Stress-strain curve for Aramid reinforced rubber, 1 direction, Ref. [6].



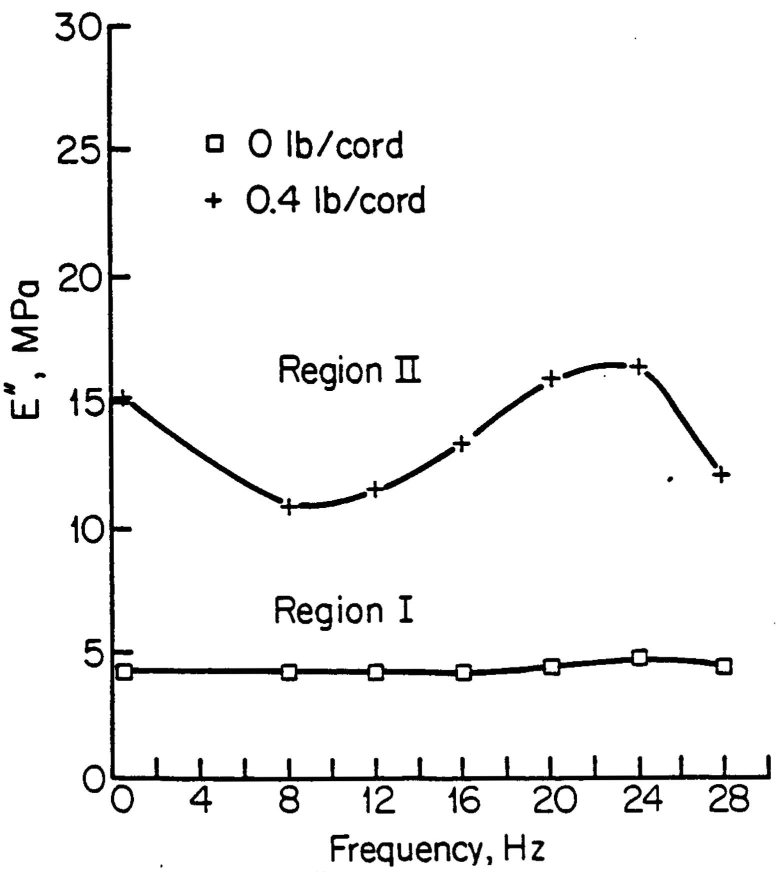
9. Generalized stress-strain curve for bimodular material, with notation.



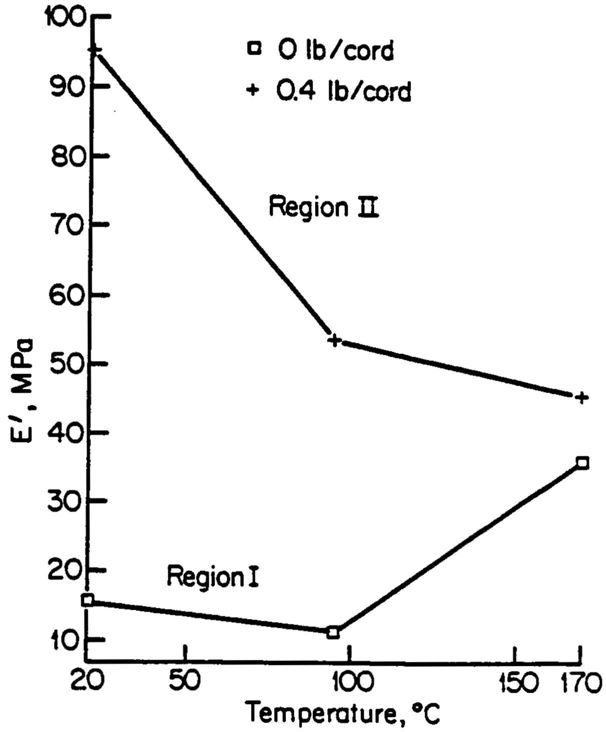
10. Typical stress-strain curves, Regions I and II, with notation.



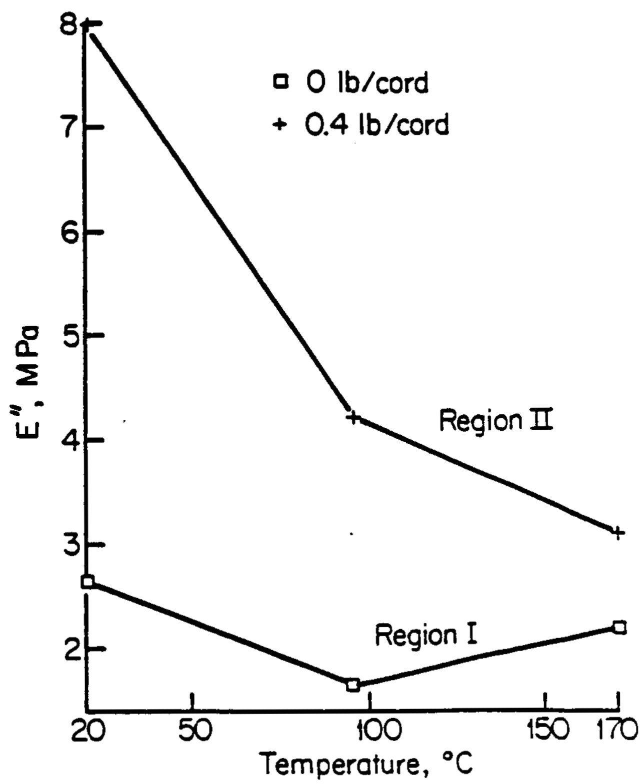
12. E'_{II} and E''_{II} values vs frequency for nylon reinforced rubber



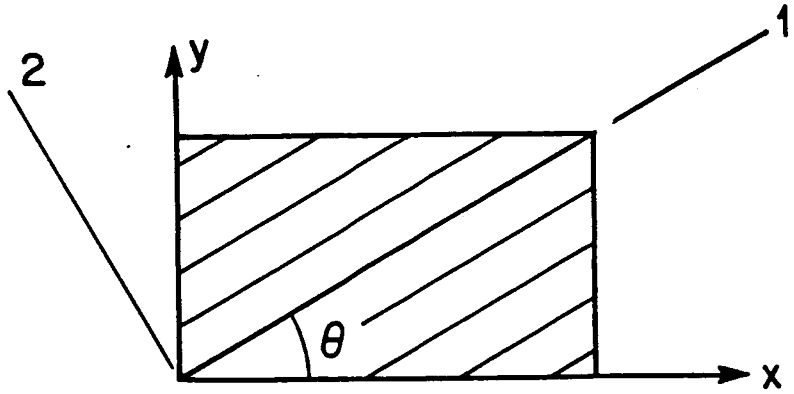
13. $E'_{2,II}$ and $E''_{2,II}$ values vs frequency for nylon reinforced rubber.



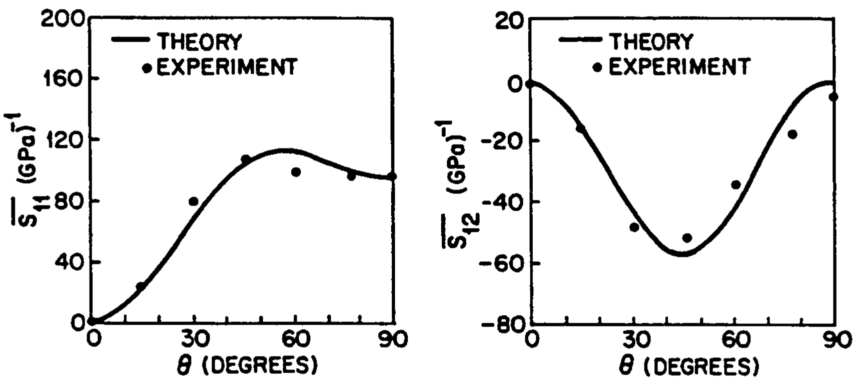
14. $E'_{1,II}$ and $E''_{1,II}$ values vs temperature for nylon reinforced rubber.



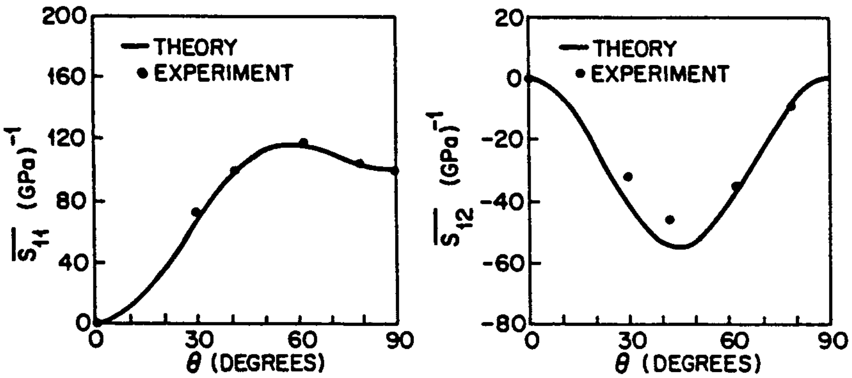
15. $E'_{2,II}$ and $E''_{2,II}$ values vs temperature for nylon reinforced rubber.



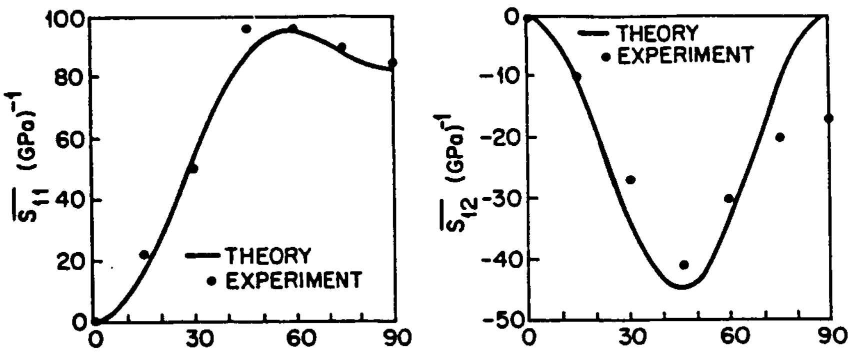
16. Notation for angular orientation of unidirectionally reinforced lamina.



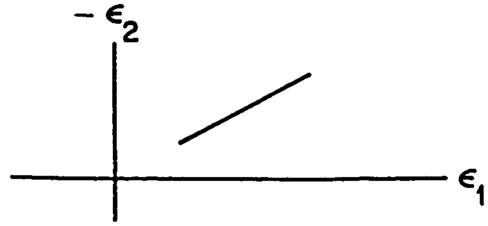
17. Variation of compliances $\bar{S}_{11}$ and $\bar{S}_{12}$ with cord angle for a polyester cord-rubber ply. Ref. [11]



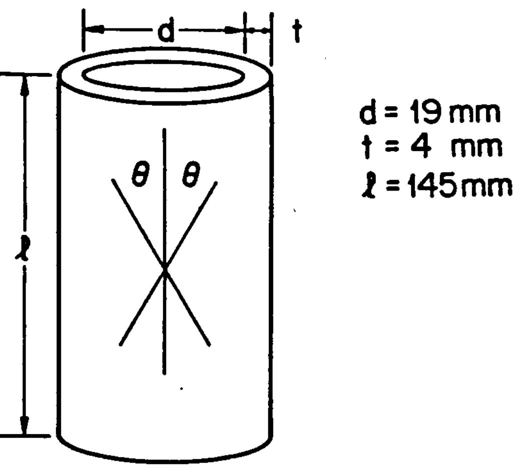
18. Variation of compliances $\bar{S}_{11}$ and $\bar{S}_{12}$ with cord angle for a steel cord- rubber ply. Ref. [11].



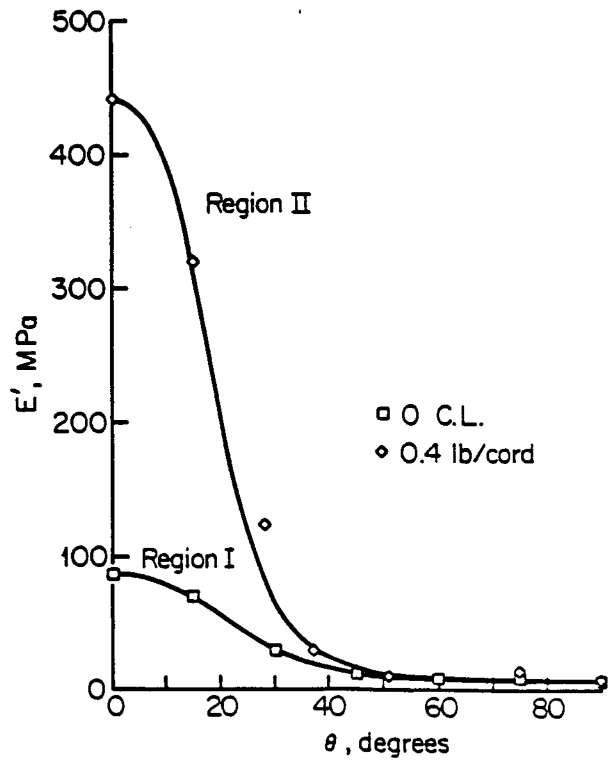
19. Variation of compliances $\bar{S}_{11}$ and $\bar{S}_{12}$ with cord angle for an Aramid cord-rubber ply. Ref. [11]



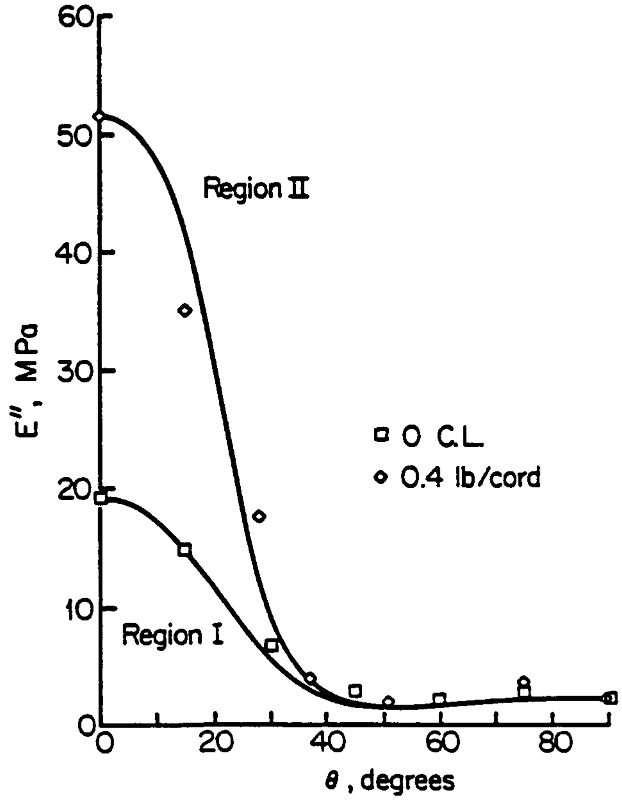
11. ε_1 vs ε_2 from experimental data on nylon reinforced rubber



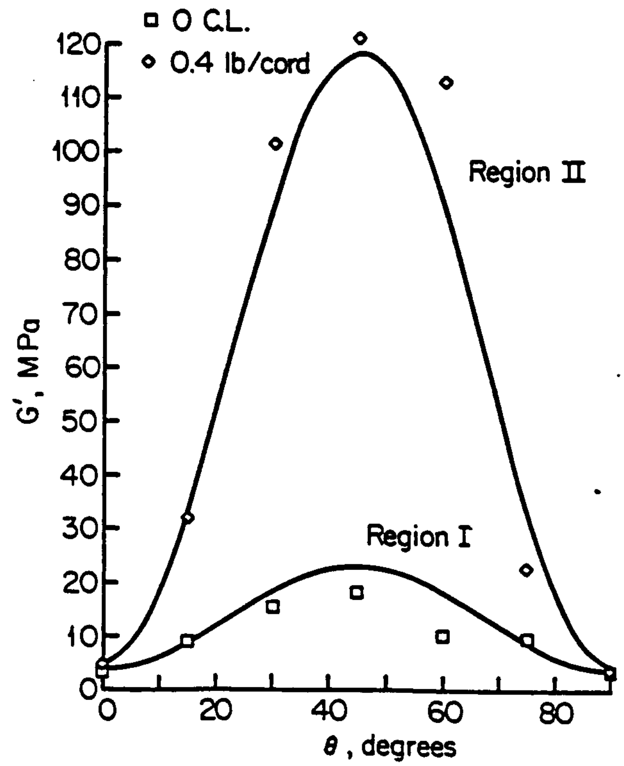
20. Tubular cord-rubber specimen.



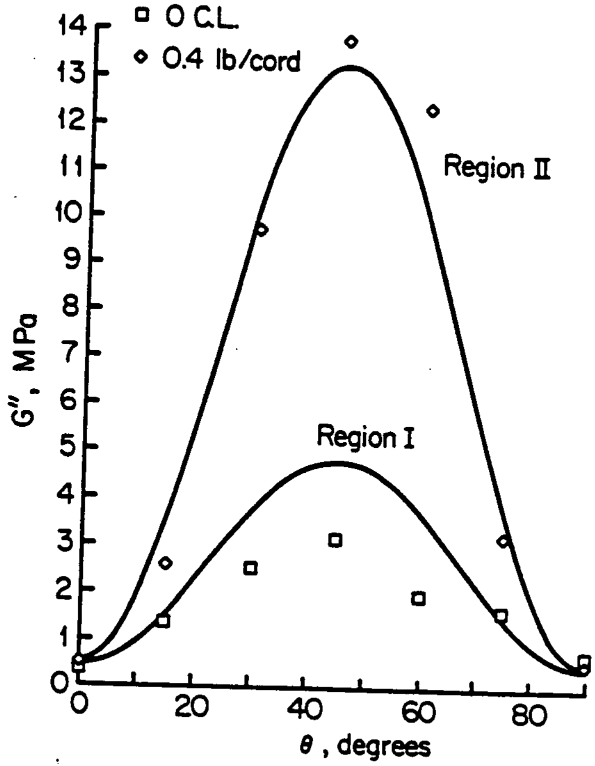
21. Variation of real Young's moduli E'_I and E'_{II} with cord angle for a nylon-rubber bias ply tube.



22. Variation of imaginary Young's moduli E''_{I} and E''_{II} with cord angle for a nylon-rubber bias ply tube.



23. Variation of real shear moduli G'_{I} and G'_{II} cord angle for a nylon-rubber bias ply tube.



24. Variation of imaginary shear modulus G''_{I} and G''_{II} with cord angle for a nylon-rubber bias ply tube.