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THERMO-ELASTO-VISCOPLASTIC ANALYSIS OF PROBLEMS IN EXTENSION AND SHEAR

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Abstract

The problems of extension and shear behavior of structural elements made of carbon steel and subjected to large thermomechanical loads are investigated. The analysis is based on nonlinear geometric and constitutive relations, and is expressed in a rate form. The material constitutive equations are capable of reproducing all nonisothermal, elasto–viscoplastic characteristics. The results of the test problems show that: (i) the formulation can accommodate very large strains and rotations, (ii) the model incorporates the simplification associated with rate–insensitive elastic response without losing the ability to model a rate–temperature dependent yield strength and plasticity, and (iii) the formulation does not display oscillatory behavior in the stresses for the simple shear problem.

I. Introduction

The prediction of inelastic behavior of metallic materials at elevated temperatures has recently increased in importance. The operating conditions within the hot section of a rocket motor or a modern gas turbine engine present an extremely harsh thermo- mechanical environment. Large thermal transients are induced each time the engine is started or shut down. Additional thermal transients from an elevated ambient occur, whenever the engine power level is adjusted to meet flight requirements. The structural elements employed to construct such hot sections, as well as any engine components located therein, must be capable of withstanding such extreme conditions. Failure of components would, due to the critical nature of the hot section, lead to an immediate and catastrophic loss in power and thus cannot be tolerated. Consequently, assuring satisfactory long term performance for such components is a major concern for the designer.

Under this kind of severe loading conditions, the structural behavior is highly nonlinear due to the combined action of geometrical and physical nonlinearities. On one side, finite deformation in a stressed structure introduces nonlinear geometric effects. On the other side, physical nonlinearities arise even in small strain regimes, whereby inelastic phenomena play a particularly important role. From a theoretical standpoint, nonlinear constitutive equations should be applied only in connection with nonlinear transformation measures (implying both deformation and rotation). However, in almost all of the works in this area [1], the two identified sources of nonlinearities are always separated. The separation yields, at one end of the spectrum, problems of large response, while at the other end, problems of isothermal and nonisothermal viscous behavior in the presence of small strain.

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The classical theories, in which the material response is characterized as a combination of distinct elastic, thermal, time independent inelastic (plastic) and time dependent inelastic (creep) deformation components cannot explain some phenomena, which can be observed in complex thermo–mechanical loading histories. This is particularly true when high temperature nonisothermal processes must be taken into account. There is a sizeable body of literature [1,2] on phenomenological constitutive equations for the rate– and temperature– dependent plastic deformation behavior of metallic materials. However, almost all of these new "unified" theories are based on small strain assumptions and several suffer from some thermodynamic inconsistencies.

In previous papers [3,4], the authors have presented an alternative constitutive law for elastic–thermo–viscoplastic behavior of metallic materials, in which the main features are: (a) unconstrained strain and deformation kinematics, (b) selection of reference space and configuration for the stress tensor, bearing in mind the rheologies of real materials, (c) an intrinsic relation which satisfies material objectivity, (d) thermodynamic consistency, and (e) proper choice of external and internal thermodynamic variables.

The present paper focuses on a general mathematical model and solution methodologies, to examine extension and shear behavior of structural elements made of a realistic (C 45) material and subjected to nonisothermal large elasto–plastic deformation.

II. Elasto–Thermo–Viscoplastic Constitutive Relations

In a previous works [3,4] following the ideas of Lehmann [5,6] the authors have presented a complete set of constitutive relations for nonisothermal, large strain, elasto–viscoplastic behavior of metals. It was shown [3,4] that the metric tensor in the convected (material) coordinate system can be linearly decomposed into elastic and (visco) plastic parts. So a yield function was assumed, which is dependent on the rate of change of stress, on the metric, on the temperature and on a set of internal variables. Moreover, a hypoelastic law was chosen to describe the thermo-elastic part of the deformation.

A time and temperature–dependent viscoplasticity model was formulated in this convected material system to account for finite strains and rotations. The history and temperature dependence were incorporated through the introduction of internal variables. The choice of these variables, as well as their evolution, was motivated by thermodynamic considerations.

The nonisothermal elasto-viscoplastic deformation process was described completely by "thermodynamic state" equations. Most investigators [1,2] (in the area of viscoplasticity) employ plastic strains as state variables. The author's previous study [3] shows that, in general, use of plastic strains as state variables may lead to inconsistencies with regard to thermodynamic considerations. Furthermore, the approach and formulation employed in previous works [1,2] lead to the condition that all plastic work is completely dissipated. This, however, is in contradiction with experimental evidence, from which it emerges that part of the plastic work is used for producing residual stresses in the lattice, which, when phenomenologically considered, causes hardening. Both limitations were excluded from this [3] formulation.

The constitutive relation will be rephrased here in some different form. For brevity we compile only some notations and fundamental relations which are used in the formulation of the intended constitutive law. For details, see Refs. 3 and 4.

Concerning the formulation of constitutive laws, it is advantageous to use a material (co-moving) coordinate system. The transformation (f^i_k) from the undeformed state (metric $\overset{\circ}{g}{}^{ir}$) to the deformed state (metric g_{rk}) can be represented by the tensor:

$$f^i_k = \overset{\circ}{g}{}^{ir}\, g_{rk} \quad or \quad (f^{-1})^i_k = g^{ir}\, \overset{\circ}{g}_{rk} \tag{1}$$

The total deformation rate is defined by

$$d^i_k = \frac{1}{2} g^{ir} \dot{g}_{rk} = -\frac{1}{2} g_{ir} \dot{g}^{rk} = \frac{1}{2} (f^{-1})^i_r (\dot{f})^r_{.k} = -\frac{1}{2} (\dot{f}^{-1})^i_{.r} f^r_k \tag{2}$$

here $(\dot{})$ denotes time material derivative. The expression

$$\overset{\nabla}{f}{}^i_k = (\dot{f})^i_{.k} + d^i_r f^r_k - d^r_k f^i_r = sym\{(\dot{f})^i_{.k}\} \tag{3}$$

represents the symmetric part of $(\dot{f})^i_{.k}$ or the covariant derivative with respect to time, also called the convected derivation, which is due to Zaremba and Jaumann. [5]

The total deformation can be decomposed according to

$$f^i_k = \overset{\circ}{g}{}^{im} \overset{*}{g}_{mr} \overset{*}{g}{}^{rs}\, g_{sk} = \overset{(i)}{f}{}^i_r \overset{(r)}{f}{}^r_k \tag{4}$$

where the superscript $\overset{*}{(\,)}$ relates to a fictitious configuration defined by a fictitious reversible process with frozen internal variables. The decomposition of Eq. (4) leads to an additive decomposition of the deformation rate

$$d^i_k = \overset{(r)}{d}{}^i_k + \overset{(i)}{d}{}^i_k \tag{5}$$

where $\overset{(r)}{d}{}^i_k$ is related to the reversible deformations, and $\overset{(i)}{d}{}^i_k$ denotes the remaining part of the deformation rate.

For the description of the stress state, we introduce the Kirchoff stress tensor s^i_k, which is connected with the real Cauchy stress tensor σ^i_k, by the relation:

$$s^i_k = \frac{\overset{\circ}{\rho}}{\rho} \sigma^i_k \tag{6}$$

where ρ is the mass density.

Assuming that the elastic behavior is not affected essentially by inelastic deformations, the following hypoelastic incremental law was chosen [3]

$$\overset{(r)}{d}{}^i_k = \frac{1}{2G} \overset{\nabla}{t}{}^i_k + \{\frac{1}{9K} \dot{s}^r_r + \alpha \dot{T}\} \delta^i_k \tag{7}$$

with

t^i_k	:	weighted stress deviator
G	:	shear modulus
K	:	bulk modulus
α	:	coefficient of thermal expansion

The following constitutive relations were established [3] for the inelastic behavior.
yield condition:

$$F = (t^i_k - c\overset{\circ}{\rho}\, g\beta^i_k)(t^k_i - c\overset{\circ}{\rho}\, g\beta^k_i) - k^2(A,T) = f^2 - k^2 > 0 \tag{8}$$

accompanying equilibrium state:

$$\overline{F} = (\bar{t}^i_k - c\overset{\circ}{\rho}\, g\beta^i_k)(\bar{t}^k_i - c\overset{\circ}{\rho}\, g\beta^k_i) - k^2(A,T) = \overline{f}^2 - k^2 = 0 \tag{9}$$

evolution law for inelastic deformations:

$$\overset{(i)}{d}{}^i_k = 2\dot{\lambda}(\bar{t}^i_k - c\overset{\circ}{\rho}\, g\beta^i_k) \tag{10}$$

with

$$\dot{\lambda} = \frac{1}{4\eta}\left(\sqrt{\frac{(t^i_k - c\overset{\circ}{\rho}\, g\beta^i_k)(t^k_i - c\overset{\circ}{\rho}\, g\beta^k_i)}{k^2}} - 1\right) \tag{11}$$

and

$$\bar{t}^i_k = \frac{1}{1+4\eta\dot{\lambda}}(t^i_k - c\overset{\circ}{\rho}\, g\beta^i_k) + c\overset{\circ}{\rho}\, g\beta^i_k \tag{12}$$

evolution laws for the internal variables:

$$\dot{A} = \frac{1}{\overset{\circ}{\rho}}\bar{t}^i_k\, \overset{(i)}{d}{}^k_i \tag{13}$$

$$\overset{\nabla}{\beta}{}^i_k = \xi\, \overset{(i)}{d}{}^i_k \tag{14}$$

if

$$F = 0 \quad and \quad \frac{\partial F}{\partial s^i_k}\overset{\nabla}{s}{}^i_k + \frac{\partial F}{\partial T}\dot{T} > 0 \tag{15}$$

then

$$d^i_k = \overset{(r)}{d}{}^i_k \tag{16}$$

$$\overset{(i)}{d}{}^i_k = 0 \quad and \quad \overset{\nabla}{d}{}^i_k = 2\ddot{\lambda}(t^i_k - c\overset{\circ}{\rho}\, g\beta^i_k) \tag{17}$$

with

$$\ddot{\lambda} = \frac{1}{8\eta k^2}\{2(t^i_k - c\overset{\circ}{\rho}\, g\beta^i_k)\,\overset{\nabla}{t}{}^k_i - \frac{\partial k^2}{\partial T}\dot{T}\} \tag{18}$$

if

$$F = 0 \quad and \quad \frac{\partial F}{\partial s^i_k}\overset{\nabla}{s}{}^i_k + \frac{\partial F}{\partial T}\dot{T} \leq 0 \tag{19}$$

or ,if

$$F < 0 \tag{20}$$

then

$$\begin{aligned} d^i_k &= \overset{(r)}{d}{}^i_k \\ \dot{A} &= 0 \\ \overset{\nabla}{\beta}{}^i_k &= 0 \end{aligned} \tag{21}$$

Within the developed frame, the elasto-viscoplastic behavior is governed by the scalar material functions $c(s^i_k, T, A, \beta^i_k)$, $k^2(A,T)$, $g(s^i_k, A, T, \beta^i_k)$, $\xi(A,T,\beta^i_k)$, and $\eta(A,T,\beta^i_k)$. These material functions can be determined from a series of monotonic and cyclic processes with proportional and nonproportional paths at different temperature levels [4].

III. General Formulation

The rate form of the constitutive equations suggests [7,8] that a rate approach be taken toward the entire problem so that flow is viewed as a history dependent process rather than an event. For this purpose, a complete true ab-initio rate theory of kinematics and kinetics for the continuum and curved thin structures, without any restriction on the magnitude of the transformation was presented in Ref. 4. It is implemented with respect to a body- fixed system of convected coordinates, and it is valid for finite strains and finite rotations. The time dependence and large strain behavior are incorporated through the introduction of the time rates of change of the metric (d_{ik}) and of the spin (ω_{ik}). The constitutive law may be applied to the conservation of momentum via an appropriate variational principle. The principle of virtual power (or of virtual velocities) reads

$$\int_V \sigma^{ij}\delta v_{j,i}dV - \int_V \rho f^j \delta v_j dV - \int_A \nu T^j \delta v_j dA = 0 \tag{22}$$

where δv_j are the virtual velocities, f^j the body forces per unit mass and νT^j the surface tractions. Total differentiation of Eq. (22) yields,

$$\begin{aligned}\int_V (\frac{d\sigma^{ij}}{dt} + \sigma^{ij}d^k_{\cdot k} - v^i_{,k}\sigma^{kj})\delta v_{j,i}dV - \int_V \rho\frac{df^j}{dt}\delta v_j dV \\ - \int_A \nu\frac{dT^j}{dt}\delta v_j dA + \int_V \sigma^{ij}(\frac{d\delta v_j}{dt}),i dV - \int_V \rho f^j \frac{d\delta v_j}{dt}dV \\ - \int_A \nu T^j \frac{d\delta v_j}{dt}dA = 0\end{aligned} \tag{23}$$

At any instant, Eq. (23) must be satisfied. The virtual velocity and its time derivative are then independent. Moreover, the last three terms of Eq. (23) are equivalent to Eq. (22). Hence, the principle of the rate of virtual power may be obtained in its concise form. For further classifications, the total derivative of the stress components will be represented by the Jaumann derivative, and the following integrals are defined by

$$I_e = \int_V \overset{\nabla}{\sigma}{}^{ij}\ \delta v_{j,i}dV \tag{24}$$

$$I_d = \int_V (\sigma^{ij}d^k_k - \sigma^{kj}d^i_{\cdot k})\delta v_{j,i}dV \tag{25}$$

$$I_r = \int_V \omega^j_{\cdot k}\sigma^{kj}\delta v_{j,i}dV \tag{26}$$

Then, substitution in Eq. (23) yields the final form of the principle of the rate of virtual power,

$$I = I_e + I_d + I_r = \int_V \rho\frac{df^j}{dt}\delta v_j dV + \int_A \nu\frac{dT^j}{dt}\delta v_j dA \tag{27}$$

The quasi-linear nature of the principle of the rate of virtual power suggests the adoption of an incremental approach to numerical integration with respect to time. The availability of the field formulation provides assurance of the completeness of the incremental equations and allows the use of any convenient procedure for spatial integration over the domain V.

IV. Simplified Constitutive Relations

One of the most challenging aspects of finite strain formulations is to locate a known solution with which to compare a proposed formulation. Typically, as a first problem, a large strain uniaxial test case is analyzed. The uniaxial tensile test is a common and simple way to characterize the stress–strain relation for a given material, since the tensor components used in the constitutive relations have to be related to this uniaxial test. This example clearly demonstrates how the general constitutive relations can be applied to a particular real material. This material law is then applied to the remaining examples.

For a carbon steel C45 (DIN 1720), in a pure tension test at a moderate temperature and strain rate, the material behavior, which is shown in Fig. 1, is obtained from Ref. 9. From this we may derive the stress–stain–temperature relations for loading in pure tension in the form

$$\sigma = \sigma(\epsilon, T) \tag{28}$$

For our purpose it is more useful to write this relation in the form

$$\sigma = \sigma(A,T) = \frac{c_1(T)A}{c_2(T)+A} + \sigma_0(T) \tag{29}$$

In our special case we get

$$\begin{aligned} c_1(T) &= 72.42 - 36.03 * 10^{-3}T \quad \frac{N}{mm^2} \\ c_2(T) &= 7.35 - 8.04 * 10^{-3}T \quad \frac{N}{mm^2} \\ \sigma_0(T) &= 47.41 - 38.9 * 10^{-3}T \quad \frac{N}{mm^2} \end{aligned} \tag{30}$$

with T in ^0K.

We may consider the carbon steel approximately as an isotropic work–hardening material obeying the Von Misses–Hill yield condition. Furthermore, we assume that a constant ratio of 90% of the plastic work is dissipated. With these assumptions, we get the following desecription for the material under consideration,

independent process variables : s^i_k, T
dependent process variables : (a) A or $k^2(A,T)$
(b) $f^i_k, q, \ldots$

yield condition :

$$\begin{aligned} F(s^i_k, T, A) &= t^i_k t^k_i - k^2(A,T) = 0, \\ k^2(A,T) &= \frac{2}{3}[\sigma_0(T) + \frac{c_1(T)A}{c_2(T)+A}]^2 \end{aligned} \tag{31}$$

loading condition :

$$\frac{\partial F}{\partial s_k^i} \overset{\nabla}{s_k^i} + \frac{\partial F}{\partial T}\dot{T} = 2t_k^i \overset{\nabla}{t_i^k} - \frac{\partial k^2}{\partial T}\dot{T} > 0 \tag{32}$$

elastic strain rate :

$$\overset{(r)}{d_k^i} = \frac{1}{2G}[\overset{\nabla}{s_k^i} - \frac{\nu}{1+\nu} \overset{\nabla}{s_j^j}\,\delta_k^i] + \alpha\dot{T}\delta_k^i \tag{33}$$

with $\alpha = 11.9 * 10^{-6}K^{-1}$,
plastic strain rate :
when Eqs. (31) and (32) are fulfilled :

$$\overset{(i)}{d_k^i} = \dot{\lambda}\frac{\partial F}{\partial s_k^i} = \frac{2t_n^m \overset{\nabla}{t_m^n} - \frac{\partial k^2}{\partial T}\dot{T}}{k^2\frac{\partial k^2}{\partial A}} \tag{34}$$

otherwise :

$$\overset{(i)}{d_k^i} = 0 \tag{35}$$

rate of "plastic work" :

$$\dot{A} = s_k^i \overset{(i)}{d_i^k} \tag{36}$$

rate of applied heat :

$$\dot{q} = c\dot{T} - \xi\dot{A} \tag{37}$$

with $\xi = 0.9 = const.$ and $c = 465\frac{j}{kg\,K}$ (heat capacity).

V. Uniaxial Examples

Uniaxial loading at different rates, loading–unloading– reloading, and jump tests are selected for the investigation of the response of the described model. Figure 2 presents the strain rate effect on the stress–strain curve. The results are presented for dimensionless stress σ/σ_0. It is clear that increasing the strain rate, in general, increases the yield stress and the plastic hardening.

Loading–unloading–reloading results are shown in Fig. 3. In this figure, the upper and the lower dotted curves represent the constant–rate stress–strain curves at 1.0 and 0.1 sec $^{-1}$, respectively. And the solid as well as the dashed curves represents the loading–unloading–reloading stress response. Dotted curves are plotted for the sake of comparison. Clearly, upon reloading at a different rate from that of loading–unloading, the yielding takes place at values that are influenced by the previous strain rate of loading–unloading.

The jump test results, where the strain rate is suddenly changed, are displayed in Fig. 4. As in the case of loading–unloading–reloading, the jump test results reveal the strain rate–history dependence inherited in the model. Upon decreasing the strain rate from 1.0 to 0.1 sec $^{-1}$ (solid line) the stress drops to values higher than that of the constant–rate loading at 0.1 sec $^{-1}$ (lower dotted curve). This is because the response after the jump is influenced by the previous strain rate of loading before the jump(1.0 sec $^{-1}$). Similarly, when the strain rate is increased from 0.1 to 1.0 sec $^{-1}$ (dashed line) the stress response rises to values lower than that of the constant–rate loading at 1.0 sec $^{-1}$ (upper dotted curve).

Finally, an example is considered, through which we show the response of the system corresponding to a given strain history. The history is depicted on Fig. 5(a), and all rates (positive or negative) are equal to 10 sec $^{-1}$. The response is shown in Fig. 5(b). The letters in Fig. 5(b) correspond to the letters of Fig. 5(a). The dotted lines in Fig. 5(b) were obtained under the assumption that the total plastic work was dissipated. Due to the viscid effect, the rate–dependent stress–strain curves (solid lines) have continuous slopes at the shifting points. Note that the transient hardening causes the subcycles not to be closed.

VI. Simple Shear Examples

The previous examples, although important, only represent a partial test because the principal stretch directions remain constant. The problem which was discussed by Nagtegaal and de Jong [10] and by Rolf and Bathe [11] as a problem which demonstrates limitations of the constitutive models in many finite strain formulations is the simple shear problem.

We take the same material as in the former examples, C 45, and consider a simple shear process, by effectively applying a shear strain, γ (see Fig. 6). We denote this material as material a.

The simple shear problem is defined by

$$X_1 = x_1 + KtX_2, \quad X_2 = x_2, \quad X_3 = x_3 \tag{38}$$

where the capital letters denote current position. This leads to the following nonzero components of deformation rates and spin

$$d_{12} = d_{21} = \frac{K}{2}, \quad \omega_{12} = -\omega_{21} = \frac{K}{2} \tag{39}$$

Please note that Eqs.(38) define simple shear for isothermal material behavior (process). This case was treated in Refs 10 and 11 and herein also. In adiabatic material behavior (process), X_3 must contain a temperature effect, which in our formulation appears through the coupling of (thermal) deformation and material behavior [constitutive law, see Eq.(33)]. The angular velocity of a line of material points depends only on its current orientation, angle γ, and is given by

$$\dot{\gamma} = -K\cos^2\gamma \tag{40}$$

Note that $K/2$ is the magnitude of the angular velocity of the material lines, when $\gamma = 0$ and $\gamma = \pi$, which instantaneously coincide with the principal directions of the rate of deformation tensor, d_{ij}. This is also the average of the angular velocities over all directions in the current configuration.

The process will be carried out, on the one hand, isothermally and, on the other hand, adiabatically. We find the solution of the problem by numerically integrating a system of first–order differential equations, originating from Eqs. (31) - (37). In the first case (isothermal process), the total deformation rate d^i_k and the temperature T_0 are given, and in the second case (adiabatic process) the total deformation rate is prescribed, and there is no heat transfer.

For comparison, we introduce, furthermore, a theoretical material whose yield condition is unaffected by temperature. This means, for this material, the hardening parameter k^2 is

$$k^2 = k^2(A, T_0)$$

In isothermal processes this material (denoted as material b) shows the same behavior as material a. But in adiabatic processes we have differences. For material a, the temperature influences the yield condition as well as the elastic part of deformation. For material b, only the elastic components of the deformation are affected by temperature changes.

Regarding this we must distinguish three cases:

(I) : isothermal processes with material a or b,

(IIA) : adiabatic processes with material a (hardening rule depending on temperature),

(IIB) : adiabatic processes with material b (hardening rule independent of temperature).

The results for the shear stresses and the temperature are shown in Fig. 7. We see that the differences between the shear stresses in the isothermal and in the adiabatic processes are mainly influenced by the dependence of the yield condition upon temperature. The differences between cases (I) and (IIB) are negligible, but not the differences between (IIA) and (IIB). It should be remarked that in case (IIA), we get a maximum shear stress for $\vartheta = 0.87$. So for larger deformation we find in this case a softening effect due to the increasing temperature. With respect to the temperature the differences between the adiabatic cases, (IIA) and (IIB) are rather small, since the differences in the plastic work, in both these cases, are not so important.

The second–order effects are more influenced by the temperature than the first–order effects. This can be seen from Fig. 8. The effects are partially changed in the opposite direction (see stress σ_{11}). This is due to the strong influence of the temperature on the elastic deformation. We may conclude this from the fact, that the differences between cases (IIA) and (IIB) are less than the differences between cases (I) and (IIA) or (IIB).

It is clear that the importance of the issue discussed here is not limited to the simple shear example, but would be of consequence in problems whenever the local shearing strain at a point or points in a structure becomes large during deformation of the structure.

VII. Conclusion

A true ab–initio rate theory has been developed by the present authors [3,4] to deal and predict the behavior of structural components made of metallic materials, when acted upon by large time–dependent thermo–mechanical loads. This rate theory is based on coupled material and deformation behavior in a highly nonlinear form, which also includes time and temperature effects.

The main thrust of this paper is to demonstrate the capabilities of this formulation, through problems of extension and shear. Among the most important findings of the test problems one may list the following: (i) the formulation can accommodate very large strains and rotations, (ii) the model incorporates the simplification associated with rate–insensitive elastic response without losing the ability to model a rate–temperature dependent yield strength and plasticity, (iii) the general constitutive relations can be applied to a particular real material, (iv) the constitutive relations are sensitive to strain rate and temperature history, (v) the formulation does not display oscillatory behavior in the stresses for the simple shear problem, and (vi) the influence of the thermo–mechanical coupling on the processes can become very large and must be considered.

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Figure Captions

Fig. 1. Carbon Steel C 45 in Tension

Fig. 2. Effect of Strain Rate on the Stress–Strain Curve

Fig. 3. Loading–Unloading–Reloading at Different Strain Rates

Fig. 4. Jump Test

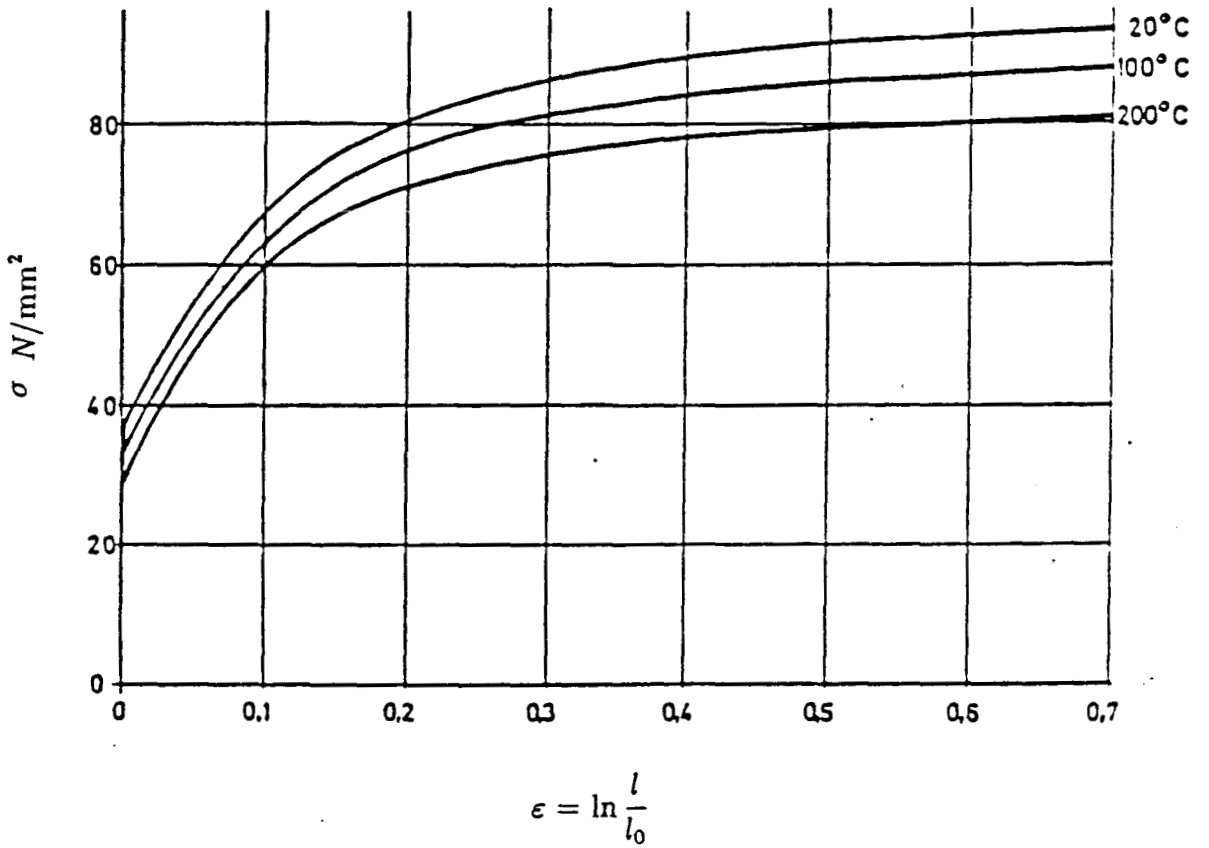
Fig. 5(a). Loading Program of Variable Strain Amplitude ($d = 10sec^{-1}$)

Fig. 5(b). Stress–Strain Response

Fig. 6. Simple Shear

Fig. 7. Response to Simple Shear Test

Fig. 8. Response to Simple Shear Test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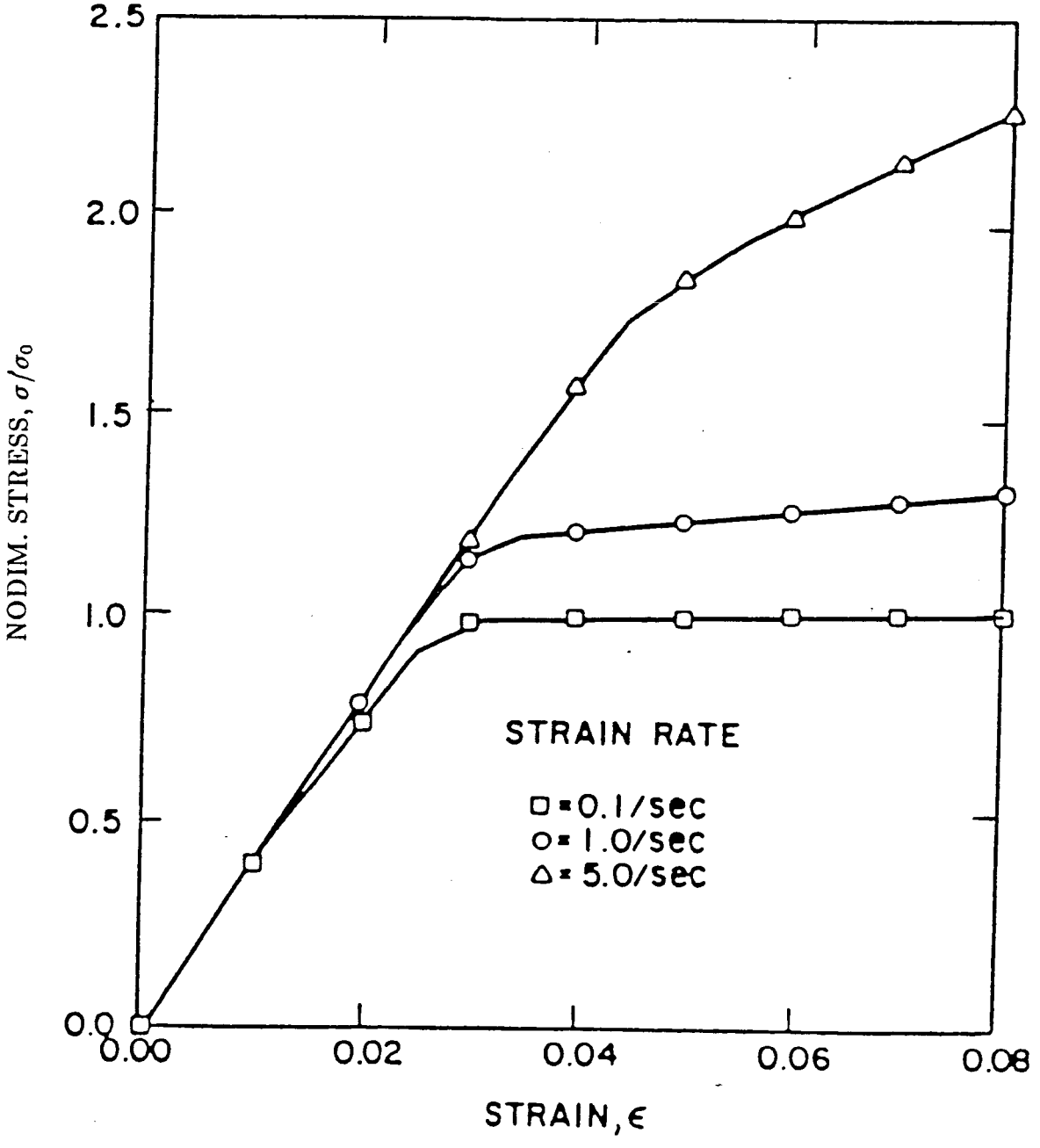
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STRAIN RATE
□ = 0.1/sec
○ = 1.0/sec
△ = 5.0/sec

Fig. 7

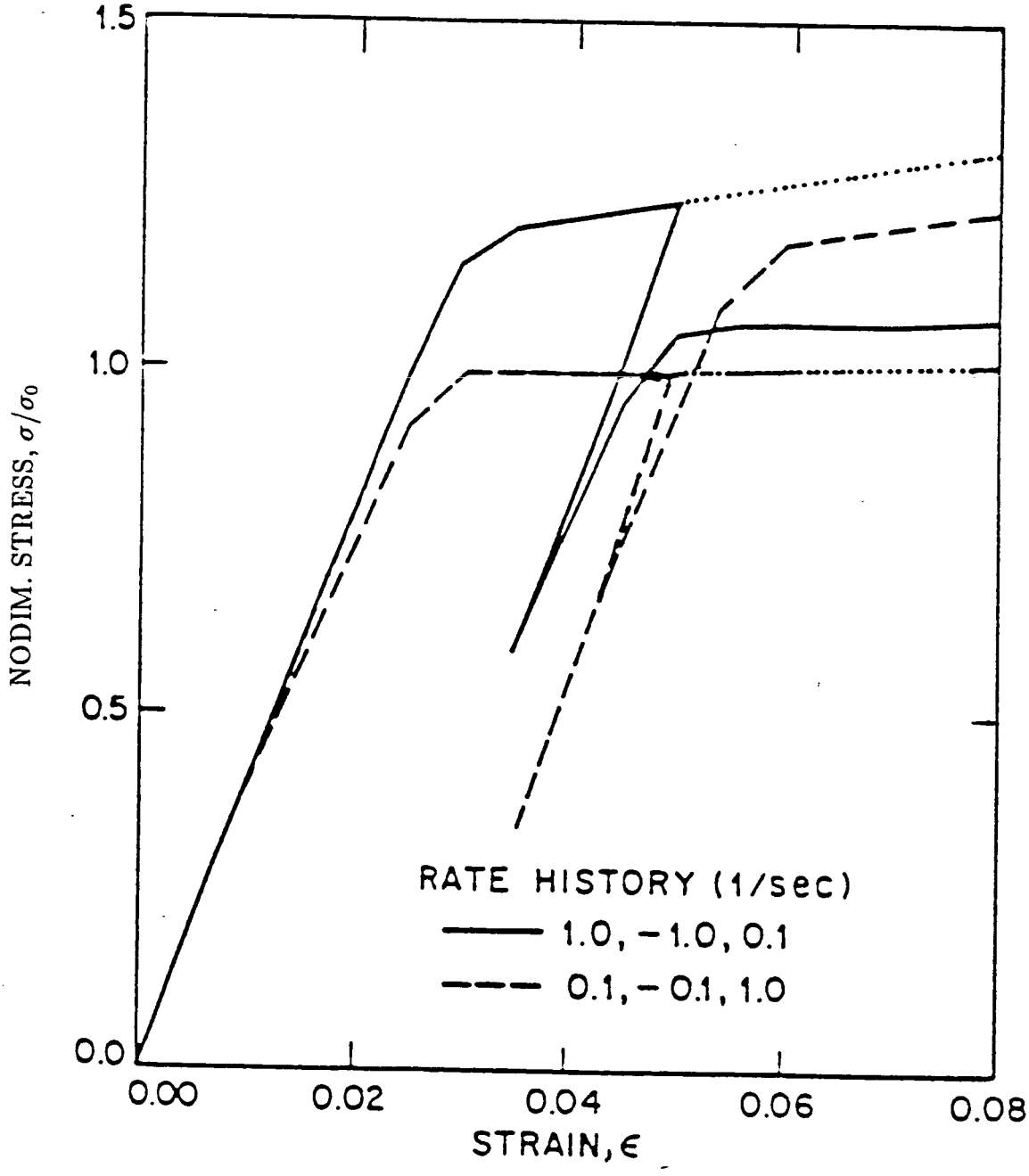
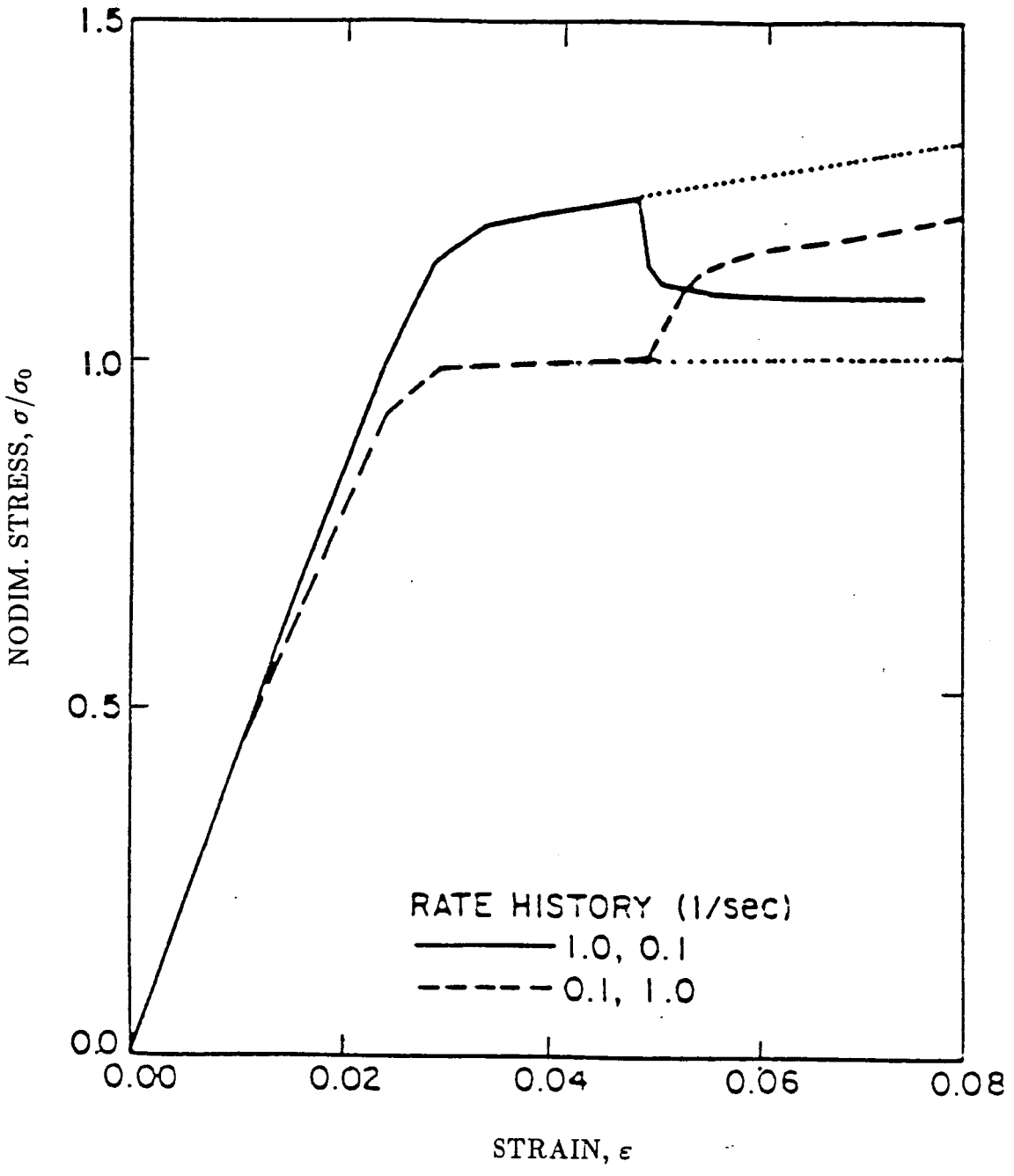
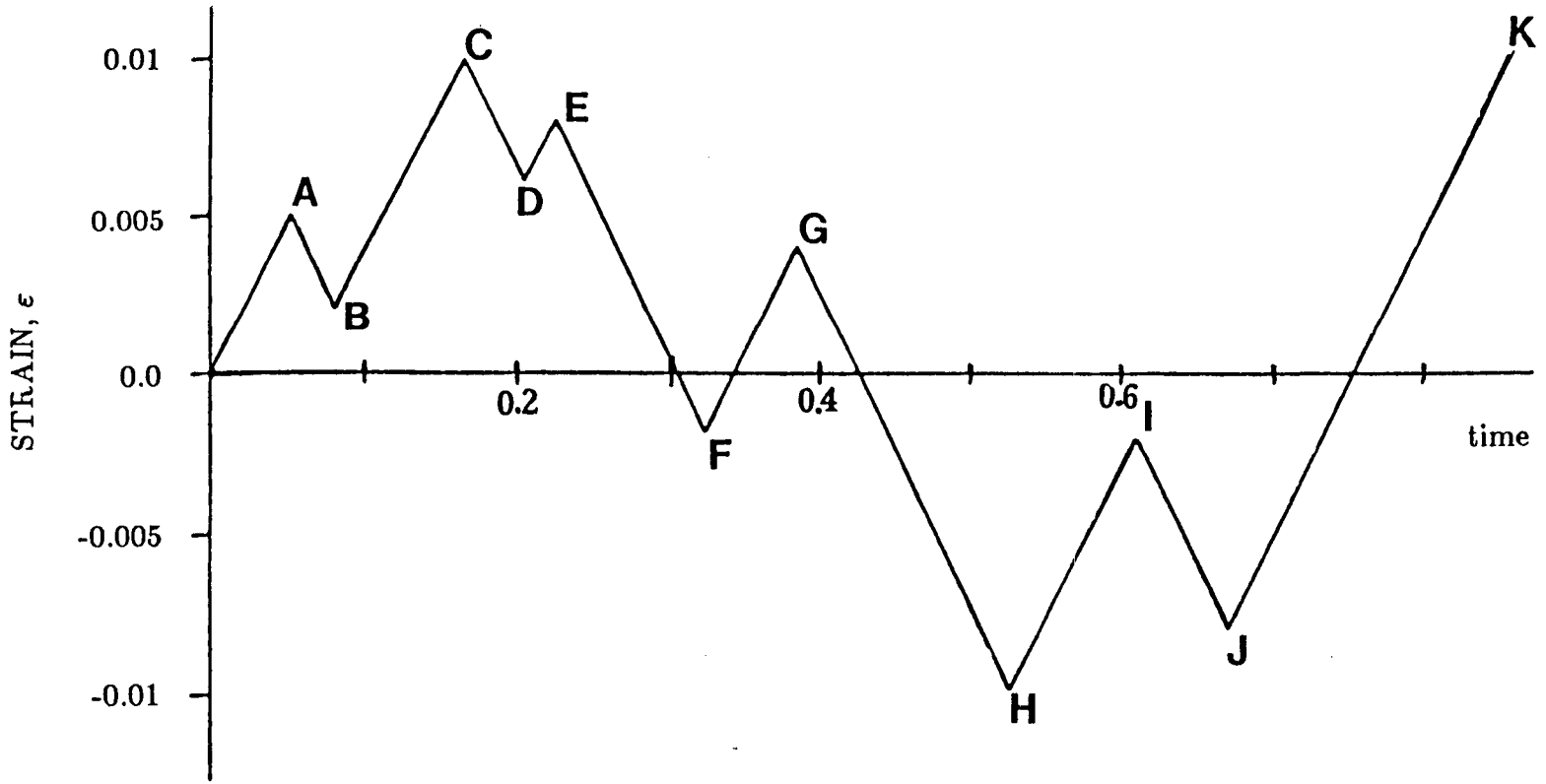
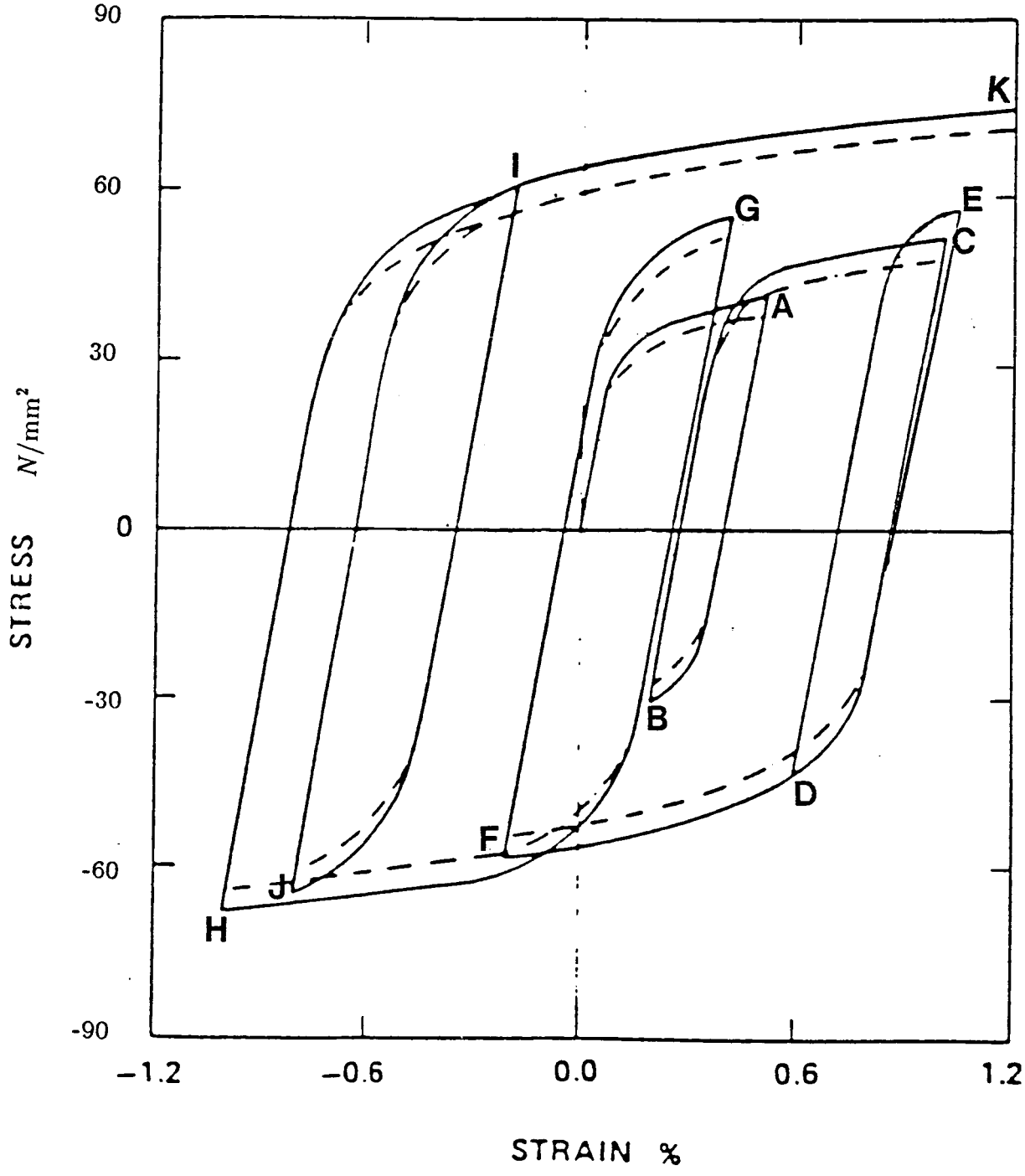
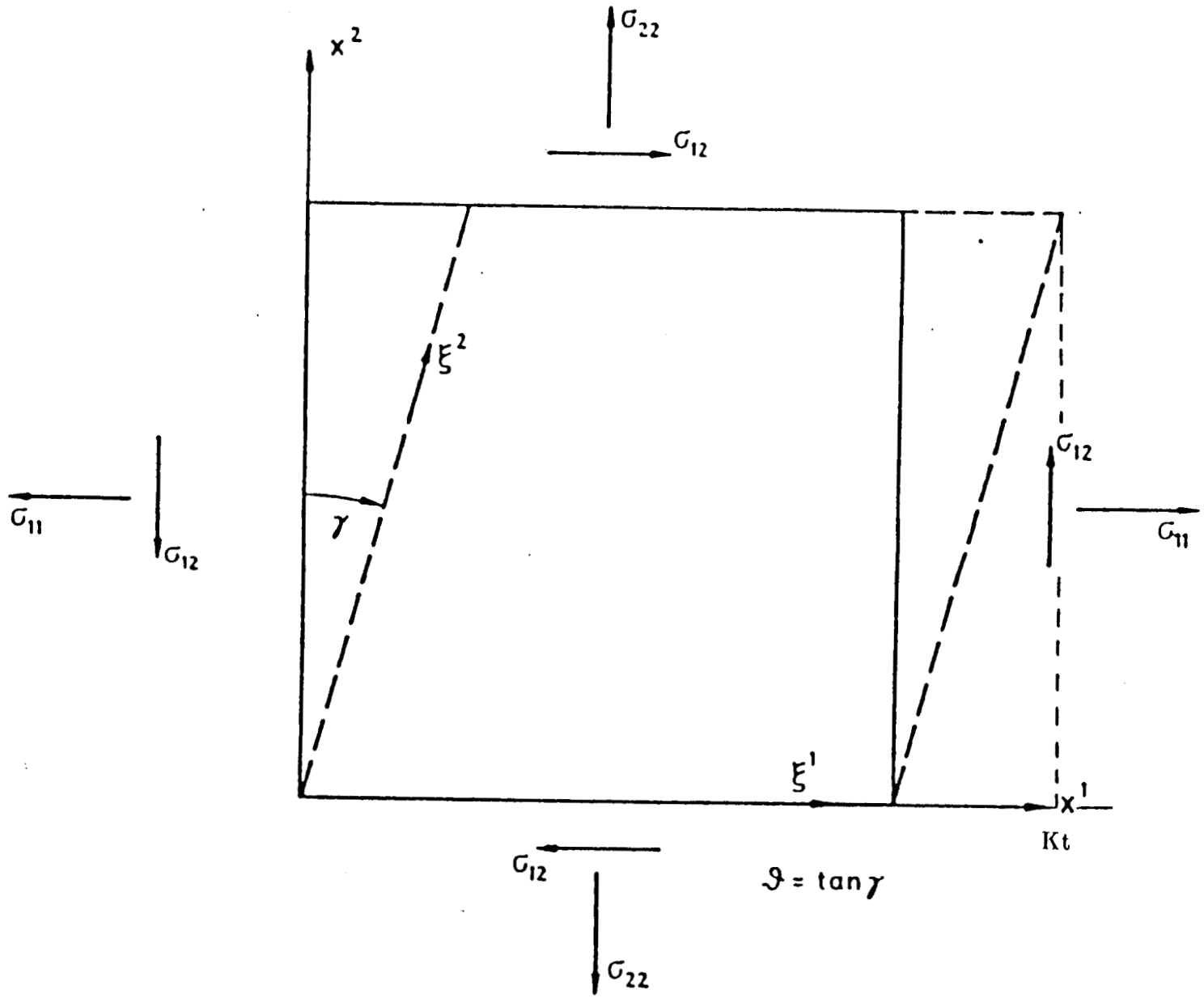

1.5
1.0
0.5
0.0
NODIM. STRESS, σ/σ_0
0.00
0.02
0.04
0.06
0.08
STRAIN, ϵ
RATE HISTORY (1/sec)
1.0, −1.0, 0.1
0.1, −0.1, 1.0

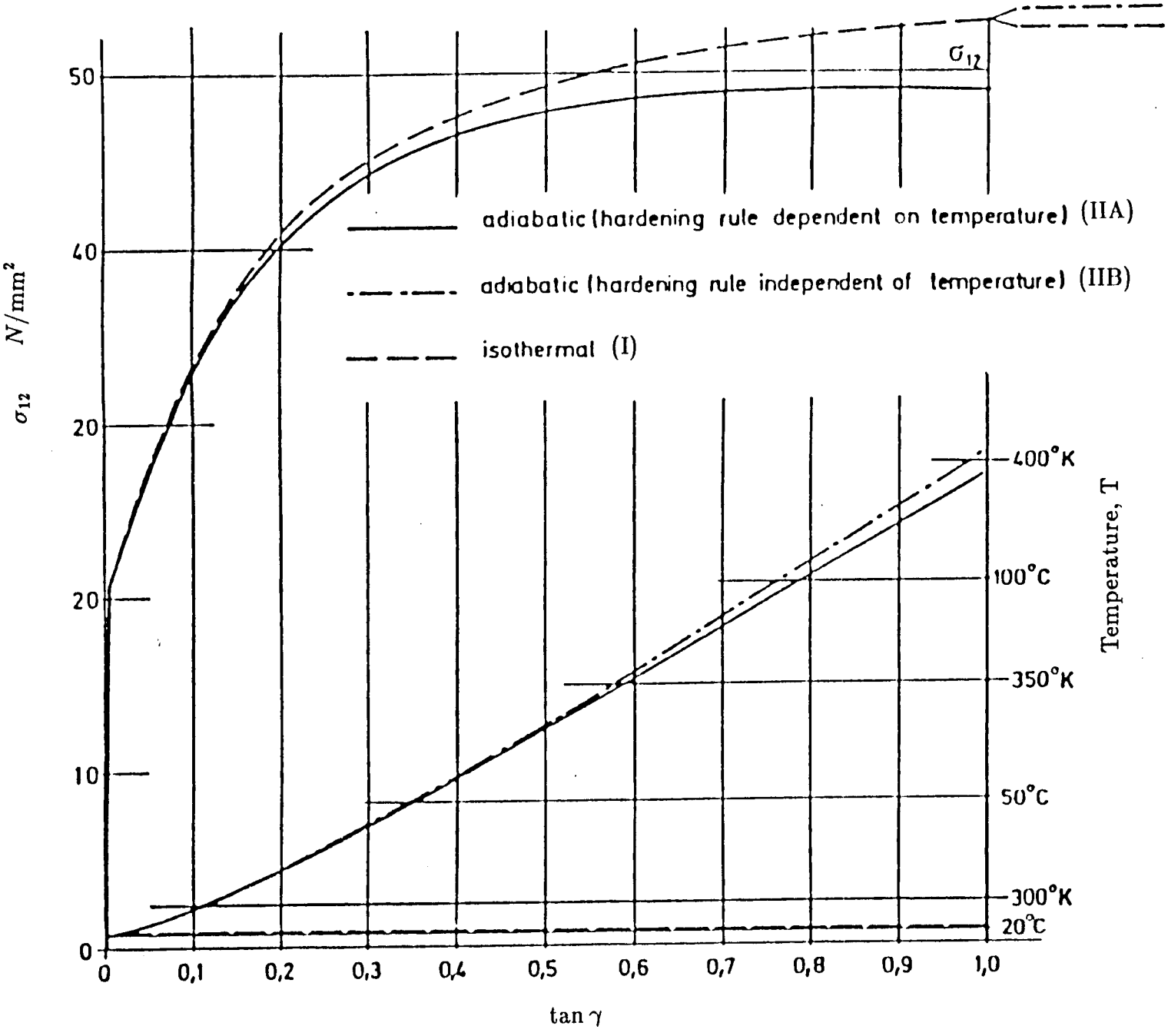
Fig. 3

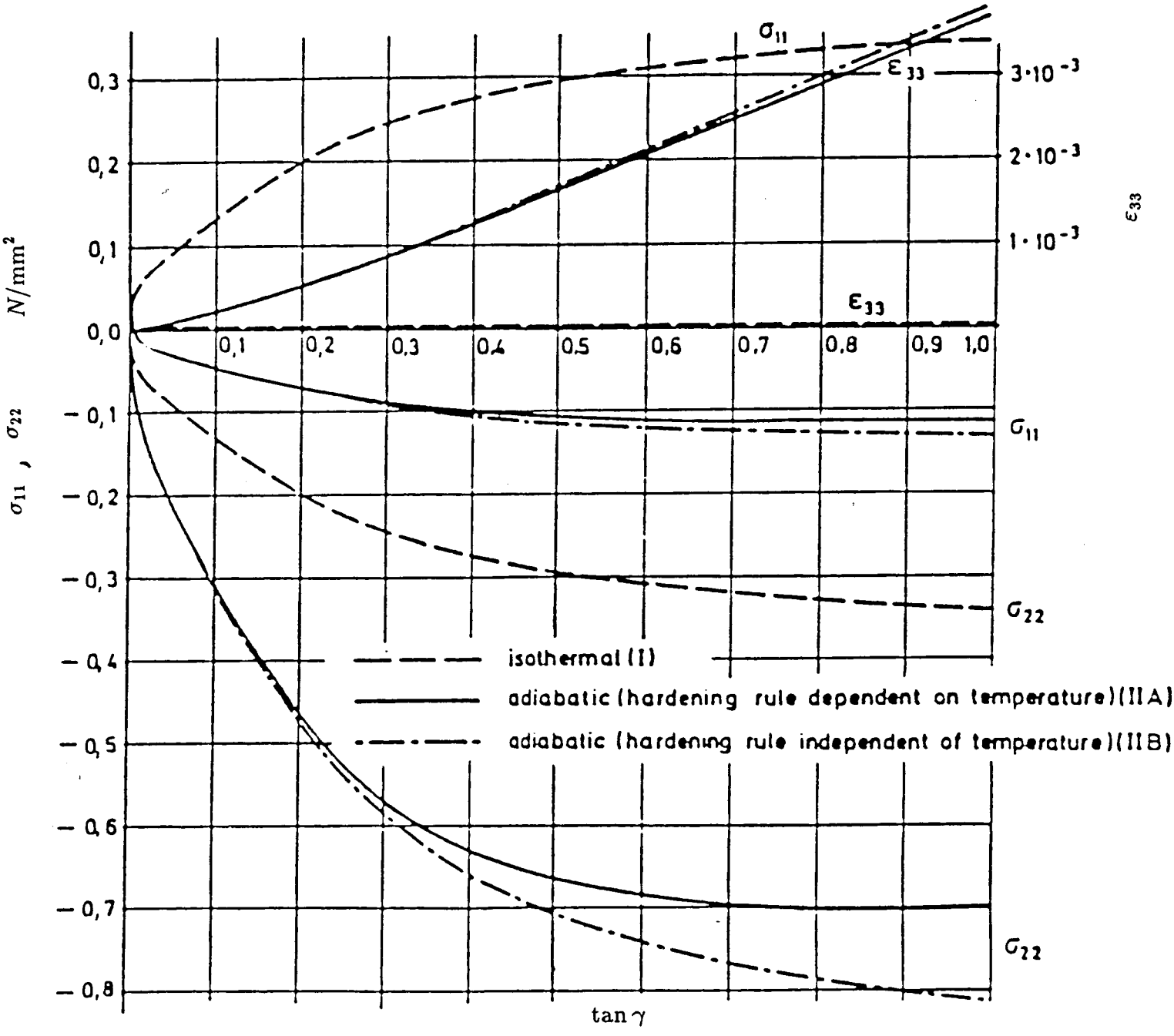

1.5
1.0
0.5
0.0
0.00
0.02
0.04
0.06
0.08
NODIM. STRESS, σ/σ_0
STRAIN, ε
RATE HISTORY (1/sec)
1.0, 0.1
0.1, 1.0


STRAIN, ε
0.01
0.005
0.0
-0.005
-0.01
0.2
0.4
0.6
time
A
B
C
D
E
F
G
H
I
J
K


90
60
30
0
-30
-60
-90
STRESS N/mm²
−1.2
−0.6
0.0
0.6
1.2
STRAIN %
K
I
G
E
C
A
B
D
F
J
H


x 2
σ22
σ12
ξ2
σ11
σ12
γ
σ12
σ11
ξ1
x 1
Kt
σ12
ϑ = tan γ
σ22


σ_{12} N/mm²
50
40
20
20
10
0
σ_{12}
adiabatic (hardening rule dependent on temperature) (IIA)
adiabatic (hardening rule independent of temperature) (IIB)
isothermal (I)
400°K
100°C
350°K
50°C
300°K
20°C
Temperature, T
0
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
tan γ


σ11
ε33
3·10⁻³
2·10⁻³
1·10⁻³
ε33
0,3
0,2
0,1
0,0
ε33
0,1
0,2
0,3
0,4
0,5
0,6
0,7
0,8
0,9
1,0
σ11
σ22
σ11 , σ22 N/mm²
−0,1
−0,2
−0,3
−0,4
−0,5
−0,6
−0,7
−0,8
isothermal (I)
adiabatic (hardening rule dependent on temperature)(IIA)
adiabatic (hardening rule independent of temperature)(IIB)
σ22
tan γ