NASA Contractor Report 178313

INFLUENCE OF LARGE DEFLECTION AND TRANSVERSE SHEAR ON RANDOM RESPONSE OF RECTANGULAR SYMMETRIC COMPOSITE LAMINATES TO ACOUSTIC LOADS

C. Mei and C. B. Prasad

OLD DOMINION UNIVERSITY RESEARCH FOUNDATION
Norfolk, Virginia

Contract NAS1-17993
June 1987

National Aeronautics and Space Administration

Langley Research Center
Hampton, Virginia 23665

SUMMARY

Nonlinear equations of motion of symmetrically laminated anisotropic plates are derived accounting for von Karman strains. The effect of transverse shear is included in the formulation and the rotatory inertia effect is neglected. Using a single-mode Galerkin procedure the nonlinear modal equation is obtained. The direct equivalent linearization method is employed for solution of this equation. The response to acoustic excitation of moderately thick composite panels is studied. Further, the effects of transverse shear on large deflection vibration of laminates under random excitation are studied. Mean-square deflections and mean-square inplane stresses are obtained for some symmetric graphite-epoxy laminates. Using equilibrium equations, and the continuity requirements, the mean-square transverse shear stresses are calculated. The results obtained will be useful in the sonic fatigue design of composite aircraft panels. The analysis is presented in detail for simply supported plates. The analogous equations for a clamped case are given in the appendix.

INTRODUCTION

Acoustically induced fatigue failures in aircraft structures have been a design consideration for the past three decades. With the advent of the jet engine which produced high intensity acoustic pressure fluctuations on aircraft surfaces, the problem acquired prominence. The number of acoustic fatigue failures have resulted in unacceptable maintenance and inspection burdens associated with the operation of aircraft. Therefore, accurate design methods are needed to determine the acoustic fatigue life of structures. Numerous analytical studies Refs. 1-13 and experimental investigations Refs. 7, 14-22 on sonic fatigue design of aircraft structures have been undertaken during the past decade to help in providing the needed information.

The majority of analytical studies on flat panels to date have been formulated within the framework of linear or classical plate theory which assumes small deflections. Current analytical design methods Refs. 7, 9, 12, 13 for sonic fatigue prevention are based essentially on linear structural theory. Test results on various aircraft panels reported in the literature, Refs. 7, 13-17, 19-22 however, have shown that high noise levels produce nonlinear large deflection behavior in such panels. Recently, analytical efforts Refs. 1-3, 5, 6, 8, 16 have demonstrated that the prediction of panel random response is greatly improved by including the large deflection effects in the formulation. In all these efforts, both analytical and experimental, the thickness of panel is very small as compared with its length ($a/h > 100$). The effect of acoustic excitation on moderately thick composite structural panels has not been investigated. Further, the effects of transverse shear on large deflection vibrations of laminates under random excitation have also not been studied.

The classical theory of plates, in which it is assumed that normals to the midplane before deformation remain straight and normal to the plane after deformation, under predicts deflections and over predicts natural frequencies and buckling loads. Such results are due to the neglect of transverse shear strains in the classical plate theory. The errors in deflections, stresses, natural frequencies and buckling loads are even higher for plates made of advanced composites like graphite-epoxy and boron-epoxy, whose elastic modulus to shear modulus ratios are very large (e.g., of the order of 25 to 40, instead of 2.6 for typical isotropic materials). These high ratios render classical theories inadequate for the analysis of composite plates. Many plate theories exist that account for transverse shear strains Refs. 23-34. Recently, the dynamic von Karman plate theory has been extended to include the shear and rotatory inertia effects by Sathyamoorthy and Chia for nonlinear free vibrations of anisotropic rectangular (Ref. 35) and Skew (Ref. 36) plates. Sivakumaran and Chia have extended this approach to generally laminated anisotropic thick plates, Ref.37.

In this report, the equations of motion are derived from plate theory which takes Von Karman large deflection strain-displacement relations into account. The transverse shear deformation effects are included and the rotatory inertia effects are neglected. The system of equations is then simplified to two coupled nonlinear differential equations in terms of transverse displacement and a stress function. Due to the complex nature of the problem, the study is restricted to a single-mode response. A deflection function that represents the first mode is assumed; and corresponding to the assumed mode, a stress function satisfying the different inplane edge boundary conditions is obtained by solving the compatibility equation. The Galerkin method is applied to the governing equation of motion in the transverse direction using the assumed displacement function as the weighting function. This yields a nonlinear, nonhomogeneous, second-order differential equation of the response in time. Slope functions that include the transverse shear deformation effects are assumed and the coefficients are evaluated by Galerkins approximation. Finally, the random responses from cases based on four formulations at various acoustic loadings are evaluated for simply supported rectangular symmetrical laminates. The four cases of formulations are: 1) linear, small deflection plate theory without transverse shear deformation, 2) linear theory with shear, 3) large deflection without shear and 4) large deflection with shear. The excitation is assumed to be stationary, ergodic and Gaussian with zero mean; its magnitude and phase are uniform over the plate surface.

The equivalent linearization method is employed. Root-mean-square (RMS) deflections, RMS inplane stresses and frequencies are calculated. Using three-dimensional equilibrium equations and the continuity requirements, the RMS transverse shear stresses in the laminate are estimated.

EQUATIONS OF MOTION

Consider an initially flat, rectangular, elastic plate of constant thickness h in the z-direction, length a in the x-direction and width b in the y-direction, see Fig. 1. The reference plane $z=0$ is located at the undeformed middle plane.

The displacement components that include the effects of transverse shear deformation are assumed in the form (Ref. 38):

$$\begin{aligned} U(x,y,z,t) &= u(x,y,t) + z\,\alpha(x,y,t) \\ V(x,y,z,t) &= v(x,y,t) + z\,\beta(x,y,t) \\ W(x,y,z,t) &= w(x,y,t) \end{aligned} \tag{1}$$

in which U, V and W are the inplane and transverse displacements in x,y and z directions, respectively, u, v and w are the values of U, V, W at the midsurface of the reference plane, and α,β are slope functions in the xz and yz planes due to bending only. These are averaged components of direction change of the normal to the undeformed middle surface. The total strains for the laminated plate can be expressed as

$$\varepsilon_x = \varepsilon_x^0 + z\,\alpha,_x$$

$$\varepsilon_y = \varepsilon_y^0 + z\beta,_y$$

$$\varepsilon_{xy} = \varepsilon_{xy}^0 + z(\alpha,_y + \beta,_x)$$

$$\tag{2}$$

$$\varepsilon_z = 0$$

$$\varepsilon_{xz} = \alpha + w,_x$$

$$\varepsilon_{yz} = \beta + w,_y$$

where ε_{xy}, ε_{xz} and ε_{yz} are the engineering shear strains.

The von Karman large deflection strain-displacement relations are given by

$$\varepsilon_x^0 = u,_x + \frac{1}{2} w,_x^2$$

$$\varepsilon_y^0 = v,_y + \frac{1}{2} w,_x^2 \tag{3}$$

$$\varepsilon_{xy}^0 = u,_y + v,_x + w,_x\, w,_y$$

As in the classical plate theory, stress resultants and stress couples are defined as

$$(N_x, N_y, N_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy})\, dz$$

$$(M_x, M_y, M_{xy}) = \int_{-h/2}^{h/2} (\sigma_x, \sigma_y, \sigma_{xy})\, zdz \qquad (4)$$

$$(Q_y, Q_x) = \int_{-h/2}^{h/2} (\sigma_{yz}, \sigma_{xz})dz$$

with

$$\begin{Bmatrix} \sigma_{yz} \\ \sigma_{xz} \end{Bmatrix} = \begin{bmatrix} \bar{Q}_{44} & \bar{Q}_{45} \\ \bar{Q}_{45} & \bar{Q}_{55} \end{bmatrix} \begin{Bmatrix} \varepsilon_{yz} \\ \varepsilon_{xz} \end{Bmatrix} \qquad (5)$$

where $\bar{Q}$'s are the transformed reduced stiffness. Thus for symmetrical laminates

$$\begin{Bmatrix} [N] \\ [M] \end{Bmatrix} = \begin{bmatrix} [A] & 0 \\ 0 & [D] \end{bmatrix} \begin{Bmatrix} [\varepsilon^o] \\ [\kappa] \end{Bmatrix} \qquad (6)$$

$$\begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} = \begin{bmatrix} A_{44} & A_{45} \\ A_{45} & A_{55} \end{bmatrix} \begin{Bmatrix} \beta + w,_y \\ \alpha + w,_x \end{Bmatrix} \qquad (7)$$

where

$$[N] = \begin{Bmatrix} N_x \\ N_y \\ N_{xy} \end{Bmatrix} \qquad [M] = \begin{Bmatrix} M_x \\ M_y \\ M_{xy} \end{Bmatrix}$$

$$[\varepsilon^o] = \begin{Bmatrix} \varepsilon_x^o \\ \varepsilon_y^o \\ \varepsilon_{xy}^o \end{Bmatrix} \qquad [\kappa] = \begin{Bmatrix} \alpha,_x \\ \beta,_y \\ \alpha,_y + \beta,_x \end{Bmatrix} \tag{8}$$

and elements of laminate stiffnesses A_{ij} and D_{ij} are defined as

$$(A_{ij}, D_{ij}) = \int_{-h/2}^{h/2} (1, z^2)\ \bar{Q}_{ij}\ dz \qquad (i,j=1,2,6)$$

and (9)

$$A_{ij} = \int_{-h/2}^{h/2} \bar{Q}_{ij} dz \qquad (i,j=4,5)$$

Partial inversion of equation (6) and inverted form of equation (7) can be written as

$$\begin{Bmatrix} [\varepsilon^o] \\ [M] \end{Bmatrix} = \begin{bmatrix} [A^*] & 0 \\ 0 & [D] \end{bmatrix} \begin{bmatrix} [N] \\ [\kappa] \end{bmatrix} \tag{10}$$

and

$$\begin{Bmatrix} \beta + w,_y \\ \alpha + w,_x \end{Bmatrix} = \begin{bmatrix} A_{44}^* & A_{45}^* \\ A_{45}^* & A_{55}^* \end{bmatrix} \begin{Bmatrix} Q_y \\ Q_x \end{Bmatrix} \tag{11}$$

where

$$[A^*] = [A]^{-1}$$

The plate equations are obtained by considering the equilibrium of an element of the kth layer of the laminate. Integrating the equilibrium relations over the plate thickness h, neglecting inplane inertia terms and retaining the nonlinear terms in accordance with the Von Karman assumptions, leads to the following equations of motion

$$N_{x,x} + N_{xy,y} = 0 \tag{12}$$

$$N_{xy,x} + N_{y,y} = 0 \tag{13}$$

$$M_{x,x} + M_{xy,y} = Q_x \tag{14}$$

$$M_{xy,x} + M_{y,y} = Q_y \tag{15}$$

$$(N_x w,_x + N_{xy} w,_y),_x + (N_{xy} w,_x + N_y w,_y),_y$$

$$+ Q_{x,x} + Q_{y,y} + p(t) = \rho h \ddot{w} \tag{16}$$

Expanding equation (11) and introducing a tracing constant T_s, we can write

$$\alpha + w,_x = T_s (A_{55}^* Q_x + A_{45}^* Q_y) \tag{17}$$

$$\beta + w,_y = T_s \, (A_{45}^* \, Q_x + A_{44}^* \, Q_y) \tag{18}$$

in which T_s takes the value of either 1 or 0. If transverse shear deformation effects are neglected $T_s = 0$ and equations (17) and (18) reduce to

$$\alpha = -w,_x \quad \text{and} \quad \beta = - w,_y \tag{19}$$

Using equations (14) and (15) and making use of constitutive relations given in equations (10), equations (17) and (18) can be shown to be

$$\alpha + w,_x = b_1\alpha,_{xx} + b_2\alpha,_{xy} + b_3\alpha,_{yy} + b_4\beta,_{xx} + b_5\beta,_{xy} + b_6\beta,_{yy} \tag{20}$$

$$\beta + w,_y = b_7\alpha,_{xx} + b_8\alpha,_{xy} + b_9\alpha,_{yy} + b_{10}\beta,_{xx} + b_{11}\beta,_{xy} + b_{12}\beta,_{yy} \tag{21}$$

where the coefficients b_i are defined as

$$b_1 = T_s \, (A_{55}^* \, D_{11} + A_{45}^* \, D_{16})$$

$$b_2 = T_s \, (2 \, A_{55}^* \, D_{16} + A_{45}^* \, (D_{12} + D_{66}))$$

$$b_3 = T_s \, (A_{55}^* \, D_{66} + A_{45}^* \, D_{26})$$

$$b_4 = T_s \, (A_{55}^* \, D_{16} + A_{45}^* \, D_{66})$$

$$b_5 = T_s \, (A_{55}^* \, (D_{12} + D_{66}) + 2 \, A_{45}^* \, D_{26})$$

$$b_6 = T_s \, (A_{55}^* \, D_{26} + A_{45}^* \, D_{22})$$

$$b_7 = T_s \, (A_{45}^* \, D_{11} + A_{44}^* \, D_{16})$$

$$b_8 = T_s \, (2A_{45}^* \, D_{16} + A_{44}^* \, (D_{12} + D_{66})) \tag{22}$$

$$b_9 = T_s \, (A_{45}^* \, D_{66} + A_{44}^* \, D_{26})$$

$$b_{10} = T_s \, (A_{45}^* \, D_{16} + A_{44}^* \, D_{66})$$

$$b_{11} = T_s \, (A_{45}^* \, (D_{12} + D_{66}) + 2 \, A_{44}^* \, D_{26})$$

$$b_{12} = T_s \, (A_{45}^* \, D_{26} + A_{44}^* \, D_{22})$$

Equations (20) and (21) can be written as

$$w,_x + J(\alpha) + K(\beta) = 0 \tag{23}$$

$$w,_y + L(\alpha) + M(\beta) = 0 \tag{24}$$

where J,K,L and M are the operators defined as

$$J = 1 - b_1 \frac{\partial^2}{\partial x^2} - b_2 \frac{\partial^2}{\partial x \partial y} - b_3 \frac{\partial^2}{\partial y^2}$$

$$K = - b_4 \frac{\partial^2}{\partial x^2} - b_5 \frac{\partial^2}{\partial x \partial y} - b_6 \frac{\partial^2}{\partial y^2}$$

$$L = - b_7 \frac{\partial^2}{\partial x^2} - b_8 \frac{\partial^2}{\partial x \partial y} - b_9 \frac{\partial^2}{\partial y^2}$$

$$M = 1 - b_{10} \frac{\partial^2}{\partial x^2} - b_{11} \frac{\partial^2}{\partial x \partial y} - b_{12} \frac{\partial^2}{\partial y^2}$$

Solving equations (23) and (24) we get

$$M(w,_x) - K(w,_y) = N(\alpha) \tag{25}$$

$$J(w,_y) - L(w,_x) = N(\beta) \tag{26}$$

where the operator $N = KL - MJ$ can be shown to be

$$N = k_1 \frac{\partial^4}{\partial x^4} + k_2 \frac{\partial^4}{\partial x^3 \partial y} + k_3 \frac{\partial^4}{\partial x^2 \partial y^2} + k_4 \frac{\partial^4}{\partial x \partial y^3} + k_5 \frac{\partial^4}{\partial y^4} +$$

$$k_6 \frac{\partial^2}{\partial x^2} + k_7 \frac{\partial^2}{\partial x \partial y} + k_8 \frac{\partial^2}{\partial y^2} -1 \tag{27}$$

with coefficients k_i defined as

$$k_1 = b_4 b_7 - b_1 b_{10}$$

$$k_2 = b_4 b_8 + b_5 b_7 - b_1 b_{11} - b_2 b_{10}$$

$$k_3 = b_4 b_9 + b_5 b_8 + b_6 b_7 - b_1 b_{12} - b_2 b_{11} - b_3 b_{10}$$

$$k_4 = b_5 b_9 + b_6 b_8 - b_3 b_{11} - b_2 b_{12} \tag{28}$$

$$k_5 = b_6 b_9 - b_3 b_{12}$$

$$k_6 = b_1 + b_{10}$$

$$k_7 = b_2 + b_{11}$$

$$k_8 = b_3 + b_{12}$$

Using equations (14) and (15) and the relationships given in equations (6) and (8) we can get

$$Q_{x,x} + Q_{y,y} = L_1 (\alpha,\beta) \tag{29}$$

where

$$\begin{aligned} L_1(\alpha,\beta) = {} & D_{11} \alpha_{,xxx} + 3 D_{16} \alpha_{,xxy} + (D_{12} + 2D_{66}) \alpha_{,xyy} \\ & + D_{26} \alpha_{,yyy} + D_{16} \beta_{,xxx} + (D_{12} + 2D_{66}) \beta_{,xxy} \\ & + 3D_{26} \beta_{,xyy} + D_{22} \beta_{,yyy} \end{aligned} \tag{30}$$

The Airy's stress function F is defined such that

$$[N]^T = [F_{,yy} \quad F_{,xx} \quad - F_{,xy}] \tag{31}$$

using equation (31) and the relationships given in equations (6) and (8), we can get

$$N_x w,_{xx} + N_y w,_{yy} + 2N_{xy} w,_{xy} = \phi(F,w) \tag{32}$$

Substituting equations (29) and (32), equation of transverse motion as given in equation (16) can be rearranged to be

$$I_1 + I_2 + I_3 = 0 \tag{33}$$

where

$$I_1 = p(t) - \rho h \ddot{w}$$

$$I_2 = \phi(F,w) \tag{34}$$

$$I_3 = L_1 (\alpha,\beta)$$

By making use of the operators defined in equations (25) and (26), α and β can be eliminated from equation (33) and can be written as

$$N(I_1 + I_2) + U_1 (w) = 0 \tag{35}$$

where N is the operator as defined in equation (27) and

$$U_1 = D_{11} \frac{\partial^4}{\partial x^4} + 4D_{16} \frac{\partial^4}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^4}{\partial x^2 \partial y^2} +$$

$$4D_{26} \frac{\partial^4}{\partial x \partial y^3} + D_{22} \frac{\partial^4}{\partial y^4} + P_1 \frac{\partial^6}{\partial x^6} + P_2 \frac{\partial^6}{\partial x \partial y^5}$$

$$+ P_3 \frac{\partial^6}{\partial x^4 \partial y^2} + P_4 \frac{\partial^6}{\partial x^3 \partial y^3} + P_5 \frac{\partial^6}{\partial x^2 \partial y^4} + P_6 \frac{\partial^6}{\partial x \partial y^5}$$

$$+ P_7 \frac{\partial^6}{\partial y^6} \tag{36}$$

where P_i are defined as

$$P_1 = b_7 D_{16} - b_{10} D_{11}$$

$$P_2 = (b_4 - b_{11}) D_{11} + (b_8 - b_1 - 3b_{10}) D_{16} + b_7(D_{12} + 2D_{66})$$

$$P_3 = (b_5 - b_{12}) D_{11} + (b_9 - b_2 + 3b_4 - 3b_{11}) D_{16}$$

$$+ (b_8 - b_1 - b_{10}) (D_{12} + 2D_{66}) + 3b_7 D_{26}$$

$$P_4 = b_6 D_{11} + (3b_5 - 3b_{12} - b_3) D_{16}$$

$$+ (b_4 - b_{11} + b_9 - b_2) (D_{12} + 2D_{66}) \tag{37}$$

$$+ (3b_8 - 3b_1 - b_{10}) D_{26} + b_7 D_{22}$$

$$P_5 = 3 b_6 D_{16} + (b_5 - b_{12} - b_3) (D_{12} + 2 D_{66})$$

$$+ (b_4 - b_{11} + 3b_9 - 3b_2)\, D_{26} + (b_8 - b_1)\, D_{22}$$

$$P_6 = b_6\,(D_{12} + 2D_{66}) + (b_5 - b_{12} - 3b_3)\, D_{26} + (b_9 - b_2)\, D_{22}$$

$$P_7 = b_6\, D_{26} - b_3\, D_{22}$$

Thus the equation of motion in the transverse direction as shown in Eq. (35) can be expressed in terms of out-of-plane deflection w, the stress function F and the operators N and U_1. Note that with tracing constant $Ts = 0$, the b_i, k_i, and P_i are zero, then the operators N and U_1 reduce to

$$N = -1 \tag{27a}$$

$$U_1 = D_{11} \frac{\partial^4}{\partial x^4} + 4D_{16} \frac{\partial^4}{\partial x^3 \partial y} + 2(D_{12} + 2D_{66}) \frac{\partial^4}{\partial x^2 \partial y^2} + 4D_{26} \frac{\partial^4}{\partial x \partial y^3} + D_{22} \frac{\partial^4}{\partial y^4} \tag{36a}$$

The compatiblity equation is derived from equation (3) and can be written as

$$\varepsilon^0_{x,yy} + \varepsilon^0_{y,xx} - \varepsilon^0_{xy,xy} - w,^2_{xy} + w,_{xx}\, w,_{yy} = 0 \tag{38}$$

Making use of equations (10) and (31), one can write Eq. (38) as

$$A_{22}^* F_{,xxxx} - 2A_{26}^*F_{,xxxy} + (2A_{12}^* + A_{66}^*)F_{,xxyy} - 2A_{16}^*F_{,xyyy} +$$

$$A_{11}^* F_{,yyyy} = w_{,xy}^2 - w_{,xx} w_{,yy} \tag{39}$$

Equations (35) and (39) are the governing equations which will be solved by employing Galerkin's approach and equivalent linearization method.

DEVELOPMENT OF SOLUTIONS

Modal Equation

Consider a simply supported†, rectangular, symmetric composite plate of dimensions $a \times b \times h$ with the origin located at the center of the plate. The out-of-plane boundary conditions† are

$$x = \pm\ a/2 \quad : \quad w = M_x = \beta = 0 \tag{40}$$

$$y = \pm\ b/2 \quad : \quad w = M_y = \alpha = 0$$

For the inplane condition of zero shear stresses at the edges, the deflection function is assumed as

$$w(x,y,t) = q(t)h \cos \frac{\pi x}{a} \cos \frac{\pi y}{b} \tag{41}$$

The slope functions α and β are assumed as

---

†The analogous equations for clamped case are shown in appendix.

$$\alpha(x,y,t) = B_1\ q(t)\ \sin\frac{\pi x}{a}\ \cos\frac{\pi y}{b} \tag{42}$$

$$\beta(x,y,t) = B_2\ q(t)\ \cos\frac{\pi x}{a}\ \sin\frac{\pi y}{b} \tag{43}$$

where the constants B_1 and B_2 will be evaluated later. Substituting equation (41) in equation (39) and solving it, the stress funciton F is obtained as

$$F = F_c + F_p \tag{44}$$

in which the particular solution is given as (Ref. 6)

$$F_p = -\frac{q^2 h^2 r^2}{32}\left(F_{10}\ \cos\frac{2\pi x}{a} + F_{01}\ \cos\frac{2\pi y}{b}\right) \tag{45}$$

where

$$F_{10} = \frac{1}{A^*_{22}} \qquad F_{01} = \frac{1}{A_{11}{}^*r^4} \qquad r = a/b \tag{46}$$

The complementary solution F_c will now be obtained such that it satisfies inplane boundary conditions. For movable edges, the inplane boundary conditions are

$$x = \pm a/2 \quad : \quad F,_{xy} = \int_{-b/2}^{b/2} F,_{yy}\ dy = 0 \tag{47}$$

$$y = \pm b/2 \quad : \quad F_{,yy} = \int_{-a/2}^{a/2} F_{,xx}\, dx = 0$$

By using the above conditions, it can be shown that F_c is zero for movable inplane edges. For immovable edges, the inplane boundary conditions of zero shear stresses and zero normal displacement at the four edges are

$$x = \pm a/2 \quad : \quad F_{,xy} = \int \int (\varepsilon_x^0 - \frac{1}{2} w_{,x}^2)\, dxdy = 0$$

$$\tag{48}$$

$$y = \pm b/2 \quad : \quad F_{,xy} = \int \int (\varepsilon_y^0 - \frac{1}{2} w_{,y}^2)\, dxdy = 0$$

The complementary solution is assumed as

$$F_c = \bar{N}_y \frac{x^2}{2} + \bar{N}_x \frac{y^2}{2} + \bar{N}_{xy}\, xy \tag{49}$$

Upon using Eqs. (10) and (41) and enforcing the conditions of Eq. (48), $\bar{N}_x$, $\bar{N}_y$ and $\bar{N}_{xy}$ in Eq. (49) are obtained as

$$\bar{N}_x = \frac{q^2 h^2 \pi^2}{8(A_{11}^* A_{22}^* - A_{12}^{*2})} \left(\frac{A_{22}^*}{a^2} - \frac{A_{12}^*}{b^2} \right)$$

$$\bar{N}_y = \frac{q^2 h^2 \pi^2}{8(A_{11}^* A_{22}^* - A_{12}^{*2})} \left(\frac{A_{11}^*}{b^2} - \frac{A_{12}^*}{a^2} \right) \tag{50}$$

$$\bar{N}_{xy} = 0$$

The particular solution F_p has been obtained and given in equation (45). The total stress function, therefore, is $F = F_p + F_c$ for immovable inplane edge conditions. With the assumed deflection w given by equation (41) and the stress function F given by Eq. (44), Eq. (35) is then satisfied by applying a modified Galerkin's method:

$$
\begin{aligned}
&\int \int [N(I_1 + I_2) + U_1(w)]w\,dx\,dy \\
&\quad + \int_{-b/2}^{b/2} (M_x)_{x=-a/2}\,(\alpha)_{x=-a/2}\,dy \\
&\quad + \int_{-b/2}^{b/2} (M_x)_{x=a/2}\,(\alpha)_{x=a/2}\,dy \\
&\quad + \int_{-a/2}^{a/2} (M_y)_{y=-b/2}\,(\beta)_{y=-b/2}\,dx \\
&\quad + \int_{-a/2}^{a/2} (M_y)_{y=b/2}\,(\beta)_{y=b/2}\,dx = 0
\end{aligned}
\tag{51}
$$

which yields a modal equation of the form

$$\ddot{q} + \omega_0^2\, q + \lambda\, q^3 = \frac{p(t)}{m} \tag{52}$$

with

$$\omega_o^2 = \frac{\pi^2 h}{16m} [D_{11} (\frac{\pi}{a})^4 + 2 (D_{12} + 2 D_{66}) (\frac{\pi}{a})^2 (\frac{\pi}{b})^2 + D_{22} (\frac{\pi}{b})^4 \tag{53}$$

$$- P_1 (\frac{\pi}{a})^6 - P_3 (\frac{\pi}{a})^4 (\frac{\pi}{b})^2 - P_5 (\frac{\pi}{a})^2 (\frac{\pi}{b})^4 - P_7 (\frac{\pi}{b})^6]$$

$$\lambda = \lambda_p + \lambda_c \tag{54}$$

$$\lambda_p = \frac{\pi^4 h}{16\rho b^4} (F_{10} + F_{01}) \tag{55}$$

$$\lambda_c = \frac{\pi^4 h}{8\rho a^4} \left(\frac{A_{22}^* - 2A_{12}^* r^2 + A_{11}^* r^4}{A_{11}^* A_{22}^* - A_{12}^{*2}} \right) \tag{56}$$

and m is the modal mass, it is given by

$$m = \frac{\rho h^2 \pi^2}{16} [1 - k_1 (\frac{\pi}{a})^4 - k_3 (\frac{\pi}{a})^2 (\frac{\pi}{b})^2 - k_5 (\frac{\pi}{b})^4 + k_6 (\frac{\pi}{a})^2$$

$$+ k_8 (\frac{\pi}{b})^2] \tag{57}$$

Note that with $T_s = 0$, k_i and P_i are zero and Eqs. (53) and (57) reduce to

$$\omega_0^2 = \frac{\pi^2 h}{16m} \left[D_{11} \left(\frac{\pi}{a}\right)^4 + 2\,(D_{12} + 2\,D_{66})\left(\frac{\pi}{a}\right)^2 \left(\frac{\pi}{b}\right)^2 + D_{22}\left(\frac{\pi}{b}\right)^4 \right] \tag{58}$$

$$m = \frac{\rho h^2 \pi^2}{16} \tag{59}$$

where ω_0 is linear radian frequency, λ_p is nonlinearity coefficient, λ_c is an addition to the nonlinearity coefficient due to immovable inplane edge conditions and m is the modal mass. For movable inplane edge conditions $\lambda_c = 0$. For linear small deflection theory, $\lambda = 0$. Equation (52) represents a single-mode, undamped, large amplitude modal equation with transverse shear effects taken into account (T_s=1). This nonlinear modal equation will be solved by employing the method of equivalent linearization.

Random Response

It is known that damping has significant effects on the response of structures. Therefore, the precise determination of the damping coefficient of a structure should be emphasized. The values of damping ratio ζ ($=c/c_{cr}$) generally range from 0.005 to 0.05 for the common type of composite panel used in aircraft construction (Refs. 12, 20, 22). Once the damping ratio is determined from experiment or from existing data, Eq. (52) can be expressed in a general form as

$$\ddot{q} + 2\zeta\,\omega_0 \dot{q} + \omega_0^2 q + \lambda q^3 = \frac{p(t)}{m} \tag{60}$$

The method of equivalent linearization is used to obtain an approximate mean-square amplitude of Eq. (60).

The basic idea of the equivalent linearization method (Refs. 39, 40) is that the approximate response can be obtained from the linearized equation

$$\ddot{q} + 2\zeta\omega_0\dot{q} + \Omega^2 q = \frac{p(t)}{m} \tag{61}$$

where Ω is an equivalent linear or nonlinear frequency. The error of linearization, a random process, is

$$\delta = (\omega_0^2 - \Omega^2)q + \lambda q^3 \tag{62}$$

which is simply the difference between Eq. (60) and Eq. (61). The method of attack is to minimize this error in a suitable way. The classical choice is to minimize the mean-square error $E[\delta^2]$, that is

$$\frac{\partial}{\partial(\Omega^2)} E[\delta^2] = 0 \tag{63}$$

If the acoustic pressure excitation $p(t)$ is stationary Gaussian, is ergodic, and has a zero mean, then the approximate displacement q computed from the linearized Eq. (61), is also Gaussian and approaches stationarity; this result is due to the fact that the panel motion is stable. Substituting Eq. (62) into Eq. (63) and interchanging the order of differentiation and expectation yields

$$(\omega_0^2 - \Omega^2)\, E[q^2] + \lambda\, E[q^4] = 0 \tag{64}$$

which leads to the relatonship between the equivalent linear frequency and the mean-square displacement as

$$\Omega^2 = \omega_0^2 + 3\lambda\, E[q^2] \tag{65}$$

where $E[q^2]$ is the mean-square maximum deflection of the laminated composite plate.

The mean-square response of the modal amplitude from Eq. (61) is

$$E[q^2] = \int_0^\infty S(\omega)\,|H(\omega)|^2\, d\omega \tag{66}$$

where $S(\omega)$ is the pressure spectral density (PSD) function of the excitation $p(t)$. The frequency response function $H(\omega)$ is given by

$$H(\omega) = \frac{1}{m(\Omega^2 - \omega^2 + 2i\zeta\omega_0\omega)} \tag{67}$$

For lightly damped $(\zeta < .05)$ structures, the frequency response curve will be highly peaked at the equivalent linear frequency Ω (not at ω_0 as in the small deflection linear theory). Integration of Eq. (66) can be greatly simplified when the spectral density of the excitation is slowly varying in the neighborhood of Ω, and $S(\Omega)$ can be treated as constant in the frequency band surrounding this nonlinear resonance peak Ω; then Eq. (66) yields

$$E[q^2] = \frac{\pi S(\Omega)}{4m^2 \zeta \omega_0 \Omega^2} \tag{68}$$

To convert the PSD function from $(Pa)^2$/radian to $(Pa)^2$/Hz, substitute

$$\Omega = 2\pi f$$

$$S(\Omega) = \frac{S(f)}{2\pi} \tag{69}$$

into Eq. (68); then the mean-square deflection becomes

$$E[q^2] = \frac{S(f)}{8m^2 \zeta \omega_0 \Omega^2} \tag{70}$$

The PSD function $S(f)$ has the units $(Pa)^2$/Hz or $(psi)^2$/Hz.

Slope Functions

In deriving the governing equations of motion, the slope functions α and β were eliminated as such, and for the determination of linear and nonlinear frequencies and the mean-square displacement, the slope functions need not be known. But for the determination of strains and stresses, slope functions α and β are to be known. The boundary conditions for a simply supported plate are given by Eq. (40). The slope functions given in Eqs. (42) and (43) satisfy the boundary conditions. The constants B_1 and B_2 are determined by applying Galerkin method. Using Eqs. (25), (26), and (41) and with slope functions α and β as the weighting functions, one can get:

$$\int_{-b/2}^{b/2} \int_{-a/2}^{a/2} [N(\alpha) - M(w_{,x}) + K(w_{,y})] \, \alpha \, dxdy = 0 \tag{71}$$

$$\int_{-b/2}^{b/2} \int_{-a/2}^{a/2} [N(\beta) - J(w_{,y}) + L(w_{,x})] \, \beta \, dxdy = 0 \tag{72}$$

From integrating Eqs. (71) and (72), one can find that the constants B_1 and B_2 of the slope functions are found to be

$$B_1 = \frac{\frac{h\pi}{a}\left[1 + b_{10}\left(\frac{\pi}{a}\right)^2 - (b_5 - b_{12})\left(\frac{\pi}{b}\right)^2\right]}{1 - k_1\left(\frac{\pi}{a}\right)^4 - k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 - k_5\left(\frac{\pi}{b}\right)^4 + k_6\left(\frac{\pi}{a}\right)^2 + k_8\left(\frac{\pi}{b}\right)^2} \tag{73}$$

$$B_2 = \frac{\frac{h\pi}{b}\left[1 + b_3\left(\frac{\pi}{b}\right)^2 - (b_8 - b_1)\left(\frac{\pi}{a}\right)^2\right]}{1 - k_1\left(\frac{\pi}{a}\right)^4 - k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 - k_5\left(\frac{\pi}{b}\right)^4 + k_6\left(\frac{\pi}{a}\right)^2 + k_8\left(\frac{\pi}{b}\right)^2} \tag{74}$$

Note that the constants B_1 and B_2 depend on plate dimensions and the coefficients b_i and k_i. With no transverse shear effects, $T_s = 0$, the coefficients b_i and k_i vanish and the slope functions reduce to

$$\alpha = -w_{,x} = \left(\frac{h\pi}{a}\right) q \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} \tag{75}$$

$$\beta = -w_{,y} = \left(\frac{h\pi}{b}\right) q \cos\frac{\pi x}{\alpha} \sin\frac{\pi y}{b} \tag{76}$$

It is obvious that the expressions for B_1 and B_2 given in Eqs. (73) and (74) are consistent with no transverse shear condition given in Eqs. (75) and (76).

Stress and Strain Response

The strains at any point of the laminate are given by equation (2) and by using the constitutive relationships given in Eq. (10). One can write Eq. (2) as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = [A^*] \begin{Bmatrix} F,_{yy} \\ F,_{xx} \\ -F,_{xy} \end{Bmatrix} + z \begin{Bmatrix} \alpha,_x \\ \beta,_y \\ \alpha,_y + \beta,_x \end{Bmatrix} \tag{77}$$

where z is thickness coordinate. Once the strains are known the stresses in kth layer can be determined from

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = [\bar{Q}]^{(k)} \begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} \tag{78}$$

where $\bar{Q}_{ij}$ are the transformed reduced stiffnesses for kth layer. Substituting for stress function F as given in Eq. (44) and slope functions α and β as given in Eqs. (42) and (43), a general expression for strains at any point in the structure is given as

$$\begin{Bmatrix} \varepsilon_x \\ \varepsilon_y \\ \varepsilon_{xy} \end{Bmatrix} = \begin{Bmatrix} C_{1x} \\ C_{1y} \\ C_{1xy} \end{Bmatrix} q + \begin{Bmatrix} C_{2x} \\ C_{2y} \\ C_{2xy} \end{Bmatrix} q^2 \tag{79}$$

where

$$\begin{aligned} C_{1x} &= z \quad S_1 \cos\frac{\pi y}{a}\cos\frac{\pi y}{b} \\ C_{1y} &= z \quad S_2 \cos\frac{\pi x}{a}\cos\frac{\pi y}{b} \\ C_{1xy} &= z \quad S_3 \sin\frac{\pi x}{a}\sin\frac{\pi y}{b} \end{aligned} \tag{80}$$

with

$$S_1 = \frac{\pi}{a} B_1 \; , \quad S_2 = \frac{\pi}{b} B_2 \; , \quad S_3 = -\left(\frac{\pi}{b} B_1 + \frac{\pi}{a} B_2\right) \tag{81}$$

and

$$\begin{aligned} C_{2x} &= A_{11}^* n_x + A_{12}^* n_y + A_{16}^* n_{xy} \\ C_{2y} &= A_{12}^* n_x + A_{22}^* n_y + A_{26}^* n_{xy} \\ C_{2xy} &= A_{16}^* n_x + A_{26}^* n_y + A_{66}^* n_{xy} \end{aligned} \tag{82}$$

with

$$n_x = \frac{1}{8}\left(\frac{hr\pi}{b}\right)^2 F_{01} \cos\frac{2\pi y}{b} + \frac{h^2\pi^2}{8(A_{11}^*A_{22}^*-A_{12}^{*2})}\left(\frac{A_{22}^*}{a^2} - \frac{A_{12}^*}{b^2}\right) \tag{83}$$

$$n_y = \frac{1}{8}\left(\frac{hr\pi}{a}\right)^2 F_{10} \cos\frac{2\pi x}{a} + \frac{h^2\pi^2}{8(A_{11}^*A_{22}^*-A_{12}^{*2})}\left(\frac{A_{11}^*}{b^2} - \frac{A_{12}^*}{a^2}\right)$$

$$n_{xy} = 0$$

The mean-square strain is then related to mean-square modal amplitude in a general expression as

$$E[\varepsilon^2] = C_1^2\, E[q^2] + 3\, C_2^2\, (E[q^2])^2 \tag{84}$$

where $E[q^2]$ is mean-square displacement of Eq. (70). By making use of Eq. (78), an expression similar to Eq. (79) can be obtained for stress at any point in the structure, as given by

$$\begin{Bmatrix} \sigma_x \\ \sigma_y \\ \tau_{xy} \end{Bmatrix} = \begin{Bmatrix} E_{1x} \\ E_{1y} \\ E_{1xy} \end{Bmatrix} q + \begin{Bmatrix} E_{2x} \\ E_{2xy} \\ E_{2xy} \end{Bmatrix} q^2 \tag{85}$$

where

$$\begin{Bmatrix} E_{1x} \\ E_{1y} \\ E_{1xy} \end{Bmatrix} = [\bar{Q}]^{(k)} \begin{Bmatrix} C_{1x} \\ C_{1y} \\ C_{1xy} \end{Bmatrix} \tag{86}$$

and

$$\begin{Bmatrix} E_{2x} \\ E_{2y} \\ E_{2xy} \end{Bmatrix} = [\bar{Q}]^{(k)} \begin{Bmatrix} C_{2x} \\ C_{2y} \\ C_{2xy} \end{Bmatrix} \tag{87}$$

It is obvious that a similar expression to Eq. (84) can be employed for computing mean-square stresses.

Transverse Shear Stresses

The transverse shear stresses can be obtained either by using the constitutive equations ((Eqs. (2) and (5)) or by integrating equilibrium equations (of three-dimensional elasticity in the absence of body forces) with respect to the thickness coordinate. Reddy (Ref. 33) felt that the second approach not only gives single-valued shear stresses at the interfaces but yields excellent results for all theories in comparison with the three-dimensional solutions. In view of its accuracy, inspite of the fact that the use of stress equilibrium equations in the analysis of laminated plates is quite cumbersome, the integration of equilibrium equations is used for the determination of shear stresses τ_{xz} and τ_{yz}. Thus, integrating the equilibrium equations with respect to the thickness coordinate z yields

$$\tau_{xz} = -\int_{-h/2}^{z} (\sigma_{x,x} + \tau_{xy,y})\, dz + f(x,y) \tag{88}$$

$$\tau_{yz} = -\int_{-h/2}^{z} (\tau_{xy,x} + \sigma_{y,y})\, dz + g(x,y) \qquad (89)$$

Using Eq. (77) and the inplane stress-strain relationships given in Eq. (78), Eqs. (88) and (89) can be written as

$$\tau_{xz} = \int_{-h/2}^{z} (-\{[(\bar{Q}_{11} A_{11}^* + \bar{Q}_{12} A_{12}^* + \bar{Q}_{16} A_{16}^*) \frac{\partial n_x}{\partial x} + (\bar{Q}_{11} A_{12}^* + \bar{Q}_{12} A_{22}^*$$

$$+ \bar{Q}_{16} A_{26}^*) \frac{\partial n_y}{\partial x} + (\bar{Q}_{11} A_{16}^* + \bar{Q}_{12} A_{26}^* + \bar{Q}_{16} A_{66}^*) \frac{\partial n_{xy}}{\partial x}] q^2 + z \frac{\pi}{a} [$$

$$- (\bar{Q}_{11} S_1 + \bar{Q}_{12} S_2) \sin \frac{\pi x}{a} \cos \frac{\pi y}{b} + \bar{Q}_{16} S_3 \cos \frac{\pi x}{a} \sin \frac{\pi y}{b}] q\} -$$

$$\{[(\bar{Q}_{16} A_{11}^* + \bar{Q}_{26} A_{12}^* + \bar{Q}_{66} A_{16}^*) \frac{\partial n_x}{\partial y} + (\bar{Q}_{16} A_{12}^* + \bar{Q}_{26} A_{22}^*$$

$$+ \bar{Q}_{66} A_{26}^*) \frac{\partial n_y}{\partial y} + (\bar{Q}_{16} A_{16}^* + \bar{Q}_{26} A_{26}^* + \bar{Q}_{66} A_{66}^*) \frac{\partial n_{xy}}{\partial y}] q^2$$

$$+ z \frac{\pi}{b} [- (\bar{Q}_{16} S_1 + \bar{Q}_{26} S_2) \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} + \bar{Q}_{66} S_3 \sin \frac{\pi x}{a}$$

$$\cos \frac{\pi y}{b}] q\})\, dz + f(x,y) \qquad (90)$$

$$\tau_{yz} = \int_{-h/2}^{z} (-\{[(\bar{Q}_{16} A_{11}^* + \bar{Q}_{26} A_{12}^* + \bar{Q}_{66} A_{16}^*) \frac{\partial n_x}{\partial x} + (\bar{Q}_{16} A_{12}^*$$

$$+ \bar{Q}_{26} A_{22}^* + \bar{Q}_{66} A_{26}^*) \frac{\partial n_y}{\partial x} + (\bar{Q}_{16} A_{16}^* + \bar{Q}_{26} A^*_{26} + \bar{Q}_{66} A_{66}^*) \frac{\partial n_{xy}}{\partial x}] q^2$$

$$+ z \frac{\pi}{a} [-(\bar{Q}_{16} S_1 + \bar{Q}_{26} S_2) \sin \frac{\pi x}{a} \cos \frac{\pi y}{b} + \bar{Q}_{66} S_3 \cos \frac{\pi x}{a} \sin \frac{\pi y}{b}] q\}$$

$$- \{[(\bar{Q}_{12} A_{11}^* + \bar{Q}_{22} A_{12}^* + \bar{Q}_{26} A_{16}^*) \frac{\partial n_x}{\partial y} + (\bar{Q}_{12} A_{12}^* +$$

$$\bar{Q}_{22} A_{22}^* + \bar{Q}_{26} A_{26}^*) \frac{\partial n_y}{\partial y} + (\bar{Q}_{12} A_{16}^* + \bar{Q}_{22} A^*_{26} + \bar{Q}_{26} A_{66}^*)$$

$$\frac{\partial n_{xy}}{\partial y}] q^2 + z \frac{\pi}{b} [- (\bar{Q}_{12} S_1 + \bar{Q}_{22} S_2) \cos \frac{\pi x}{a} \sin \frac{\pi y}{b} + \bar{Q}_{26} S_3$$

$$\sin \frac{\pi x}{a} \cos \frac{\pi y}{b}] q\}) \, dz + g(x,y) \tag{91}$$

where $\bar{Q}_{ij} = \bar{Q}_{ij}^{(k)}$ but for brevity the superscript k is omitted in Eqs. (90) and (91) and hereafter. Expressions for n_x, n_y and n_{xy} are given in Eq. (83) and S_i are defined in Eq. (81). Equations (90) and (91) are integrated with respect to the thickness coordinates z and rearranged to give

$$\tau_{xz} = D_{1xz} \, q + D_{2xz} \, q^2 + f(x,y) \tag{92}$$

$$\tau_{yz} = D_{1yz} \, q + D_{2yz} \, q^2 + g(x,y) \tag{93}$$

where the functions $f(x,y)$ and $g(x,y)$ have to be determined from continuity considerations and

$$D_{1xz} = 0.5z^2 \left[(F_1-F_2) \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} + (F_3-F_4) \cos\frac{\pi x}{a} \sin\frac{\pi y}{b}\right] \tag{94}$$

$$D_{2xz} = -z\left[R_1 \bar{Q}_{11} + R_2 \bar{Q}_{12} + (R_3 + R_4) \bar{Q}_{16} + R_5 \bar{Q}_{26} + R_6 \bar{Q}_{66}\right] \tag{95}$$

$$D_{1yz} = 0.5z^2\left[(F_5-F_6) \sin\frac{\pi x}{a} \cos\frac{\pi y}{b} + (F_7-F_8) \cos\frac{\pi x}{a} \sin\frac{\pi y}{b}\right] \tag{96}$$

$$D_{2yz} = -z\left[R_1 \bar{Q}_{16} + (R_2 + R_6) \bar{Q}_{26} + R_3 \bar{Q}_{66} + R_4 \bar{Q}_{12} + R_5 \bar{Q}_{22}\right] \tag{97}$$

with

$$F_1 = \frac{\pi}{a}(\bar{Q}_{11} S_1 + \bar{Q}_{12} S_2)$$

$$F_2 = \frac{\pi}{b}(\bar{Q}_{66} S_3)$$

$$F_3 = \frac{\pi}{b}(\bar{Q}_{16} S_1 + \bar{Q}_{26} S_2)$$

$$F_4 = \frac{\pi}{a}(\bar{Q}_{16} S_3) \tag{98}$$

$$F_5 = \frac{\pi}{a}(\bar{Q}_{16}\ S_1 + \bar{Q}_{26}\ S_2)$$

$$F_6 = \frac{\pi}{b}(\bar{Q}_{26}\ S_3)$$

$$F_7 = \frac{\pi}{b}(\bar{Q}_{12}\ S_1 + \bar{Q}_{22}\ S_2)$$

$$F_8 = \frac{\pi}{a}(\bar{Q}_{66}\ S_3)$$

$$\begin{Bmatrix} R_1 \\ R_2 \\ R_3 \end{Bmatrix} = [A_{ij}^*] \begin{Bmatrix} \frac{\partial n_x}{\partial x} \\ \frac{\partial n_y}{\partial x} \\ \frac{\partial n_{xy}}{\partial x} \end{Bmatrix} \tag{99}$$

$$\begin{Bmatrix} R_4 \\ R_5 \\ R_6 \end{Bmatrix} = [A_{ij}^*] \begin{Bmatrix} \frac{\partial n_x}{\partial y} \\ \frac{\partial n_y}{\partial y} \\ \frac{\partial n_{xy}}{\partial y} \end{Bmatrix} \tag{100}$$

Equations (92) and (93) provide general expressions for the transverse shear stresses at any location (x,y,z). Since the excitation is random and the interest is in determining the mean-square τ_{xz} and τ_{yz}. Before the mean-square stresses are calculated, the functions $f(x,y)$ and $g(x,y)$ are to be evaluated.

The following scheme explains the procedure for evaluating the functions $f(x,y)$ and $g(x,y)$ for a four layered symmetrical laminate with the stacking sequence of (0/90/90/0), see Fig. 2. Because of the symmmetry only two layers are considered. Numbers 1, 2, 3 refer to the thickness coordinate z. Superscripts I, II refer to layer numbers as well as lamina material properties. Considering layer I and the point z (1), one can get from Eq. (92)

$$\tau^{I}_{xz(1)} = D^{I}_{1xz(1)} q + D^{I}_{2xz(1)} q^2 + f^{I}(x,y) \tag{101}$$

Now imposing the condition that $\tau_{xz}=0$ on top and bottom of the laminate results in

$$f^{I}(x,y) = -D^{I}_{1xz(1)} q - D^{I}_{2xz(1)} q^2 \tag{102}$$

By considering the point 2 of layer I (which is an interface point for layer I and layer II), from Eq. (92) one gets

$$\tau^{I}_{xz(2)} = D^{I}_{1xz(2)} q + D^{I}_{2xz(2)} q^2 + f^{I}(x,y) \tag{103}$$

The substitution of Eq. (102) in Eq. (103) results in

$$\tau^{I}_{xz(2)} = (D^{I}_{1xz(2)} - D^{I}_{1xz(1)}) q + (D^{I}_{2xz(2)} - D^{I}_{2xz(1)}) q^2 \tag{104}$$

The condition that $\tau_{xz} = 0$ at point 1 can be expressed as

$$\tau^{I}_{xz(1)} = G_{1xz(1)}\, q + G_{2xz(1)}\, q^2 \tag{105}$$

with $G_{1xz(1)} = G_{2xz(2)} = 0.$

Now Eq. (104) can be written as

$$\tau^{I}_{xz(2)} = G_{1xz(2)}\, q + G_{2xz(2)}\, q^2 \tag{106}$$

where

$$G_{1xz(2)} = D^{I}_{1xz(2)} - D^{I}_{1xz(1)} \tag{107}$$

$$G_{2xz(2)} = D^{I}_{2xz(2)} - D^{I}_{2xz(1)} \tag{108}$$

Now considering the same point 2, but in layer II one can write

$$\tau^{II}_{xz(2)} = D^{II}_{1xz(2)}\, q + D^{II}_{2xz(2)}\, q^2 + f(\overset{II}{x},y) \tag{109}$$

since continuity consideration requires this

$$\tau^{II}_{xz(2)} = \tau^{I}_{xz(2)} \tag{110}$$

using Eqs. (106), (109) and (110), one can show that

$$\overset{II}{f(x,y)} = (G_{1xz(2)} - D^{II}_{1xz(2)})\, q + (G_{2xz(2)} - D^{II}_{2xz(2)})\, q^2 \tag{111}$$

Next considering point 3 in layer II, one gets

$$\tau^{II}_{xz(3)} = D^{II}_{1xz(3)}\, q + D^{II}_{2xz(3)}\, q^2 + f^{II}(x,y) \tag{112}$$

substituting for $f^{II}(x,y)$, Eq. (112) becomes

$$\begin{aligned}\tau^{II}_{xz(3)} = {} & \left(D^{II}_{1xz(3)} - D^{II}_{1xz(2)} + G_{1xz(2)}\right) q \\ & + \left(D^{II}_{2xz(3)} - D^{II}_{2xz(2)} + G_{2xz(2)}\right) q^2\end{aligned} \tag{113}$$

Equation (113) can be rewritten as

$$\tau^{II}_{xz(3)} = G_{1xz(3)}\, q + G_{2xz(3)}\, q^2 \tag{114}$$

Once the constants G_1xz, G_2xz are evaluated at points 1, 2, 3 etc., along thickness, the mean-square τ_{xz} can be evaluated using an expression similar to Eq. (84). In similar manner the functions $g(x,y)$ is evaluated and the constants $G1_{yz}$, $G2_{yz}$ are determined for calculating mean-square τ_{yz}.

NUMERICAL RESULTS AND DISCUSSION

Numerical results for a symmetric cross-ply plate are presented first. A four-ply square laminate (12x12 in.) with layers of equal thickness and subjected to a uniform random pressure is considered. The plate is simply supported on all four edges. For the examples presented, a representative high-modulus graphite-epoxy with the following material properties is used.

$$E_1/E_2 = 40$$

$$G_{23}/E_2 = 0.5$$

$$G_{12}/E_2 = G_{13}/E_2 = 0.6$$

$$\nu_{12} = 0.25$$

$$E_2 = 0.75\text{x}10^6 \text{ psi}$$

$$\rho = 2.4\text{x}10^{-4} \text{ lb-sec}^2/\text{in.}^4$$

Table 1 shows the nondimensional fundamental frequency $\bar{\omega}_0 = (\omega_0 a^2/h)\sqrt{\rho/E_2}$ as a function of plate length to thickness ratio (a/h). The linear small deflection plate theory with and without transverse shear deformation is used. The Navier series solutions obtained by Reddy and Phan (Ref. 41) are also given in Table 1 for comparison. It clearly indicates that the present method gives good frequency predictions.

Table 2 shows the RMS nondimensional maximum deflection $\bar{w}_{max}$, and RMS nondimensional maximum stress in the major material direction $\bar{\sigma}_1$, versus plate length to thickness ratio for the same cross-ply laminate at 130 dB (Ref. $2\text{x}10^{-5}$ N/m^2) sound spectrum level (SSL). Stress $\bar{\sigma}_1$ is measured at (0,0,h/2). Table 3 shows the nondimensional equivalent linear frequency $\bar{\Omega} = (\Omega a^2/h)\sqrt{\rho/E_2}$ versus plate length to thickness ratio. Examination of Tables 2 and 3 reveals that for moderately thick plates (a/h<20) the small deflection theory with shear deformation ($\lambda = 0$ and $T_s = 1$) and for thin plates (a/h>50) the large deflection theory without shear deformation ($\lambda \neq 0$ and $T_s = 0$) would give accurate predictions as indicated by agreement with the theory including both nonlinear and shear effects ($\lambda \neq 0$ and $T_s = 1$) on maximum deflections, inplane stresses and equivalent linear or nonlinear frequencies. This is clearly evident from Figs. 3 to 5, where the values given in Tables 2 and 3 for RMS $\overline{w}_{max}$, RMS stresses and frequency $\overline{\Omega}$ are shown plotted against a/h.

The RMS nondimensional transverse shear stresses $\overline{\sigma}_4$ and $\overline{\sigma}_5$ are determined by integrating three dimensional equilibrium equations as explained earlier. $\overline{\sigma}_4$ is measured at (0,b/2,0) and $\overline{\sigma}_5$ is measured at (a/2,0,0). Table 4 shows $\overline{\sigma}_4$ and $\overline{\sigma}_5$ values calculated for different a/h ratios at 130 dB SSL. Figure 6 shows the transverse shear stresses $\overline{\sigma}_4$ and $\overline{\sigma}_5$ across the plate thickness for the laminate under consideration with a/h = 10 and at 130 dB sound spectrum level. It is evident from Fig. 6, that for thick laminates, the plate theory without transverse shear over predicts the RMS transverse shear stresses compared to plate with shear deformation.

Figures 7, 8 and 9 show the RMS (maximum deflection/h), RMS maximum normal stress σ_1 and equivalent linear frequency Ω, respectively, for the simply supported four-layer cross-ply square plate with a/h = 200 at sound spectrum level varying from 90 to 130 dB (ref. 2×10^{-5} N/m^2). Results shown are using the four formulations discussed earlier and there is no appreciable difference between the results using theories with and without transverse shear. The linear and nonlinear solutions agree at low values of SSL, but disagree at high values. For acoustic excitations of sound spectrum levels less than 90 dB, the small deflection assumption will give good predictions for the composite panels studied as the linear and nonlinear solutions coalesce. At 130 dB SSL, however, the small deflection theory predicts that the plate would deflect to a value of 6.6 times of plate thickness; whereas, the large deflection theory (with or without shear) gives the much smaller value of 1.6h. This smaller value seems more reasonable, intuitively. Similarly, for the RMS stresses and equivalent linear frequency, at high SSL values (for thin laminates) the small deflection theory over predicts the RMS deflection and stresses, and under predicts the frequency as compared with large deflection plate theory.

Finally, the nondimensionalized fundamental frequencies $\overline{\omega}_0$ for a symmetrical angle-ply ($\theta=\pm45$) square plate (12x12in.) for different a/h ratios and number of layers are shown in Table 5. The material is graphite-epoxy and the material properties are those that were given in Ref. 42. The nondimensionalized frequencies compare very well with the values given in Ref. 42.

SUMMARY AND CONCLUSIONS

The main objective of this study is to predict mean-square inplane stresses and transverse shear stresses that develop in symmetrical composite laminates when they are subjected to acoustic excitation. For moderately thick laminates, the transverse shear effects are considerable. In this report, equations of motion are developed which include geometric large amplitude nonlinear effects (von Karman theory). The transverse shear deformation effects are included. By various operations, the slope functions are eliminated from the equations of motion, which are expressed in terms of stress function F and displacement function w. These equations can be considered as an extension of von Karman's nonlinear equations of plates.

In view of the complexity of the equations, a single-mode Galerkin procedure is employed to obtain a nonlinear modal amplitude equation for the forced vibration of plate. The excitation is assummed to be stationary, ergodic and Gaussian with zero-mean. The equivalent linearization method is employed. The fundamental frequencies that are obtained for both cross-ply and angle-ply laminates found to be in good agreement with those that are available in the literature. Root-mean-square deflections and RMS inplane stresses are calculated. Using the three-dimensional equilibrium equations and continuity considerations, the RMS transverse shearing stresses in the laminate are determined.

The effects of transverse shear are considerable if the plate lengths are less than 20 times the thickness ($a/h<20$). These effects should not be ignored in moderately thick and thick plates and small-deflection theory with shear deformation would give accurate predictions of maximum deflection, frequency and inplane stresses. For thin plates ($a/h>50$), the linear and nonlinear solutions agree at low values of SSL, but disagree at high values. The small deflection theory over predicts the RMS deflection and stresses, and under predicts the frequency as compared with large deflection plate theory. The large deflection theory with transverse shear effects neglected would give accurate predictions for thin panels at high SSL values. For a particular value of a/h, therefore, one of the three simpler theories can be chosen that provides accuracy equal to the more cumbersome theory that includes both shear and large deflection effects.

The prediction of transverse shearing stresses is required for understanding sonic fatigues of composite laminates, especially the unique inter-laminar failures.

ACKNOWLEDGEMENT

The work was supported by NASA-Langley Research Center under Grant NAS-1-17993-22. The authors, Chuh Mei and C. B. Prasad, would like to thank their program monitor Dr. John S. Mixson of Acoustics Division for many technical suggestions and discussions.

REFERENCES

1. Mei, C. and Wolfe, H. F., "On Large Deflection Analysis in Acoustic Fatigue Design," in Random Vibration-Status and Recent Developemtns, The Stephen Harry Crandall Festschrift, edited by I. Elishafkoff and R. H. Lyon, Elsevier Science, 1986 pp. 279-302.

2. Mei, C. and Paul, D. B., "Large Deflection Multimode Response of Clamped Rectangular Panels to Acoustic Excitation," AIAA J., Vol. 24, April 1986, pp. 643-648.

3. Gray, C. E., Jr., Decha-Umphai, K. and Mei, C., "Large Deflection, Large Amplitude Vibrations and Random Response of Symmetrically Laminated Rectangular Plates," J. Aircraft, Vol. 22, Nov. 1985, pp. 929-930.

4. Maymon, G., "Response of Geometrically Nonlinear Elastic Structures to Acoustic Excitation-An Engineering Oriented Computational Procedure," Computers and Structures, Vol. 18, No. 4, 1984, pp. 647-652.

5. Mei, C. and Wentz, K. R., "Large Amplitude Random Response of Angle-Ply Laminated Composite Plates," AIAA J., Vol. 20, Oct. 1982, pp. 1450-1458.

6. Wentz, K. R., Paul, D. B. and Mei, C., "Large Deflection Random Response of Symmetric Laminated Composite Plates," Shock and Vib. Bulletin, Bulletin 52, May 1982, pp. 99-111.

7. Holehouse, I., "Sonic Fatigue Design Techniques for Advanced Composite Aircraft Structures," Wright-Patterson Air Force Base, Ohio, AFWAL-TR-80-3019, April 1980. Also Ph.D. Dissertation, Univrsity of Southampton, 1984.

8. Mei, C., "Response of Nonlinear Structural Panels Subjected to High Intensity Noise," Wright-Patterson Air Force Base, Ohio, AFWAL-TR-80-3018, March 1980.

9. Rudder, F. F., Jr. and Plumblee, H. E., Jr., "Sonic Fatigue Design Guide for Military Aircraft," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-74-112, May 1975.

10. Volmir, A. S., "The Nonlinear Dynamics of Plates and Shells," Wright-Patterson Air Force Base, Ohio, Foreign Technology Div., AD-781338, April 1974, Chap. X.

11. Fox, H. L., Smith, P. W. Jr., Pyle, R. W., and Nayak, P. R., "Contributions to the Theory of Randomly Forced, Nonlinear, Multiple-Degree-of-Freedom, Coupled Mechanical Systems," Wright Patterson Air Force Base, Ohio, AFFDL-TR-72-45, Aug. 1973.

12. Thomson, A. G. R. and Lambert, R. F., "Acoustic Fatigue Design Data," NATO Advisory Group for Aeronautics Research and Development, AGARD-AG-162, Part I and II. 1972.

13. Jacobs, L. D. and Lagerquist, D. R., "Finite Element Analysis of Complex Panel to Random Loads," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-68-44, Oct. 1968.

14. Soovere, J., "The Effect of Acoustic-Thermal Environment on Advanced Composite Fuselage Panels," AIAA/ASME/ASCE/AHS 24th Structures, Structural Dynamics and Materials Conference, Lake Tahoe, NV, May 1983, pp. 466-472.

15. Soovere, J., "Sonic Fatigue Testing an Advanced Composite Aileron," J. Aircraft, Vol. 19, April 1982, pp. 304-310.

16. Mei, C. and Wentz, K. R., "Analytical and Experimental Nonlinear Response of Rectangular Panels to Acoustic Excitation," AIAA/ASME/ASCE/AHS 23rd Structures, Structural Dynamics and Materials Conference, New Orleans, LA, May 1982. pp. 514-520.

17. White, R. G., "Comparison of the Statistical Properties of the Aluminum Alloy and CFRP Plates to Acoustic Excitation," Composites, Oct. 1978, pp. 251-258.

18. Wentz, K. R. and Wolfe, H. F., "Development of Random Fatigue Data for Adhesively Bonded and Weldbonded Structures Subjected to Dynamics Excitation," ASME J. Engineering Materials and Technology, Vol. 100, Jan. 1978, pp. 70-76.

19. Jacobson, M. J., "Sonic Fatigue Design Data for Bonded Aluminum Aircraft Structures," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-77-45, June 1977.

20. Van der Heyde, R. C. W. and Wolf, N. D., "Comparison of the Sonic Fatigue Characteristics of Four Structural Designs," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-76-66, Sept. 1976.

21. Van der Heyde, R. C. W. and Smith, D. L., "Sonic Fatigue Resistance of Skin-Stringer Panels," Wright-Patterson Air Force Base, Ohio, AFFDL-TM-73-149-FYA, April 1974.

22. Jacobson, M. J., "Advanced Composite Joints: Design and Acoustic Fatigue Characteristics," Wright-Patterson Air Force Base, Ohio, AFFDL-TR-71-126, April 1972.

23. Bolle, L., "Contribution an Probleme Lineare de Flexion d'une Pleaque Elastique," Bull. Technique de la Suisse Romade, Parts 1 and 2, Vol. 73, 1947, pp. 281-285: 293-298.

24. Hildebrand, F. B., Reissner, E. and Thomas, G. B., "Note on the Foundations of the Theory of Small Displacement of Orthotropic Shells," IVACA Technical Note No. 1833, Mar. 1949.

25. Hencky, H., Uber die Berucksichitung der Schubverzerrungen in ebenen Platten," Ing-Arch,. Vol. 16, 1947.

26. Mindlin, R. D., "Influence of Rotary Inertia and Shear on Flexural Motions of Isotropic, Elastic Plates," ASME J. APPLIED MECHANICS, Vol. 18, 1951.

27. Librescu, L., Elastostatics and Kinetics of Anisotropic and Heterogeneous Shell-Type Structures, Noordhoff, Leyden, GDR, 1975.

28. Whitney, J. M. and Pagano, N. J., "Shear Deformation in Heterogeneous Anisotropic Plates, ASME J. Applied Mechanics, Vol. 37, Dec. 1970, pp. 1031-1036.

29. Whitney, J. M. and Sun, C. T., "A High Order Theory for Extensional Motion of Laminated Composites," J. Sound and Vibration, Vol. 30, Sept. 1973, pp. 85-97.

30. Lo, K. H. Christensen, R. M. and Wu, E. M., A Higher-Order Theory of Plate Deformation Part 2; Laminated Plates," ASME J. Applied Mechanics, Vol. 44, 1977, pp. 669-676.

31. Levinson, M., "An Accurate Simple theory of the Statics and Dynamics of Elastic Plates," Mechanics Research Communications, Vol. 7, 1980. pp. 343-350.

32. Murthy, M. V. V., "An Improved Transverse Shear Deformation Theory for Laminated Anistropic Plates," NASA Technical Paper 1903. Nov. 1981.

33. Reddy, J. N., "A Simple Higher-Order Theory for Laminated Composite Plates," ASME J. Applied Mechanics, Vol. 45, December 1984, pp. 745-752.

34. Reddy, J. N., "A Refined Nonlinear Theory of Plates with Transverse Shear Deformation," Int. J. Solids and Struct., Vol. 20, 1984, pp. 881-896.

35. Sathyamoorthy, M. and Chia, C. Y., "Nonlinear Vibration of Anisotropic Rectangular Plates Including Shear and Rotatory Inertia," Fibre Science and Technology, Vol. 13,. 1980, pp. 337-361.

36. Sathyamoorthy, M. and Chia, C. Y., "Effects of Transverse Shear and Rotatory Inertia on Large Amplitude of Anisotropic Skew Plates, Part I Theory Part II - Numerical Results," ASME J. Applied Mechanics, Vol. 47, 1980, p.p. 128-138.

37. Sivakumaran, K. S. and Chia, C. Y., "Nonlinear Vibration of Generally Laminated Anisotropic Thick Plates," Ingenieur-Anchiv, Vol. 54, 1984, pp. 220-231.

38. Reissner, E., "The Effect of Transverse Shear Deformation on the Bending of Elastic Plates," ASME J. Applied Mechanics, Vol. 12, 1945, A69-A77.

39. Caughy, T. K., "Equivalent Linearization Techniques," JASA, Vol. 35, 1963, pp. 1706-1711.

40. Atalik, T. S. and Utku, S., "Stochastic Linearization of Multi-Degree-of Freedom Nonlinear Systems," Earthquake Engineering and Structural Dynamics, Vol. 4, 1976, pp. 411-420.

41. Reddy, J. N. and Phan, N. D., "Stability and Vibration of Isotropic, Orthotropic and Laminated Plates According to a Higher-Order Shear Deformation Theory, "J. Sound and Vibration, Vol. 98, 1985, pp. 157-170.

42. Sivakumaran, K. S., 'A Refined Theory of Generally Laminated Plates', Ph.D. Thesis, University of Calgary, Oct. 1982.

Table 1. Nondimensional Fundamental Frequency $(\omega_0 a^2/h)\sqrt{\rho/E_2}$ of a Simply Supported Four-Layer Cross-Ply Square Plate.

$\frac{a}{h}$	No Shear		Shear	
	Navier[41] Solution	Present Result	Navier[41] Solution	Present Result
5	18.215	18.891	10.820	11.554
10	18.652	18.891	15.083	15.662
20	18.767	18.891	17.583	17.872
50	18.799	18.891	18.590	18.715
100	18.804	18.891	18.751	18.846
200	-	18.891	-	18.880
400	-	18.891	-	18.888
1000	-	18.891	-	18.890

Table 2. Nondimensional RMS Maximum Deflection $10(\text{RMS } w)E_2h^3/(a^4\sqrt{(\text{PSD})f_0})$ and RMS Maximum Stress $(\text{RMS } \sigma_1)\ h^2/(10a^2\sqrt{(\text{PSD})\ f_0})$ of a Simply Supported Four-Layer Cross-Ply Square Plate at 130 dB Sound Spectrum Level.

$\frac{a}{h}$	Small Deflection		Large Deflection	
	No Shear	Shear	No Shear	Shear
	RMS Maximum Deflection			
5	0.4026	0.2471	0.4026	0.2471
10	0.4026	0.3478	0.4026	0.3478
20	0.4026	0.3873	0.4026	0.3873
50	0.4026	0.4001	0.3993	0.3968
100	0.4026	0.4019	0.2810	0.2806
200	0.4026	0.4024	0.0973	0.0972
400	0.4026	0.4025	0.0293	0.0293
1000	0.4026	0.4026	0.0059	0.0059
	RMS Maximum Stress			
5	0.8009	0.1521	0.8009	0.1521
10	0.8009	0.4467	0.8009	0.4467
20	0.8009	0.6780	0.8008	0.6780
50	0.8009	0.7789	0.7951	0.7732
100	0.8009	0.7953	0.5923	0.5885
200	0.8009	0.7995	0.3284	0.3280
400	0.8009	0.8005	0.2788	0.2788
1000	0.8009	0.8008	0.2739	0.2739

Table 3. Nondimensional Equivalent Linear Frequency $(\Omega\ a^2/h)\sqrt{\rho/E_2}$ of a Simply Supported Four-Layer Cross-Ply Square Plate at 130 dB Sound Spectrum Level.

$\frac{a}{h}$	Small Deflection		Large Deflection	
	No Shear	Shear	No Shear	Shear
5	18.891	11.554	18.891	11.554
10	18.891	15.662	18.891	15.662
20	18.891	17.872	18.900	17.872
50	18.891	18.715	19.047	18.870
100	18.891	18.846	27.063	26.995
200	18.891	18.880	78.196	78.146
400	18.891	18.888	259.463	259.396
1000	18.891	18.890	1287.297	1287.264

Table 4. Nondimensional RMS Maximum Transverse Shear Stress (RMS τ_{yz})h/10a $\sqrt{(PSD)\ f_o}$ and (RMS τ_{xz})h/10a $\sqrt{(PSD)\ f_o}$ of a Simply Supported Four-Layer Cross-Ply Square Plate at 130 dB Sound Spectrum Level.

a/h	Small Deflection		Large Deflection	
	No Shear	Shear	No Shear	Shear
		RMS Maximum τ_{yz} ($\bar{\sigma}_4$)		
5	0.1906	0.0797	0.1906	0.0797
10	0.1906	0.1459	0.1906	0.1459
20	0.1906	0.1774	0.1906	0.1774
50	0.1906	0.1883	0.1890	0.1868
100	0.1906	0.1900	0.1330	0.1327
200	0.1906	0.1905	0.0461	0.0460
400	0.1906	0.1906	0.0139	0.0139
1000	0.1906	0.1906	0.0028	0.0028
		RMS Maximum τ_{xz} ($\bar{\sigma}_5$)		
5	0.4954	0.0966	0.4954	0.0966
10	0.4954	0.2786	0.4954	0.2786
20	0.4954	0.4203	0.4953	0.4203
50	0.4954	0.4819	0.4913	0.4779
100	0.4954	0.4919	0.3458	0.3434
200	0.4954	0.4945	0.1197	0.1195
400	0.4954	0.4951	0.0361	0.0361
1000	0.4954	0.4953	0.0073	0.0073

Table 5. Nondimensional Fundamental Frequency $(\omega_0 a^2/h)\sqrt{\rho/E_2}$ of a Simply Supported n-Layer angle-ply ($\theta=\pm45$) square plate.

		No Shear		Shear	
a/h	No. of Layers	Ref. 42	Present Result	Ref. 42	Present Result
5	3	25.82	25.82	12.78	12.21
10	3	25.82	25.82	19.38	18.89
10	5	25.82	25.82	19.23	19.06
10	7	25.82	25.82	19.19	19.12
20	3	25.82	25.82	23.62	23.40

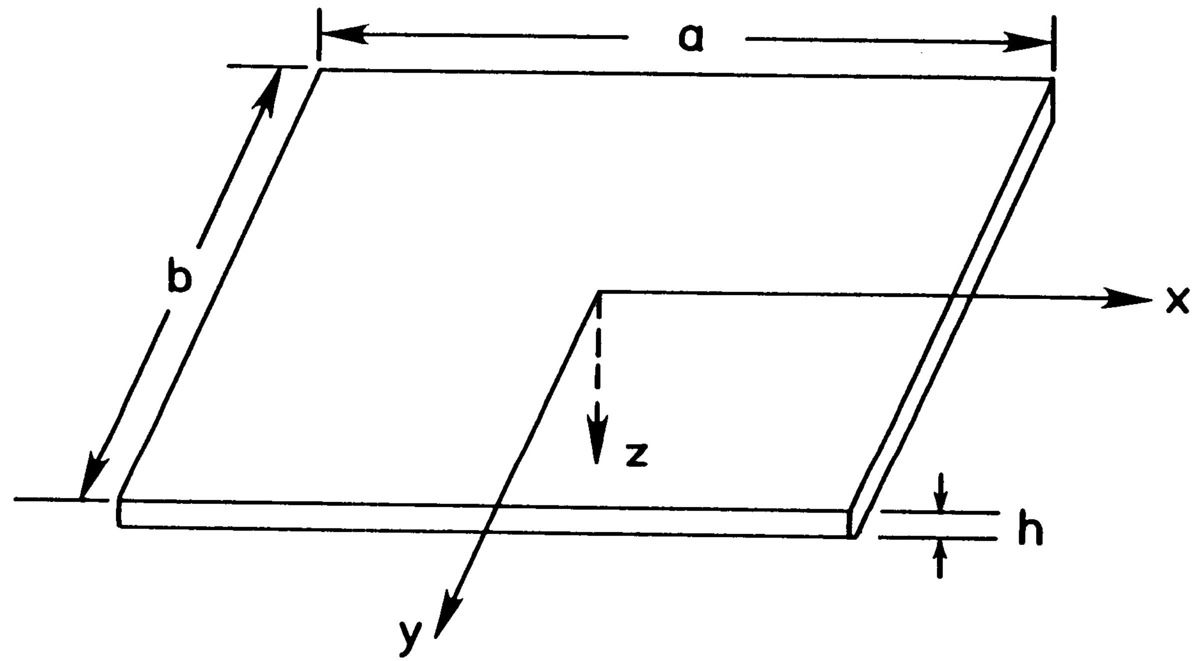

Fig. 1. Plate coordinates and geometry.

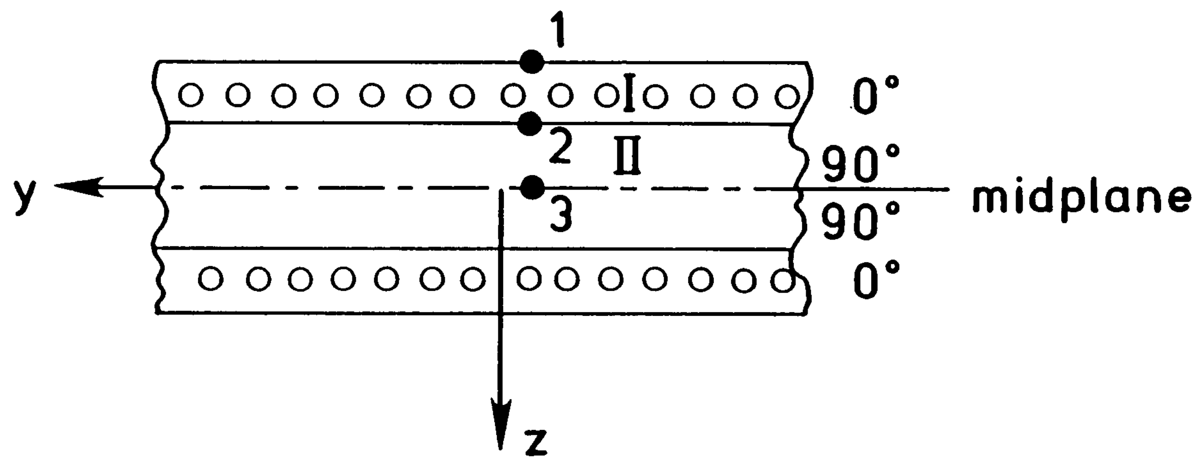

Fig. 2. Laminate cross-section.

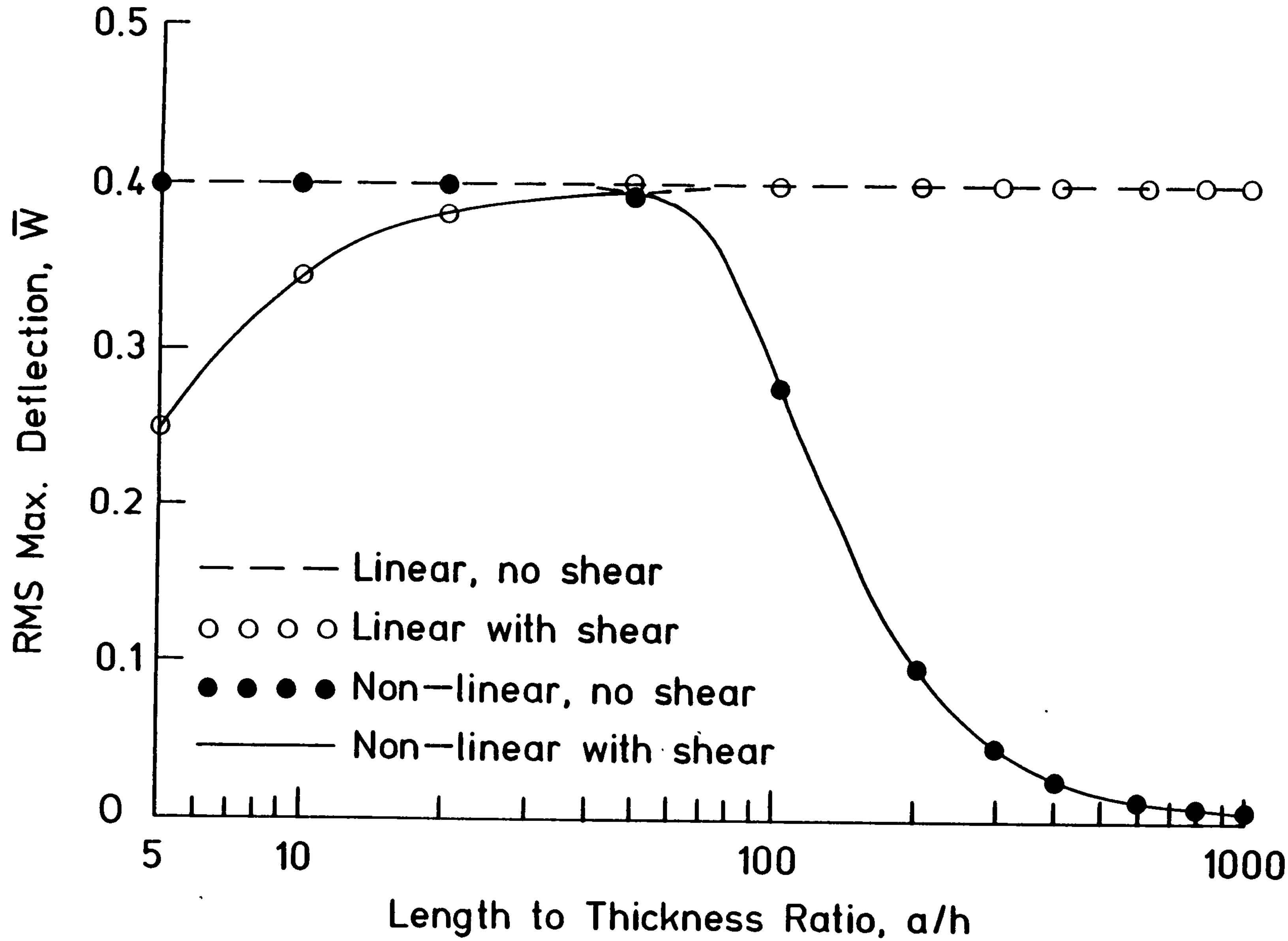

Fig. 3. Nondimensional RMS maximum deflection of a simply supported four-layer cross-ply square plate at 130 dB sound spectrum level.

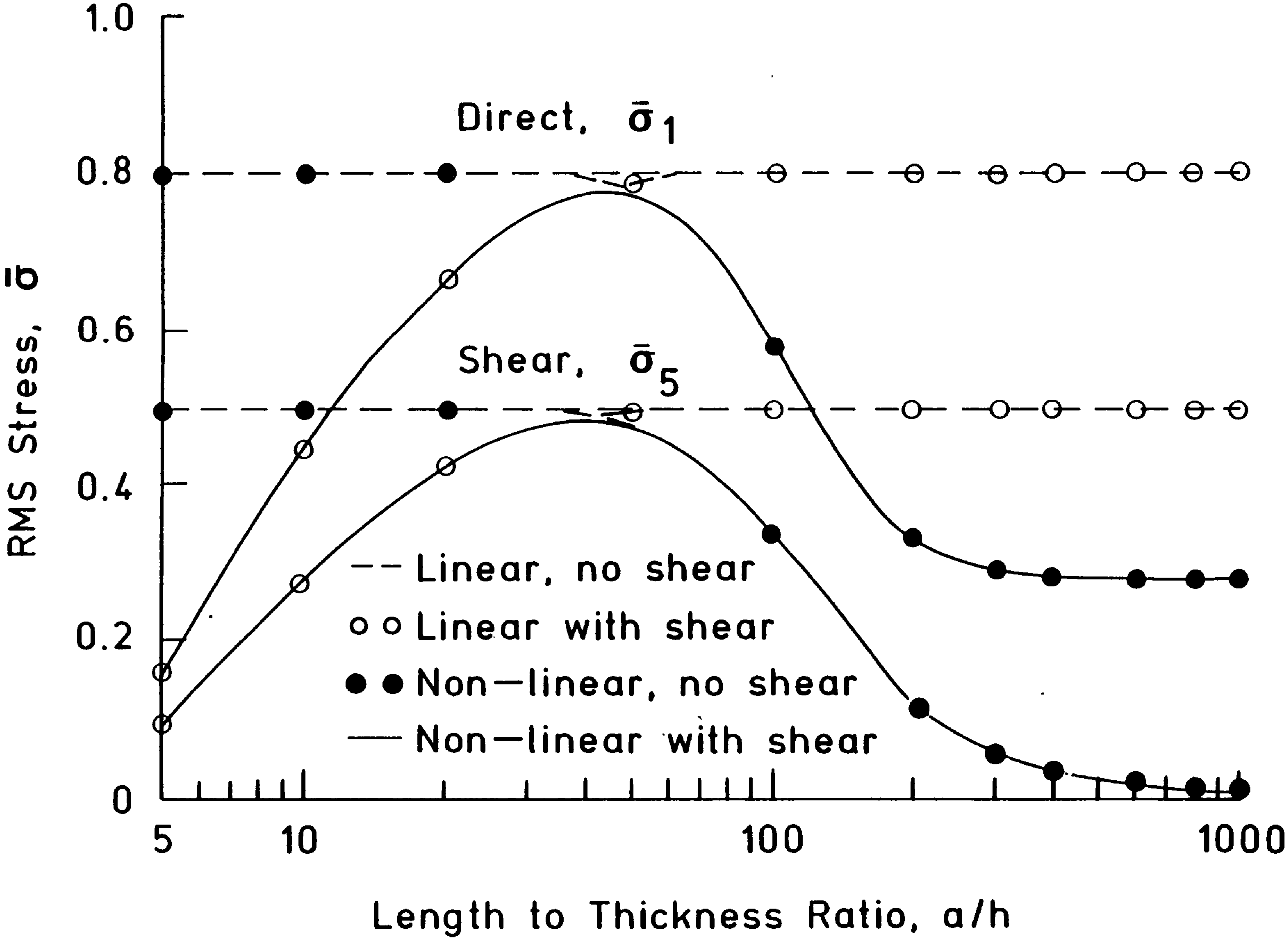

Fig. 4. Nondimensional RMS maximum stresses of a simply supported four-ply cross-ply square plate at 130 dB sound spectrum level.

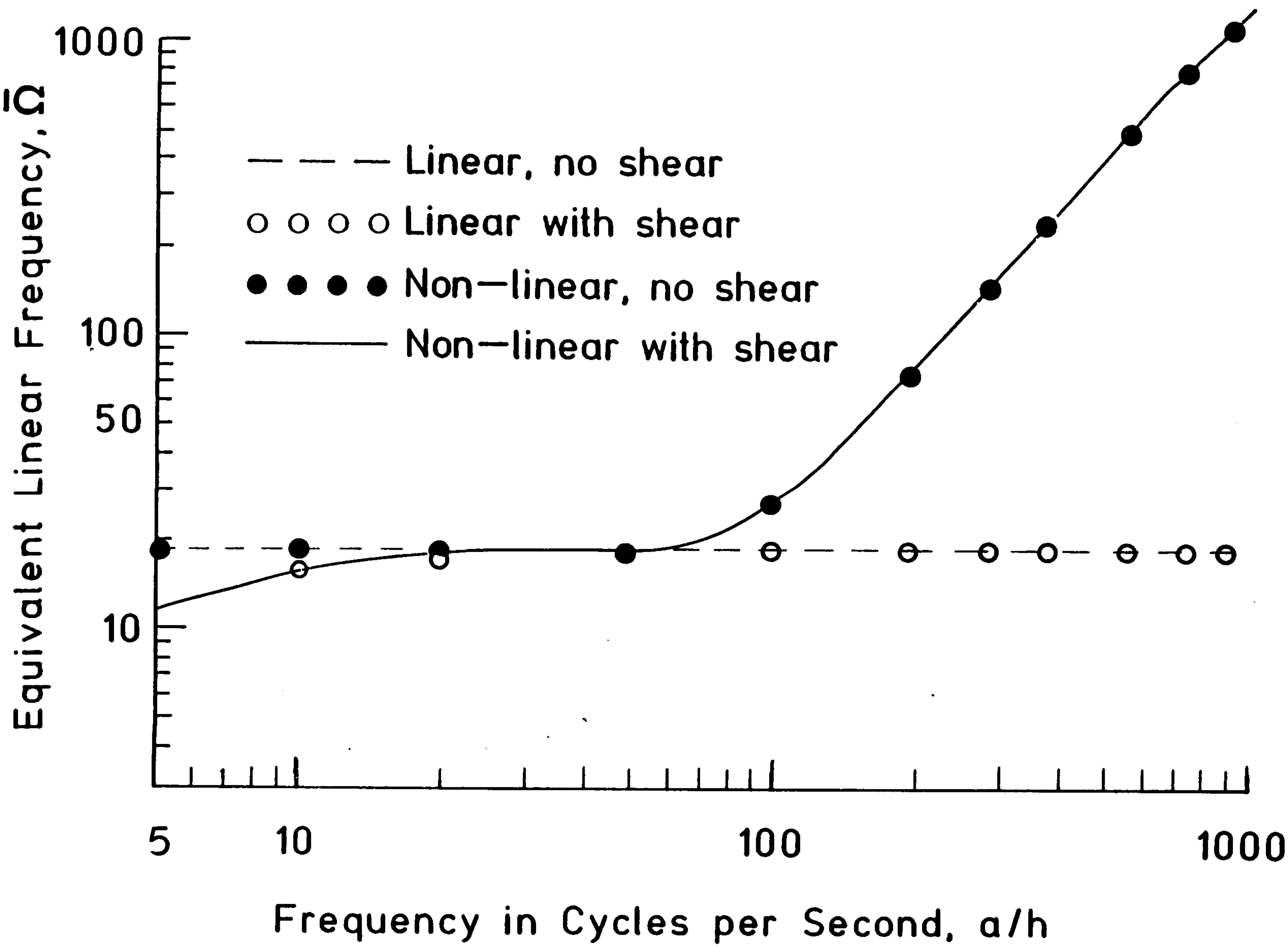

Fig. 5. Nondimensional equivalent linear frequency of a simply supported four—layer cross—ply square plate at 130 dB sound spectrum level.

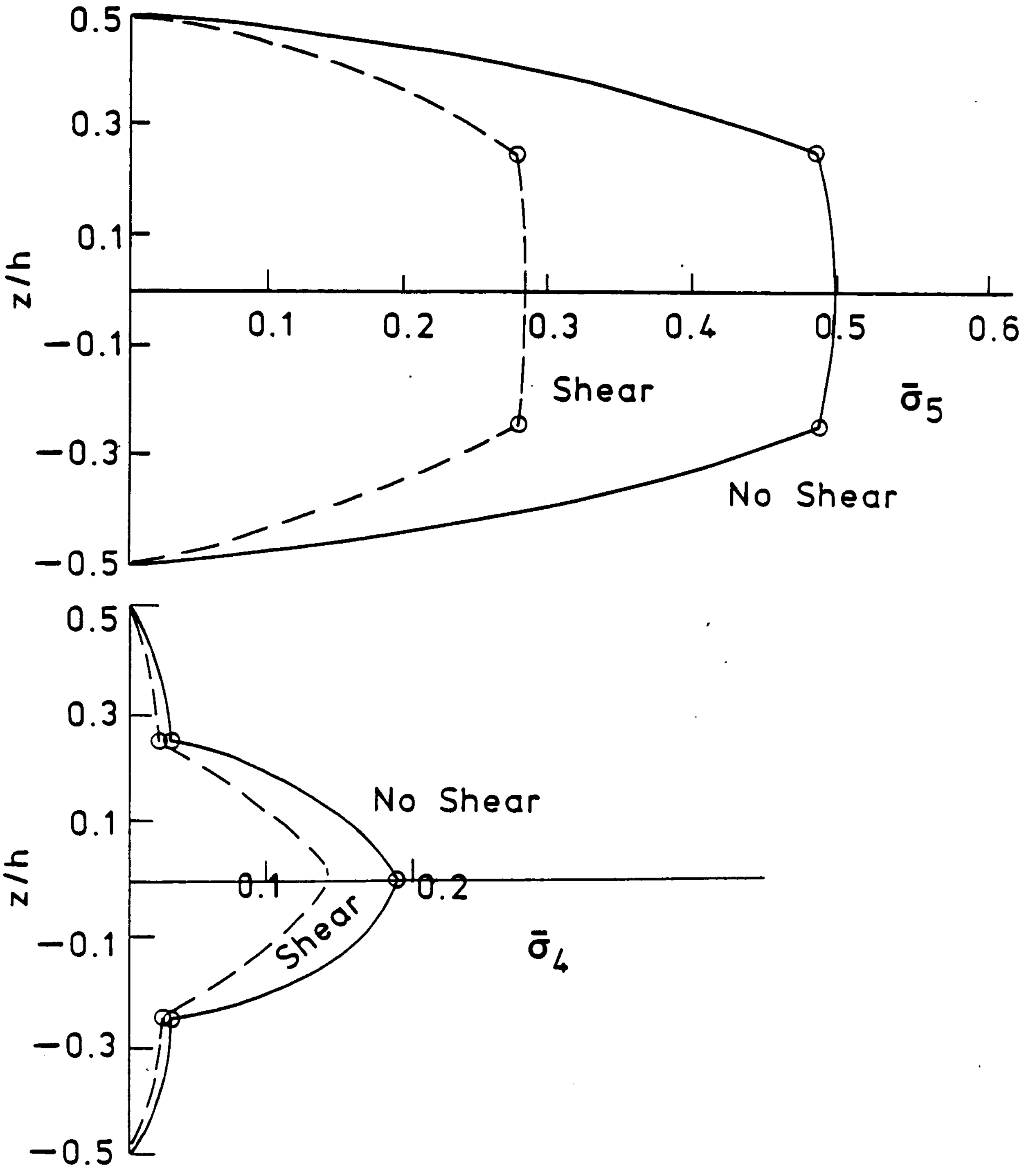

Fig. 6. Nondimensional RMS transverse shearing stresses of a simply supported four-layer cross-ply of a /h = 10 at 130 dB sound spectrum level.

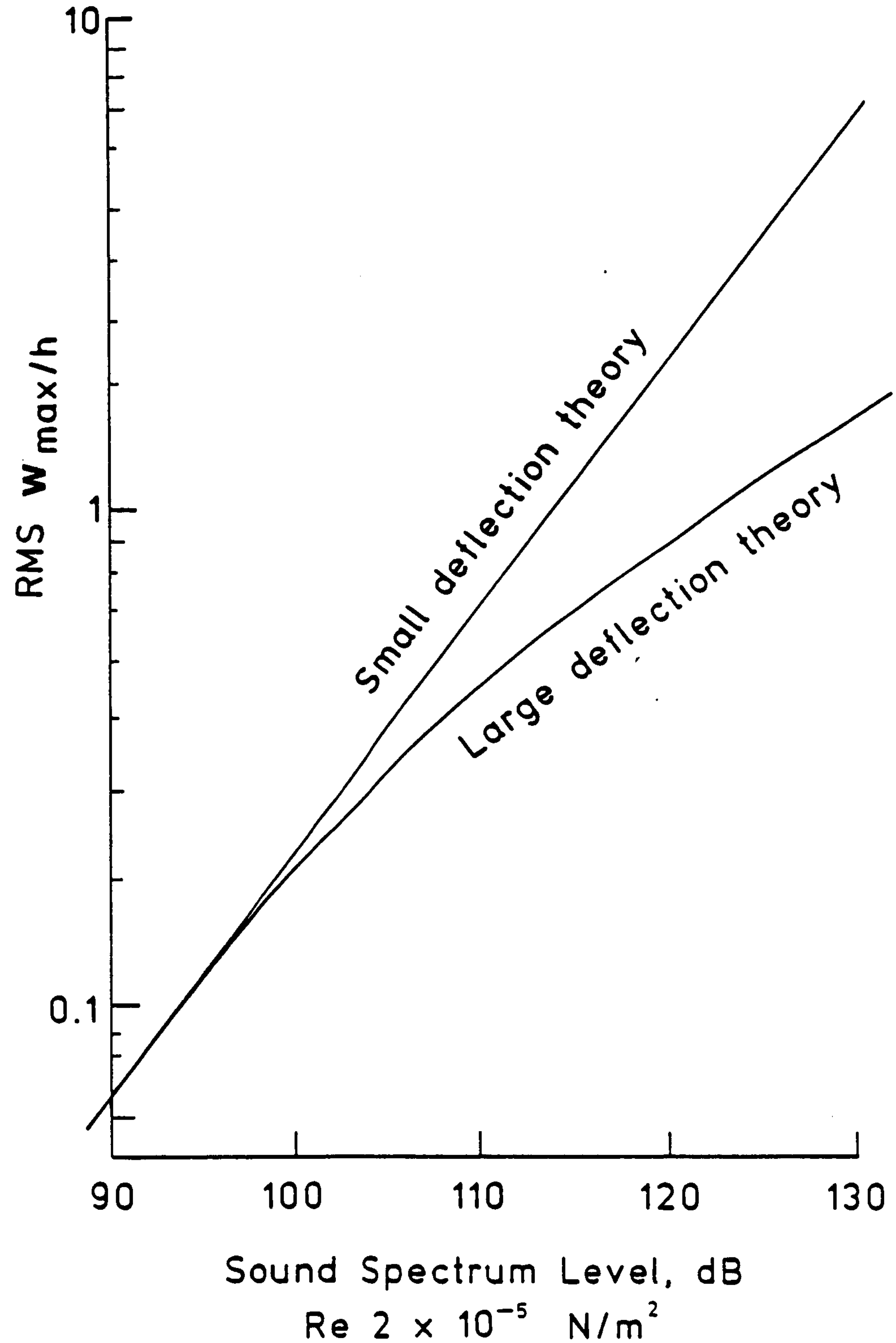

Fig. 7. RMS maximum deflection versus sound spectrum level for a simply supported four–layer cross–ply square plate with a/h = 200.

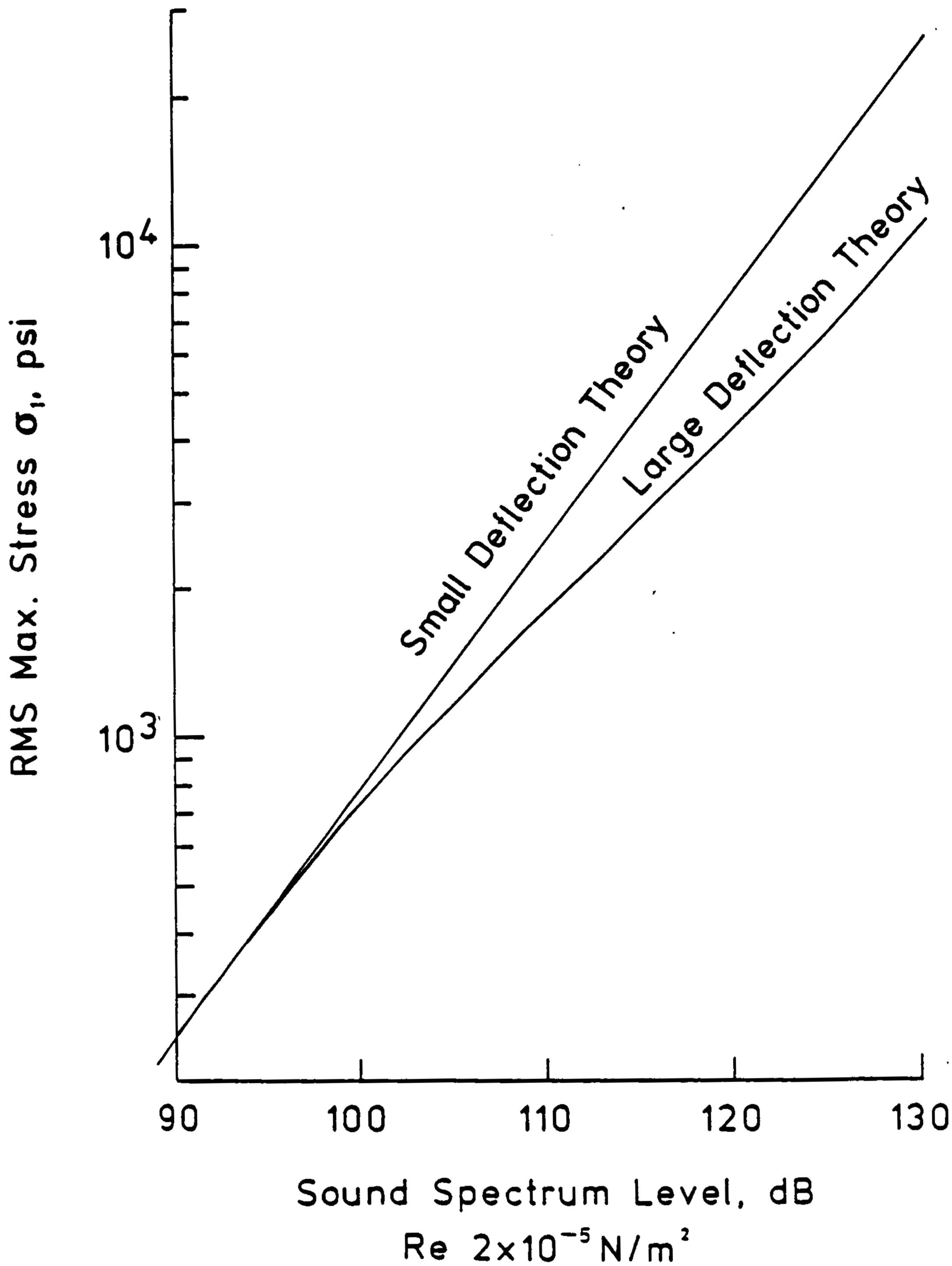

Fig. 8. RMS maximum stress versus sound spectrum level for a simply supported four-layer cross-ply square plate with $a/h = 200$.

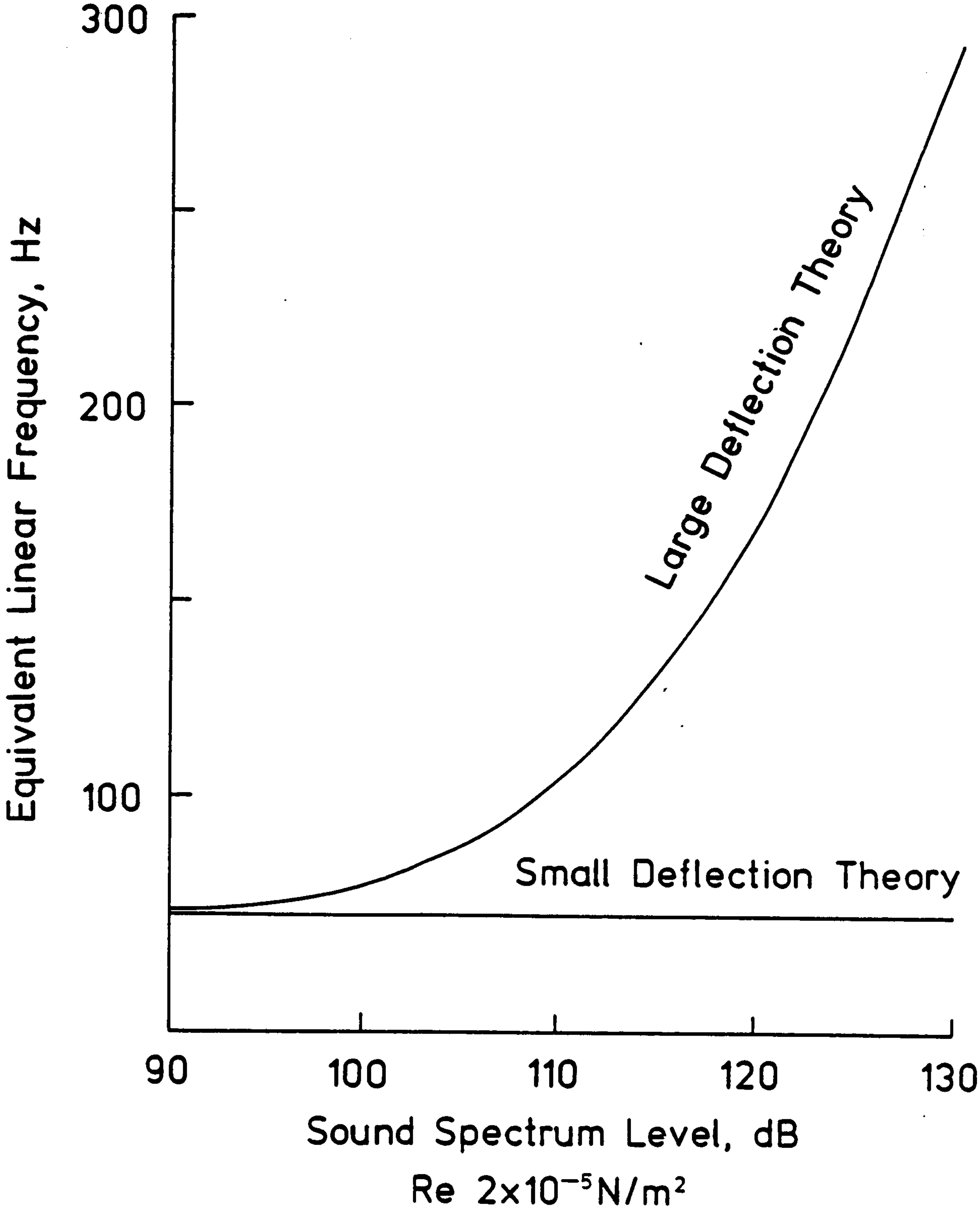

Fig. 9. Equivalent linear frequency versus sound spectrum level for a simply supported four-layer cross-ply square plate with $a/h = 200$.

APPENDIX

The boundary conditions for clamped plate are

$$x = \pm\ a/2 \ :\ w = \alpha = \beta = 0$$

$$y = \pm\ b/2 \ :\ w = \alpha = \beta = 0 \tag{A1}$$

The deflection function which satisfy the boundary conditions on all four edges is assumed as

$$w = \frac{q(t)h}{4} \left(1 + \cos \frac{2\pi x}{a}\right) \left(1 + \cos \frac{2\pi y}{b}\right) \tag{A2}$$

The stress function is of the following form

$$F = F_c + F_p \tag{A3}$$

in which the particular solution is

$$F_p = - \frac{q^2 h^2 r^2}{32} [C_{10} \cos X + C_{01} \cos Y$$

$$+ C_{11} \cos X \cos Y + C_{20} \cos 2X$$

$$+ C_{02} \cos 2Y + C_{21} \cos 2X \cos Y$$

$$+ C_{12} \cos X \cos 2Y + S_{11} \sin X \sin Y$$

$$+ S_{21} \sin 2X \sin Y$$

$$+ S_{12} \sin X \sin 2Y] \tag{A4}$$

where

$$X = \frac{2\pi x}{a}, \; Y = \frac{2\pi y}{b}, \; r = a/b \tag{A5}$$

and the constants C_{ij} and S_{ij} can be expressed in terms of the inverted extensional laminate stiffness and length-to-width ratio, r, of the panel as

$$C_{10} = 1/A^*_{22}$$

$$C_{01} = 1/(r^4 A^*_{11})$$

$$C_{11} = 2G_5/(G_5^2 - G_6^2)$$

$$C_{20} = 1/(16 A^*_{22})$$

$$C_{02} = 1/(16 A^*_{11} r^4)$$

$$C_{12} = G_3/(G_3^2 - G_4^2)$$

$$C_{21} = G_1/(G_1^2 - G_2^2)$$

$$S_{11} = -\, 2G_6/(G_5^2 - G_6^2)$$

$$S_{12} = -\, G_4/(G_3^2 - G_4^2)$$

$$S_{21} = -\, G_2/(G_1^2 - G_2^2)$$

$$G_1 = 16\ A^*_{22} + 4(2A^*_{12} + A^*_{66})r^2 + A^*_{11}\ r^4$$

$$G_2 = 16\ A^*_{26}\ r + 4\ A^*_{16}\ r^3$$

$$G_3 = A^*_{22} + 4\ (2A^*_{12} + A^*_{66})\ r^2 + 16\ A^*_{11}\ r^4$$

$$G_4 = 4\ A^*_{26}\ r + 16\ A^*_{16}\ r^3$$

$$G_5 = A^*_{22} + (2A^*_{12} + A^*_{66})\ r^2 + A^*_{11}\ r^4$$

$$G_6 = 2A^*_{26}\ r + 2A^*_{16}\ r^3 \qquad \text{(A6)}$$

The complementary solution is of the same form as given by Eq. (49) with

$$\mathcal{N}_x = \frac{3q^2 h^2 \pi^2}{32(A_{11}^* A_{22}^* - A_{12}^{*2})} \left(\frac{A_{22}^*}{a^2} - \frac{A_{12}^*}{b^2} \right) \qquad \text{(A7)}$$

$$\overline{N}_y = \frac{3q^2h^2\pi^2}{32(A_{11}^* A_{22}^* - A_{12}^{*2})}\left(\frac{A_{11}^*}{b^2} - \frac{A_{12}^*}{a^2}\right)$$

$$\overline{N}_{xy} = 0$$

The modal amplitude equation is of the same form as that of Eq. (52) with

$$\omega_0^2 = \frac{h}{m}\{3D_{11}(\frac{\pi}{a})^4 + 2(D_{12} + 2D_{66})(\frac{\pi}{a})^2(\frac{\pi}{b})^2 + 3D_{22}(\frac{\pi}{b})^4$$

$$- 4[3P_1(\frac{\pi}{a})^6 + P_3(\frac{\pi}{a})^4(\frac{\pi}{b})^2 + P_5(\frac{\pi}{a})^2(\frac{\pi}{b})^4 + 3P_7(\frac{\pi}{b})^6]\} \quad \text{(A8)}$$

and the modal mass m is given by

$$m = \frac{9\rho h^2}{16}\{1 - \frac{4}{3}[4k_1(\frac{\pi}{a})^4 + \frac{4}{3}k_3(\frac{\pi}{a})^2(\frac{\pi}{b})^2 + 4k_5(\frac{\pi}{b})^4 - k_6(\frac{\pi}{a})^2 - k_8(\frac{\pi}{b})^2]\} \quad \text{(A9)}$$

The nonlinearity coefficient is of the same form as given in Eq. (54) with

$$\lambda_p = \frac{\pi^4h^3}{16mb^4}\{C_{10}[1 - 8k_1(\frac{\pi}{a})^4 - 8k_3(\frac{\pi}{a})^2(\frac{\pi}{b})^2 - 16k_5(\frac{\pi}{b})^4 + 2k_6(\frac{\pi}{a})^2 + 4k_8(\frac{\pi}{b})^2]$$

$$+ C_{01}[1 - 16k_1(\frac{\pi}{a})^4 - 8k_3(\frac{\pi}{a})^2(\frac{\pi}{b})^2 - 8k_5(\frac{\pi}{b})^4 + 4k_6(\frac{\pi}{a})^2 + 2k_8(\frac{\pi}{b})^2]$$

$$+ C_{11}[1 - 8k_1(\frac{\pi}{a})^4 - 8k_5(\frac{\pi}{a})^2(\frac{\pi}{b})^2 + 2k_6(\frac{\pi}{a})^2 + 2k_8(\frac{\pi}{b})^2]$$

$$+(C_{20}+C_{02})\left[1-16\,k_1\left(\frac{\pi}{a}\right)^4 - 16k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 - 16k_5\left(\frac{\pi}{a}\right)^4 + 4k_6\left(\frac{\pi}{a}\right)^2 + 4k_8\left(\frac{\pi}{b}\right)^2\right]$$

$$+\frac{1}{2}\,C_{21}\left[1-16k_1\left(\frac{\pi}{a}\right)^4 - 8k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 - 8k_5\left(\frac{\pi}{b}\right)^4 + 4k_6\left(\frac{\pi}{a}\right)^2 + 2k_8\left(\frac{\pi}{b}\right)^2\right)]$$

$$+\frac{1}{2}\,C_{12}\left[1-8k_1\left(\frac{\pi}{a}\right)^4 - 8k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 - 16k_5\left(\frac{\pi}{b}\right)^4 + 2k_6\left(\frac{\pi}{a}\right)^2 + 4k_8\left(\frac{\pi}{b}\right)^2\right)]$$

$$+\ (S_{21}+S_{12})\left[4k_2\left(\frac{\pi}{a}\right)^3\left(\frac{\pi}{b}\right) + 4k_4\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)^3 - k_7\left(\frac{\pi}{a}\right)\left(\frac{\pi}{b}\right)\right]\} \qquad \text{(A10)}$$

$$\lambda_c = \frac{9\pi^4 h^3}{128\ m\ (A_{11}{*}A_{22}{*}-A_{12}{*}^2)b^4 r^4}\ \{(A_{22}{*} - A_{12}{*}r^2)\ (1-4[4k_1\left(\frac{\pi}{a}\right)^4$$

$$+\frac{4}{3}\,k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 + \frac{4}{3}\,k_5\left(\frac{\pi}{b}\right)^4 - k_6\left(\frac{\pi}{a}\right)^2 - \frac{1}{3}\,k_8\left(\frac{\pi}{b}\right)^2])+(A_{11}{*}r^4-A_{12}{*}r^2)$$

$$(1-4[\frac{4}{3}\,k_1\left(\frac{\pi}{a}\right)^4 + \frac{4}{3}\,k_3\left(\frac{\pi}{a}\right)^2\left(\frac{\pi}{b}\right)^2 + 4k_5\left(\frac{\pi}{b}\right)^4 - \frac{1}{3}\,k_6\left(\frac{\pi}{a}\right)^2 - k_8\left(\frac{\pi}{b}\right)^2])\} \qquad \text{(A11)}$$

with transverse shear effects neglected, ie $T_s = 0$. The coefficients k_i, P_i are zero, Eqs. (A8)-(A11) reduce to

$$\omega_0^2 = \frac{16\pi^4}{9\rho h a^4}\,[3\,D_{11} + 2(D_{12} + 2D_{66})\ r^2 + 3\ D_{22}\ r^4] \qquad \text{(A12)}$$

$$m = \frac{9\rho h^2}{16} \qquad \text{(A13)}$$

$$\lambda_p = \frac{\pi^4 h}{9\rho b^4}\,[C_{10} + C_{01} + C_{11} + C_{02} + C_{20} + \frac{1}{2}\,(C_{21} + C_{12})] \qquad \text{(A14)}$$

$$\lambda_c = \frac{\pi^4 h}{8\rho a^4}\left(\frac{A_{22}^* - 2A_{12}^* r^2 + A_{11}^* r^4}{A_{11}^* A_{22}^* - A_{12}^* r^2}\right) \tag{A15}$$

Equations (A12) - (A15) are the same as in Ref. 3. The slope functions α and β for the clamped case are assumed to be

$$\alpha\,(x,y,t) = B_1\, q(t)\, \sin\frac{2\pi x}{a}\,(1 + \cos\frac{2\pi y}{b}) \tag{A16}$$

$$\beta\,(x,y,t) = B_2\, q(t)\, (1 + \cos\frac{2\pi y}{a})\, \sin\frac{2\pi y}{b} \tag{A17}$$

with

$$B_1 = \frac{\frac{h}{2}\frac{\pi}{a}\,[1 + 4b_{10}\,(\frac{\pi}{a})^2 - \frac{4}{3}\,(b_5 - b_{12})\,(\frac{\pi}{b})^2]}{\{1 - 4\,[4k_1(\frac{\pi}{a})^4 + \frac{4}{3}\,k_3(\frac{\pi}{a})^2(\frac{\pi}{b})^2 + \frac{4}{3}\,k_5(\frac{\pi}{b})^4 - k_6(\frac{\pi}{a})^2 - \frac{1}{3}\,k_8(\frac{\pi}{b})^2]\}} \tag{A18}$$

$$B_2 = \frac{\frac{h\pi}{2b}\,[1 + 4\,b_3(\frac{\pi}{b})^2 - \frac{4}{3}\,(b_8 - b_1)\,(\frac{\pi}{a})^2]}{\{1 - 4\,[\frac{4}{3}\,k_1(\frac{\pi}{a})^4 + \frac{4}{3}\,k_3(\frac{\pi}{a})^2\frac{\pi}{b})^2 + 4k_5(\frac{\pi}{b})^4 - \frac{1}{3}\,k_6(\frac{\pi}{a})^2 - k_8(\frac{\pi}{b})^2]\}} \tag{A19}$$

The S_i of Eq. (80) are defined as

$$S_1 = \frac{2\pi}{a}\,B_1$$

$$S_2 = \frac{2\pi}{b} B_2 \tag{A20}$$

$$S_3 = -2\left(\frac{\pi}{b} B_1 + \frac{\pi}{a} B_2\right)$$

The n_i of Eq. (82) are defined as

$$n_x = \frac{1}{8}\left(\frac{hr\pi}{b}\right)^2 (4S_{12} \sin X \sin 2Y + S_{21} \sin 2X \sin Y + S_{11} \sin X \sin Y$$

$$+ 4C_{12} \cos X \cos 2Y + C_{21} \cos 2X \cos Y + 4C_{02} \cos 2Y$$

$$+ C_{11} \cos X \cos Y + C_{01} \cos Y) + \frac{3h^2\pi^2}{32(A_{11}^* A_{22}^* - A_{12}^{*2})}\left(\frac{A_{22}^*}{a^2} - \frac{A_{12}^*}{b^2}\right) \tag{A21}$$

$$n_y = \frac{1}{8}\left(\frac{hr\pi}{a}\right)^2 (S_{12} \sin X \sin 2Y + 4S_{21} \sin 2X \sin Y + S_{11} \sin X \sin Y$$

$$+ C_{12} \cos X \cos 2Y + 4C_{21} \cos 2X \cos Y + 4C_{20} \cos 2X + C_{11} \cos X \cos Y$$

$$+ C_{10} \cos X) + \frac{3h^2\pi^2}{32(A_{11}^* A_{22}^* - A_{12}^{*2})}\left(\frac{A_{11}^*}{b^2} - \frac{A_{12}^*}{a^2}\right) \tag{A22}$$

$$n_{xy} = \frac{1}{8ab}(hr\pi)^2 (2S_{12} \cos X \cos 2Y + 2S_{21} \sin 2X \cos Y$$

$$+ S_{11} \cos X \cos Y + 2C_{12} \sin X \sin 2Y + C_{21} \sin 2X \sin Y$$

$$+ C_{11} \sin X \sin Y) \tag{A23}$$

The analagons equations for Eqs. (94) - (97) are

$$D_{1xz} = 0.5z^2 \left[(F_1 - F_2) \sin X \cos Y + (F_3 - F_4) \cos X \sin Y + E_{10} \sin X + E_{30} \sin Y \right] \tag{A24}$$

$$D_{2xz} = - z[R_1 \bar{Q}_{11} + R_2 \bar{Q}_{12} + (R_3 + R_4) \bar{Q}_{16} + R_5 \bar{Q}_{26} + R_6 \bar{Q}_{66}] \tag{A25}$$

$$D_{1yz} = 0.5Z^2[(F_5 - F_6) \sin X \cos Y + (F_7 - F_8) \cos X \sin Y + E_{50} \sin X + E_{70} \sin Y] \tag{A26}$$

$$D_{2yz} = - z[R_1 \bar{Q}_{16} + (R_2 + R_6) \bar{Q}_{26} + R_3 \bar{Q}_{66} + R_4 \bar{Q}_{12} + R_5 \bar{Q}_{22}] \tag{A27}$$

where

$$F_1 = \frac{2\pi}{a} (\bar{Q}_{11} S_1 + \bar{Q}_{12} S_2)$$

$$F_2 = \frac{2\pi}{b} (\bar{Q}_{66} S_3)$$

$$F_3 = \frac{2\pi}{b} (\bar{Q}_{16} S_1 + \bar{Q}_{26} S_2)$$

$$F_4 = \frac{2\pi}{a} (\overline{Q}_{16} \; S_3) \tag{A28}$$

$$F_5 = \frac{2\pi}{a} (\overline{Q}_{16} \; S_1 + \overline{Q}_{26} \; S_2)$$

$$F_6 = \frac{2\pi}{b} (\overline{Q}_{26} \; S_3)$$

$$F_7 = \frac{2\pi}{b} (\overline{Q}_{12} \; S_1 + \overline{Q}_{22} \; S_2)$$

$$F_8 = \frac{2\pi}{a} (\overline{Q}_{66} \; S_3)$$

$$E_{10} = \frac{2\pi}{a} (\overline{Q}_{11} \; S_1)$$

$$E_{30} = \frac{2\pi}{b} (\overline{Q}_{26} \; S_2)$$

$$E_{50} = \frac{2\pi}{a} (\overline{Q}_{16} \; S_1)$$

$$E_{70} = \frac{2\pi}{b} (\overline{Q}_{22} \; S_2)$$

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA CR-178313		

4. Title and Subtitle	5. Report Date
INFLUENCE OF LARGE DEFLECTION AND TRANSVERSE SHEAR ON RANDOM RESPONSE OF RECTANGULAR SYMMETRIC COMPOSITE LAMINATES TO ACOUSTIC LOADS	June 1987
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
Chuh Mei and C. B. Prasad	
	10. Work Unit No. 505-63-11-02
9. Performing Organization Name and Address Old Dominion University Research Foundation P. O. Box 6369 Norfolk, Virginia 23508-0369	11. Contract or Grant No. NAS1-17993, Task 22
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Langley Research Center Hampton, Virginia 23665	13. Type of Report and Period Covered Contractor Report
	14. Sponsoring Agency Code

15. Supplementary Notes

Langley Technical Monitor: John S. Mixson

16. Abstract

Nonlinear equations of motion of symmetrically laminated anisotropic plates are derived accounting for Von Karman strains. The effect of transverse shear is included in the formulation and the rotatory inertia effect is neglected. Using a single-mode Galerkin procedure the nonlinear modal equation is obtained. Direct equivalent linearization is employed. The response of acoustic excitation on moderately thick composite panels is studied. Further, the effects of transverse shear on large deflection vibration of laminates under random excitation are studied. Mean-square deflections and mean-square inplane stresses are obtained for some symmetric graphite-epoxy laminates. Using equillibrium equations and the continuity requirements, the mean-square transverse shear stresses are calculated. The results obtained will be useful in the sonic fatigue design of composite aircraft panels. The analysis is presented in detail for simply supported plate. The analogous equations for a clamped case are given in the appendix.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Sonic Fatigue Large Amplitude Random Response Composite Panels	Unclassified - Unlimited Subject Category - 71

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of Pages	22. Price*
Unclassified	Unclassified	69	A04

* For sale by the National Technical Information Service, Springfield, Virginia 22161