NASA Technical Memorandum 100502

AERODYNAMIC SENSITIVITIES FROM SUBSONIC, SONIC, AND
SUPERSONIC UNSTEADY, NONPLANAR LIFTING-SURFACE THEORY

(NASA-TM-100502) AERODYNAMIC SENSITIVITIES FROM SUBSONIC, SONIC AND SUPERSONIC UNSTEADY, NONPLANAR LIFTING-SURFACE THEORY (NASA) 18 p CSCL 01A

N88-12459
Unclas
G3/02 0107337

E. Carson Yates, Jr. *

September 1987

NASA

National Aeronautics and
Space Administration

Langley Research Center
Hampton, Virginia 23665

Summary

The technique of implicit differentiation has been used in combination with linearized lifting-surface theory to derive analytical expressions for aerodynamic sensitivities (i.e., rates of change of lifting pressures with respect to general changes in aircraft geometry, including planform variations) for steady or oscillating planar or nonplanar lifting surfaces in subsonic, sonic, or supersonic flow. The geometric perturbation is defined in terms of a single variable, and the user need only provide simple expressions or similar means for defining the continuous or discontinuous global or local perturbation of interest. Example expressions are given for perturbations of the sweep, taper, and aspect ratio of a wing with trapezoidal semispan planform. In addition to direct computational use, the analytical method presented here should provide benchmark criteria for assessing the accuracy of aerodynamic sensitivities obtained by approximate methods such as finite geometry perturbation and differencing. The present process appears to be readily adaptable to more general surface-panel methods.

Nomenclature[†]

a_{nm}	coefficient of pressure-mode function $f_n(\hat{\xi})g_m(\hat{\eta})$ in pressure expansion (Eq. (10))
B	$= \sqrt{M^2 - 1}$
$b(y)$	streamwise semichord at spanwise station y
b_o	reference length (typically root semichord)
$\Delta\bar{C}_p$	complex amplitude of lifting-pressure coefficient
$f_n(\hat{\xi})$	nth chordwise pressure-mode function
$g_m(\hat{\eta})$	mth spanwise pressure-mode function
$I_j(k_1), K_j(k_1)$	modified Bessel functions of first and second kind, respectively
K	kernel function
K_V, K_W	kernel functions for sidewash and upwash, respectively
k	$= b_o\omega/U$, reduced frequency
k_1	$= kr_o = kr/\beta$, frequency parameter
$L_j(k_1)$	modified Struve function
ℓ	arc length of lifting surface, root to tip, in cross-stream direction
M	free-stream Mach number

[†] All coordinates are normalized with respect to b_o.

Q	perturbation of lifting surface
R	$= (x_o^2 + r^2)^{1/2}$
r	$= \beta r_o$
r_o	$= (y_o^2 + z_o^2)^{1/2}$
U	free-stream speed (in X direction)
u_1	$= (MR - x_o)/\beta r$
$\bar{W}_N$	complex amplitude of normalwash at point on lifting surface
X,Y,Z	fixed streamwise, lateral, and vertical coordinates, respectively
x,y,z	Cartesian coordinates
x_o,y_o,z_o	$x-\xi$, $y-\eta$, $z-\zeta$, respectively
x_m	streamwise location of mean chord line
x_L	streamwise location of leading edge
$\hat{x},\hat{y},\hat{\xi},\hat{\eta}$	coordinates on lifting surface transformed so that $\hat{x} = \hat{\xi} = -1$ at leading edge, $\hat{x} = \hat{\xi} = +1$ at trailing edge, and $\hat{y} = \hat{\eta} = \pm 1$ at tips
β	$= \sqrt{1 - M^2}$
γ	lateral inclination (local dihedral) of lifting surface
Λ	sweep angle of lifting-surface midchord line
ξ,η,ζ	Cartesian coordinates
ρ	$= r/\beta$
ω	circular frequency of oscillation

Introduction

Accurate and efficient computation of aerodynamic sensitivities (i.e., rates of change of surface pressures and weighted integrals of pressures with respect to changes in aircraft geometry) is of growing importance for aerodynamic shape optimization as well as for multidisciplinary design synthesis (ref. 1). Such sensitivities can, of course, be obtained approximately with any aerodynamic method by simply differencing the results of a solution for a base configuration and a solution for a geometrically perturbed configuration. The resulting sensitivity values, however, are dependent upon the size of the perturbation (step size) selected and are subject to cancellation errors if the step size is too small. Calculating sensitivities by analytical methods avoids such problems.

Some capabilities to generate sensitivities and to solve the inverse (aerodynamic-design) problem have been developed from surface-panel methods (e.g., refs. 2 to 6), but these have been limited to steady state, and applications appear to have been generally limited to geometric perturbations of lifting surfaces and only in the direction normal to the chord plane.

This paper presents expressions for aerodynamic sensitivities derived from subsonic, sonic, and supersonic unsteady, nonplanar lifting-surface theory (refs. 7 and 8) for general perturbations, including planform variations. The geometric perturbation is defined in terms of a single variable. Moreover, without loss of generality, perturbations are performed by varying only the normalwash distribution and/or the streamwise planform coordinates. Performing the perturbations in this way accomplishes considerable simplification of the resulting expressions and avoids a third-order singularity along the line $r = 0$. Note that aspect ratio and even the spanwise location of a leading-edge or trailing-edge break point can be varied by perturbing only streamwise coordinates. The present formulation requires only that the user provide simple analytical expressions or similar means for defining the continuous or discontinuous global or local perturbations of interest (e.g., perturbation of sweep angle). In contrast, the method of reference 6 requires that the perturbation be made up of individual perturbations of each of the panel nodes - a computationally expensive operation.

In addition to the steady-state limiting condition, the expressions derived here can be used, for example, to track flutter and other dynamic-response characteristics in multidisciplinary design processes. Such computations of aerodynamic sensitivities, both steady and unsteady, should be useful in both conceptual and preliminary design. On the other hand, lifting-surface-theory calculations are themselves quite fast so that computation of sensitivities by differencing is not prohibitively expensive. Even so, the analytical expressions presented here provide convenient means for computation as well as accurate benchmark criteria for assessing the accuracy of differencing (e.g., as a function of perturbation size) so that, when needed, differencing can be used with better understanding and confidence. Hopefully, the present development will serve heuristic purposes as well.

Theoretical Development

Subsonic Flow

The integral equation relating lifting-pressure and normalwash distribution on a nonplanar lifting surface (Fig. 1) oscillating in subsonic flow (Eq. (1) of ref. 7) can be written

$$8\pi \frac{b_o}{\ell} \frac{\bar{W}_N(x,y,z)}{U} = \oint_{-1}^{1} \int_{-1}^{1} \Delta\bar{C}_p \; b(\eta) K \; d\hat{\xi} \; d\hat{\eta} \tag{1}$$

where

$$K = K_W \cos[\gamma(y) - \gamma(\eta)] - K_V \sin[\gamma(y) - \gamma(\eta)]$$

and K_V and K_W are given, respectively, by

$$K_V(kx_o, ky_o, kz_o, M) = e^{-ikx_o}\, \partial^2 E/\partial y \partial z \tag{2}$$

and

$$K_W(kx_o, ky_o, kz_o, M) = e^{-ikx_o}\, \partial^2 E/\partial z^2 = e^{-ikx_o}\left(\frac{1}{z_o}\frac{\partial E}{\partial z} + \frac{z_o}{y_o}\frac{\partial^2 E}{\partial y \partial z}\right) \tag{3}$$

and

$$\begin{aligned} E(kx_o, kr, M) &= \int_{u_1}^{\infty} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}}\, du \\ &= K_o(k_1) - i\frac{\pi}{2}\left[I_o(k_1) - L_o(k_1)\right] - \int_0^{u_1} \frac{e^{-ik_1 u}}{\sqrt{1+u^2}}\, du \end{aligned} \tag{4}$$

(see Eqs. (4) to (6) of ref. 7). Carrying out the differentiation indicated in Eqs. (2) and (3) leads to

$$\begin{aligned} K_V = e^{-ikx_o}\frac{y_o z_o}{r_o^3}\Bigg\{ & k\left[P_1(k_1) + Q_1(k_1) - i2\int_0^{u_1}\frac{ue^{-ik_1 u}}{\sqrt{1+u^2}}\, du - k_1\int_0^{u_1}\frac{e^{-ik_1 u}}{\sqrt{1+u^2}}\, du\right] \\ & + \left[\frac{x_o}{Rr_o}\left(2 + \frac{\beta^2 r_o^2}{R^2}\right) + i\frac{k_1}{R}\left(1 + M\frac{x_o}{R}\right)\right]e^{-ik_1 u_1}\Bigg\} \end{aligned} \tag{5}$$

and

$$K_W = \frac{z_o}{y_o}K_V - e^{-ikx_o}\left\{\frac{x_o}{Rr_o^2}e^{-ik_1 u_1} + \frac{k}{r_o}\left[P_1(k_1) - i\int_0^{u_1}\frac{ue^{-ik_1 u}}{\sqrt{1+u^2}}\, du\right]\right\} \tag{6}$$

$$P_1(k_1) = K_1(k_1) + i\frac{\pi}{2}\left[I_1(k_1) - L_1(k_1)\right] - i \tag{7}$$

$$\begin{aligned} Q_1(k_1) = {} & K_1(k_1) + k_1 K_o(k_1) + i\frac{\pi}{2}\left[I_1(k_1) - k_1 I_o(k_1)\right. \\ & \left. - L_1(k_1) + k_1 L_o(k_1)\right] - i \end{aligned} \tag{8}$$

(see Eqs. (8) to (11) of ref. 7). The physical variables are related to the transformed variables in Eq. (1) by

$$\left.\begin{aligned} x &= x_L(y) + (\hat{x} + 1)b(y) = x_m(y) + \hat{x}b(y) \\ \xi &= x_L(\eta) + (\hat{\xi} + 1)b(\eta) = x_m(\eta) + \hat{\xi}b(\eta) \\ y &= \frac{\ell}{b_o}\hat{y} \\ \eta &= \frac{\ell}{b_o}\hat{\eta} \end{aligned}\right\} \qquad (9)$$

The normalwash $\bar{W}_N$ is defined by the shape and/or motion of the lifting surface.

It is customary to represent the unknown pressure distribution $\Delta\bar{C}_p$ by a linear combination of chosen shape functions. Thus

$$\Delta\bar{C}_p(\hat{\xi},\hat{\eta}) = \sqrt{1 - \hat{\eta}^2}\sqrt{\frac{1 - \hat{\xi}}{1 + \hat{\xi}}}\, b^{-1}(\hat{\eta}) \sum_n \sum_m a_{nm} f_n(\hat{\xi}) g_m(\hat{\eta}) \qquad (10)$$

and the basic aerodynamic sensitivity is

$$\frac{\partial \Delta\bar{C}_p}{\partial Q} = \sqrt{1 - \hat{\eta}^2}\sqrt{\frac{1 - \hat{\xi}}{1 + \hat{\xi}}}\left[\frac{\partial b^{-1}(\hat{\eta})}{\partial Q} \sum_n \sum_m a_{nm} f_n(\hat{\xi}) g_m(\hat{\eta}) + b^{-1}(\hat{\eta}) \sum_n \sum_m \frac{\partial a_{nm}}{\partial Q} f_n(\hat{\xi}) g_m(\hat{\eta})\right] \qquad (11)$$

where Q represents the geometric perturbation of the lifting surface and should not be confused with the function $Q_1(k_1)$ in equations (5) and (8). Now $\frac{\partial b^{-1}(\hat{\eta})}{\partial Q}$ is purely geometrical; consequently, attention is focused on evaluating $\frac{\partial a_{nm}}{\partial Q}$. Moreover, without loss of generality, the perturbation Q will be performed by varying only the normalwash distribution $\bar{W}_N$ and/or the streamwise coordinates x and ξ in order to avoid having to deal with spanwise derivatives of K_V and K_W which involve a third-order singularity at $r = 0$. Variations of x and ξ may be defined in terms of changes in b and x_L or x_m (Eq. (9)). Note that even a geometrical parameter such as the spanwise location of a leading-edge break point may be varied by changes in $x_L(y)$ only.

By use of Eq. (10), Eq. (1) may be expressed as

$$\sum_n \sum_m a_{nm} F_{nm} - 8\pi \frac{b_o}{\ell} \frac{\bar{W}_N}{U} = 0 \tag{12}$$

where

$$F_{nm} = \oint_{-1}^{1} \int_{-1}^{1} \sqrt{1 - \hat{\eta}^2} \sqrt{\frac{1 - \hat{\xi}}{1 + \hat{\xi}}} f_n(\hat{\xi}) g_m(\hat{\eta}) \{K_W \cos[\gamma(y) - \gamma(\eta)]$$

$$- K_V \sin[\gamma(y) - \gamma(\eta)]\} \, d\hat{\xi} \, d\hat{\eta} \tag{13}$$

Solution of Eq. (12) requires, of course, that the equation be written for at least as many points x,y,z on the lifting surface as there are values of a_{nm} to be calculated. Differentiating Eq. (12) with respect to Q gives

$$\sum_n \sum_m \left(a_{nm} \frac{\partial F_{nm}}{\partial Q} + \frac{\partial a_{nm}}{\partial Q} F_{nm} \right) - 8\pi \frac{b_o}{\ell} \frac{\partial}{\partial Q}\left(\frac{\bar{W}_N}{U}\right) = 0 \tag{14}$$

in which the values of F_{nm} and a_{nm} are obtained from solution of Eq. (12) for the base configuration, and $\partial(\bar{W}_N/U)/\partial Q$ is geometrically determined and known a priori. Equation (14) shows that if the perturbation Q does not involve a planform change, then $\partial F_{nm}/\partial Q = 0$, and the required values of $\partial a_{nm}/\partial Q$ may be obtained from a direct solution of Eq. (12) with the perturbation normalwash $\partial(\bar{W}_N/U)/\partial Q$ input in place of $\bar{W}_N/U$.

Differentiating Eq. (13) with respect to Q gives

$$\frac{\partial F_{nm}}{\partial Q} = \int_{-1}^{1} \int_{-1}^{1} \sqrt{1 - \hat{\eta}^2} \sqrt{\frac{1 - \hat{\xi}}{1 + \hat{\xi}}} f_n(\hat{\xi}) g_m(\hat{\eta}) \left\{ \frac{\partial K_W}{\partial x_o} \cos[\gamma(y) - \gamma(\eta)] \right.$$

$$\left. - \frac{\partial K_V}{\partial x_o} \sin[\gamma(y) - \gamma(\eta)] \right\} \frac{\partial x_o}{\partial Q} \, d\hat{\xi} \, d\hat{\eta} \tag{15}$$

Note that

$$\frac{\partial x_o}{\partial Q} = \frac{\partial(x - \xi)}{\partial Q} = \left.\frac{\partial x}{\partial Q}\right|_{x,y,z} - \left.\frac{\partial x}{\partial Q}\right|_{\xi,\eta,\zeta} \tag{16}$$

is the only quantity in Eq. (15) that depends on the particular perturbation Q.

Geometrical Example

The example of a planar wing with trapezoidal semispan planform serves to illustrate the nature of $\partial x/\partial Q$ for perturbations in aspect ratio A, taper ratio λ, and midchord sweep angle Λ. The semispan ℓ is invariant. The first of Eqs. (9) gives

$$x = 1 + \hat{y}\,\frac{A}{2}\,(1 + \lambda)\tan\Lambda + \hat{x}[1 - (1 - \lambda)\hat{y}]$$

Then with λ and Λ invariant,

$$\frac{\partial x}{\partial Q} \equiv \frac{\partial x}{\partial A} = \hat{y}\left(\frac{1 + \lambda}{2}\right)\tan\Lambda$$

With A and Λ invariant,

$$\frac{\partial x}{\partial Q} \equiv \frac{\partial x}{\partial \lambda} = \hat{y}\left(\hat{x} + \frac{A}{2}\tan\Lambda\right)$$

With A and λ invariant,

$$\frac{\partial X}{\partial Q} \equiv \frac{\partial X}{\partial \Lambda} = \hat{y}\,\frac{A}{2}\,(1 + \lambda)(1 + \tan^2\Lambda)$$

For use in Eq. (15) these expressions lead to

$$\frac{\partial x_o}{\partial A} = (\hat{y} - \hat{\eta})\,\frac{1 + \lambda}{2}\tan\Lambda$$

$$\frac{\partial x_o}{\partial \lambda} = \hat{y}\hat{x} - \hat{\eta}\hat{\xi} + (\hat{y} - \hat{\eta})\,\frac{A}{2}\tan\Lambda$$

$$\frac{\partial x_o}{\partial \Lambda} = (\hat{y} - \hat{\eta})\,\frac{A}{2}\,(1 + \lambda)(1 + \tan^2\Lambda)$$

and these may be combined into a single expression for use in Eq. (15).

$$\frac{\partial x_o}{\partial Q} = c_o \hat{y}\hat{x} + c_1(\hat{y} - \hat{\eta}) + c_2 \hat{\eta}\hat{\xi}$$

where for $Q = A$,

$$c_o = 0, \; c_1 = \frac{1 + \lambda}{2} \tan \Lambda, \; c_2 = 0$$

for $Q = \lambda$,

$$c_o = 1, \; c_1 = \frac{A}{2} \tan \Lambda, \; c_2 = -1$$

for $Q = \Lambda$,

$$c_o = 0, \; c_1 = \frac{A}{2} (1 + \lambda)(1 + \tan^2 \Lambda), \; c_2 = 0$$

Derivative of the Kernel Function

The required expressions for $\partial K_V/\partial x_o$ and $\partial K_W/\partial x_o$ in Eq. (15) are obtained by differentiating Eqs. (5) and (6), respectively. Thus

$$\frac{\partial K_V}{\partial x_o} = e^{ikx_o M^2/\beta^2} \, 3\beta^4 \frac{y_o z_o}{R^5} \left\{ \left[1 + i \frac{kMR}{\beta^2} + \frac{1}{3}\left(i \frac{kMR}{\beta^2} \right)^2 \right] e^{-i\frac{kMR}{\beta^2}} \right\} - ikK_V \tag{17}$$

$$\frac{\partial K_W}{\partial x_o} = e^{ikx_o M^2/\beta^2} \, \beta^2 \left\{ \frac{-1}{R^3}\left(1 + i \frac{kMR}{\beta^2} \right) + 3 \frac{\beta^2 z_o^2}{R^5} \left[1 + i \frac{kMR}{\beta^2} \right.\right.$$

$$\left.\left. + \frac{1}{3}\left(i \frac{kMR}{\beta^2} \right)^2 \right] \right\} e^{-i\frac{kMR}{\beta^2}} - ikK_W \tag{18}$$

The singularities of K_V and K_W (Eqs. (5) and (6)), and in particular the r_o^{-2} singularity, remain in $\partial K_V/\partial x_o$ and $\partial K_W/\partial x_o$ (Eqs. (17) and (18)) only for unsteady flow. For both steady and unsteady flow, however, $\partial K_W/\partial x_o$ contains a third-order singularity (R^{-3}) at the point $x_o = y_o = z_o = 0$. As indicated earlier, a third-order singularity along the line $r_o = 0$ has been avoided by not perturbing in y_o direction. The R^{-3} singularity may be dealt with in Eq. (15) by the classical analytical extraction technique. The remaining numerical quadrature, however, converges slowly. Results can be improved if second-derivative terms are also included in the extraction.

Eqs. (17) and (18) contain, respectively, the factors $y_o z_o/R^5$ and z_o^2/R^5 which also appear to indicate third-order singularity. However, if the normalwash control points (x,y,z) are not located at points of normalwash discontinuity (as, indeed, they should not be!), then as y_o goes to zero, z_o behaves as y_o^2 (Fig. 1). Consequently, $y_o z_o/R^5$ behaves as R^{-2}, and z_o^2/R^5 behaves as R^{-1}. Thus these terms require no special attention in Eq. (15) beyond the aforementioned extraction of the third-order singularity.

The use of Eqs. (15) to (18) permits Eq. (14) to be solved for the required values of $\partial a_{nm}/\partial Q$ which, in turn, permit the evaluation of $\partial \Delta\bar{C}_p/\partial Q$ from Eq. (11). If this procedure is employed for a number of different perturbations, only the simple geometric expressions $\partial b^{-1}/\partial Q$ in Eq. (11) and $\partial x_o/\partial Q$ in Eq. (15) need be reevaluated for each.

The expressions for K_V and K_W as well as for $\partial K_V/\partial x_o$ and $\partial K_W/\partial x_o$ simplify considerably for steady flow (k = 0) or for planar geometry (z_o = 0). Thus for steady flow, k = 0, and

$$K_V\Big|_{k=0} = \frac{y_o z_o}{r_o^4}\left[2 + \frac{x_o}{R}\left(2 + \frac{\beta^2 r_o^2}{R^2}\right)\right] \tag{19}$$

and

$$K_W\Big|_{k=0} = \frac{1}{r_o^2}\left\{\frac{z_o^2}{r_o^2}\left[2 + \frac{x_o}{R}\left(2 + \frac{\beta^2 r_o^2}{R^2}\right)\right] - \left(1 + \frac{x_o}{R}\right)\right\} \tag{20}$$

so that

$$\frac{\partial K_V}{\partial x_o}\Big|_{k=0} = 3\beta^4 y_o z_o/R^5 \tag{21}$$

and

$$\frac{\partial K_W}{\partial x_o}\Big|_{k=0} = \frac{\beta^2}{R^3}\left(3\,\frac{\beta^2 z_o^2}{R^2} - 1\right) \tag{22}$$

For planar geometry, $z_o = 0$, and

$$K_V\Big|_{z_o=0} = \frac{\partial K_V}{\partial x_o}\Big|_{z_o=0} = 0 \tag{23}$$

and

$$K_W\Big|_{z_o=0} = -e^{-ikx_o}\left\{\frac{x_o}{Ry_o^2} e^{-ik_1u_1} + \frac{k}{|y_o|}\left[P_1(k_1) - i\int_0^{u_1} \frac{ue^{-ik_1u}}{\sqrt{1+u^2}} du\right]\right\} \tag{24}$$

so that

$$\frac{\partial K_W}{\partial x_o}\Big|_{z_o=0} = -e^{ikx_oM^2/\beta^2} \frac{\beta^2}{R^3}\left(1 + i\frac{kMR}{\beta^2}\right)e^{-i\frac{kMR}{\beta^2}} - ikK_W\Big|_{z_o=0} \tag{25}$$

Finally, for planar steady flow, $k = z_o = 0$, and

$$K_W\Big|_{k=z_o=0} = \frac{-1}{y_o^2}\left(1 + \frac{x_o}{R}\right) \tag{26}$$

and

$$\frac{\partial K_W}{\partial x_o}\Big|_{k=z_o=0} = \frac{-\beta^2}{R^3} \tag{27}$$

Alternative Formulation for Subsonic, Sonic, and Supersonic Speeds

An alternative form of the kernel function has been used to define the kernel for sonic and supersonic speeds as well as for subsonic (ref. 8). Thus in the present notation

$$K = e^{-ikx_o}(K_1T_1/\rho^2 + K_2T_2^*/\rho^4) \tag{28}$$

where

$$T_1 = \cos[\gamma(y) - \gamma(\eta)] \tag{29}$$

$$T_2^* = [z_o \cos\gamma(y) - y_o \sin\gamma(y)][z_o \cos\gamma(\eta) - y_o \sin\gamma(\eta)] \tag{30}$$

$$K_1 = \rho(\partial I_o/\partial\rho) \tag{31}$$

$$K_2 = \rho^3(\partial/\partial\rho)[(1/\rho)\partial I_o/\partial\rho] \tag{32}$$

and

$$I_o = \int_{s_1}^{s_2} \frac{e^{-iks}}{\sqrt{\rho^2 + s^2}}\, ds \tag{33}$$

K_1 in Eq. (31) should not be confused with the modified Bessel function $K_1(k_1)$ in Eqs. (7) and (8). The limits of integration s_1 and s_2 are determined by the time required for transmission of small disturbances from sending point (ξ,η,ζ) to receiving point (x,y,z) with consideration of the limits imposed by Mach lines, if any.

The derivative $\partial K/\partial x_o$ required for the generation of aerodynamic sensitivities may be obtained from Eq. (28).

$$\frac{\partial K}{\partial x_o} = e^{-ikx_o}\left(\frac{T_1}{\rho^2}\frac{\partial K_1}{\partial x_o} + \frac{T_2^*}{\rho^4}\frac{\partial K_2}{\partial x_o}\right) - ikK \tag{34}$$

and

$$\frac{\partial K_1}{\partial x_o} = \left[\frac{\rho e^{-iks}}{\sqrt{\rho^2 + s^2}}\left\{\frac{\partial}{\partial x_o}\left(\frac{\partial s}{\partial\rho}\right) - ik\frac{\partial s}{\partial x_o}\frac{\partial s}{\partial\rho} - \frac{1}{\rho^2 + s^2}\left(s\frac{\partial s}{\partial\rho} + \rho\right)\frac{\partial s}{\partial x_o}\right\}\right]_{s=s_1}^{s=s_2} \tag{35}$$

$$\begin{aligned}\frac{\partial K_2}{\partial x_o} = \Bigg[\frac{\rho e^{-iks}}{\sqrt{\rho^2 + s^2}}\Bigg\{&\left(\frac{s}{\rho^2 + s^2} + ik\right)\left[\left(\frac{\partial s}{\partial\rho} - \rho\frac{\partial^2 s}{\partial\rho^2} + ik\rho\left(\frac{\partial s}{\partial\rho}\right)^2\right)\frac{\partial s}{\partial x_o} - \rho\frac{\partial}{\partial x_o}\left(\frac{\partial s}{\partial\rho}\right)^2\right]\\ &+ \frac{\rho}{\rho^2 + s^2}\left(\frac{3s}{\rho^2 + s^2} + ik\right)\left[2\rho\frac{\partial s}{\partial\rho} + s\left(\frac{\partial s}{\partial\rho}\right)^2\right]\frac{\partial s}{\partial x_o} - \left(1 + \frac{2\rho^2}{\rho^2 + s^2}\right)\frac{\partial}{\partial x_o}\left(\frac{\partial s}{\partial\rho}\right)\\ &+ \rho\frac{\partial}{\partial x_o}\left(\frac{\partial^2 s}{\partial\rho^2}\right) + \frac{\rho}{\rho^2 + s^2}\left[\frac{3\rho^2}{\rho^2 + s^2} - \left(\frac{\partial s}{\partial\rho}\right)^2\right]\frac{\partial s}{\partial x_o}\Bigg\}\Bigg]_{s=s_1}^{s=s_2}\end{aligned} \tag{36}$$

The derivatives of s_1 and s_2 indicated in Eqs. (35) and (36) may be obtained from the expressions for s_1 and s_2 that are pertinent to subsonic, sonic, or supersonic flow (ref. 8). Thus for subsonic flow

$$\left.\begin{aligned}
&s_1 = (MR - x_o)/\beta^2, \quad \frac{\partial s_1}{\partial \rho} = M\,\frac{\rho}{R}, \quad \left(\frac{\partial s_1}{\partial \rho}\right)^2 = M^2\,\frac{\rho^2}{R^2}, \quad \frac{\partial^2 s_1}{\partial \rho^2} = \frac{M}{R}\left(1 - \beta^2\,\frac{\rho^2}{R^2}\right),\\
&\frac{\partial s_1}{\partial x_o} = \left(M\,\frac{x_o}{R} - 1\right)\Big/\beta^2, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_1}{\partial \rho}\right) = -\,\frac{M\rho x_o}{R^3}, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_1}{\partial \rho}\right)^2 = \frac{-2M^2\rho^2 x_o}{R^4},\\
&\frac{\partial}{\partial x_o}\left(\frac{\partial^2 s_1}{\partial \rho^2}\right) = \frac{Mx_o}{R^3}\left(3\beta^2\,\frac{\rho^2}{R^2} - 1\right)
\end{aligned}\right\} \tag{37}$$

$$s_2 = \infty, \quad \frac{\partial s_2}{\partial \rho} = \left(\frac{\partial s_2}{\partial \rho}\right)^2 = \frac{\partial^2 s_2}{\partial \rho^2} = \frac{\partial s_2}{\partial x_o} = \frac{\partial}{\partial x_o}\left(\frac{\partial s_2}{\partial \rho}\right) = \frac{\partial}{\partial x_o}\left(\frac{\partial s_2}{\partial \rho}\right)^2 = \frac{\partial}{\partial x_o}\left(\frac{\partial^2 s_2}{\partial \rho^2}\right) = 0 \tag{38}$$

For sonic flow,

$$\left.\begin{aligned}
&s_1 = (\rho^2 - x_o^2)/2x_o, \quad \frac{\partial s_1}{\partial \rho} = \rho/x_o, \quad \left(\frac{\partial s_1}{\partial \rho}\right)^2 = \rho^2/x_o^2, \quad \frac{\partial^2 s_1}{\partial \rho^2} = 1/x_o,\\
&\frac{\partial s_1}{\partial x_o} = -2(\rho^2 + x_o^2)/x_o^2, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_1}{\partial \rho}\right) = -\rho/x_o^2, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_1}{\partial \rho}\right)^2 = -2\rho^2/x_o^3,\\
&\frac{\partial}{\partial x_o}\left(\frac{\partial^2 s_1}{\partial \rho^2}\right) = -1/x_o^2
\end{aligned}\right\} \tag{39}$$

$s_2 = \infty$, and all the derivatives are zero as in Eq. (38).
For supersonic flow,

$$\left.\begin{aligned}
&s_1 = (x_o - MR)/B^2 = \left(M^2\rho^2 - x_o^2\right)/(MR + x_o), \quad \frac{\partial s_1}{\partial \rho} = M\rho/R, \\
&\left(\frac{\partial s_1}{\partial \rho}\right)^2 = M^2\rho^2/R^2, \quad \frac{\partial^2 s_1}{\partial \rho^2} = \frac{M}{R}\left(1 + \frac{B^2\rho^2}{R^2}\right), \quad \frac{\partial s_1}{\partial x_o} = \left(1 - M\frac{x_o}{R}\right)\Big/B^2, \\
&\frac{\partial}{\partial x_o}\left(\frac{\partial s_1}{\partial \rho}\right) = -M\rho x_o/R^3, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_1}{\partial \rho}\right)^2 = -2M^2\rho^2 x_o/R^4, \\
&\frac{\partial}{\partial x_o}\left(\frac{\partial^2 s_1}{\partial \rho^2}\right) = -M\frac{x_o}{R^3}\left(3B^2\frac{\rho^2}{R^2} + 1\right)
\end{aligned}\right\} \tag{40}$$

$$\left.\begin{aligned}
&s_2 = (x_o + MR)/B^2, \quad \frac{\partial s_2}{\partial \rho} = -M\rho/R, \quad \left(\frac{\partial s_2}{\partial \rho}\right)^2 = M^2\rho^2/R^2, \quad \frac{\partial^2 s_2}{\partial \rho^2} = \frac{-M}{R}\left(1 + B^2\frac{\rho^2}{R^2}\right), \\
&\frac{\partial s_2}{\partial x_o} = \left(1 + M\frac{x_o}{R}\right)\Big/B^2, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_2}{\partial \rho}\right) = M\rho x_o/R^3, \quad \frac{\partial}{\partial x_o}\left(\frac{\partial s_2}{\partial \rho}\right)^2 = -2M^2\rho^2 x_o/R^4, \\
&\frac{\partial}{\partial x_o}\left(\frac{\partial^2 s_2}{\partial \rho^2}\right) = \frac{Mx_o}{R^3}\left(3B^2\frac{\rho^2}{R^2} + 1\right)
\end{aligned}\right\} \tag{41}$$

Substitution of the appropriate expressions from Eqs. (37) to (41) into Eqs. (35) and (36) and the latter two equations into Eq. (34) produces the kernel-function derivative needed for subsonic, sonic, or supersonic speeds. The kernel derivative expression is used, in turn, to evaluate $\partial F_{nm}/\partial Q$ from

$$\frac{\partial F_{nm}}{\partial Q} = \fint_{-1}^{1} \int_{-1}^{1} \sqrt{1 - \hat{\eta}^2} \sqrt{\frac{1 - \hat{\xi}}{1 + \hat{\xi}}}\, f_n(\hat{\xi}) g_m(\hat{\eta}) \frac{\partial K}{\partial x_o} \frac{\partial x_o}{\partial Q}\, d\hat{\xi}\, d\hat{\eta} \tag{42}$$

which corresponds to Eq. (15).

Note that the weight factors (two radicals) in Eq. (42), as well as in Eqs. (10), (11), (13), and (15), are appropriate for subsonic speeds. Different factors and different pressure-mode functions $f_n(\hat{\xi})$ and $g_m(\hat{\eta})$ are used for sonic or supersonic speeds (see, e.g., ref. 9), and the domain of integration is limited to the portions of lifting surface lying within the Mach forecone with vertex at the receiving point (x,y,z).

Concluding Remarks

The technique of implicit differentiation has been used in combination with linearized lifting-surface theory to derive analytical expressions for aerodynamic sensitivities (i.e., rates of change of lifting pressures with respect to general changes in aircraft geometry, including planform variations) for steady or oscillating planar or nonplanar lifting surfaces in subsonic, sonic, or supersonic flow. The geometric perturbation is defined in terms of a single variable, and the user need only provide simple expressions or similar means for defining the continuous or discontinuous global or local perturbations of interest. Example expressions are given for perturbations of the sweep, taper, and aspect ratio of a wing with trapezoidal semispan planform. In addition to direct computational use, the analytical method presented here should provide benchmark criteria for assessing the accuracy of aerodynamic sensitivities obtained by approximate methods such as finite geometry perturbation and differencing. The present process appears to be readily adaptable to more general surface-panel methods.

Acknowledgement

The author extends his sincere appreciation to Robert N. Desmarais for checking the derivation of equations and for valuable and stimulating discussions relative to numerical implementation.

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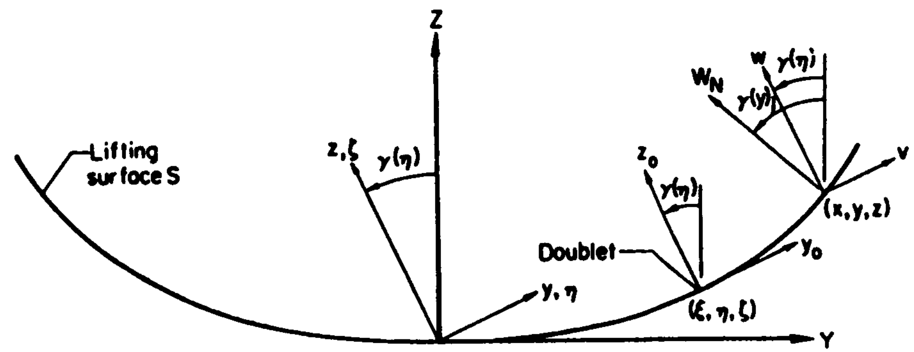

Fig. 1 Nonplanar lifting surface as seen from downstream.

Report Documentation Page

National Aeronautics and Space Administration

1. Report No.	2. Government Accession No.	3. Recipient's Catalog No.
NASA TM-100502		

4. Title and Subtitle	5. Report Date
Aerodynamic Sensitivities From Subsonic, Sonic, and Supersonic Unsteady, Nonplanar Lifting-Surface Theory	September 1987
	6. Performing Organization Code
7. Author(s)	8. Performing Organization Report No.
E. Carson Yates, Jr.	10. Work Unit No. 505-63-11
9. Performing Organization Name and Address	11. Contract or Grant No.
NASA Langley Research Center Hampton, VA 23665-5225	13. Type of Report and Period Covered Technical Memorandum
12. Sponsoring Agency Name and Address National Aeronautics and Space Administration Washington, DC 20546-0001	14. Sponsoring Agency Code

15. Supplementary Notes

16. Abstract

The technique of implicit differentiation has been used in combination with linearized lifting-surface theory to derive analytical expressions for aerodynamic sensitivities (i.e., rates of change of lifting pressures with respect to general changes in aircraft geometry, including planform variations) for steady or oscillating planar or nonplanar lifting surfaces in subsonic, sonic, or supersonic flow. The geometric perturbation is defined in terms of a single variable, and the user need only provide simple expressions or similar means for defining the continuous or discontinuous global or local perturbation of interest. Example expressions are given for perturbations of the sweep, taper, and aspect ratio of a wing with trapezoidal semispan planform. In addition to direct computational use, the analytical method presented here should provide benchmark criteria for assessing the accuracy of aerodynamic sensitivities obtained by approximate methods such as finite geometry perturbation and differencing. The present process appears to be readily adaptable to more general surface-panel methods.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
Aerodynamic sensitivities Unsteady aerodynamics Lifting-surface theory	Unclassified - Unlimited Subject Category - 02

19. Security Classif. (of this report)	20. Security Classif. (of this page)	21. No. of pages	22. Price
Unclassified	Unclassified	17	A02