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Exact State Reconstruction in Deterministic Digital Control Systems

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I. INTRODUCTION

Books on modern digital control systems usually address the problem of controlling a continuous-time plant driven by a zero-order-hold with a sampled output as shown in Figure 1 [e.g., Ref. 1, p. 126]. A common solution to this problem is to reconstruct the state of the system at the sampling instant using a state observer and then to feed back the reconstructed state [1, p. 195]. However, the state observer has two undesirable characteristics. First, it is a dynamical system in itself and, hence, adds additional states and eigenvalues to the system which can affect system stability. Second, as a consequence, the reconstructed state is normally an approximation to the true state and is usually not a good one early in the state reconstruction process. This paper presents a new state reconstructor for deterministic digital control systems which has neither of these problems. The new state reconstructor adds no new states, eigenvalues, or dynamics to the system. In fact, it does not affect the plant equation for the system in any way; it affects only the measurement equation. Furthermore, if the plant parameters are known exactly, the output of this new state reconstructor exactly equals the true state of the system. For these reasons, this new state reconstructor is herein called the ideal state reconstructor. Useful in the development of this ideal state reconstructor are some results to date for continuous-time plants driven by a zero-order-hold with sampled outputs. These are reviewed in Section II, prior to the development of the ideal state reconstructor presented in Section III. Section IV describes two methods for choosing the reconstructor parameters. Section V presents an example illustrating the procedures to completely design the reconstructor using the methods described in Section IV. Section VI contains the conclusions and final comments.

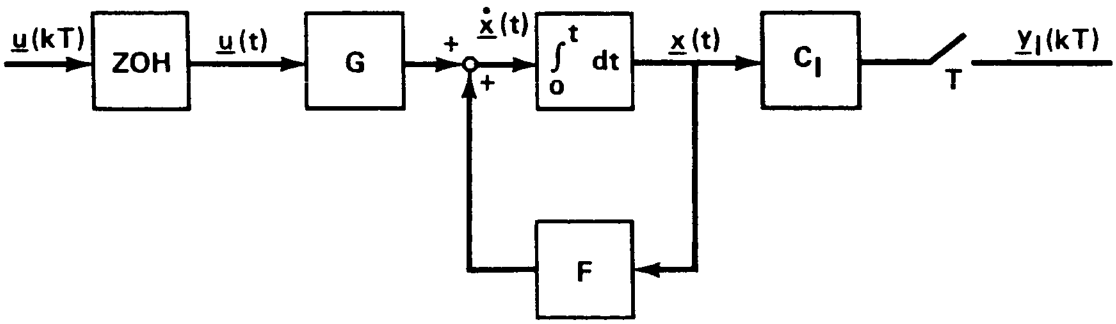

Figure 1. Continuous-time plant driven by a zero-order-hold with instantaneous measurements.

II. PRELIMINARY

For the plant in Figure 1, $\underline{x}(t)$ is an $n \times 1$ state vector, $\underline{u}(k)$ is an $r \times 1$ control input vector, $\underline{y}_I(k)$ is an $m \times 1$ output or measurement vector, F is an $n \times n$ system matrix, G is an $n \times r$ control matrix, and C_I is an $m \times n$ output matrix. Since $\underline{y}_I(k) = C_I\,\underline{x}(k)$, where k is the usual shorthand notation for time kT, $\underline{y}_I(k)$ represents an instantaneous measure of the system at the sampling instant kT. Hence, the plant in Figure 1 can be regarded as having instantaneous measurements for outputs. It is well known that this system can be modeled at the sampling instants by the discrete state equations [1, p. 126]

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{1}$$

$$\underline{y}_I(k) = C_I\,\underline{x}(k) \quad , \tag{2}$$

where

$$\phi(t) = \mathcal{L}^{-1}\,[(sI-F)^{-1}] \quad , \tag{3}$$

$$A = \phi(T) \quad , \tag{4}$$

and

$$B = \left[\int_0^T \phi(\lambda)\,d\lambda\right] G \quad . \tag{5}$$

$\phi(t)$ is the $n \times n$ state transition matrix. A and B are the $n \times n$ system matrix and the $n \times r$ control matrix, respectively, for the discrete state equations (1) and (2).

A and B can be determined analytically using equations (3) through (5). An alternative approach, which is also quite suitable for numerical computation, is as follows [2]: $\phi(t)$ and $\int_0^t \phi(\lambda)d\lambda$ can be expressed in the form of matrix exponential series as

$$\phi(t) = \sum_{i=0}^{\infty} \frac{F^i\,t^i}{i!} \tag{6}$$

and

$$\int_0^t \phi(\lambda)d\lambda = \sum_{i=0}^{\infty} \frac{F^i t^{i+1}}{(i+1)!} \quad , \tag{7}$$

respectively. From equations (6) and (7),

$$\phi(t) = I + F\left[\int_0^t \phi(\lambda)d\lambda\right] \quad , \tag{8}$$

where I is an $n \times n$ identity matrix. Hence, $\int_0^T \phi(\lambda)d\lambda$ can be determined using equation (7) with $t = T$ and this result substituted into equation (8) to get $\phi(T)$. With these results, A and B can be found using equations (4) and (5).

Now consider the plant in Figure 2, which is a generalization of the one in Figure 1. In addition to the instantaneous measurement vector $\underline{y}_I(kT)$, the plant in Figure 2 has the measurement vector $\underline{y}_F'(kT)$ generated as follows. First, the continuous-time output $\underline{z}(t)$ is sampled every T/N seconds. Every N samples are multiplied by the weighting matrices H_j, $j = 0,1,\ldots,N\text{-}1$, and then summed to generate the output $\underline{y}_F(kT)$ every T seconds. Functionally, this is equivalent to passing the discrete measurements generated every T/N seconds through a multi-input/multi-output moving average (MA) process with coefficient matrices H_j, $j = 0,1,\ldots,N\text{-}1$ [3]. The output of the MA prefilter is sampled every T seconds to generate $\underline{y}_F(kT)$. Then $\underline{y}_F(kT)$ has subtracted from it $E_{-}\,\underline{u}[(k\text{-}1)T]$, where E_{-} is a constant matrix, to produce the modified MA-prefiltered measurement vector $\underline{y}_F'(kT)$. In Figure 2, C_F is a $p \times n$ output matrix and $\underline{z}(t)$ is a $p \times 1$ vector. The weighting matrices H_j, $j = 0,1,\ldots,N\text{-}1$ are each $q \times p$. Hence, $\underline{y}_F(kT)$ and $\underline{y}_F'(kT)$ are $q \times 1$ vectors. Since $\underline{u}[(k\text{-}1)T]$ is an $r \times 1$ delayed input vector, E_{-} is a $q \times r$ matrix.

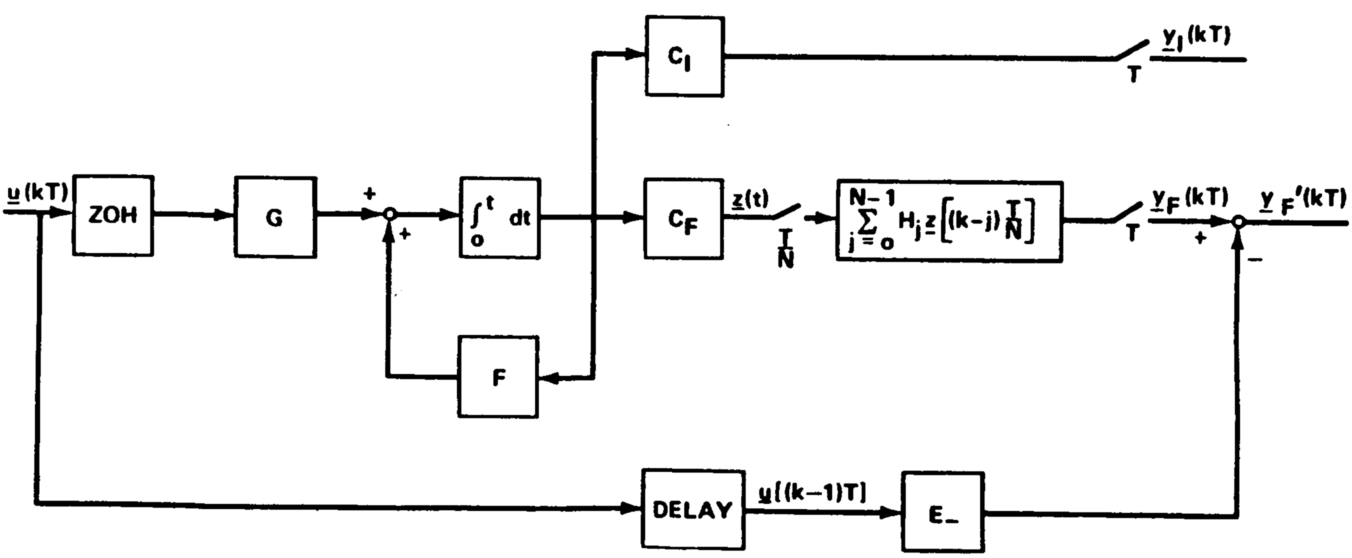

Figure 2. Continuous-time plant driven by a zero-order-hold with instantaneous and modified MA-prefiltered measurements.

From Figure 2, $\underline{y}_F(kT)$ can be written as

$$\underline{y}_F(kT) = [H] \begin{bmatrix} \underline{z}(kT) \\ \underline{z}(kT - \frac{T}{N}) \\ \vdots \\ \underline{z}[kT-(N-1)\frac{T}{N}] \end{bmatrix} , \tag{9}$$

where H is a q $\times$ (Np) matrix given by

$$H = [H_0 \mid H_1 \mid \dots \mid H_{N-1}] \quad . \tag{10}$$

Previously, Polites [4] showed that when

$$E\_ = H\beta \quad , \tag{11}$$

where β is the (Np) $\times$ r matrix

$$\beta = \begin{bmatrix} C_F \left[\int_0^0 \phi(\lambda)d\lambda\right] G \\ C_F \left[\int_0^{-(T/N)} \phi(\lambda)d\lambda\right] G \\ \vdots \\ C_F \left[\int_0^{-(N-1)(T/N)} \phi(\lambda)d\lambda\right] G \end{bmatrix} , \tag{12}$$

the discrete state equations for the plant in Figure 2 become

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{13}$$

$$\begin{bmatrix} \underline{y}_I(k) \\ \underline{y}_F'(k) \end{bmatrix} = \begin{bmatrix} C_I \\ D\_ \end{bmatrix} \underline{x}(k) \quad , \tag{14}$$

where D_- is a q $\times$ n matrix given by

$$D_- = H\alpha \tag{15}$$

and α is the (Np) $\times$ n matrix

$$\alpha = \begin{bmatrix} C_F\ \phi(0) \\ C_F\ \phi(-\frac{T}{N}) \\ \vdots \\ C_F\ \phi\left[-(N-1)\ \frac{T}{N}\right] \end{bmatrix} \quad . \tag{16}$$

E_- and D_- can be evaluated analytically using equations (3), (10) through (12), (15), and (16). An alternative approach, which can be either analytical or numerical, is as follows. Let $t = -j(T/N)$, where $j = 0,1,\ldots,N-1$, and use equation (7) to determine $\int_0^{-j(T/N)} \phi(\lambda)d\lambda$, $j = 0,1,\ldots,N-1$. Substitute these results into equation (8) to get $\phi[-j(T/N)]$, $j = 0,1,\ldots,N-1$. At this point, E_- and D_- can be found using equations (10) through (12), (15), and (16).

III. THE IDEAL STATE RECONSTRUCTOR

A general block diagram of the plant and the ideal state reconstructor is shown in Figure 3. The state reconstructor in Figure 3 is a generalization of the one presented in Reference 5. Observe the similarities, and differences, between Figures 2 and 3. In Figure 3, if E_- is given by equation (11), then

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{17}$$

and

$$\underline{y}_F'(k) = D_-\ \underline{x}(k) \quad , \tag{18}$$

where D_- is given by equation (15). This follows from Figures 2 and 3 and equations (13) and (14). Also, in Figure 3,

$$\underline{y}_F''(k) = (D_-^T\ D_-)^{-1}\ D_-^T\ \underline{y}_F'(k) \tag{19}$$

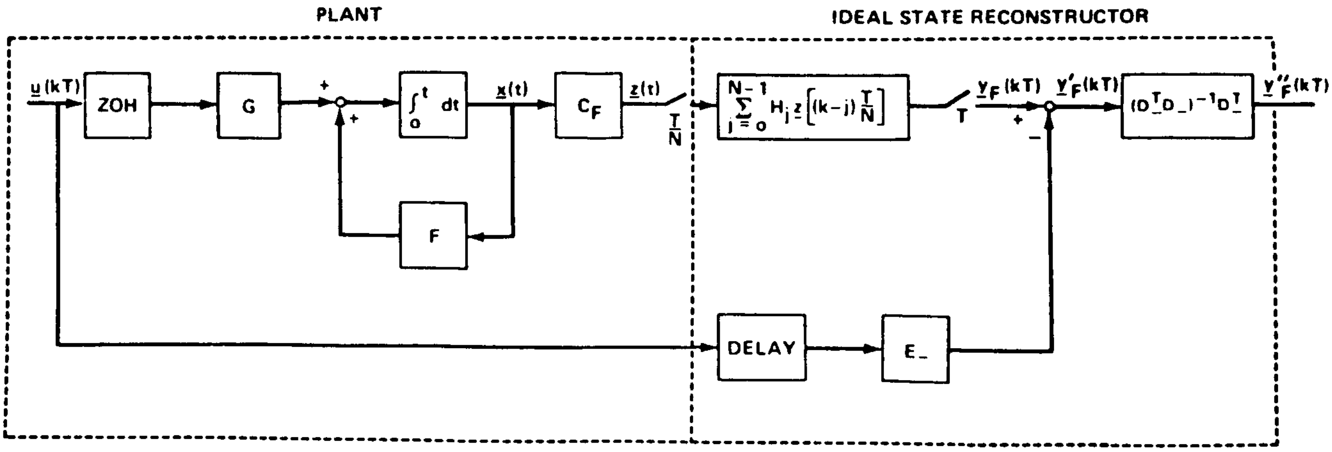

Figure 3. General block diagram of the plant and the ideal state reconstructor.

However, for equation (19) to be meaningful, $(D_-^T D_-)^{-1}$ must exist, and this occurs only when $(D_-^T D_-)$ is nonsingular. Recall that D_- is a q $\times$ n matrix. If $q \geqslant n$ and D_- has maximal rank (i.e., rank n), then $(D_-^T D_-)$ is positive definite and therefore nonsingular [6]. Hence, equation (19) requires that $q \geqslant n$ and rank $(D_-) = n$ for it and the ideal state reconstructor to be meaningful. Assuming this is the case, it follows from equations (17) through (19) that the discrete state equations for the system in Figure 3 are

$$\underline{x}(k+1) = A\underline{x}(k) + B\underline{u}(k) \tag{20}$$

$$\underline{y}_F''(k) = \underline{x}(k) \quad .$$

Hence, the output of the state reconstructor, $\underline{y}_F''(k)$, equals the true state, $\underline{x}(k)$, exactly. Moreover, this is achieved without adding any new states, eigenvalues, or dynamics to the plant, since the plant equation (20) for the system in Figure 3 is identical to the plant equation (1) for the plant in Figure 1.

In summary, the requirements which must be satisfied for the ideal state reconstructor in Figure 3 to exactly reconstruct the state of the plant in Figure 3 are as follows. The plant matrices F, G, and C_F must be known exactly in order to determine E_- and D_- according to equations (11) and (15), respectively. The number of rows, q, in the weighting matrices H_j, j = 0,1,...,N-1 must be greater than or equal to the number of states in the plant, n. The q $\times$ n matrix D_- in equation (15) must have rank n. So long as these requirements are met, there are no other restrictions on the ideal state reconstructor, including its weighting matrices. This is very desirable because it means there are a multitude of ways to choose the weighting matrices and still achieve exact state reconstruction. Two distinct methods for doing this are presented in Section IV.

IV. CHOOSING THE PARAMETERS IN THE IDEAL STATE RECONSTRUCTOR

Once the measurements which are inputs into the ideal state reconstructor have been specified, the $p \times n$ output matrix C_F is defined. To complete the definition of the reconstructor, the following parameters must be chosen: the number of weighting matrices in the state reconstructor N, the number of rows in each weighting matrix q, and the elements of each $q \times p$ weighting matrix H_j, $j = 0,1,\ldots,N-1$. While there are countless ways of choosing these, two distinct methods are presented.

Method A. Let $H = (\alpha^T\alpha)^{-1} \alpha^T$

Consider H, the $q \times (Np)$ matrix formed by catenating the weighting matrices H_j, $j = 0,1,\ldots,N-1$ as in equation (10). Consider also α, the $(Np) \times n$ matrix defined by equation (16). If $Np \geq n$ and $\text{rank}(\alpha) = n$, then $(\alpha^T\alpha)$ is positive definite and therefore nonsingular [6]. If this is the case, then H can be given by the pseudo inverse of α or

$$H = (\alpha^T\alpha)^{-1} \alpha^T \quad . \tag{21}$$

Hence, N must be chosen so that $Np \geq n$, or equivalently $N \geq n/p$. Having chosen N, α can be determined using equation (16). If it has rank n, then let H be given by equation (21). This makes H an $n \times (Np)$ matrix and means that $q = n$ in this case. Substituting equation (21) into equation (15) yields $D_- = I$, where I is an $n \times n$ identity matrix. Consequently,

$$(D_-^T D_-)^{-1} D_-^T = I \tag{22}$$

also. This simplifies the state reconstructor by eliminating the need to solve equation (19) in it. From equations (11) and (21), E_- is found to be

$$E_- = (\alpha^T\alpha)^{-1} \alpha^T \beta \quad , \tag{23}$$

where β is defined by equation (12). From equations (10) and (21),

$$[H_0 \mid H_1 \mid \ldots \mid H_{N-1}] = (\alpha^T\alpha)^{-1} \alpha^T \quad , \tag{24}$$

which reveals the N $n \times p$ weighting matrices. The state reconstructor is now completely defined for this method. It is equivalent to the one described in Reference 5.

Method B. Let H = I

In this method, let

$$H = I \quad , \tag{25}$$

where I is an (Np) $\times$ (Np) identity matrix. Since H is a q $\times$ (Np) matrix in general, this means that q = Np in this case. Recall from Section III that one requirement for the reconstructor to be meaningful is for $q \geqslant n$. Consequently, N must be chosen so that $Np \geqslant n$ or, equivalently, $N \geqslant n/p$, just as in Method A. Having chosen N, α can be determined using equation (16). Substituting equation (25) into equation (15) yields

$$D_{-} = \alpha \quad . \tag{26}$$

Recall also from Section III that another requirement for the reconstructor to be meaningful is for $\text{rank}(D_{-}) = n$. By virtue of equation (26), it is necessary for $\text{rank}(\alpha) = n$, just as in Method A. It follows from equation (26) that

$$(D_{-}^T D_{-})^{-1} D_{-}^T = (\alpha^T \alpha)^{-1} \alpha^T \quad . \tag{27}$$

From equations (21), (22), (25), and (27), one can see the relationships which exist between H and $(D_{-}^T D_{-})^{-1} D_{-}^T$ in Methods A and B. From equations (11) and (25), E_{-} is found to be

$$E_{-} = \beta \tag{28}$$

where β is defined by equation (12). From equations (10) and (25),

$$[H_0 \mid H_1 \mid \dots \mid H_{N-1}] = I \quad , \tag{29}$$

which reveals the N (Np) $\times$ p weighting matrices. However, recognize from equations (9) and (25) that

$$\underline{y}_F(kT) = \begin{bmatrix} \underline{z}(kT) \\ \underline{z}(kT - \frac{T}{N}) \\ \vdots \\ \underline{z}[kT - (N-1)\frac{T}{N}) \end{bmatrix} \quad . \tag{30}$$

Hence, in this method, the intermediate output $\underline{y}_F(kT)$ can be formed directly by catenating the measurements $\underline{z}[kT - j(T/N)]$, $j = 0,1,\ldots,N\text{-}1$ as in equation (30). This is the beauty of this method.

This description completes the definition of the ideal state reconstructor for this method. In Section V, an example is presented which illustrates the procedures to completely design the ideal state reconstructor using both methods presented in this section.

V. AN EXAMPLE

Consider the double integrator plant driven by a zero-order-hold as shown in Figure 4. The continuous-time output z(t) is sampled every T/N seconds and input into the ideal state reconstructor together with the control input u(kT). The parameters in the state reconstructor will be chosen to achieve exact state reconstruction using both of the methods described in Section IV.

Manipulating the plant in Figure 4 into the format of Figure 3 yields

$$[F] = \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} \quad , \tag{31}$$

$$[G] = \begin{bmatrix} 0 \\ 1 \end{bmatrix} \quad , \tag{32}$$

and

$$[C_F] = [1 \quad 0] \quad . \tag{33}$$

Using equations (31) and (32) and the formulas presented in Section II,

$$\phi(t) = \begin{bmatrix} 1 & t \\ 0 & 1 \end{bmatrix} \quad , \tag{34}$$

$$\int_0^t \phi(\lambda)d\lambda = \begin{bmatrix} t & \frac{t^2}{2} \\ 0 & t \end{bmatrix} \quad , \tag{35}$$

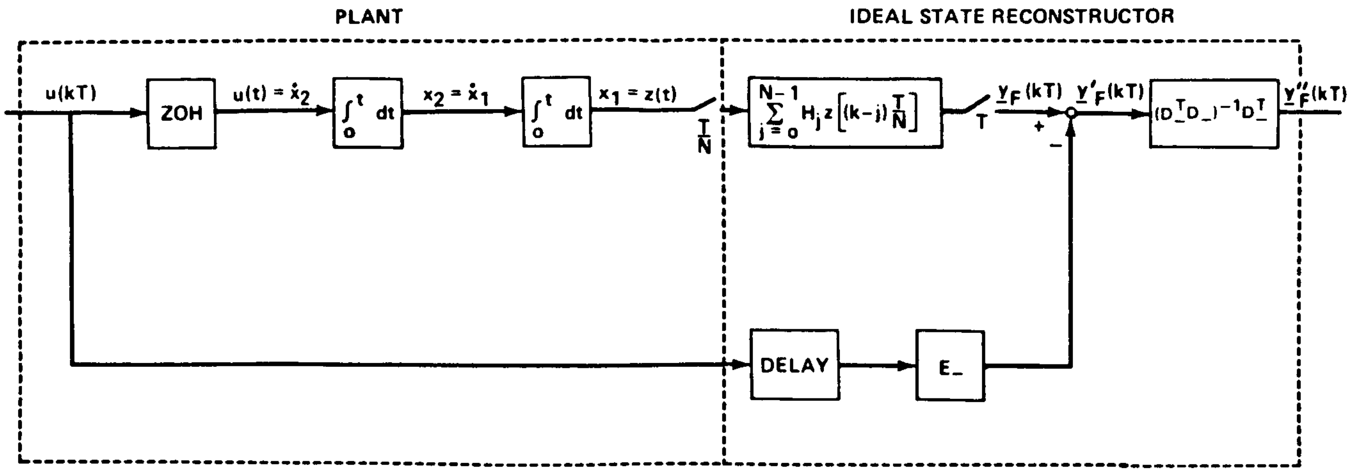

Figure 4. Plant and ideal state reconstructor
for the example.

$$A = \begin{bmatrix} 1 & T \\ 0 & 1 \end{bmatrix} \quad ,$$

and

$$B = \begin{bmatrix} \frac{T^2}{2} \\ T \end{bmatrix} \quad .$$

Since C_F is a $p \times n$ matrix, it follows from equation (33) that $p = 1$ and $n = 2$. Hence, the requirement to select N so that $N \geqslant n/p$ can be satisfied by letting

$$N = 4 \quad . \tag{36}$$

Using equations (12), (16), (32) through (35) and (36), α and β are found to be

$$\alpha = \begin{bmatrix} 1 & 0 \\ 1 & -\frac{T}{4} \\ 1 & -\frac{2T}{4} \\ 1 & -\frac{3T}{4} \end{bmatrix} \tag{37}$$

and

$$\beta = \begin{bmatrix} 0 \\ \frac{T^2}{32} \\ \frac{4T^2}{32} \\ \frac{9T^2}{32} \end{bmatrix} , \tag{38}$$

respectively. In equation (37), eliminating any 2 rows forms a 2×2 matrix with nonzero determinant, assuming of course that $T > 0$. Hence, $\mathrm{rank}(\alpha) = 2 = n$ and so $(\alpha^T\alpha)$ is nonsingular. Consequently, $(\alpha^T\alpha)^{-1}\,\alpha^T$ exists and is found to be

$$(\alpha^T\alpha)^{-1}\,\alpha^T = \begin{bmatrix} \left(\frac{7}{10}\right) & \left(\frac{4}{10}\right) & \left(\frac{1}{10}\right) & \left(-\frac{2}{10}\right) \\ \left(\frac{6}{5T}\right) & \left(\frac{2}{5T}\right) & \left(-\frac{2}{5T}\right) & \left(-\frac{6}{5T}\right) \end{bmatrix} , \tag{39}$$

using equation (37).

Method A. Let $H = (\alpha^T\alpha)^{-1}\,\alpha^T$

From equations (21), (24), and (39),

$$H = [H_0 \mid H_1 \mid H_2 \mid H_3] = \left[\begin{array}{c|c|c|c} \left(\frac{7}{10}\right) & \left(\frac{4}{10}\right) & \left(\frac{1}{10}\right) & \left(-\frac{2}{10}\right) \\ \left(\frac{6}{5T}\right) & \left(\frac{2}{5T}\right) & \left(-\frac{2}{5T}\right) & \left(-\frac{6}{5T}\right) \end{array}\right] ,$$

which reveals the weighting matrices H_j, $j = 0,1,2,3$. From equation (22) and the fact that $n = 2$,

$$(D\_^T D\_)^{-1}\, D\_^T = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$$

From equations (23), (38), and (39),

$$E_- = \begin{bmatrix} \left(-\frac{T^2}{32}\right) \\ \left(-\frac{3T}{8}\right) \end{bmatrix} .$$

This completely defines the ideal state reconstructor for this example and this method.

Method B. Let H = I

Since N = 4 and p = 1, it follows that q = Np = 4. From equations (25) and (29),

$$H = [H_0 \mid H_1 \mid H_2 \mid H_3] = \left[\begin{array}{c|c|c|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{array}\right] ,$$

which reveals the weighting matrices H_j, j = 0,1,2,3. However, from equation (30), the intermediate output $\underline{y}_F(kT)$ can be related directly to the measurements $z[kT - j(T/4)]$, j = 0,1,2,3 and is

$$\underline{y}_F(kT) = \begin{bmatrix} z(kT) \\ z(kT - \frac{T}{4}) \\ z(kT - \frac{2T}{4}) \\ z(kT - \frac{3T}{4}) \end{bmatrix} .$$

From equations (27) and (39),

$$(D_-^T D_-)^{-1} D_-^T = \begin{bmatrix} \left(\frac{7}{10}\right) & \left(\frac{4}{10}\right) & \left(\frac{1}{10}\right) & \left(-\frac{2}{10}\right) \\ \left(\frac{6}{5T}\right) & \left(\frac{2}{5T}\right) & \left(-\frac{2}{5T}\right) & \left(-\frac{6}{5T}\right) \end{bmatrix} .$$

From equations (28) and (38),

$$E_- = \begin{bmatrix} 0 \\ \frac{T^2}{32} \\ \frac{4T^2}{32} \\ \frac{9T^2}{32} \end{bmatrix} .$$

This completes the definition of the ideal state reconstructor for this example and this method.

VI. CONCLUSIONS AND RECOMMENDATIONS

A new state reconstructor for deterministic digital control systems has been presented which offers two distinct advantages over the widely used state observer. First, it adds no new states, eigenvalues, or dynamics to the system and, consequently, will not alter the stability of the system. In fact, adding the new state reconstructor to the system will not affect the plant equation in any way. It only affects the measurement equation. Second, if the plant parameters are known exactly, the reconstructed state will exactly equal the true state of the system, not just approximate it. For these reasons, this new state reconstructor has herein been called the ideal state reconstructor. Its disadvantages are that the plant output must be sampled and calculations must be performed more frequently than with the state observer. Fortunately, this is not the problem it was 20 years ago, considering the speed of today's digital computers.

If the research in this report is extended, two approaches are recommended. One is to explore other methods for choosing the parameters in the ideal state reconstructor. Two distinct methods have been presented here, each with attractive features of its own. However, these do not exhaust all the possibilities, by any means. For example, the weighting matrices could be selected so the MA prefilter acts as a multi-input/multi-output low-pass filter for the case where measurement noise is present. The other approach is to investigate the robustness of the ideal state reconstructor and see how it compares with the state observer's. Specifically, the following questions should be addressed. What effect do modeling errors in the plant have on the ideal state reconstructor, and how does this compare with the state observer? What effect do plant process and measurement noise have on the ideal state reconstructor and how does this compare with the state observer, or even the Kalman filter? How can the robustness of the ideal state reconstructor be improved? Increasing the number of weighting matrices, N, may be one possibility. Catenating the ideal state reconstructor with a state observer, or a Kalman filter, may be another. This might produce a composite estimator which is better than either the ideal state reconstructor, the state observer, or the Kalman filter alone.

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15. SUPPLEMENTARY NOTES

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16. ABSTRACT

This report presents a new state reconstructor for deterministic digital control systems which is ideal in the following sense: if the plant parameters are known exactly, the output of the new state reconstructor will exactly equal the true state of the plant, not just approximate it. Furthermore, this ideal state reconstructor adds no additional states or eigenvalues to the system. Nor does it affect the plant equation for the system in any way; it affects only the measurement equation. While there are countless ways of choosing the ideal state reconstructor parameters, two distinct methods are described here. An example is presented which illustrates the procedures to completely design the ideal state reconstructor using both methods.

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