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Analytical Modeling of Helicopter Static and Dynamic Induced Velocity in GRASP

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Abstract. This paper describes the methodology used by the General Rotorcraft Aeromechanical Stability Program (GRASP) to model the characteristics of the flow through a helicopter rotor in hovering or axial flight. Since the induced flow plays a significant role in determining the aeroelastic properties of rotorcraft, the computation of the induced flow is an important aspect of the program. Because of the combined finite-element/multibody methodology used as the basis for GRASP, the implementation of induced velocity calculations presented an unusual challenge to the developers. To preserve the modeling flexibility and generality of the code, it was necessary to depart from the traditional methods of computing the induced velocity. This is accomplished by calculating the actuator disc contributions to the rotor loads in a separate element called the air mass element, and then performing the calculations of the aerodynamic forces on individual blade elements within the aeroelastic beam element.

Keywords. Aeroelasticity; Finite Elements; Helicopters; Induced Velocity; Rotary Wings.

INTRODUCTION

In September 1980, work began on developing the General Rotorcraft Aeromechanical Stability Program (GRASP). While numerous analyses (Ormiston and Hodges, 1972; Friedmann, 1973; Hodges, 1976, 1979; Warmbrodt and Friedmann, 1979; Friedmann and Straub, 1980; Davis *et al.*, 1974; Bielawa, 1976; Johnson, 1977, 1980; Sivaneri and Chopra, 1982) are available to perform aeroelastic analyses for rotorcraft, all of them are subject to major limitations (Johnson, 1986) in generality, flexibility, or theoretical consistency. The purpose for which GRASP has been developed is to provide a tool with enhanced capabilities that can be used to perform aeroelastic calculations for helicopters in hover and axial flight.

The implementation of the hybrid finite-element/multibody methodology (Hodges *et al.*, 1987a, 1987bb) in GRASP allows a structure to be modeled as a collection of rigid bodies and flexible elements that can be connected in a completely arbitrary manner. While this methodology presents the analyst with a great deal of generality and flexibility in structural modeling, it also presents the developer with some challenges in implementing an appropriate representation of the helicopter flow field. Since the treatment of the flow around and through the rotor disk is an important part of any aeroelastic analysis of rotorcraft, it is vital that the induced velocity be calculated in a consistent manner.

In this paper, current methods used to calculate the inflow will first be described. Then, the methodology used in GRASP will be discussed and the differences with the more traditional methods highlighted. Finally, the theoretical basis of the approach implemented in GRASP will be outlined.

METHODOLOGY

Current aeroelastic stability analyses for helicopters use a variety of methods to calculate the steady-state and dynamic induced inflow. These range from simple, linear models for uniform inflow to sophisticated, nonuniform inflow models using free-wake analyses. While not breaking any new ground with respect to developing new models, GRASP does take a different approach with regard to its calculations of induced velocity. Therefore, before describing the methodology used in GRASP, and its rationale, it will be instructive to look at some representative examples of the approaches taken in current analyses.

Traditional Methods

In many analyses (Ormiston and Hodges, 1972; Friedmann, 1973; Hodges, 1976, 1979; Warmbrodt and Friedmann, 1979; Friedmann and Straub, 1980), the steady-state induced velocity is calculated from a single, linear, closed-form expression that combines both momentum and blade-element contributions to the rotor forces and moments. This expression is a function of the rotor collective pitch angle (usually at the three-quarter rotor radius). Assuming uniform inflow, one takes the induced inflow velocity over the entire rotor disk to be constant with the same value as the theoretical value at the three-quarter rotor radius. Alternatively, one could assume that the inflow angle, which is the inflow velocity divided by the local blade speed, is constant over the rotor radius with the same value as the theoretical value at the three-quarter rotor radius.

Another approach to calculating the steady-state inflow is demonstrated by Davis *et al.* (1974), Bielawa (1976), Johnson (1977), and Sivaneri and Chopra (1982). In this method the inflow velocity is calculated as a function of the thrust coefficient, which is usually given. However, the blade pitch angle required to produce the desired thrust is also a function of the inflow velocity. Thus, the computation of the induced velocity is nonlinear, and requires an iterative solution. The distribution of induced velocity over the rotor disk is then either assumed to be uniform, or specified by a set of assumed functions such as the Glauert induced velocity (Bielawa, 1976).

The method used in CAMRAD (Johnson, 1980) to calculate the induced velocity is more sophisticated than any of the preceding analyses. CAMRAD can use any of three methods to determine the induced flow. First, as above, a uniform inflow distribution is computed as nonlinear function of the thrust coefficient. Then, if desired, a nonlinear distribution can be determined from a prescribed-wake analysis, using the uniform inflow as an initial guess. If further refinement is needed, a free-wake analysis is performed using the prescribed-wake solution as the initial guess.

Of the analyses just discussed. only a few (Johnson, 1977, 1980) consider the effects of inflow dynamics. Basic to this type of dynamic inflow analysis is the assumption that total forces on the rotor vary slowly enough that actuator disk theory is applicable to perturbation velocities. Comparisons with experimental results (Johnson, 1986) have shown that dynamic inflow can have a significant effect on aeroelastic phenomena.

A common feature of all of the analyses discussed earlier is that the calculation of the steady-state inflow velocities is performed separately from the main calculation of the steady-state deformation of the structure. Thus, the inflow generalized coordinates are not included in the state vector with the structural degrees of freedom. However, when inflow dynamics are included in the dynamic problem (Johnson, 1977, 1980), those generalized coordinates are (and must be) included in the state vector. Although this is a somewhat inconsistent treatment of the steady-state and dynamic inflow generalized coordinates, it does not result in any significant analytical problems. This is a result of the coupling between the steady-state inflow and the structural deformations being very weak.

To separate the steady-state induced velocity calculations from the structural calculations, contributions to the rotor loads from individual blade elements must be calculated at the same time as the flow field contributions. Since the exact geometry of each blade may not be known, it is necessary to assume a relationship between one or more blade parameters and the forces on the rotor. For example, in Sivaneri and Chopra (1982) the induced velocity is calculated as a function of thrust, and thrust is a function of the blade pitch angle at the three-quarter radius and the induced velocity. It is apparent in this case that there is some implied relationship between the blade pitch angle and the blade geometry and section aerodynamics.

GRASP Methodology

The axisymmetric flow field for a helicopter in hover or axial flight is represented in GRASP by an element called the air mass element. The inflow generalized coordinates associated with this element are then included in the steady-state model state vector as well as in the dynamic model state vector. This means the steady-state inflow velocity is calculated in parallel with the structural deformations, and that those velocities are fully coupled to the deformed state of the rotor blades. The inflow generalized coordinates are introduced into the model in a manner similar to that used to introduce structural degrees of freedom. That is, an air node is introduced to represent the flow field at a point on the axisymmetric axis of that flow field.

One of the difficulties that arises from integrating the flow-field model into the structural model is related to the specification of the motion of the flow field relative to the structure. It is known that if the rotor disk undergoes large deformations, the flow field will also undergo changes that follow and lag the disk deformations. This occurs because the flow field is not physically attached to the rotor, but is highly dependent (to say the least) on its location. However, since such large motions would result in periodic forces and moments for which GRASP will not account, that situation may be ignored. There are, therefore, two possible implementations that may be used. The first associates the flow field with an inertially fixed frame of reference. In this case, it is understood that large motions of the rotor (which are not allowed) will have no effect on the location of the flow field. The other option is to attach the flow field to the structure with the understanding that large deformations will violate the flow-field model assumptions.

For GRASP, the former option was chosen since it is closer to the actual physics of the phemonenon.

Another difficulty with integrating the air mass element into the structural model arises because of the multilevel substructuring capabilities, which enhance the flexibility and generality of GRASP in modeling complex structures. One of the concepts fundamental to the use of multilevel substructuring is that no substructure is required to have any specific knowledge of any substructures other than its parent. In the context of the flow-field calculations, the air mass element has no access to information on the geometry of the rotor. This makes it virtually impossible to make any assumptions that would allow the blade-element contributions to the inflow calculations to be included in the air mass element. Any assumptions that might be made would be to the detriment of the generality of the code. Therefore, the calculations of the momentum contributions from the actuator disk are separated from the blade-element calculations. The air mass element represents only the flow-field aerodynamics, while the blade-element aerodynamics are isolated in the aeroelastic beam element.

THEORETICAL DEVELOPMENT

The theoretical development of the inflow equations is dependent on three components: the air node, the air mass element, and the aeroelastic beam element. The generalized coordinates that are used by GRASP to describe the static state and dynamic perturbations of the induced velocities are supplied by the air node. The air mass element performs the calculations of the actuator-disk contributions to the inflow equations, while the aeroelastic beam element calculates the blade-element contributions.

Under the assumptions used for this development, there are noncirculatory, blade-element contributions to the apparent-mass terms in the dynamic inflow. Some recent, but as yet unpublished work indicates that the dynamic inflow, apparent-mass terms result solely from circulatory effects. If this can be verified, some of the assumptions used in this analysis would have to be revised.

Air Node

The induced velocity generalized coordinates are introduced into GRASP via the air node. These generalized coordinates are defined relative to an inertial frame of reference I, and they define the inertial air velocity at any point in the rotor flow field. Given that $\underline{\hat{b}}_1^A$ is an inertially fixed unit vector and A is also an inertial coordinate system with its origin at the center of the flow field (Fig. 1), the induced velocity $\underline{U}^{QI}$ at a point Q

$$\underline{U}^{QI} = -(U_1^A + r\gamma_{1r}^A + R_{A2}^{QA}\gamma_{12}^A + R_{A3}^{QA}\gamma_{13}^A)\underline{\hat{b}}_1^A \qquad (1)$$

where r is the flow-field radial coordinate, and R_{Ai}^{QA} is the position of Q relative to A in the $\underline{\hat{b}}_i^A$ direction. U_1^A, γ_{1r}^A, γ_{12}^A, and γ_{13}^A are the air node generalized coordinates.

For the case of static inflow, generalized coordinates U_1^A and γ_{1r}^A are used to represent uniform inflow velocity and radial velocity gradient at the center of the flow field. The other two coordinates are not used. Dynamic inflow uses only generalized coordinates U_1^A, γ_{12}^A, and γ_{13}^A to represent the vertical and cyclic velocity perturbations.

Air Mass Element

The air mass element is implemented in GRASP to model the momentum flow of air through the disk of a helicopter rotor. In this element, the rotor is assumed to be an actuator disk, and the flow field a cylindrical region surrounding the disk (Fig. 1). The state vector for the air mass element is made up of the generalized coordinates for a single air node. In the following subsections the static and dynamic inflow models developed for the air mass element are discussed.

Static Inflow. In the static case, the air is considered to be flowing steadily through the rotor disk. From momentum theory (Gessow and Myers, 1967), the differential thrust dT acting on a differential annulus of the rotor disk is related to the induced velocity U by the equation

$$dT = 4\pi\rho_a rU\,|\mathcal{V} + U|\,dr \qquad (2)$$

where ρ_a is the air density, r is the rotor radial coordinate, and V is axial velocity of the rotor relative to still air (positive up). The total virtual work $\delta\mathcal{W}$ done by the thrust on the air is

$$\delta\mathcal{W} = 4\pi\rho_a \int_e^{\mathcal{R}} U\,\delta P\,|\mathcal{V} + U|\,r dr \qquad (3)$$

where e is the root cutout radius, $\mathcal{R}$ is the rotor radius, and δP is the virtual displacement of the air. The expression for virtual work is discretized by assuming that the induced velocity can be divided into a uniform velocity $\bar{U}_1^A$ and a radial gradient $\bar{\gamma}_{1r}^A$ so that

$$U = \bar{U}_1^A + \bar{\gamma}_{1r}^A r \qquad (4)$$

The virtual displacement of the air is discretized identically. Thus,

$$\delta P = \delta P_1^A + \delta\phi_{1r}^A r \qquad (5)$$

When these expressions are substituted into the expression for the virtual work, the coefficient of δP_1^A is equal to the rotor thrust while the coefficient of $\delta\phi_{1r}^A$ has the units of moment, but no real physical significance.

Dynamic Inflow. The model for the inflow dynamics is taken from Pitt and Peters (1981). It is assumed that the freestream velocity of the rotor relative to still air is spatially and temporally uniform. This freestream velocity is augmented within the cylindrical region of the flow field by the steady-state inflow velocity components just described. Then, infinitesimal dynamic perturbations to the inflow are induced by dynamic perturbations of the rotor thrust, roll moment, and pitch moment.

The virtual work for the unsteady flow of air through the rotor disk is

$$\delta W = \int_e^R \int_0^{2\pi} 2\rho_a U \left| \mathcal{V} + U \right| \delta P \, d\psi \, dr + \iiint_{V_{\text{eff}}} \rho_a \dot{U} \, \delta P \, dV_{\text{eff}} \tag{6}$$

where ψ is the rotor azimuth and V_{eff} is the effective volume of the cylindrical flow field. This statement of virtual work produces a system of first-order differential equations may be converted to a set of second-order equations and discretized by assuming

$$U = \bar{U}_1^A + \bar{\gamma}_{1r}^A r + \dot{P}_1^A - \dot{\phi}_{12}^A r \sin\psi + \dot{\phi}_{13}^A r \cos\psi \tag{7}$$

$$\delta P = \delta P_1^A - \delta\phi_{12}^A r \sin\psi + \delta\phi_{13}^A r \cos\psi \tag{8}$$

where $\dot{P}_1^A$ is the vertical perturbation of the induced velocity at the center of flow, $\dot{\phi}_{12}^A$ and $\dot{\phi}_{13}^A$ are the cyclic perturbation gradients at the center of flow. δP_1^A is the vertical virtual displacement of the air at the center of flow, and $\delta\phi_{12}^A$ and $\delta\phi_{13}^A$ are the cyclic virtual displacement components at the center of flow.

Aeroelastic Beam Element

The aeroelastic beam element is the primary structural element in GRASP. It represents a slender beam that is subject to elastic, inertial, gravitational, and aerodynamic forces. Hodges (1985) derives the elastic, inertial, and gravitational forces in detail. This section will discuss the derivation of the aerodynamic forces as they apply to the induced velocity calculations.

In the following discussion, the symbol Q (for quarter chord) is used to denote the aerodynamic center. The static position of any quantity is identified as $(\ )'$, while $(\ )''$ refers to the instantaneous position of the dynamic motions of the blade. As just mentioned, vectors are denoted by the underlined symbol. Measure numbers of vectors associated with a particular set of unit base vectors are subscripted with the identifier(s) for that set of unit base vectors. The unit base vectors used in the following discussion are shown in Fig. 2.

The wind velocity vector $\underline{W}^{Q''}$ at the aerodynamic center is calculated by subtracting the inertial structural velocity at Q'' ($\underline{V}^{Q''I}$) from the inertial air velocity at Q'' ($\underline{U}^{Q''I}$). In terms of the inflow generalized coordinates and $\underline{V}^{Q''I}$, the relative wind velocity measure numbers associated with the zero-lift-line basis vectors are

$$W_{Z''_i}^{Q''} = -C_{i1}^{Z''A}\left(\bar{U}_1^A + r\bar{\gamma}_{1r}^A + \dot{P}_1^A + R_{A2}^{Q''A}\dot{\phi}_{12}^A + R_{A3}^{Q''A}\dot{\phi}_{13}^A\right) - V_{Z''_i}^{Q''I} \tag{9}$$

The relative virtual displacement of an element of air with respect to the structure $\delta S_{Z''_i}^{Q''}$ can then be obtained by applying Kirchhoff's kinetic analogy to Eq. 9. All $(\dot{\ })$ quantities are replaced with $\delta(\ )$, and all velocity, angular velocity, and velocity gradient symbols are replaced by identically labelled virtual displacement, virtual rotation, and virtual displacment gradient symbols, respectively. All other terms are then discarded from Eq. 9.

The magnitude of the relative wind velocity W at the aerodynamic center and the angle of attack α are time-dependent quantities that can be written in terms of the measure numbers of the relative wind velocity vector. Since this theory is two-dimensional, the relative wind velocity and angle of attack depend only on the measure numbers in the plane of the blade airfoil cross section; thus

$$W = \sqrt{\left(W_{Z''_1}^{Q''}\right)^2 + \left(W_{Z''_2}^{Q''}\right)^2} \tag{10}$$

and

$$\tan\alpha = \frac{W_{Z''_1}^{Q''}}{W_{Z''_2}^{Q''}} \tag{11}$$

The local airflow velocity gradient $G_{Z''12}^{Q''}$ is also a time-dependent quantity that depends on the relative wind velocity. The subscripts 1 and 2 denote the gradient in the x_2 direction of the velocity measure number in the x_1 direction. This velocity gradient can be shown to be

$$G_{Z''12}^{Q''} = \frac{\partial W_{Z''1}^{Q''}}{\partial R_{Z2}^{QP}} \tag{12}$$

which, in terms of the inflow generalized coordinates and $\underline{V}^{Q''I}$, is

$$G_{Z''12}^{Q''} = -C_{11}^{Z''A}\left(\frac{C_{22}^{Z''A}R_{A2}^{Q''A} + C_{23}^{Z''A}R_{A3}^{Q''A}}{r}\bar{\gamma}_{1r}^A + C_{22}^{Z''A}\dot{\phi}_{12}^A + C_{23}^{Z''A}\dot{\phi}_{13}^A\right) - \frac{\partial V_{Z''1}^{Q''I}}{\partial R_{Z2}^{QP}} \tag{13}$$

Like the virtual displacement, the virtual rotation of a structural element relative to the air $\delta\Upsilon_{Z''3}^{Q''}$ can be obtained by applying Kirchhoff's kinetic analogy to Eq. 13. This is accomplished by replacing all $(\dot{\ })$ quantities with $\delta(\ )$, replacing all velocity, angular velocity, and flow gradient symbols by identically labelled virtual displacement, virtual rotation, and virtual displacement gradient symbols, respectively, and discarding all other terms from Eq. 13.

Static Inflow. Since the relative wind velocity, the velocity gradient, and the angle of attack are all time-dependent quantities, contributions to the static part of the virtual work are obtained by separating out the static terms from Eqs. 9, 13, and 11. The static magnitude of the relative wind is then

$$\bar{W} = \sqrt{\left(\bar{W}^{Q'}_{Z'1}\right)^2 + \left(\bar{W}^{Q'}_{Z'2}\right)^2} \tag{14}$$

where the overbars indicate the static part. Similarly, the static value of the angle of attack is

$$\tan\bar{\alpha} = \frac{\bar{W}^{Q'}_{Z'1}}{\bar{W}^{Q'}_{Z'2}} \tag{15}$$

The expression for the virtual work $\delta\mathcal{W}$ done by the aerodynamic forces over the length of the beam element is

$$\delta\mathcal{W} = \int_0^{\ell} (-\delta S^{Q''}_{Z''i} F_{Z''i} + \delta\Upsilon^{Q''}_{Z''3}\mathcal{M})dx_3 \tag{16}$$

where $\delta S^{Q''}_{Z''i}$ is the virtual displacement of the structure relative to the air, $\delta\Upsilon^{Q''}_{Z''3}$ is the virtual rotation of the structure relative to the air, and $F_{Z''i}$ and $\mathcal{M}$ are the applied forces and moments at the aerodynamic center, respectively. The applied aerodynamic force vector $\underline{F}$ on a blade section (a distributed force per unit length of blade) is assumed to be

$$\underline{F} = \mathcal{L}_c\hat{\underline{b}}_1^{W''} + \mathcal{D}\hat{\underline{b}}_2^{W''} + \mathcal{L}_{nc}\hat{\underline{b}}_1^{Z''} \tag{17}$$

where $\mathcal{L}_c$ is the circulatory lift, $\mathcal{D}$ is the drag, and $\mathcal{L}_{nc}$ is the noncirculatory lift.

The equations that define the aerodynamic force components act on the aeroelastic beam element at Q and are determined from a quasi-steady adaptation of Greenberg's thin-airfoil theory (Greenberg, 1947).

$$\begin{aligned}
\mathcal{L}_c &= \frac{1}{2}\rho_a W^2 c c_l + \frac{\pi}{2}\rho_a c^2 W G^{Q''}_{Z''12} \\
\mathcal{D} &= \frac{1}{2}\rho_a W^2 c c_d \\
\mathcal{M} &= \frac{1}{2}\rho_a W^2 c^2 c_m \\
&\quad - \frac{\pi}{16}\rho_a c^3\left(W G^{Q''}_{Z''12} + \dot{W}^{Q''}_{Z''1} + \frac{3c}{8}\dot{G}^{Q''}_{Z''12}\right) \\
\mathcal{L}_{nc} &= \frac{\pi}{4}\rho_a c^2\left(\dot{W}^{Q''}_{Z''1} + \frac{c}{4}\dot{G}^{Q''}_{Z''12}\right)
\end{aligned} \tag{18}$$

where c is the local blade chord, W is the magnitude of the relative wind velocity, $G^{Q''}_{Z''12}$ is the flow velocity gradient, and $W^{Q''}_{Z''1}$ is the flow velocity normal to the zero-lift line (Fig. 2). The lift, drag, and moment coefficients (c_l, c_d, and c_m), respectively, are nonlinear functions of the blade angle of attack α.

Dynamic Inflow. After the static quantities in Eqs. 9, 11, and 13, have been removed, only the dynamic terms remain. The dynamic part of the magnitude of the relative wind is written as

$$\check{W} = \frac{\bar{W}^{Q'}_{Z'1}\check{W}^{Q''}_{Z''1} + \bar{W}^{Q'}_{Z'2}\check{W}^{Q''}_{Z''2}}{\bar{W}} \tag{19}$$

and the dynamic part of the angle of attack is

$$\check{\alpha} = \frac{\bar{W}^{Q'}_{Z'2}\check{W}^{Q''}_{Z''1} - \bar{W}^{Q'}_{Z'1}\check{W}^{Q''}_{Z''1}}{\bar{W}^2} \tag{20}$$

where the checks indicate the dynamic part. Then, the dynamic terms in the virtual work can be put into the form

$$\begin{aligned}
-\delta\mathcal{W} &= \begin{Bmatrix} \delta P^A_1 \\ \delta\phi^A_{1r} \\ \delta\phi^A_{12} \\ \delta\phi^A_{13} \\ \delta q_S \end{Bmatrix}^T \left\{ [A]\begin{Bmatrix} \check{\mathcal{L}}_c \\ \check{\mathcal{D}} \\ \check{\mathcal{L}}_{nc} \\ \check{\mathcal{M}} \end{Bmatrix} + [B]\begin{Bmatrix} \check{W}^Q_{Z\alpha} \\ \check{G}^Q_{Z12} \end{Bmatrix} \right. \\
&\qquad \left. + [D]\begin{Bmatrix} \check{P}^A_1 \\ \check{\phi}^A_{1r} \\ \check{\phi}^A_{12} \\ \check{\phi}^A_{13} \\ \check{q}_S \end{Bmatrix} - \{Q\} \right\} \\
\begin{Bmatrix} \check{\mathcal{L}}_c \\ \check{\mathcal{D}} \\ \check{\mathcal{L}}_{nc} \\ \check{\mathcal{M}} \end{Bmatrix} &= [E]\begin{Bmatrix} \check{W}^Q_{Z\alpha} \\ \check{G}^Q_{Z12} \end{Bmatrix} + [F]\begin{Bmatrix} \dot{\check{W}}^Q_{Z\alpha} \\ \dot{\check{G}}^Q_{Z12} \end{Bmatrix} \\
\begin{Bmatrix} \check{W}^Q_{Z\alpha} \\ \check{G}^Q_{Z12} \end{Bmatrix} &= [G]\begin{Bmatrix} \check{P}^A_1 \\ \check{\phi}^A_{1r} \\ \check{\phi}^A_{12} \\ \check{\phi}^A_{13} \\ \check{q}_S \end{Bmatrix} + [H]\begin{Bmatrix} \dot{\check{P}}^A_1 \\ \dot{\check{\phi}}^A_{1r} \\ \dot{\check{\phi}}^A_{12} \\ \dot{\check{\phi}}^A_{13} \\ \dot{\check{q}}_S \end{Bmatrix}
\end{aligned} \tag{21}$$

where $\check{q}_S$ and $\dot{\check{q}}_S$ represent the displavement and velocity perturbations of all of the *structural* generalized coordinates. From this expression for the virtual work, the aerodynamic contributions to the aeroelastic beam element mass, damping, and stiffness matrices, M, C, and K, respectively, can be determined to be

$$
\begin{aligned}
M &= AFH \\
C &= AEH + AFG + BH \\
K &= AEG + BG + D
\end{aligned} \tag{22}
$$

Here M turns out to be symmetric, but neither C nor K are. Explicit expressions for the elements of M, C, and K can obviously be obtained by substitution. Such expressions are quite long and complicated: however, in view of GRASP's method of evaluation of these matrices numerically from Gauss-Legendre quadrature, it is not necessary to obtain them.

CONCLUDING REMARKS

The method used in GRASP to model the rotor flow field for a helicopter has been described. The primary feature of this implementation that differentiates it with other approaches is the separation of the blade-element calculations from the actuator-disk calculations. Also, this method incorporates the inflow generalized coordinates in the state vector for the steady-state problem, which guarantees full coupling with the structural deformations.

Because of the approach used to implement the inflow calculations in GRASP, it is also possible to use improved flow-field models without having to develop an entirely new blade element. The analyst would then have at his disposal a prescribed- or free-wake flow field representation as in Johnson (1980), or perhaps an unsteady flow-field model like that recently developed by Peters and He (1987). However, since the current version of GRASP does not have an air node that is general enough to accomodate the different sets of generalized coordinates that would be required for these flow-field models, an improved, generalized air node would need to be developed.

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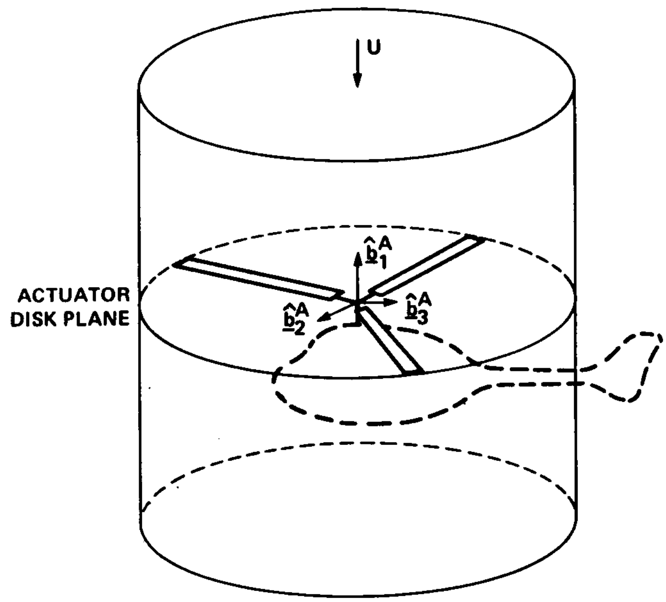

FIG. 1. Rotor flow-field model.

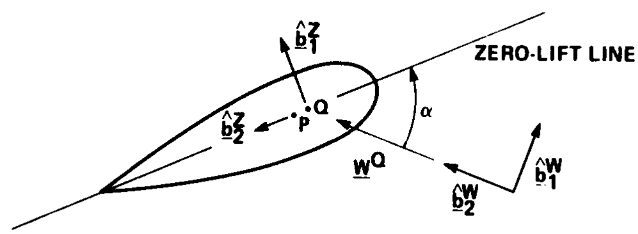

FIG. 2. Blade cross section.

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16. Abstract

This paper describes the methodology used by the General Rotorcraft Aeromechanical Stability Program (GRASP) to model the characteristics of the flow through a helicopter rotor in hovering or axial flight. Since the induced flow plays a significant role in determining the aeroelastic properties of rotorcraft, the computation of the induced flow is an important aspect of the program. Because of the combined finite-element/multibody methodology used as the basis for GRASP, the implementation of induced velocity calculations presented an unusual challenge to the developers. To preserve the modeling flexibility and generality of the code, it was necessary to depart from the traditional methods of computing the induced velocity. This is accomplished by calculating the actuator disc contributions to the rotor loads in a separate element called the air mass element, and then performing the calculations of the aerodynamic forces on individual blade elements within the aeroelastic beam element.

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