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EFFICIENT NUMERICAL TECHNIQUES FOR COMPLEX FLUID FLOWS*

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INTRODUCTION

Numerical calculation methods for multi-dimensional recirculating flow have been developed over the last 15-20 years. As a result, it has become possible to predict complex flows and heat transfer in combustion chambers, gas turbines, rotating machinery, heat exchangers, and many other practical devices. As the predictive methods have become more powerful, the researchers and designers have applied them to even more challenging problems. Although the computational prediction is far less expensive than the full-scale testing of the equipment, the cost of computational run for a complex problem is still quite substantial. Therefore, attempts are continually being made for improving the accuracy and efficiency of numerical techniques so that the predictions of a given accuracy can be obtained at a modest cost.

A crucial consideration in the calculation of fluid flow is the treatment of the coupling between the velocity components and pressure as expressed by the momentum and continuity equations. A very widely used method for the coupled solution is SIMPLE. Also, its many variants have been developed in recent years. These methods provide an iterative scheme in which the momentum equations are sequentially solved and the pressure is obtained from a special equation derived from the continuity equation. Although the methods are on the whole satisfactory, they do exhibit, on occasion, slow convergence, divergence, and sensitivity to under-relaxation factors.

The aim of the present research program is the development of more efficient and reliable calculation schemes for the coupled momentum and continuity equations. The resulting schemes would significantly reduce the expense of computing complex flows such as those in combustion chambers, gas turbines, and heat exchangers.

METHODS CHOSEN FOR STUDY

It is first realized that the coupling between the momentum and continuity equations is best handled by a simultaneous solution of their (linearized) discrete forms. For a flow at a very small Reynolds number, for which the equations are truly linear, such a direct method gives the solution instantly, without the need for iterations. For nonlinear problems, however, the direct solution of the linearized equations must be repeated many times until convergence is reached. The following methods are currently being investigated for the handling of the nonlinearity.

(i) Successive substitution
At any given iteration, the nonlinear eofficients are calculated simply from the values available from the previous iteration.

(ii) Newton-Raphson method
The nonlinear terms in the discrete equations are differentiated with respect to the unknowns. Thus, the new solution is obtained as a Newton-Raphson extrapolation along the derivatives evaluated at the previous iteration. In general, this method requires the evaluation of a large number of cross derivations. The storage requirements are also correspondingly high.

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(iii) The Broyden method

The expense of computing the many derivatives in the Newton-Raphson method can be reduced by employing the Broyden method described in references 1-5. The essence of the Broyden method is that the inverse of the Jacobian matrix is replaced by a suitable approximation. At each iteration, this approximate inverse is updated so as to promote convergence.

(iv) Norm minimization methods

The changes in the dependent variables predicted by the successive-substitution or Newton-Raphson methods do not always lead to convergence. Therefore, underrelaxation may be necessary. Instead of employing the underrelaxation in an arbitrary manner, the norm minimization methods seek an optimum underrelaxation so that the norm of the residual vector would be minimized.

TESTING OF THE METHODS

The above-mentioned methods are being applied to a number of two-dimensional problems such as the flow in a driven cavity, a sudden expansion in a duct, and the natural convection in an enclosure. The early indication is that these direct methods perform very well. Especially with methods (ii), (iii), and (iv), it has been possible to obtain solutions to highly nonlinear problems within a few (10-20) iterations. Methods like SIMPLE require about 500 iterations for the same problems.

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