ICIAM'87

First international conference on industrial and applied mathematics

Applications of Classical and Zero-Total-Pressure-Loss Sets of Euler Equations to Delta Wings

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JUNE 29 - JULY 3 - 1987

OUTLINE OF THE TALK

1. MOTIVATION AND OBJECTIVES

2. FORMULATION
   - CLASSICAL AND ZERO-TOTAL-PRESSURE-LOSS SETS
   - SUPERSONIC CONICAL FLOW EQUATIONS
   - RELATIVE MOTION IN A ROTATING FRAME OF REFERENCE

3. HIGHLIGHTS OF METHOD OF SOLUTION

4. APPLICATIONS:
   - CONICAL FLOW, SHARP-EDGED WINGS (CLASSICAL AND ZTPL SETS)
   - CONICAL FLOW, ROUND-EDGED WINGS (CLASSICAL AND ZTPL SETS)
   - THREE-DIMENSIONAL FLOWS; TRANSONIC AND LOW-SPEED FLOWS

   Numerical Examples (1)

   - UNIFORM ROLLING IN A CONICAL FLOW
   - ROLLING OSCILLATION IN A LOCALLY CONICAL FLOW

   Numerical Examples (2)

5. CONCLUDING REMARKS

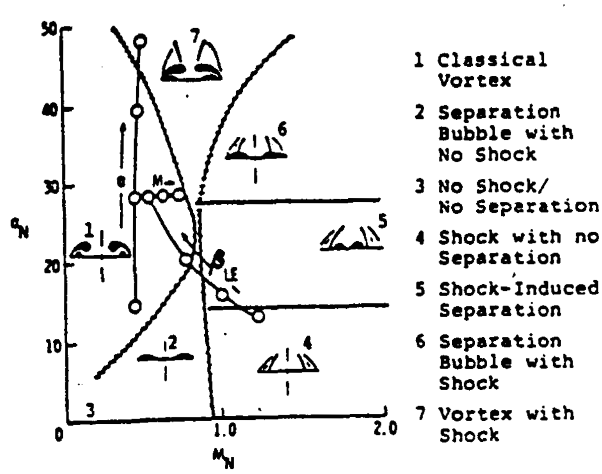


Figure 1. Miller and Wood[1] Classification Diagram.

CLASSICAL EULER EQUATIONS

- CONSERVATION FORM OF EULER EQUATIONS IN A SPACE-FIXED FRAME OF REFERENCE

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial \bar{E}}{\partial x} + \frac{\partial \bar{F}}{\partial y} + \frac{\partial \bar{G}}{\partial Z} = 0 \tag{1}$$

$$\bar{q} = [\rho,\ \rho u,\ \rho v,\ \rho w,\ \rho e]^t \tag{2}$$

$$\bar{E} = [\rho u,\ \rho u^2 + p,\ \rho uv,\ \rho uw,\ \rho uh]^t \tag{3}$$

$$\bar{F} = [\rho v,\ \rho uv,\ \rho v^2 + p,\ \rho vw,\ \rho vh]^t \tag{4}$$

$$\bar{G} = [\rho w,\ \rho uw,\ \rho vw,\ \rho w^2 + p,\ \rho wh]^t \tag{5}$$

$$e = p/\rho(\gamma-1) + (u^2 + v^2 + w^2)/2 \tag{6}$$

$$h = e + p/\rho \tag{7}$$

SUPERSONIC CONICAL FLOW EQUATIONS

CONICAL VARIABLES

$$\xi = x,\ \eta = y/x,\ \zeta = z/x \tag{6}$$

CONICAL FLOW EQUATIONS

$$\frac{\partial \bar{q}}{\partial t} + \frac{\partial \tilde{\bar{F}}}{\partial \eta} + \frac{\partial \tilde{\bar{G}}}{\partial \zeta} + 2\ \bar{E} = 0 \tag{7}$$

WHERE

$$\tilde{\bar{F}} = \bar{F} - \eta\ \bar{E} \tag{8}$$

$$\tilde{\bar{G}} = \bar{G} - \zeta\ \bar{E} \tag{9}$$

Zero-Total-Pressure-Loss Euler Equations

- Replace the energy equation by either one of the isentropic gas equations

$$p/\rho^{\gamma} = \text{const. or } \frac{\partial}{\partial t}(\rho s) + \nabla\cdot(\rho s\bar{v}) = 0$$

Set (1): • Replace the x-momentum equation (second elements in the vectors given in Eqs. (2) and (3)) by the steady energy equation (total constant enthalpy)

$$h = \text{const} = \frac{\gamma p}{(\gamma-1)\rho} + \frac{1}{2}(u^2 + v^2 + w^2)$$

Set (2): • Replace the continuity equation (first elements in the vectors given in Eqs. (2) and (3)) by the steady energy equation given in Set (1)

EXPLANATION OF TOTAL-PRESSURE CHANGE FOR CLASSICAL AND ZTPL SETS OF EULER EQUATIONS

<u>Differential Euler Equations</u>

Crocco's Theorem

$$T\nabla S = \bar{\omega}\times\bar{V} + \frac{\partial\bar{V}}{\partial t} + \nabla h \qquad (1)$$

Definition of Entropy Change

$$\Delta S = R\,\ln\frac{P_{T_\infty}}{P_T} + C_p\,\ln\frac{T_o}{T_{o_\infty}} \qquad (2)$$

(A) <u>Classical Set</u>

Steady flow $\frac{\partial\bar{V}}{\partial t} = 0$, $h = \text{const}$ and

$$T\nabla S = \bar{\omega}\times\bar{V},\quad \Delta S = R\,\ln\frac{P_{T_\infty}}{P_T}$$

For a free-sheet $\bar{\omega}$ is parallel to $\bar{V}$! $\nabla S = 0 \rightarrow P_{T_\infty} = P_T \rightarrow$ Zero-Total-Pressure Loss

(B) Zero-total-Pressure-Loss Set (Shock-Free and Weak Shocks)

$$h = \text{const},\ \nabla S = 0$$

and $\bar{\omega}$ must be parallel to $\bar{V}$, $P_{T_\infty} = P_T \rightarrow$ Zero-Total-Pressure Loss

COMPUTATONAL EULER EQUATIONS

CROCCO'S THEOREM

$T \Delta S = \bar{\omega} \times \bar{V} + \frac{\partial \bar{V}}{\partial t} + \nabla h - \frac{1}{\rho} \nabla \cdot \bar{\tau}$ VISCOUS-FORM OF THE EQUATION (1)

DEFINITION OF ENTROPY CHANGE

$$\Delta S = R \ln \frac{P_{T_\infty}}{P_T} + C_p \ln \frac{T_o}{T_{o_\infty}} \qquad (2)$$

(A) CLASSICAL SET

FOR STEADY FLOW $\frac{\partial \bar{V}}{\partial t} = 0$, $h = \text{const}$ AND

$$T \nabla S = \bar{\omega} \times \bar{V} + \text{Numerical Dissipation}, \ \Delta S = R \ln \frac{P_{T_\infty}}{P_T}$$

EVEN IF $\bar{\omega}$ IS PARALLEL TO $\bar{V}$, $\nabla S \neq 0 \rightarrow P_{T_\infty} \neq P_T \rightarrow$ NON-ZERO TPL

(B) ZERO-TOTAL-PRESSURE-LOSS SET (SHOCK-FREE AND WEAK SHOCKS)

$$h = \text{const}, \ \Delta S = 0 \quad \text{AND}$$

$$0 = \bar{\omega} \times V + \text{Numerical Dissipation}, \ P_{T_\infty} = P_T \rightarrow \text{Zero TPL}$$

CLASSICAL EULER EQUATIONS FOR THE RELATIVE MOTION IN A ROTATING FRAME OF REFERENCE

- THE CONSERVATION FORM OF THE CLASSICAL EULER EQUATIONS FOR THE ABSOLUTE MOTION OF THE FLOW IN A SPACE-FIXED FRAME OF REFERENCE

$$\frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \bar{V}) = 0 \qquad (1)$$

$$\frac{\partial (\rho \bar{V})}{\partial t} + \nabla \cdot (\rho \bar{V} \bar{V} + p \bar{I}) = 0 \qquad (2)$$

$$\frac{\partial (\rho e)}{\partial t} + \nabla \cdot (\rho h \bar{V}) = 0 \qquad (3)$$

$$e = p/\rho(\gamma-1) + \frac{V^2}{2} \qquad (4)$$

$$h = e + p/\rho \qquad (5)$$

- To express these equations in terms of a moving frame of reference, we use the following relations of the substantial and local derivatives for a scalar "a" and a vector "$\bar{A}$":

$$\frac{Da}{Dt} = \frac{D'a}{Dt'} \qquad (6.a)$$

$$\frac{\partial a}{\partial t} = \frac{\partial' a}{\partial t'} - \bar{V}_t \cdot \nabla a \qquad (6.b)$$

$$\frac{D\bar{A}}{Dt} = \frac{D'\bar{A}}{Dt'} + \bar{\omega} x \bar{A} \qquad (7.a)$$

$$\frac{\partial \bar{A}}{\partial t} = \frac{\partial' \bar{A}}{\partial t'} - \bar{V}_t \cdot \nabla \bar{A} + \bar{\omega} x \bar{A} \qquad (7.b)$$

- The transformation velocity $\bar{V}_t$ is a function of the moving frame of reference translation and rotation

$$\bar{V}_t = \bar{V} - \bar{V}_r = \bar{V}_o + \bar{\omega} x \bar{r} \qquad (8)$$

- Restricting the motion of the frame of reference to the rotational motion,

$$\bar{V}_o = 0 \text{ and } \frac{D\bar{V}_o}{Dt} = 0,$$

- the equations of relative motion in the rotating frame of reference

$$\frac{\partial' \rho}{\partial t'} + \nabla \cdot (\rho \bar{V}_r) = 0 \qquad (9)$$

$$\frac{\partial'(\rho \bar{V}_r)}{\partial t'} + \nabla \cdot [\rho \bar{V}_r V_r + p \bar{I}] = - \rho[\dot{\bar{\omega}} x \bar{r} + 2\bar{\omega} x \bar{V}_r + \bar{\omega} x (\bar{\omega} x \bar{r})] \qquad (10)$$

$$\frac{\partial'(\rho e_r)}{\partial t'} + \nabla \cdot [\rho h_r \bar{V}_r] = - \rho[\bar{V}_r \cdot (\dot{\bar{\omega}} x \bar{r}) + (\bar{\omega} x \bar{r}) \cdot (\dot{\bar{\omega}} x \bar{r})] \qquad (11)$$

where

$$e_r = \frac{p}{\rho(\gamma-1)} + \frac{V_r^2}{2} - \frac{1}{2} |\bar{\omega} x \bar{r}|^2 = e - \bar{V} \cdot (\bar{\omega} x \bar{r}) \qquad (12)$$

$$h_r = \frac{\gamma p}{\rho(\gamma-1)} + \frac{V_r^2}{2} - \frac{1}{2} |\bar{\omega} x \bar{r}|^2 = h - \dot{\bar{V}} \cdot (\bar{\omega} x \bar{r}) \qquad (13)$$

- The abstract conservative form of the relative motion in terms of the rotating coordinates (x', y', z') is given by

$$\frac{\partial' \bar{q}_r}{\partial t'} + \frac{\partial' \bar{E}_r}{\partial x'} + \frac{\partial' \bar{F}_r}{\partial y'} + \frac{\partial' \bar{G}_r}{\partial z'} = \bar{S} \tag{14}$$

where

$$\bar{q}_r = [\rho, \rho u_r, \rho v_r, \rho w_r, \rho e_r]^t \tag{15}$$

$$\bar{E}_r = [\rho u_r, \rho u_r^2 + p, \rho u_r v_r, \rho u_r w_r, \rho u_r h_r]^t \tag{16}$$

$$\bar{F}_r = [\rho v_r, \rho u_r v_r, \rho v_r^2 + p, \rho v_r w_r, \rho v_r h_r]^t \tag{17}$$

$$\bar{G}_r = [\rho w_r, \rho u_r w_r, \rho v_r w_r, \rho w_r^2 + p, \rho w_r h_r]^t \tag{18}$$

$$\bar{S} = [0, 0, \rho(\dot{\omega} z + 2\omega w_r + \omega^2 y), - \rho(\dot{\omega} y + 2\omega v_r - \omega^2 z), - \rho(-v_r \dot{\omega} z + w_r \dot{\omega} y + \omega\dot{\omega} y^2 + \omega\dot{\omega} z^2)]^t \tag{19}$$

- Since only the rolling motion is solved, the source term $\bar{S}$ has been written for $\bar{\omega} = \omega \bar{e}_{x'}$, and $\dot{\bar{\omega}} = \dot{\omega} \bar{e}_{x'}$.

HIGHLIGHTS OF METHOD OF SOLUTION

1. We use the Central-Difference Finite-Volume Scheme with Four-Stage Runge Kutta time stepping and explicit second- and fourth-order dissipation terms.

2. For steady flows, local-time stepping is used, and for unsteady flows minimum global time stepping is used.

3. A three-dimensional computer program is used to solve for:

   - Conical flows (using 3 conical planes, we enforce the absolute flow vector to be equal on these planes)

   - Direct solution of the three-dimensional flow problem.

4. Depending on the problem under consideration, different initial conditions are used.

5. Depending on the problem under consideration, different surface, farfied and symmetry conditions are used. For supersonic flows, the outer bow shock is captured as part of the solution.

NUMERICAL EXAMPLES (I)

- SHARP-EDGED WINGS (CLASSICAL EULER EQS. & ZERO-TOTAL-PRESSURE-LOSS SETS)

- ROUND-EDGED WINGS (CLASSICAL EURER EQS. & ZERO-TOTAL-PRESSURE-LOSS SETS)
  - NUMERICAL BOUNDARY CONDITION (COARSE AND FINE GRIDS)
  - CLOSED FORM BOUNDARY CONDITION (COARSE AND FINE GRIDS)

- THREE-DIMENSIONAL TRANSONIC AND SUBSONIC FLOWS

Figure 1. Standard Euler Set, Sharp-edged Wing, 64X64 Cell, $M_\infty = 2.0$, $\alpha=10^0$, $\beta=70^0$, $\epsilon_2=0.12$, $\epsilon_4=0.005$, 1. Surface pressure, 2. Crossflow Mach number, 3. Crossflow Velocity.

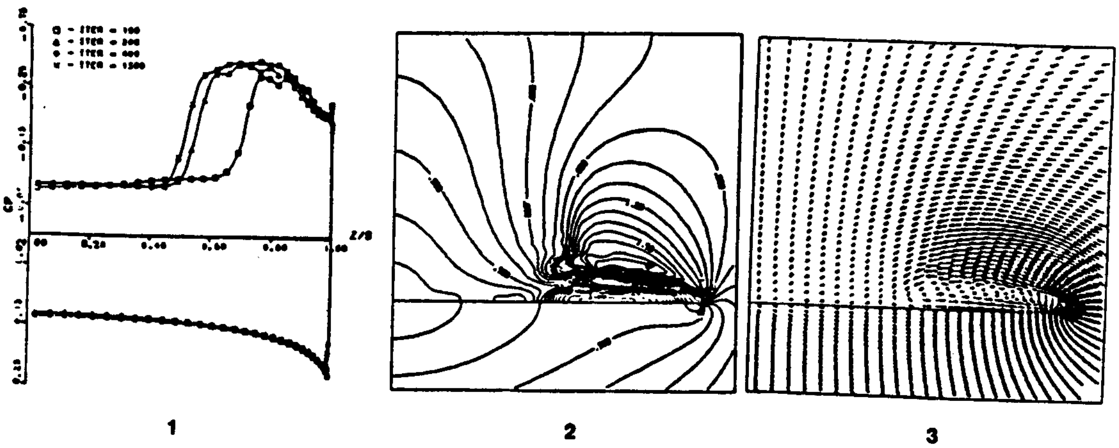
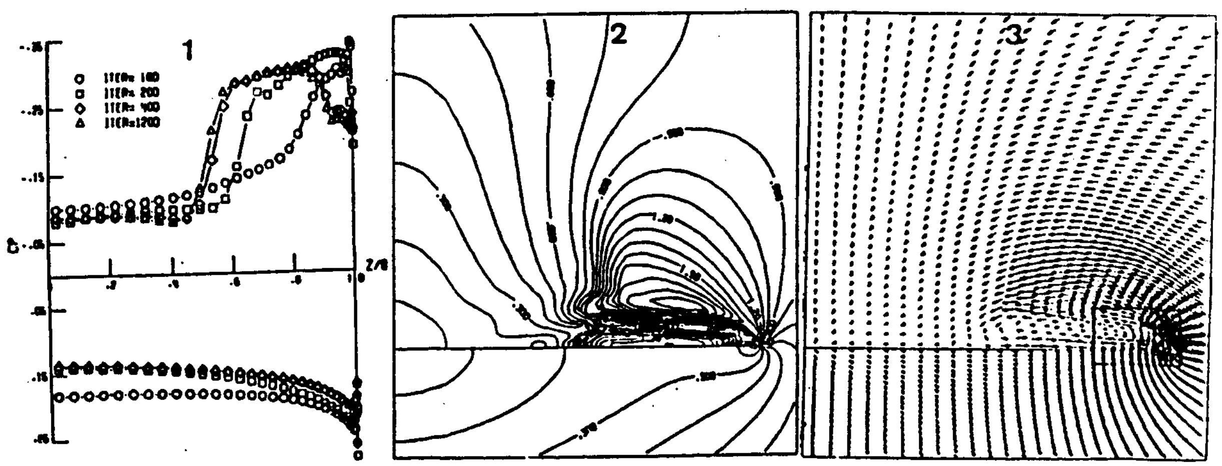
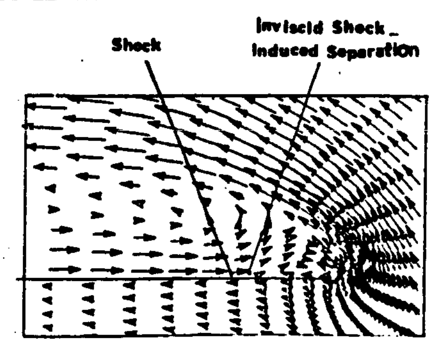


Figure 2. Zero-Total-Pressure-Loss Euler, Set (1), Sharp-edged Wing, 64x64 Cell, $M_\infty=2.0$, $\alpha=10^0$, $\beta=70^0$, $\epsilon_2=0.12$, $\epsilon_4=0.005$,1. Surface Pressure, 2. Crossflow Mach number, 3. Crossflow Velocity.





Details of Leading-Edge Flow (Fig. 2.3)