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A TAYLOR-GALERKIN FINITE ELEMENT ALGORITHM FOR TRANSIENT NONLINEAR THERMAL-STRUCTURAL ANALYSIS

By

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Progress Report
For the period January 1 to June 1, 1985

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Under
Research Grant NSG-1321
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By

Earl A. Thornton[1] and Pramote Dechaumphai[2]

INTRODUCTION

The determination of the structural response induced by thermal effects is an important factor in many aerospace structural designs. Extreme aerodynamic heating on advanced aerospace vehicles may produce severe thermal stresses that can reduce operational performance or even damage structures. The performance of laser devices can be degraded by thermal distortions of mirror surfaces. The thermal environment in space may cause orbiting structures to distort beyond operational tolerances. To predict the structural response accurately, effective numerical techniques capable of both thermal and structural analyses are required. One technique, the finite element method, has been found to be particularly suited for such analyses due to its capability to model complex geometry and to perform both thermal and structural analyses.

In the most common approach to determining structural responses induced by thermal effects, the thermal-structural analyses are assumed uncoupled, and the structural analysis is assumed quasistatic. The uncoupled assumption means that mechanical deformation terms in the heat transfer energy equation are neglected. The quasistatic assumption means that inertia terms in the structural equations of motion are neglected. The practical effect of these assumptions is that the heat transfer analysis can be performed

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first, and the resulting temperatures can be used as input to a subsequent stress analysis. This approach works well when temperatures change slowly as occurs, for example, in an orbiting space structure subject to solar heating. Under these circumstances, the uncoupled, quasistatic idealization provides an effective approach for finite element thermal-structural analysis. The principal difficulty encountered in uncoupled analyses is achieving compatability between different thermal and structural models. A hierarchical finite element approach for integrating uncoupled thermal and structural analyses is described in Ref. 1.

When changes in temperature occur rapidly, the assumptions that justify the uncoupled, quasistatic idealization are no longer justifiable. Temperature changes can occur rapidly due to propagation of thermoelastic waves, during vibrations induced by periodic variations of temperature fields, due to thermal shocks and in similar circumstances. These types of problems may involve resolving wave-like details of the time dependent response for complex structures. Moreover, if the mechanical coupling terms are retained, the equations are inherently nonlinear even in the material's elastic range. The finite element method remains a logical solution approach because of its capability to represent complex geometries. There is a need, however, for effective finite element solution algorithms that can solve large nonlinear transient problems efficiently.

The purpose of this paper is to present a Taylor-Galerkin finite element method for solving large, nonlinear transient thermal-structural problems. The method is a thermal-structural application of a Taylor-Galerkin algorithm recently developed to solve the conservation equations of inviscid, compressible flow (Ref. 2). In the flow problem, the algorithm is used to solve the highly nonlinear Euler equations that includes capturing shock

discontinuities in the flow field. Finite element models of flow problems usually are quite large with the number of equations typically in the range from 3,000 to 30,000 or more. Thus, the algorithm appears to have desirable attributes that will make it effective for large nonlinear, transient thermal-structural problems.

The formulation of a nonlinear thermal-structural problem will be presented first, then the Taylor-Galerkin algorithm will be described. Next, an explicit evaluation of the finite element integrals is described. Finally, a programming strategy for a vector computer implementation of the algorithm is described.

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THERMAL-STRUCTURAL FORMULATION

The nonlinear coupled thermal-structural equations for a two-dimensional continuum (ref. 3) are written in the form

$$\frac{\partial\{U\}}{\partial t} + \frac{\partial\{E\}}{\partial x} + \frac{\partial\{F\}}{\partial y} = \{H\} \tag{1}$$

where $\{U\}$ is the vector of unknowns, $\{E\}$ and $\{F\}$ are vectors of stresses and heat fluxes, and $\{H\}$ is a "load" vector. These vectors are given by

$$\{U\} = \begin{Bmatrix} u \\ v \\ \dot{\rho u} \\ \dot{\rho v} \\ \rho c_v T \end{Bmatrix}, \quad \{E\} = \begin{Bmatrix} 0 \\ 0 \\ -\sigma_x \\ -\tau_{xy} \\ q_x \end{Bmatrix}, \quad \{F\} = \begin{Bmatrix} 0 \\ 0 \\ -\tau_{xy} \\ -\sigma_y \\ q_y \end{Bmatrix} \tag{2a}$$

$$\{H\} = \begin{Bmatrix} \dot{u} \\ \dot{v} \\ \rho f_x \\ \rho f_y \\ T\left[\beta_{11}\frac{\partial \dot{u}}{\partial x} + \frac{1}{2}\beta_{12}\left(\frac{\partial \dot{u}}{\partial y} + \frac{\partial \dot{v}}{\partial x}\right) + \beta_{22}\frac{\partial \dot{v}}{\partial y}\right] + Q \end{Bmatrix} \quad (2b)$$

where u and v are the displacements, ρ is the density, $\dot{u}$ and $\dot{v}$ are the velocities, c_v is the specific heat and T is the temperature; σ_x, σ_y, τ_{xy} are the stress components; q_x, q_y are the heat fluxes; f_x, f_y are body force components per unit volume; β_{11}, β_{12}, β_{22} are coefficients that depend on the coefficients of thermal expansion, and Q is the internal heat generation rate per unit volume. The first two equations in Eq. (2) define the velocity components, the third and fourth equations are the equations of motion, and the fifth equation represents conservation of energy. The term in the square brackets in the last line of $\{H\}$ represents the thermal-structural coupling.

The structural and thermal equations are written in the form of Eq. (1) to resemble the conservation equations of fluid flow. In the fluid context, the components of $\{U\}$ are called the conservation variables, and the components of $\{E\}$ and $\{F\}$ are fluxes of mass, momentum and energy across the faces of a control volume. In the thermal-structural context, the components of $\{U\}$ may also be regarded as conservation variables. The stress components of $\{E\}$ and $\{F\}$ now represent tractions on surfaces of the control volume; however, q_x and q_y still represent heat fluxes across surfaces of the control volume.

In formulating the constitutive equations, highly nonlinear relations

between stresses, strains and temperatures are permitted as well as nonlinear relations between heat fluxes, temperatures and temperature gradients. Anisotropic materials can be accommodated as well. For simplicity, simple constitutive relations for linear, elastic orthotropic materials will be presented herein. The stress-strain relations for an orthotropic material are expressed as

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{bmatrix} C_{11} & C_{12} & o \\ C_{21} & C_{22} & o \\ o & o & C_{33} \end{bmatrix} \left\{ \begin{matrix} \dfrac{\partial u}{\partial x} \\ \dfrac{\partial v}{\partial y} \\ \dfrac{1}{2}\left(\dfrac{\partial u}{\partial y} + \dfrac{\partial v}{\partial x}\right) \end{matrix} \right\} + (T-T_o) \begin{Bmatrix} \beta_{11} \\ \beta_{22} \\ \beta_{12} \end{Bmatrix} \qquad (3a)$$

where C_{ij} are elastic constants, and T_o is the reference temperature for zero stress. The heat fluxes are expressed by Fourier's law,

$$q_x = -k_x \frac{\partial T}{\partial x}$$

$$q_y = -k_y \frac{\partial T}{\partial y} \qquad (3b)$$

where k_x, and k_y are the thermal conductivities.

Equation (1) is solved subject to appropriate initial and boundary conditions. The initial conditions consist of specifying the distributions of the conservation variables $\{U\}$ at time zero. The structural boundary conditions consist of specifying the displacements or surface tractions at all points on the boundary. The thermal boundary conditions consist of

specifying temperatures or heat fluxes at all points on the boundary. Convective and radiation boundary conditions are incorporated through heat fluxes.

TAYLOR-GALERKIN ALGORITHM

The solution domain D is divided into an arbitary number of elements of r nodes each. Fig. 1 shows typical quadrilateral elements ($r=4$) used in this paper. For simplicity, the finite element formulation will be given for a single scalar equation.

$$\frac{\partial u}{\partial t} + \frac{\partial E}{\partial x} + \frac{\partial F}{\partial y} = H \tag{4}$$

where the variables u, E, F and H are analogous to the corresponding vector quantities in Eq. (1). Let $\{u\}^n$ denote the element nodal values of the variable $u(x,y,t)$ at time t_n. The time step Δt spans two typical times t_n and t_{n+1} in the transient response. The computation proceeds through two time levels, $t_{n+1/2}$ and t_{n+1}. At time level $t_{n+1/2}$, values for u that are constant within each element are computed explicitly. At time level t_{n+1}, the constant element values computed at the first time level are used to compute nodal values for u. In the time level t_{n+1} computations, element contributions are assembled to yield the global equations for nodal unknowns. The resulting equations are approximately diagonalized to yield an explicit algorithm.

Time Level $t_{n+1/2}$

To advance the solution to time level $t_{n+1/2}$, a truncated Taylor series yields

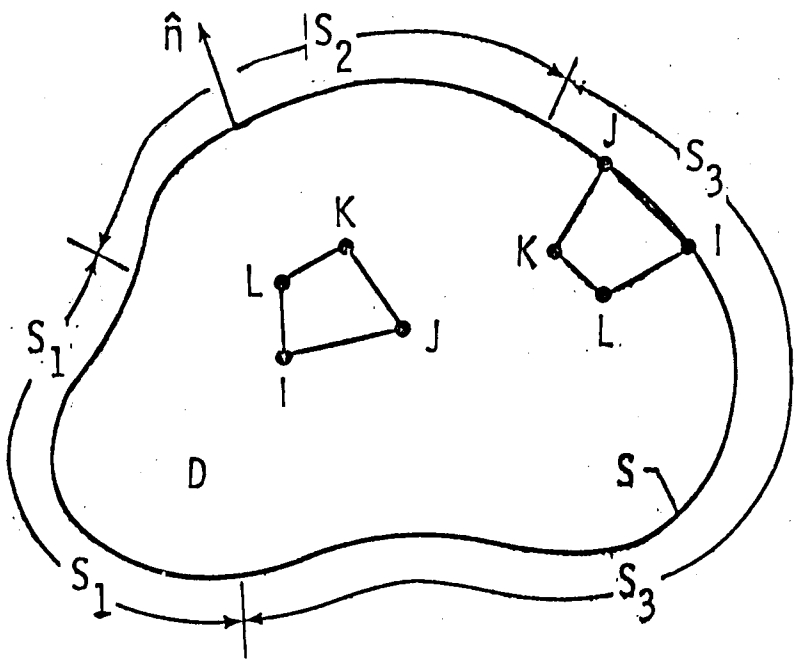

Fig. 1 Two dimensional finite element computational domain.

$$u(x,y,t_{n+1/2}) = u(x,y,t_n) + \frac{1}{2}\Delta t \frac{\partial u}{\partial t}(x,y,t_n) \quad . \tag{5}$$

Then Eq. (4) is introduced on the right hand side of Eq. (5) so that

$$u(x,y,t_{n+1/2}) = u(x,y,t_n) - \frac{1}{2}\Delta t \left[\frac{\partial E}{\partial X}(x,y,t_n) + \frac{\partial F}{\partial y}(x,y,t_n)\right] + \frac{1}{2}\Delta t\, H(x,y,t) \quad . \tag{6}$$

At time level $t_{n+1/2}$, the dependent variable $u(x,y,t_{n+1/2})$ is assumed to have a constant value $u_D{}^{n+1/2}$ within an element.

At time level t_n in the response u, E, F and H vary within an element and are interpolated from nodal values. Thus, the following spatial approximations are used within an element.

$$u(x,y,t_{n+1/2}) = U_D{}^{n+1/2} \tag{7a}$$

$$u(x,y,t_n) = [N(x,y)]\{u\}^n \tag{7b}$$

$$E(x,y,t_n) = [N(x,y)]\{E\}^n \tag{7c}$$

$$F(x,y,t_n) = [N(x,y)]\{F\}^n \tag{7d}$$

$$H(x,y,t_n) = [N(x,y)]\{H\}^n \tag{7e}$$

where $[N(x,y)]$ denotes element interpolation functions and $\{u\}^n$ is a

vector of the element nodal quantities. For a typical quadrilateral element the interpolation functions are bilinear, and in a local natural coordinate system (see Ref. 4) have the form

$$N_i = \frac{1}{4}(1 + \xi_i\xi)(1 + \eta_i\eta) \qquad i = 1, 2, 3, 4 \tag{8}$$

where ξ_i and η_i denote the nodal coordinates $(\xi_i,\eta_i = \pm 1)$ of the element in the ξ-η plane. The equations for $u_D^{n+1/2}$ for each element are derived by the method of weighted residuals. The spatial approximations given in Eq. (7) are introduced into Eq. (6) to give a residual; the result is multiplied by a weighting function which in this case is unity. Finally, the weighted residual is integrated over the area A of the element. The result is,

$$\begin{aligned} A\, u_D^{n+1/2} &= \int_A [N]\, dA\, \{u\}^n \\ &\quad - \frac{\Delta t}{2}\int_A \left[\frac{\partial N}{\partial x}\right] dA\, \{E\}^n - \frac{\Delta t}{2}\int_A \left[\frac{\partial N}{\partial y}\right] dA\, \{F\}^n \\ &\quad + \frac{\Delta t}{2}\int_A [N]\, dA\, \{H\}^n \quad . \end{aligned} \tag{9}$$

With Eq. (9), the dependent variable $u_D^{n+1/2}$ for each element can be computed explicitly using nodal values for u, E, F and H from the previous time t_n. The constant element variable $u_D^{n+1/2}$ may be interpreted as a weighted average of an element's nodal values at time t_n. A later section will discuss the evaluation of the three integrals that appear on the right

hand side of Eq. (9).

In advancing the solution to the next time level, the values of the dependent variables on the surface may be required also. Let $u_s^{n+1/2}$ denote the surface value on a typical element edge IJ on the surface, S_3 in Fig. 1. Following the approach used previously, $u_s^{n+1/2}$ is assumed constant on edge IJ at time $t_{n+1/2}$, but at time t_n, u, E and F vary along the edge. Thus the following approximations are used on an element edge on S_3,

$$u(x,y,t_{n+1/2}) = u_s^{n+1/2} \tag{10a}$$

$$u(x,y,t_n) = [N(s)]\{u\}^n \tag{10b}$$

$$E(x,y,t_n) = [N(s)]\{E\}^n \tag{10c}$$

$$F(x,y,t_n) = [N(s)]\{F\}^n \tag{10d}$$

$$H(x,y,t_n) = [N(s)]\{H\}^n \tag{10e}$$

where $[N(s)]$ denotes the interpolation functions along an element edge. Using the method of weighted residuals, the values for $u_s^{n+1/2}$ are derived by integrating over the length L of an element edge. Hence

$$L\, u_s^{n+1/2} = \int_L [N]\, ds\{u\}^n - \frac{\Delta t}{2}\int_L [N]\, ds\, \{E\}^n$$

$$- \frac{\Delta t}{2}\int_L \left[\frac{\partial N}{\partial y}\right] ds\, \{F\}^n + \frac{\Delta t}{2}\int_L [N]\, ds\, \{H\}^n \quad . \tag{11}$$

Thus, Eqs. (9) and (11) can be used to advance explicitly the element and surface values of the dependent variables to $t_{n+1/2}$. Beginning with nodal values of $\{u\}^n$, $\{E\}^n$, $\{F\}^n$ and $\{H\}^n$, Eq. (9) is used to compute constant values $u_D^{n+1/2}$ for each element. In a similar way, Eq. (11) is used to compute constant surface values $u_s^{n+1/2}$ for element edges on boundaries that require these values at $t_{n+1/2}$. For example, $u_s^{n+1/2}$ are required on element edges with specified stresses or heat fluxes. These values are computed explicitly by looping through all elements and appropriate element edges.

Time Level t_{n+1}

To advance the solution to t_{n+1}, forward and backward truncated Taylor series expansions at $t_{n+1/2}$ are used to write the approximation

$$u(x,y,t_{n+1}) = u(x,y,t_n) + \Delta t \frac{\partial u}{\partial t}(x,y,t_{n+1/2}) \quad . \tag{12}$$

Then, following the approach used previously, Eq. (4) is introduced on the right hand side to yield

$$u(x,y,t_{n+1}) = u(x,y,t_n)$$

$$- \Delta t \left[\frac{\partial E}{\partial x}(x,y,t_{n+1/2}) + \frac{\partial F}{\partial y}(x,y,t_{n+1/2})\right] \quad . \tag{13}$$

$$+ \Delta t \, H(x,y,t_{n+1/2}) \quad .$$

If the flux gradients $\partial E/\partial x$ and $\partial F/\partial y$ are evaluated using values from the midstep, the gradients will be zero because E and F are constant within the element at midstep. To avoid this undesirable situation, the flux gradients are interpolated from their values at t_n and t_{n+1}. Thus

$$\frac{\partial E}{\partial X}(x,y,t_{n+1/2}) = \theta \frac{\partial E}{\partial X}(x,y,t_n) + (1-\theta)\frac{\partial E}{\partial X}(x,y,t_{n+1})$$

$$\frac{\partial F}{\partial y}(x,y,t_{n+1/2}) = \theta \frac{\partial F}{\partial y}(x,y,t_n) + (1-\theta)\frac{\partial F}{\partial y}(x,y,t_{n+1}) \qquad (14)$$

where the interpolation parameter θ varies from zero to one. The equations for the nodal values of $\{u\}^{n+1}$ can next be derived by the method of weighted residuals in the standard way (ref. 4) using the interpolation functions N_i as weighting functions. In the process, Eq. (14) is substituted into Eq. (13), and the terms containing derivatives are integrated by parts. These operations yield the equations for the nodal values of a single element,

$$[M]\{u\}^{n+1} = [M]\{u\}^n + \theta\{R_1\}^n + \theta\{R_2\}^n + (1-\theta)\{R_3\}^{n+1} + (1-\theta)\{R_4\}^{n+1} + \{R_5\}^{n+1/2} \qquad (15)$$

where

$$[M] = \int_A \{N\}[N]\,dA \qquad (16a)$$

$$\{R_1\}^n = \Delta t \int_A \left\{\frac{\partial N}{\partial X}\right\} [N]\, dA\, \{E\}^n + \Delta t \int_A \left\{\frac{\partial N}{\partial y}\right\} [N]\, dA\, \{F\}^n \tag{16b}$$

$$\{R_2\}^n = -\Delta t \int_S \{N\}[N]\, ds\, (\ell\, \{E\}_S^n + m\, \{F\}_S^n) \tag{16c}$$

$$\{R_3\}^{n+1} = \Delta t \int_A \left\{\frac{\partial N}{\partial X}\right\} [N]\, dA\, \{E\}^{n+1} + \Delta t \int_A \left\{\frac{\partial N}{\partial y}\right\} [N]\, dA\, \{F\}^{n+1} \tag{16d}$$

$$\{R_4\}^{n+1} = -\Delta t \int_S \{N\}\, [N]\, ds\, (\ell\, \{E\}_S^{n+1} + m\, \{F\}_S^{n+1}) \tag{16e}$$

$$\{R_5\}^{n+1/2} = \Delta t \int_A \{N\}\, dA\, H^{n+1/2} \quad . \tag{16f}$$

In Eqs. (16c) and (16e) the coefficients ℓ and m are the components of a unit vector normal to the boundary. Following usual finite element procedures, the element matrices given in Eq. (16) can be assembled to form system (global) equations.

The matrix [M] defined by Eq. (16a) is the element consistent mass matrix. The term, consistent, is used to distinguish these matrices from diagonal mass matrices that arise from other discretization methods. A consistent mass matrix has off-diagonal terms that couple the element variables on the left hand side of Eq. (15). The algorithm with consistent mass matrices and arbitrary θ is an implicit scheme. If the mass matrices are diagonalized and θ is taken as 1, the algorithm becomes an explicit scheme.

EXPLICIT EVALUATION OF ELEMENT INTEGRALS

Element integrals for the Taylor-Galerkin algorithm shown in Eqs. (9), (11), and (16) were evaluated in closed form in Ref. 2 to avoid expensive

numerical integrations that are customary for quadrilateral elements. For two dimensional problems, these element integrals are either in the form of integration over the element area or along an element edge. As an example, the element mass matrix for a quadrilateral element, Eq. (15), is given by

$$[M] = \int_{-1}^{1} \int_{-1}^{1} \{N\} [N] |J| \, d\xi \, d\eta \tag{17}$$

where $\{N\}$ is the vector of the element interpolation functions in terms of the element natural coordinates ξ and η. The determinant of the Jacobian $|J|$ that appears in the above equation represents the transformation from the element global coordinates x-y to natural coordinates ξ-η. The transformation permits the element integration to be evaluated over a square.

The approach developed in Ref. 2 was also used for the evaluation of three dimensional hexahedral element integrals. Typical element integrals derived using the Taylor-Galerkin algorithm are in the same form as obtained for the quadrilateral element where the element integrations are either performed over the element volume or the element surface areas. As an example, the mass matrix for a hexahedral element is given by

$$[M] = \int_V \{N\} [N] \, dV$$

$$= \int_{-1}^{1} \int_{-1}^{1} \int_{-1}^{1} \{N\} [N] |J| \, d\xi \, d\eta \, d\zeta \tag{18}$$

where ξ, η, ζ are the element natural coordinates.

The CPU time used by the closed form solution of the mass matrix has

been investigated and compared with CPU times required for different orders of Gauss numerical integration for quadrilateral and hexahedral elements. Although CPU time savings for quadrilateral elements are large (a factor of 50 for all integrals), the CPU time savings for hexahedral elements is even more significant. Typical CPU times for evaluation of a hexahedral element mass matrix are compared in Fig. 2. The figure shows that the closed form solution reduces CPU time significantly for an element mass matrix. Time savings in excess of an order of magnitude are obtained from the closed form solution in comparison with the popular 8 points Gauss integration method. Computational savings from explicit evaluation of other hexahedral element matrices are shown in Fig. 3.

The time savings gained by explicit evaluation of the finite element integrals is an important step toward developing efficient finite element computations for large three-dimensional problems.

VECTOR PROGRAMMING STRATEGIES

The Taylor-Galerkin algorithm used in Ref. 2 was implemented with vectorization strategies specifically for the Langley VPS 32 (a Cyber 205 with 16 million words of central memory). This computer achieves high computational speed when performing operations on long vectors. Vector lengths of at least 60 are required to justify vectorization efforts with maximum payoff achieved for vector lengths of 1000 or more. The predominant vector lengths in the vectorization scheme are the number of elements in the finite element model with occasional operations using vector lengths equal to the number of nodes.

The critical vectorization tasks are for those operations that are repetitive and performed at every time step. For finite element algorithms,

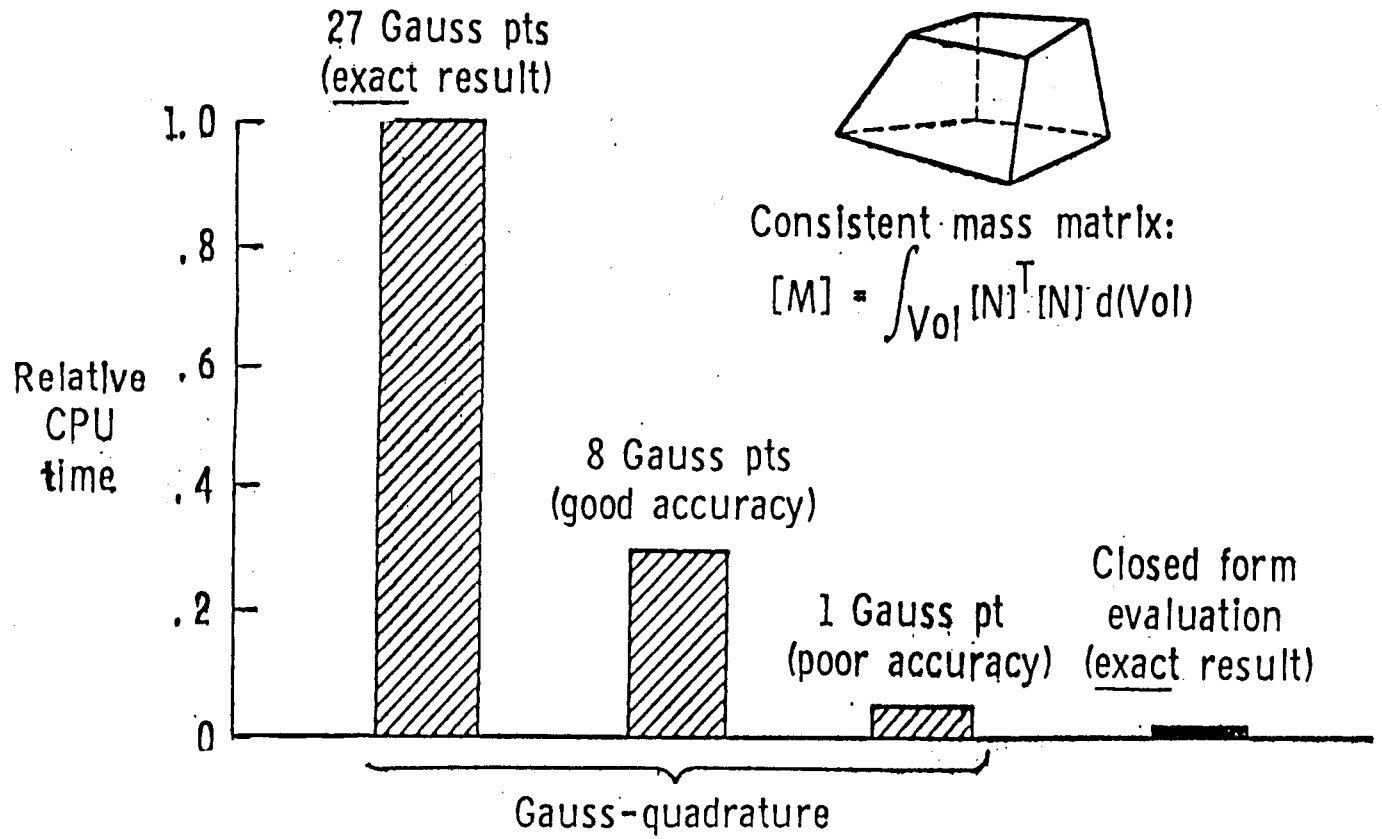

Fig. 2 Comparison of execution times for evaluation of hexahedral element mass matrix.

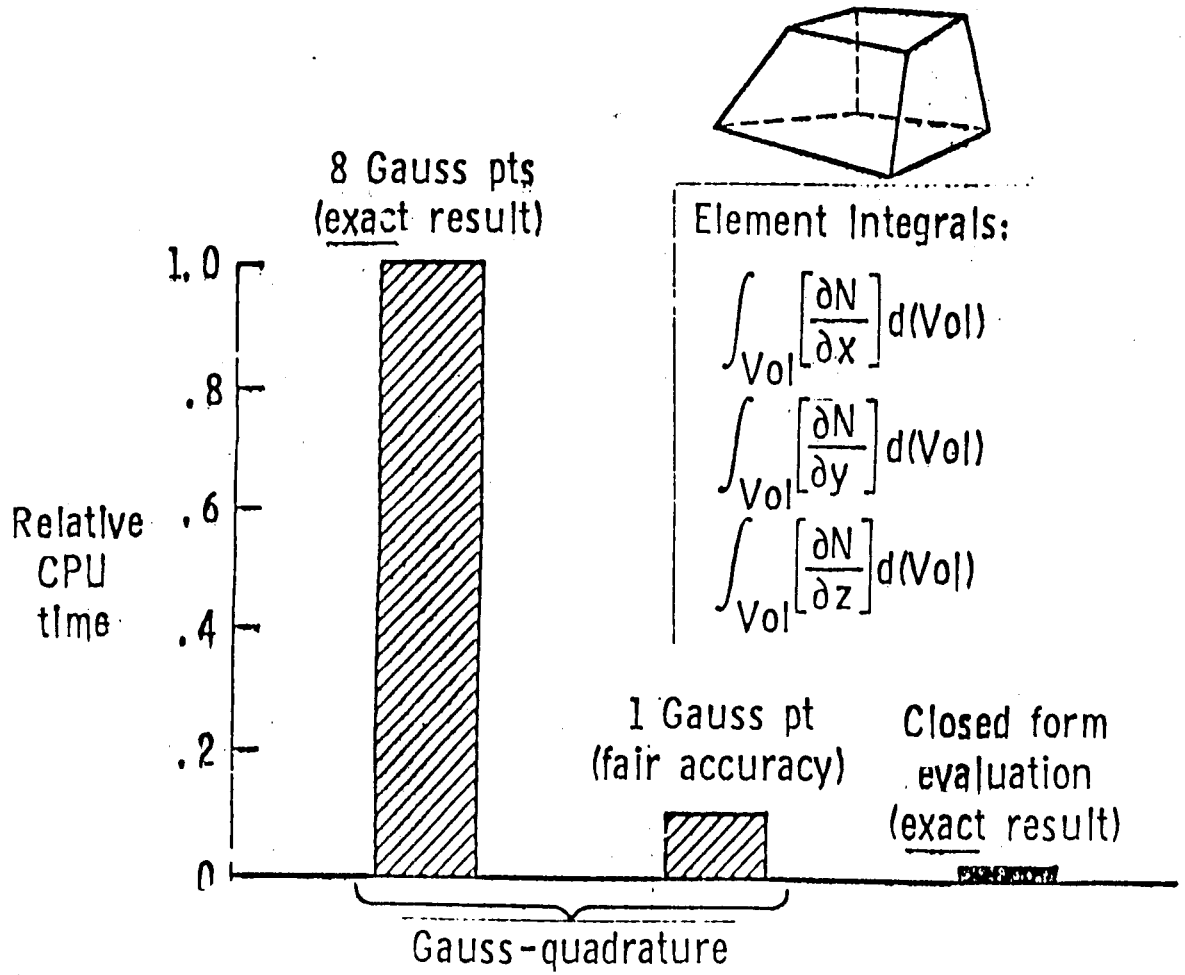

Fig. 3 Comparison of execution times for evaluation of hexahedral element matrices.

these operations are: (1) assembly of element contributions into the global system of equations, (2) solution of the global system of equations, and (3) application of boundary conditions.

The assembly of element contributions into the global system of equations is the process in finite elements which differs most from other numerical techniques and requires special routines for vectorization. Nodal unknowns are stored in one dimensional arrays from 1 to the number of nodes in the model, and in general, node numbering may be arbitary throughout the mesh. Assembly of element contributions is performed using the VPS 32 FORTRAN-supplied scatter routine which places an element contribution into the proper location in the system of equations based on the element connectivity. Every element that contains a particular node in its connectivity provides its own contribution to the system equations, therefore, the assembly is an additive operation. "Scattering" alone would merely overwrite a previous element contribution. The special vector routine, then does an "additive scatter."

For an explicit scheme, solutions are obtained directly, so that operation (2) vectorizes naturally. Operation (3), the application of boundary conditions, is an intrinsically scalar operation and difficult to vectorize. However, use of bit vectors to flag boundary nodes and use of VPS 32 FORTRAN supplied routines enables full vectorization of this operation.

A program flow chart for the Taylor-Galerkin algorithm is shown in Fig. 4.

CONCLUDING REMARKS

A Taylor-Galerkin finite element solution algorithm for transient nonlinear thermal-structural analyses of large, complex structural problems

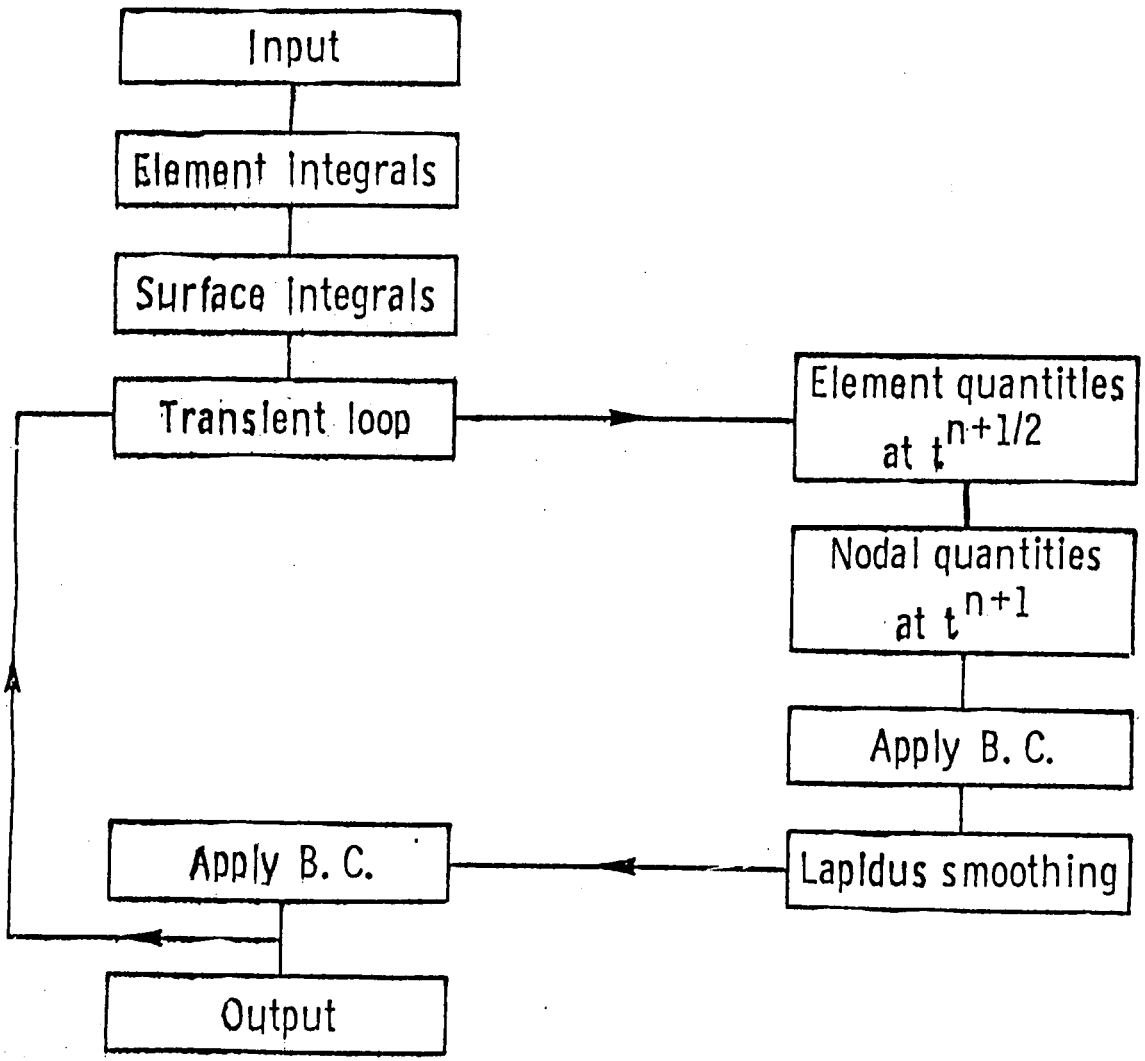

Fig. 4 Program flowchart for Taylor-Galerkin algorithm.

subjected to rapidly applied thermal-structural loads is described. The two-step Taylor-Galerkin algorithm is an application of an algorithm recently developed for problems in compressible fluid dynamics. The element integrals that appear in the algorithm can be evaluated in closed form for two and three dimensional elements. Numerical calculations show that computational times are reduced significantly by the closed form integral evaluation. The algorithm has been implemented on the NASA Langley VPS 32 vector computer with special programming strategies to yield very high computational speeds for the solution of large problems. The many desirable attributes of the algorithm indicate that it will be quite effective for the solution of large, nonlinear, transient thermal-structural problems.

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