

## The Design of Worm Gear Sets

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### ABSTRACT

The purpose of this paper is to present a method for designing worm gear sets to meet torque multiplication requirements. First, the fundamentals of worm gear design are discussed, covering worm gear set nomenclature, kinematics and proportions, force analysis, and stress analysis. Then, a suggested design method is discussed, explaining how to take a worm gear set application, and specify a complete worm gear set design. The discussions in this paper will be limited to cylindrical worm gear sets that have a  $90^\circ$  shaft angle between the worm and the mating gear.

### INTRODUCTION

Designing worm gear sets to meet torque multiplication requirements is a challenge because of the high friction in worm gearing. Friction is higher in worm gearing than in more conventional types of gearing, such as spur, bevel and conical, due to the sliding that occurs between the worm and the mating gear. Friction is very difficult to quantify because there are so many factors that affect it. The coefficient of friction depends on the material combinations, surface roughness, operating speeds, and the pressure, temperature, and viscosity of the lubricant.

The curves in Figure 1 show the dependence of the coefficient of friction on the sliding velocity between the worm and the mating gear, the material combination, and the lubrication. Curve A is for a cast-iron worm and mating gear. Curve B is for a case-hardened worm mating with a phosphor-bronze gear. Both curves are based on good lubrication. The curves indicate that the coefficient of friction increases as the sliding velocity decreases. Many more curves could be added to this graph for more combinations of materials and different lubrication. In aerospace applications where the sliding velocities are slow, and non-standard materials and lubricants are used, it is easy to see how the coefficient of friction for a particular application would be difficult to determine.

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## WORM GEAR SET NOMENCLATURE

Figure 2 shows the nomenclature for worm gear sets as defined below.

### Pitch Diameter

The pitch diameters of the worm,  $D_w$ , and of the gear,  $D_g$ , are tangent to each other and represent where the curved surfaces of the gear teeth and worm threads contact each other during operation.  $D_g$  is the diameter of the gear's pitch circle, and  $D_w$  is the diameter of the worm's pitch cylinder.

### Center Distance

The center distance,  $C_d$ , is the distance between the center of the worm and the center of the gear when in mesh.

### Root Diameter

The root diameter of the worm,  $R_{dw}$ , is the diameter to the root of the worm's threads; and of the gear,  $R_d$ , is the diameter to the root of the gear's teeth.

### Outside Diameter

The outside diameter of the worm,  $O_{dw}$ , is the diameter to the tips of the threads; and of the gear,  $O_d$ , is the diameter to the tips of the teeth.

### Circular Pitch

The circular pitch,  $P_c$ , is the spacing of gear teeth measured along the gear's pitch circle from a point on one tooth to a corresponding point on an adjacent tooth.

### Normal Circular Pitch

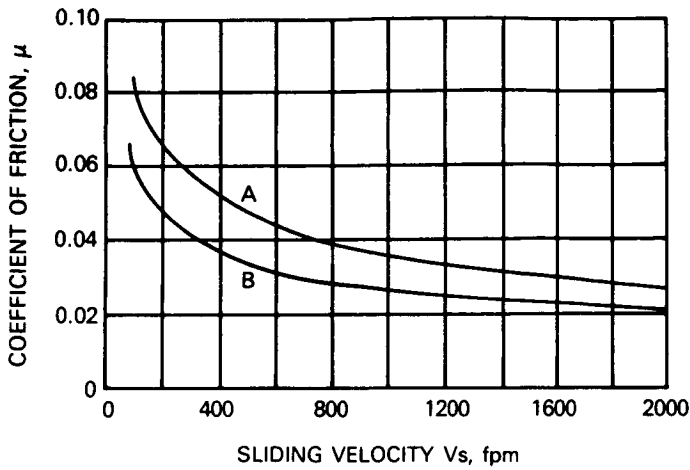
The normal circular pitch,  $P_n$ , is the circular pitch in the normal plane.

### Tooth Thickness

The tooth thickness,  $T$ , is the thickness of the tooth measured along the pitch circle.

### Addendum

The addendum,  $A_d$ , is the height of the gear tooth beyond the pitch circle.

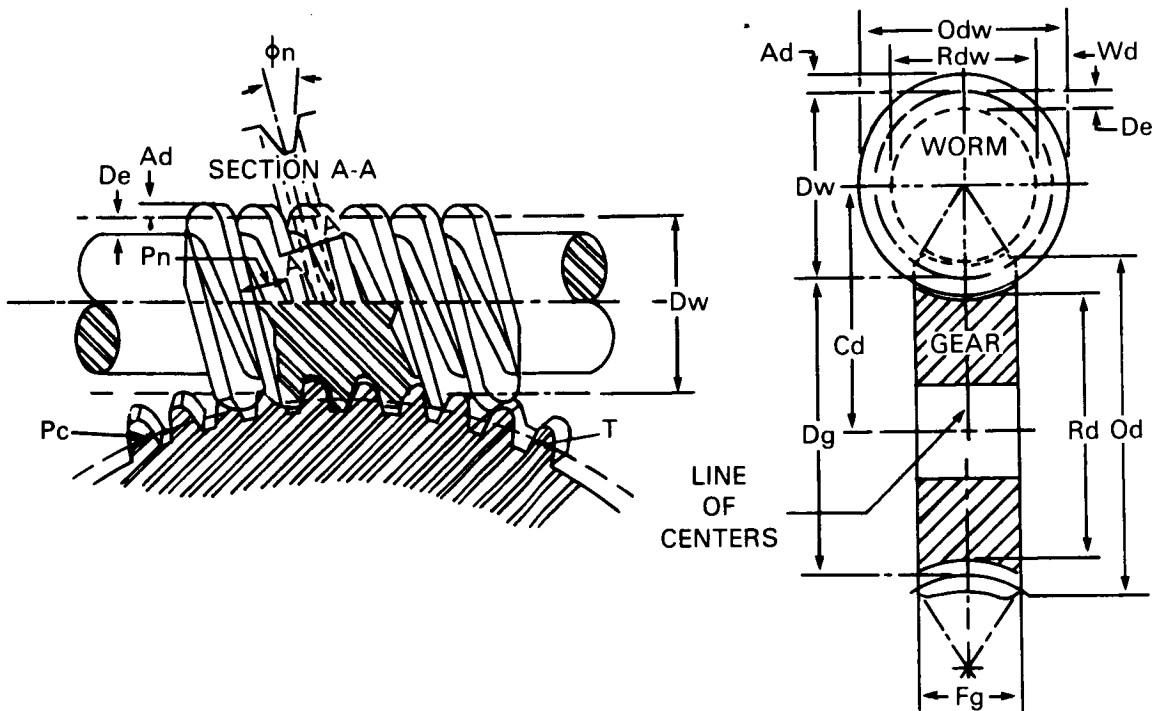


CURVE A: CAST-IRON WORM AND GEAR WITH GOOD LUBRICATION

CURVE B: CASE-HARDENED WORM AND PHOSPHOR-BRONZE GEAR WITH GOOD LUBRICATION

**FIGURE 1: REPRESENTATIVE VALUES OF COEFFICIENT OF FRICTION FOR WORM GEARING**

SHIGLEY, JOSEPH EDWARD, *MECHANICAL ENGINEERING DESIGN* THIRD EDITION, McGRAW-HILL BOOK COMPANY, 1977



**FIGURE 2: WORM GEAR SET NOMENCLATURE**

### Dedendum

The dedendum,  $D_e$ , is the depth of the gear tooth below the pitch circle.

### Whole Depth

The whole depth,  $W_d$ , is the sum of the addendum and the dedendum.

### Face Width

The face width of the gear tooth,  $F_g$ , is the width of the gear tooth measured on the pitch circle.

### Normal Pressure Angle

The normal pressure angle,  $\phi_n$ , is the angle, in the normal plane, between the line-of-action (common tangent to the base circle of the gear and base cylinder of the worm) and a perpendicular to the line of centers.

### Gear's Helix Angle

The gear's helix angle,  $\psi_g$ , measured at the pitch diameter, is the angle between the teeth and the axis of rotation.

Figure 3 illustrates the developed pitch cylinder of a double-threaded worm, where A and B represent the two threads.

### Lead

The lead of the worm,  $L$ , is the axial advance per revolution of the worm.

### Lead Angle

The lead angle of the worm,  $\lambda$ , is the angle measured at the pitch cylinder, between the helix of the worm and the plane of rotation. When the shaft angle between the worm and gear is  $90^\circ$ , then  $\lambda$  is equal to  $\psi_g$ .

### Axial Pitch

The axial pitch of the worm,  $P_a$ , is the linear pitch in the axial plane on the pitch cylinder. When the shaft angle is  $90^\circ$ , then  $P_a$  is equal to  $P_c$ .

The worm gear set is characterized by the gear ratio, and the gear is often characterized by the diametral pitch.

### Gear Ratio

The gear ratio,  $Z_g$ , is the number of teeth on the gear,  $N_g$ , divided by the number of threads on the worm,  $N_w$ . The gear ratio is also the speed ratio and is equal to the input speed of the worm,  $S_w$ , divided by the output speed of the gear,  $S_g$ .

### Diametral Pitch

The diametral pitch of a gear,  $D_p$ , is the number of teeth per inch on the pitch circle.

## KINEMATICS AND PROPORTIONS

When designing a worm gear set, the designer's application will dictate certain parameters. The remaining parameters can be calculated using the following kinematic relations.

The diametral pitch of a gear is the number of teeth per inch on the pitch circle:

$$D_p = \frac{N_g}{D_g}$$

The circular pitch is the circular distance the gear advances on the pitch circle per turn of the worm:

$$P_c = \frac{\pi}{D_p}$$

The gear tooth thickness and the worm thread thickness are not necessarily equal, but for the purposes of this paper, will be assumed to be equal. The tooth thickness and the space between teeth will also be assumed equal. Their sum is  $P_c$ , so:

$$T = 1/2 P_c$$

The lead of the worm is the axial distance the worm advances per turn:

$$L = N_w P_a$$

When the shaft angle is  $90^\circ$ , the usual case, then:

$$P_c = P_a \text{ and}$$

$$L = N_w P_c = \frac{\pi N_w}{D_p}$$

Referring to Figure 3:

$$\tan \lambda = \frac{L}{\pi D_w}$$

$$\lambda = \tan^{-1} \left( \frac{L}{\pi D_w} \right)$$

Substituting in for L,  $\lambda$  can also be expressed:

$$\lambda = \tan^{-1} \left( \frac{N_w}{D_p D_w} \right)$$

The normal circular pitch is:

$$P_n = P_a \cos \lambda$$

Table 1 gives recommended pressure angles and tooth proportions for various lead angle ranges.

Table 1<sup>1</sup>

RECOMMENDED PRESSURE ANGLES AND TOOTH PROPORTIONS FOR WORM GEARING

Lead angle, $\lambda$ degrees	Pressure angle, $\phi_n$ degrees	Addendum, $A_d$	Dedendum, $D_e$
0-15	14 1/2	.3683 Pa	.3683 Pa
15-30	20	.3683 Pa	.3683 Pa
30-35	25	.2865 Pa	.3314 Pa
35-40	25	.2546 Pa	.2947 Pa
40-45	30	.2228 Pa	.2578 Pa

The whole depth of the gear tooth and of the worm thread is the sum of the addendum and the dedendum.

$$W_d = A_d + D_e$$

<sup>1</sup>Shigley, Joseph Edward, Mechanical Engineering Design, Third Edition, McGraw-Hill Book Company, 1977.

The outside diameter is the sum of the pitch diameter and twice the addendum.

$$O_d = D_g + 2A_d$$

$$O_{d_w} = D_w + 2A_d$$

The root diameter is the pitch diameter minus twice the dedendum.

$$R_d = D_g - 2D_e$$

$$R_{d_w} = D_w - 2D_e$$

The recommended minimum face width of the gear tooth is, as illustrated in Figure 4, equal to the length of a tangent to the worm's pitch circle between its points of intersection with the outside diameter. Expressing  $F_g$  in terms of  $D_w$  and  $O_{d_w}$ :

$$F_g \geq \sqrt{O_{d_w}^2 - D_w^2}$$

#### FORCE ANALYSIS

Figure 5 shows the forces acting at the worm's pitch cylinder.  $W$  represents the force exerted by the gear. The gear tooth contacts the worm thread at their pitch diameters, at the normal pressure angle relative to the worm's axis of rotation, and at the gear's helix angle (equal to the worm's lead angle) relative to the gear's axis of rotation. The relative motion between the worm and gear teeth is pure sliding. The force  $W$  acting normal to the worm-thread profile produces a frictional force,  $W_f = \mu W$ , with component  $\mu W \cos \lambda$  in the negative X-direction and  $\mu W \sin \lambda$  in the positive Z direction.

The X-axis is tangent to the worm's pitch cylinder and parallel to the gear's axis of rotation. The Y-axis is parallel to the mutual radial axis between the worm and the gear. The Z-axis is tangent to the gear's pitch circle and parallel to the worm's axis of rotation. The X, Y and Z components of  $W$ , respectively, are:

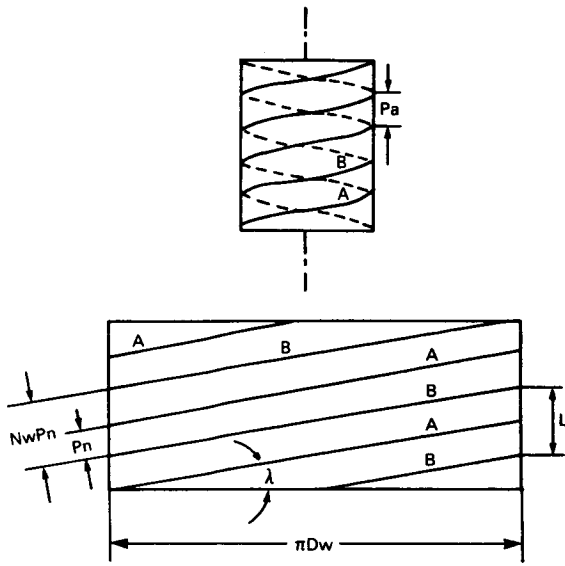
$$W^X = W (\cos \phi_n \sin \lambda + \mu \cos \lambda)$$

$$W^Y = W \sin \phi_n$$

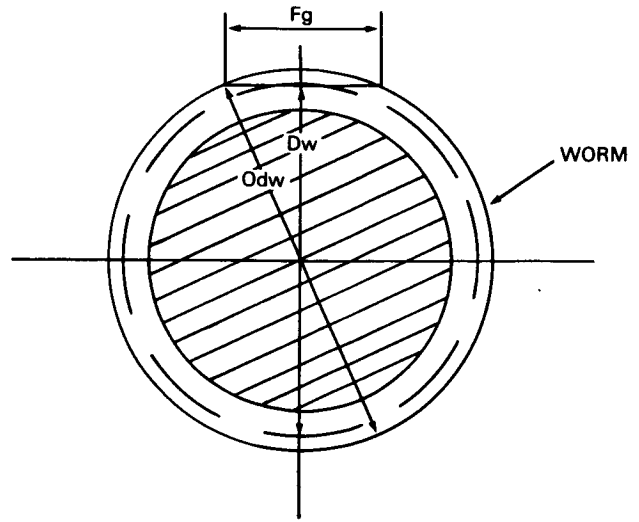
$$W^Z = W (\cos \phi_n \cos \lambda - \mu \sin \lambda)$$

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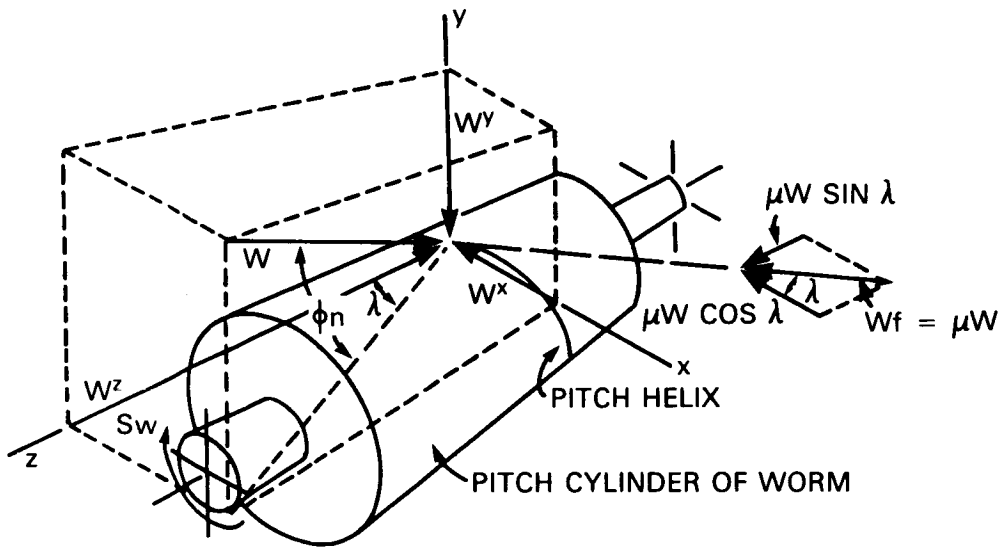
<sup>2</sup>Shigley, Joseph Edward, Mechanical Engineering Design, Third Edition, McGraw-Hill Book Company, 1977.



**FIGURE 3: DEVELOPED PITCH CYLINDER OF WORM**



**FIGURE 4: MINIMUM GEAR FACE WIDTH RECOMMENDATION**



**FIGURE 5: PITCH CYLINDER OF A WORM SHOWING THE FORCES ACTING UPON IT BY THE MATING GEAR**



The tangential force on the worm is  $W^x$ , the radial force on the worm is  $W^y$  and the axial force on the worm is  $W^z$ . Since the gear forces are opposite the worm forces, the forces can be summarized as:

$$Wwt = - Wga = W^x$$

$$Wwr = - Wgr = W^y$$

$$Wwa = - Wgt = W^z$$

where g denotes the forces acting on the gear, w denotes the forces acting on the worm, a denotes the axial direction, r denotes the radial direction and t denotes the tangential direction.

The torque input from the worm is the product of the worm's tangential force and its pitch radius :

$$T_w = \frac{Wwt D_w}{2}$$

Similarly, the torque output from the gear is:

$$T_g = \frac{Wgt D_g}{2}$$

The torque multiplication ratio is:

$$Z_t = \frac{T_g}{T_w} = \frac{Wgt D_g}{Wwt D_w}$$

Substituting the expressions for  $Wgt$  and  $Wwt$  into this expression:

$$\begin{aligned} Z_t &= \frac{-W (\cos \phi_n \cos \lambda - \mu \sin \lambda) D_g}{W (\cos \phi_n \sin \lambda + \mu \cos \lambda) D_w} \\ &= \frac{-D_g (\cos \phi_n \cos \lambda - \mu \sin \lambda)}{D_w (\cos \phi_n \sin \lambda + \mu \cos \lambda)} \end{aligned}$$

The efficiency of the worm gear set can be expressed as the absolute value of the torque ratio divided by the gear ratio:

$$Eff = \left| \frac{Z_t}{Z_g} \right|$$

Substituting  $N_g/N_w$  in for  $Z_g$  and the expression for  $Z_t$ :

$$\begin{aligned} \text{Eff} &= \left| \frac{N_w [-D_g (\cos \phi_n \cos \lambda - \mu \sin \lambda)]}{N_g [D_w (\cos \phi_n \sin \lambda + \mu \cos \lambda)]} \right| \\ &= \left| \frac{N_w D_g}{N_g D_w} \frac{(\cos \phi_n \cos \lambda - \mu \sin \lambda)}{(\cos \phi_n \sin \lambda + \mu \cos \lambda)} \right| \end{aligned}$$

Dividing the numerator and denominator by  $\cos \lambda$  :

$$\text{Eff} = \left| \frac{N_w D_g (\cos \phi_n - \mu \tan \lambda)}{N_g D_w (\cos \phi_n \tan \lambda + \mu)} \right|$$

Substituting  $D_g D_p$  for  $N_g$ :

$$\text{Eff} = \left| \frac{N_w D_g}{D_g D_p D_w} \frac{(\cos \phi_n - \mu \tan \lambda)}{(\cos \phi_n \tan \lambda + \mu)} \right|$$

Substituting  $\tan \lambda$  for  $\frac{N_w}{D_p D_w}$  :

$$\text{Eff} = \left| \frac{\tan \lambda (\cos \phi_n - \mu \tan \lambda)}{(\cos \phi_n \tan \lambda + \mu)} \right|$$

Simplifying:

$$\text{Eff} = \left| \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda} \right|$$

The efficiency equation is a function of three parameters: pressure angle, lead angle, and coefficient of friction. Two of these are design parameters,  $\phi_n$  and  $\lambda$ . The coefficient of friction, however, is dependent upon many factors and is very difficult to set in the design. Figure 6 shows the effect of the coefficient of friction on worm gear efficiency for the standard pressure angles, between  $14\frac{1}{2}^\circ$  and  $30^\circ$ . The lead angle was held constant at  $18^\circ 26'$ . The graph shows a clear deterioration of efficiency as the friction increases, but very little change over the standard range of pressure angles. In Figure 7, the lead angle was varied within the range  $5^\circ$  to  $45^\circ$ , at  $5^\circ$  increments, and pressure angle was held constant at  $20^\circ$ . This graph shows that the efficiency drops off more drastically at the lower end of the lead angle range as friction increases. The optimum lead angle for maximizing efficiency is in the  $35^\circ$  to  $40^\circ$  range, but in actual practice lead angles above  $25^\circ$  are rarely used because they are difficult to manufacture.

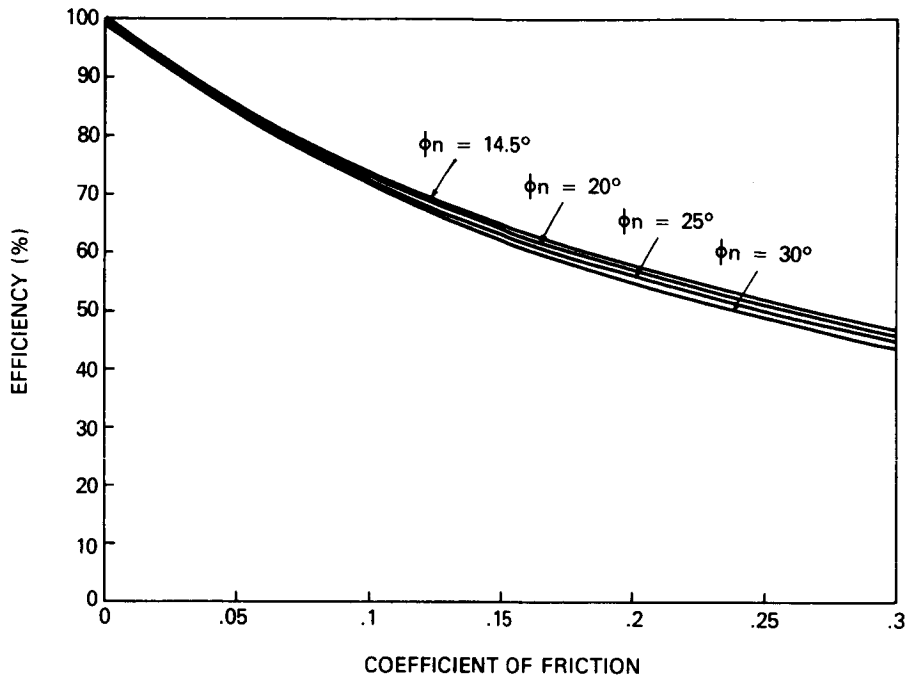


FIGURE 6: THE EFFECT OF THE COEFFICIENT OF FRICTION AND THE PRESSURE ANGLE ON WORM GEAR EFFICIENCY ( $\lambda = 18^\circ 26'$ )

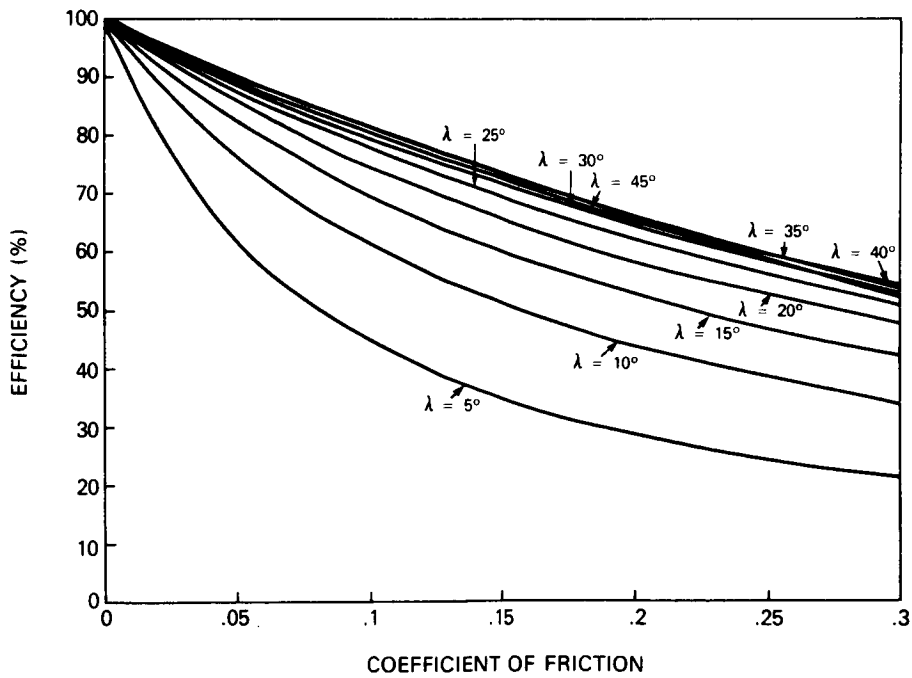


FIGURE 7: THE EFFECT OF THE COEFFICIENT OF FRICTION AND THE LEAD ANGLE ON WORM GEAR EFFICIENCY ( $\phi_n = 20^\circ$ )

The torque multiplication ratio can be expressed as the product of the efficiency and the gear ratio:

$$Z_t = Z_g \text{ Eff} = Z_g \left| \frac{\cos \phi_n - \mu \tan \lambda}{\cos \phi_n + \mu \cot \lambda} \right|$$

Figure 8 shows the effect of the coefficient of friction on the torque multiplication ratio for various gear ratios, with  $\phi_n = 20^\circ$  and  $\lambda = 18^\circ 26'$ . As the gear ratio increases, the slopes of the curves increase proportionately. When friction is negligible,  $Z_g = Z_t$ . This, however, can never be assumed because friction is always present and difficult to estimate. Figure 9 shows an improvement over Figure 8 where  $\lambda$ , at  $40^\circ$ , is close to the optimum for high efficiency. Figure 10 shows the other extreme with  $\lambda = 5^\circ$ .

#### STRESS ANALYSIS

When worm gear sets are run at slow speeds, the bending strength of the gear tooth may become a principal design factor. Especially when using non-standard materials, the stress on the tooth should be checked to assure a good factor of safety. Since it is customary to make the worm threads out of a stronger material than the gear teeth, the worm threads aren't usually considered. Bending stress is difficult to determine because worm gears are thick and short at the two edges of the face and thin in the central plane. The Lewis stress equation, as follows, is usually used to approximate bending stress in worm gear teeth:

$$\sigma = \frac{Wgt}{F_g Y P_n}$$

where, Y, the form factor can be obtained from Table 2.

Table 2

#### LEWIS FORM FACTORS

Normal Pressure angle, $\phi_n$ , degrees	Lewis Form Factor, Y
14 1/2	.100
20	.125
25	.150
30	.175

## POWER RATING

The velocity components in a worm gear set are shown in Figure 11. The worm velocity:

$$V_w = \pi \frac{D_w S_w}{12}$$

the gear velocity:

$$V_g = \pi \frac{D_g S_g}{12}$$

and the sliding velocity:

$$V_s = \frac{V_w}{\cos \lambda}$$

where  $V_w$ ,  $V_g$  and  $V_s$  are expressed in fpm,  $D_w$  and  $D_g$  are in inches and  $S_w$  and  $S_g$  are in rpm. The AGMA equation for input - horsepower rating of worm gearing is (where  $W_{gt}$  and  $W_f$  are in lbs.):

$$H_p = \frac{W_{gt} D_g S_w}{12,000 Z_g} + \frac{V_s W_f}{33,000}$$

## DESIGN METHOD

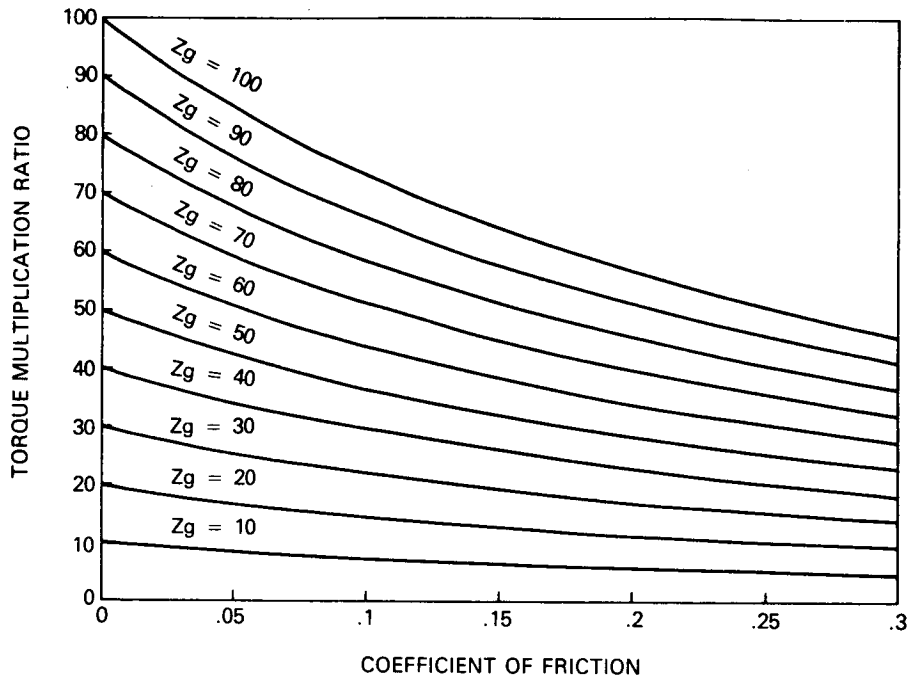
This section presents a suggested design method for worm gear sets when the primary requirement is torque output. Figure 12 shows a flow chart that summarizes the following discussion.

The designer must first look at the worm gear set application to determine the dimensional requirements. The designer usually starts with a center distance requirement. The American Gear Manufacturer's Association (AGMA) recommends the following minimum worm pitch diameter based on center distance:

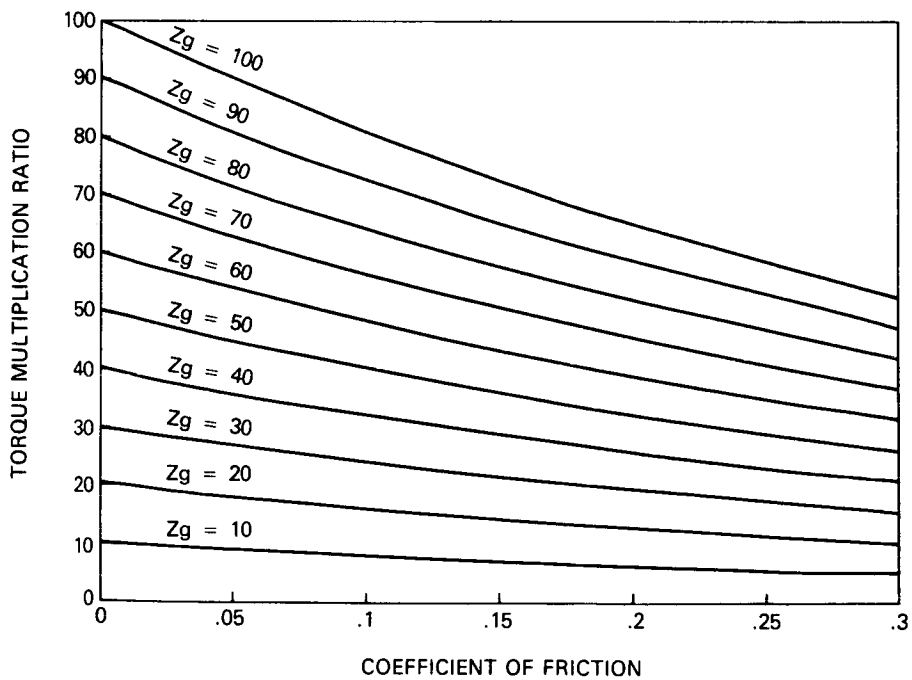
$$D_w \geq \frac{C_d}{2.2} \cdot 0.875$$

Select a center distance, use the above relation to select a worm pitch diameter (rounding up a standard size), then calculate the gear's pitch diameter using:

$$D_g = 2 C_d - D_w$$



**FIGURE 8: THE EFFECT OF THE COEFFICIENT OF FRICTION AND THE GEAR RATIO ON THE TORQUE MULTIPLICATION RATIO ( $\phi_n = 20^\circ, \lambda = 18^\circ 26'$ )**



**FIGURE 9: THE EFFECT OF THE COEFFICIENT OF FRICTION AND THE GEAR RATIO ON THE TORQUE MULTIPLICATION RATIO ( $\phi_n = 20^\circ, \lambda = 40^\circ$ )**

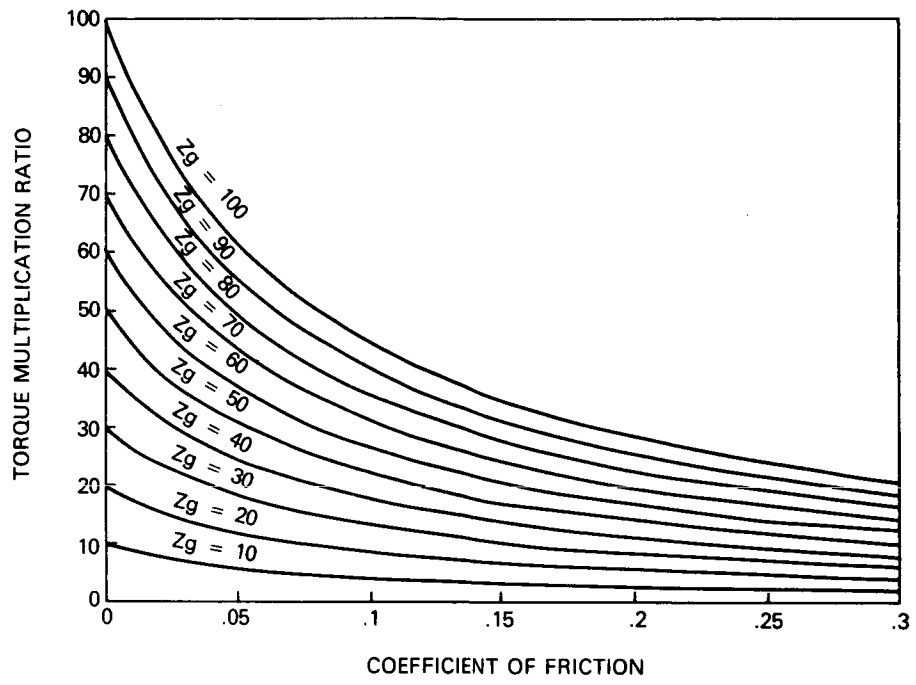


FIGURE 10: THE EFFECT OF THE COEFFICIENT OF FRICTION AND THE GEAR RATIO ON THE TORQUE MULTIPLICATION RATIO ( $\phi_n = 20^\circ$ ,  $\lambda = 5^\circ$ )

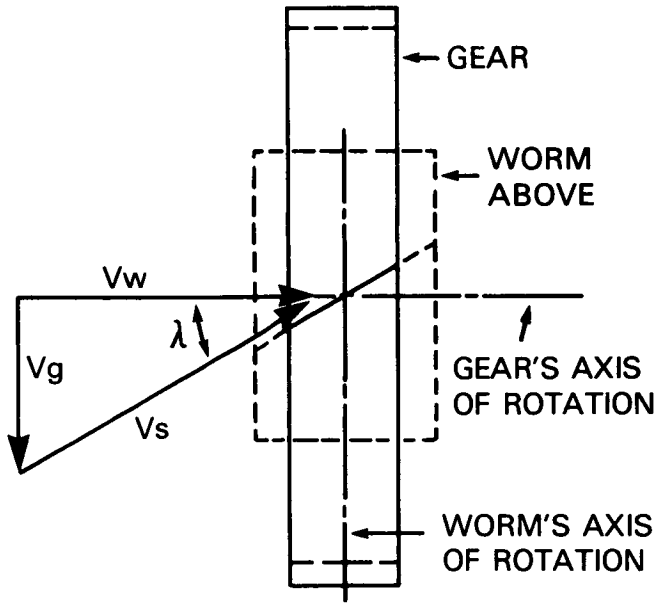


FIGURE 11: VELOCITY COMPONENTS IN WORM GEARING

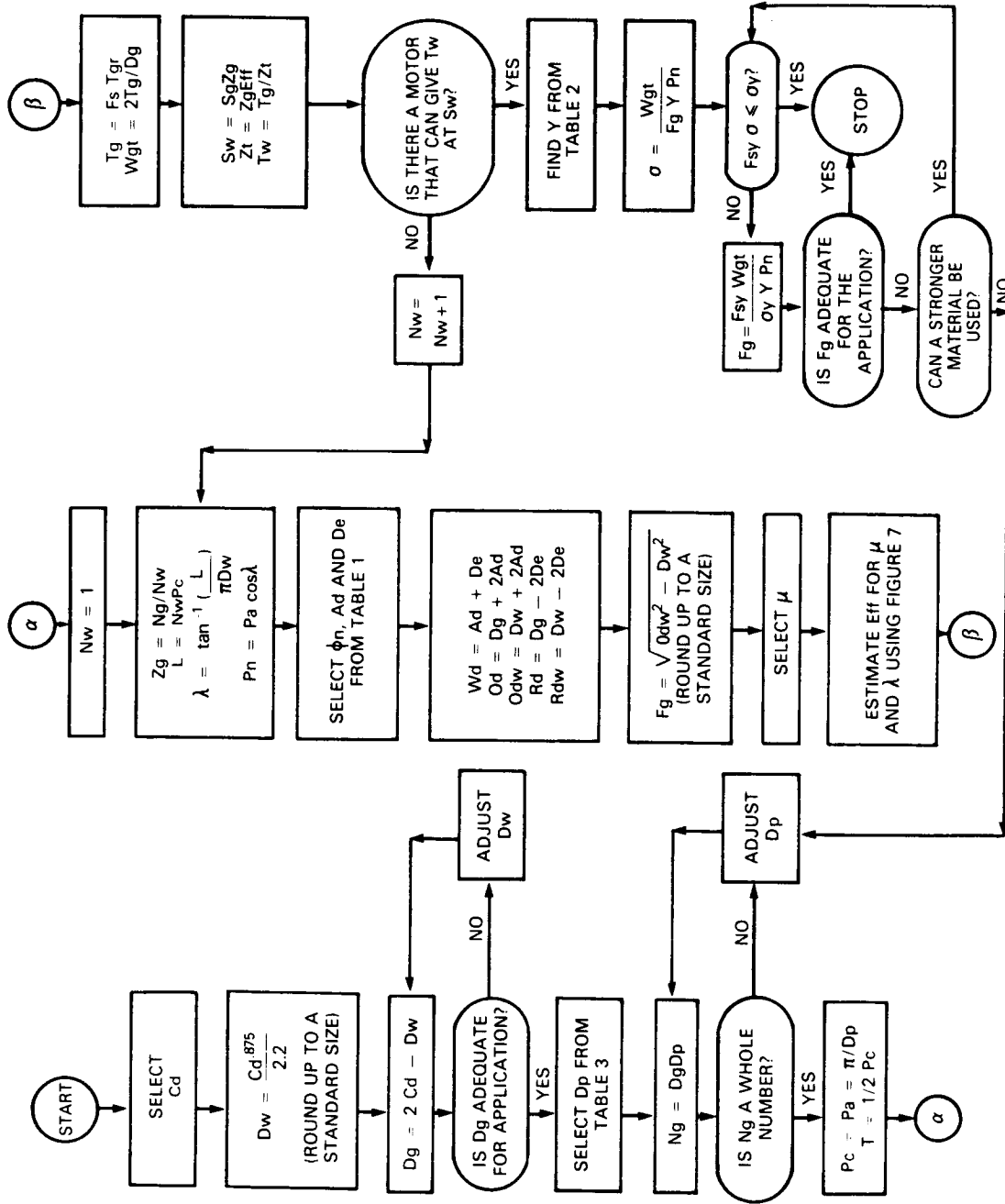


FIGURE 12: WORM GEAR DESIGN FLOW CHART



In any application, out of the three dimensions  $C_d$ ,  $D_w$  and  $D_g$ , the magnitude of at least one will be dictated by the gear set application; the other two can be calculated using the above equations.

The designer must now select a diametral pitch for the gear. The diametral pitch determines how smooth the gear set runs. Although there are no standards established, a preference has developed among gear producers and users. Table 3 classifies preferred diametral pitches from coarse to ultra-fine. Use this table to make a selection appropriate for the worm gear set application.

Table 3

PREFERRED DIAMETRAL PITCHES

Class	Diametral Pitch $D_p$	Class	Diametral Pitch $D_p$	Class	Diametral Pitch $D_p$	Class	Diametral Pitch $D_p$
Coarse	1/2	Medium-Coarse	12	Fine	20	Ultra-Fine	150
	1		14		24		180
	2		16		32		200
	4		18		48		
	6				64		
	8		72				
	10		80				
			96				
			120				
			128				

Once the diametral pitch has been selected, the required number of teeth on the gear is calculated. Since  $N_g$  must be a whole number,  $D_p$  may have to be slightly adjusted. Now calculate the circular pitch, axial pitch, and tooth thickness.

Initially set the number of threads on the worm to one. Next, calculate the gear ratio, the lead, the lead angle, and the normal circular pitch. Use Table 1 to select pressure angle, addendum and dedendum; then calculate the whole depth, outside diameters, and root diameters. Finally, calculate the recommended minimum face width of the gear tooth (rounding up to a standard size).

At this point the designer must make the assumption of worst-case friction. Based on a knowledge of the materials, lubrication, operating speeds and environment, a conservative estimate of the worst-case coefficient of friction must be made. Once a coefficient of friction has been assumed, use the graph in Figure 7 to estimate the efficiency for the particular lead angle. Now, a safety factor,  $F_s$ , must be selected. When torque output of the gear set is the primary design parameter, and knowledge of the friction is limited, a conservative factor of safety must be used.

The required torque output,  $T_{gr}$ , must be adjusted by applying the factor of safety:

$$T_g = F_s T_{gr}$$

The required input speed is the product of the required output speed and the gear ratio:

$$S_w = S_g Z_g$$

To determine the required input torque, first calculate the required torque multiplication ratio:

$$Z_t = Z_g \text{ Eff};$$

then divide the adjusted required output torque by the torque multiplication ratio:

$$T_w = T_g / Z_t$$

Now the designer has established the requirement that the drive motor must output a torque of  $T_w$  at a rpm speed of  $S_w$ . If this requirement is unsatisfactory, increase the number of threads on the worm and iterate until a reasonable motor requirement has been established.

Once the worm gear set has been sized, the designer must check the bending stress in the gear tooth. Use the Lewis equation to calculate the bending stress,  $\sigma$ . Select a factor of safety,  $F_{sy}$ , on the yield stress,  $\sigma_y$ , and compare  $F_{sy} \sigma$  to  $\sigma_y$ . If  $F_{sy} \sigma$  exceeds  $\sigma_y$ , calculate the required face width of the gear tooth to assure at least a factor of safety

of  $F_{sy}$  over  $\sigma_y$ . If this increase in  $F_g$  is unacceptable for the application, either consider a coarser gear and decrease  $D_p$  or consider using a stronger gear material.

#### CONCLUSIONS

The design of worm gear sets to meet torque multiplication requirements is a difficult task. The sliding that occurs between the worm and the mating gear causes high friction. When the sliding velocity is slow, as with many aerospace applications, the friction is high. Friction is dependent on many other factors including material combinations and lubrication. In aerospace applications of worm gearing, friction is especially difficult to characterize due to the use on non-standard materials and lubricants. Because the friction is difficult to quantify for a particular worm gear set application, the importance of conservative safety factors on torque output and material yield are strongly emphasized.

The suggested design method presented in this paper demonstrates how a worm gear set designer can start with the dimensional limitations of the application, and a torque output requirement, and develop a complete worm gear set design. The designer will likely perform several iterations before finding a design that is appropriate for the application, and also has attainable requirements for the drive motor. If the designer keeps the requirements and limitations in mind, and uses conservative factors of safety, this design method can be used to design a worm gear set able to deliver the required output torque.