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SURVEY ON LARGE SCALE SYSTEM CONTROL METHODS

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The problems inherent to large scale systems have been of interest for a long time now. The areas which have motivated the study have been principally power network, communication network and economic or ecological systems. The increase in size and flexibility of future spacecraft has put those dynamical systems into the category of large scale systems, and tools specific to the class of large systems are being sought to design control systems that can guarantee more stability and better performance.

Among several survey papers, reference [1] was found to be a thorough investigation on decentralized control methods. Especially helpful in this paper was the classification made of the different existing approaches to deal with large scale systems. A very similar classification is used in the following, even though the papers surveyed here are somehow different from the ones reviewed in [1]. Special attention is brought to the applicability of the existing methods to controlling large mechanical systems like large space structures. Furthermore, some recent developments are added to this survey.

Simplification methods

Aggregation method

The first way to tackle the problem of size for large scale systems is to simplify the model. The first class of simplification methods recognized in [1] is the class of aggregation methods. Those methods were first derived in the economic field. The motivation was that there is an important number of economic agents which act independently and have the

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same dynamics. Those agents can be individuals in the economy and the dynamics describe the way they spend, invest, or save, their income. As long as the dynamics are similar and there is no interaction between the agents, only one average individual is necessary to describe the behavior of the whole and the different agents can be aggregated into one single state, thus reducing tremendously the analysis of economic equilibria.

More mathematical grounds have since been brought to the idea. Starting from a very general and theoretical basis, the aggregation of a system can be considered as a particular form of contraction. The mathematics of system contraction, system expansion and the inclusion principle are presented in [2]. It is shown there that the principle of aggregation is to find a reduced order dynamical system which can describe some projection of the overall state at every time for any initial conditions. That is, the trajectory of the aggregate system for initial conditions being the projection of the entire initial state vector, and receiving the same inputs as the entire system, will be the projection of the overall system's trajectory. In the case of redundant states and redundant equations, as it is the case when independent agents are acting in a similar fashion, the averaging over the agents is the same as starting with the initial average and propagating it using the common dynamics of the different agents.

The control aspect using aggregated models is presented in [3] for a general linear time invariant system. First shown in [3] is that the derivation of an aggregated model corresponds effectively to the selection of some modes of the system. The issue of efficiently selecting the modes is addressed. A good aggregate model retains the dominant modes appearing in the output of the system.

Considering the LQR problem, it is shown in [3] that suboptimal control laws can be derived using the simplified model. The idea is to solve the LQR problem for the reduced order system using an aggregated cost functional which is as close as possible to the cost functional chosen for the complete system. Then, implementing the reduced order control law to the complete system will yield a stable system whose poles are 1) the modes not retained in the aggregation, 2) the closed-loop poles of the reduced-order system.

The modes not retained in the aggregate model do not change since they are not contained in the aggregate state vector which is fed back.

The degree of suboptimality of such a control law can be estimated, that is, a lower bound for the optimal cost that one would obtain by designing the optimal regulator problem for the overall system can be evaluated ([2,4]). Therefore, the difference between this lower bound estimate and the cost obtained by using the suboptimal law provides some information on how well the simplified control performs.

Output feedback is also investigated in [3], the approach being to use a control law as close as possible to the reduced order control law presented above. There is unfortunately in that case no guarantee of stability. Finally the problem of control with an observer is presented. In that case the measurements used in the aggregated observer are influenced by the modes not retained in the model and spill-over results from it. However results for global stability of the complete system can be derived that show that, as long as the coupling via the measurements and the feedback law between the modes retained in the simplified model and those which were not, remains within some bounds which depend on the closed-loop dynamics of the aggregate system, the complete system remains stable. The proof uses vector Lyapunov type techniques.

Optimal projection for model reduction

A very recent development in the field of system reduction has been the method of optimal projection. The method systematized the way to represent a large order dynamical system by a reduced order model. The optimal projection equations for model reduction are presented in [26]. A comparison between this technique and other more intuitive techniques is also made in this paper, which leads to the unsurprising conclusion that the value of the criterion for which the reduction was optimized is greater when those alternative techniques are used. The optimization criterion is a quadratic function of the error between the output of the complete system and the output of the reduced order system when both are excited by a white noise of chosen intensity. The optimal projection equations take the form of two modified Lyapunov equations having the order of the complete system

which are found to be coupled by an oblique projection matrix whose rank is the dimension of the reduced-order system. The projection has to be determined as part of the solution. There exist projections which lead to local rather than global minima for the cost functional, since the equations correspond to first order necessary conditions. Nevertheless, there appear to be promising numerical techniques for actually solving the optimal projection equations and converging toward the global minimum, which makes this method very appealing.

Nonsingular perturbation techniques

The second type of approximation method singled out in [1] are the perturbation methods. A distinction has to be made between singular and nonsingular perturbations since they apply for very different classes of systems.

The nonsingular perturbations occur in the case of weak coupling in the system. The overall system can in fact be described as a set of interconnected dynamical subsystems where the coupling between the subsystems is supposedly small. Each subsystem is described by its own local state variables. The overall system's state vector is made by regrouping all the local state vectors. It is also supposed that the local sensors attributed to one subsystem can only sense the subsystem's state variables and that actuators attributed to one subsystem do not directly influence other subsystems.

In that case, the intuitive approach is to neglect completely the interaction. A more rigorous approach is to design local optimal control systems for each subsystem as if they were isolated and then try to determine how the global stability of the coupled system is ensured. This corresponds to the approach by Siljak [5-7] or Singh [4] and to the notion of connective stability. The design procedure presented in [5] is the following: 1) solve for each subsystem, as if it were isolated, the LQR problem with guaranteed degree of stability (meaning the cost functional is of the form $e^{2\alpha t} (X^T Q X + U^T R U)$) then 2) adjust the parameter α so that the system will be connectively stable. A system is said to be connectively stable if the system remains stable for all admissible values of the

coupling. The property translates into an algebraic criterion involving the internal dynamics of the subsystems and the coupling. Such a criterion is derived for example in [4], and as intuitively foreseen, compares the speed of the internal dynamics to the speed of the loops closed via the coupling. Thus, increasing the parameter α should make the internal dynamics fast enough to guarantee connective stability, and indeed, a limit is computed in [5-7], that depends on the maximum allowable coupling between the subsystems, such that having the parameter α above the limit guarantees stability for all admissible values of the coupling. The computation of the limit as well as the derivation of the connective stability criterion in [4] involves vector Lyapunov type techniques ([6]).

The design obtained through this method is a decentralized control: each subsystem is controlled by its local actuators using local state variables. It yields very good robustness characteristics, since the system remains stable for a large class of structural changes.

This approach can be qualified as non-cooperative since the system is broken down into subsystems which are made as independent as possible. Therefore, the dynamics of the system, and especially the coupling existing between the subsystems, is not fully used by the local controllers which only have a limited knowledge of the structure of the overall system.

The same philosophy is used in the design of a hierarchical control system presented in [4]. Local LQR problems are solved for each subsystem. The perturbation entering each subsystem in the form of coupling is reduced by a global controller which tries to reduce the interaction as much as possible. In the best case, the design decouples the subsystems via the global controller, and then implements local optimal regulators for each subsystem.

Such a controller is of course suboptimal, but bounds on suboptimality can be computed as shown in [4]. The approach presents some advantages in the simplicity of the design: finding the gains to decouple the subsystems is nothing more than an algebraic manipulation; the remaining task is to solve a number of reduced-order Riccati equations for the subsystems considered as isolated, with order much smaller than that of the complete system. However, one can see that it is not always smart in that no best

use of the dynamical properties is made. The subsystems do not cooperate, neither do the controllers. This leads to relying on higher control gains. The robustness property in the case of the decentralized control law is however very appealing if one is concerned with reliability. It must also be noted that the interconnection can be nonlinear.

An interesting way to augment the cooperation between the local controllers while still using the methodology of [5] is shown in [9]. The idea is to make the subsystems overlap: the system state variables are partitioned into subsets which define the state vectors for the subsystems; making an overlapping partition of the system is to permit the same state variable to be in the state vector of two different subsystems. The dynamics of such a variable will therefore be taken into account by different local controllers. Based on the results of [2] about system expansion and system contraction, it is shown in [9] that the problem considered is similar to that of [5] and the design methodology is in fact similar: for each subsystem considered independent, the LQR problem with guaranteed degree of stability is solved. The bound for α is however less conservative when overlapping decomposition is used [11]. [8] shows that more freedom exists to build vector Lyapunov functions with overlapping decomposition, thus sometimes succeeding in proving stability where Lyapunov functions based on the disjoint decomposition of the system have failed.

Nyquist array method and diagonal dominance

The Nyquist array method should be included in the category of nonsingular perturbation techniques, even though the approach is not a state space approach but rather a frequency domain approach. Rosenbrock in [10] develops the method which can be regarded as an extension to Multi-Input-Multi-Output systems of the use of the Nyquist diagram or the inverse Nyquist diagram in the design of compensators for Single Input Single Output systems. A row (column) diagonally dominant matrix is a matrix for which the norm of each diagonal element is greater than the sum of the norms of the off-diagonal elements situated on the corresponding row (column). If this property is satisfied by the matrix transfer function of the system, then the Nyquist stability criterion which involves the determinant of the

matrix transfer function can be broken down to several Nyquist stability criteria each of which involves only one diagonal element of the matrix transfer function - a SISO transfer function. Compensation can thus be undertaken in a SISO way. The first step of the design procedure shown in [10] is to tailor the matrix transfer function on which gain feedback will be used. Starting from a physical input-output matrix transfer function, one uses pre and post compensation as well as recombination of the physical inputs and outputs to obtain some matrix transfer function as diagonally dominant as possible. The inputs of the transfer function are thus the inputs to the precompensator, some electronics box whose outputs drive the real plant, and the outputs are those of the postcompensator, some electronic box whose inputs are the plant outputs. Some inner loop might also be closed to modify the input-output characteristics of the plant. The whole purpose of the operation is to minimize the sum of the norms of the off-diagonal elements of the rows (or the columns) of the matrix transfer function defined between the new inputs and the new outputs to enforce diagonal dominance. Then a set of feedback gains is chosen so that the extended Nyquist stability criteria are not violated. The method can handle nonlinearity for the Popov's circle criterion can be extended in the same manner as the Nyquist stability criterion. The control law is connectively stable, meaning that the actual values of the off-diagonal elements of the closed-loop matrix transfer function are not important as long as the matrix satisfies the diagonal dominance property. One drawback of the method is that there is no really straightforward clever way to achieve diagonal dominance. Pseudo-diagonalization, presented in [10], where one tries to make the plant transfer function diagonal using compensation, is very similar in essence to the idea of [4] to use a global controller to decouple the subsystems constituting the overall system, and the same restrictions apply.

A similar design procedure is considered in [11] with relaxed dominance conditions. The property required there is called quasi-block diagonal dominance. A diagonally dominant matrix always satisfies the quasi-block diagonal dominance criterion but the reverse is not true. The methodology presented in [11] includes the possibility to decompose the matrix transfer

function into overlapping blocks. The restrictions about the non-cooperativeness of the connectively stable decentralized control applied for the methodology of [5]. But again, benefits are to be expected by making an overlapping decomposition of the system. In that case, the system input vector as well as the output vector are partitioned into subsets of inputs and subsets of outputs. The reason for expecting better performance with an overlapping decomposition is similar whether the approach is from the state space viewpoint or the frequency domain viewpoint: it is that the local controllers are built using more structural information.

The local LQG/LTR design methodology presented in [12] uses block-diagonal dominance properties even though the problem is presented in a stability robustness setting which is becoming more widely understood in engineering. In this paper, the overall system is first being modeled in state space. The overall state vector is then partitioned into possibly overlapping subsets to define the subsystems. For every subsystem, the coupling with the rest of the system is translated into a multiplicative transfer function error which is then bounded by some function of the frequency $e(\omega)$. Then, a standard LQG/LTR procedure is applied to each subsystem. The plant which is being controlled is the local subsystem dynamics and the effect of the remaining state variables are considered to be perturbations. Thus, for every subsystem, the stability robustness test is to compare the maximum singular value of the local closed-loop transfer function with the inverse of its multiplicative error $1/e(\omega)$ which bounds the intercoupling effects.

Singular perturbation method: the multi time-scale approach

The second category of perturbations are singular perturbations. The class of systems for which this theory is applicable is the class of systems with well separated spectra. In that case, the overall system is not taken to be a collection of subsystems with their particular input, output and state variables. The system is rather broken down into a slow system and a fast system [1]. When the time constants of the fast and the slow systems are well separated simplifications occur. First, the fast system is considered to be infinitely fast. The dynamical equation becomes an

algebraic equation and the fast modes (or the fast state variables) can be eliminated in favor of the slow modes. Substituting into the dynamics of the slow system, one gets a reduced-order system. A control system can be designed for the reduced-order slow system. The fast modes being related algebraically to the slow modes, their slow part is specified and represents a desired trajectory. The next step is to consider the slow modes as infinitely slow. The dynamics of the fast modes then describe the error of fast modes about their desired trajectory specified by their slow part. A second controller is then developed to give suitable dynamics to the error. The derivation of the method is presented in [13] for the deterministic case, and in [14] for the stochastic case. Such composite controllers are of course suboptimal. The degree of suboptimality is estimated in the deterministic case in [13]. In [14], it is shown that as the perturbation tends toward zero the suboptimal closed-loop system tends asymptotically toward the optimum. The multi time-scale approach presents many nice features. First, the two time-scale case can be extended to a multi time-scale case (with more than two time-scales) as shown in [15]. The design method can be used iteratively to design controllers operating with different bandwidth. This should improve the degree of suboptimality, as more structural information is used to derive the control system. Second, the architecture is naturally that of a hierarchical system: The slow modes are controlled with a reduced-order controller and with a relatively small bandwidth. Some directives are passed from the slow controller to the fast controller and the fast dynamics error is driven to zero so that the system follows the prescribed trajectory.

Restrictions apply to the use of multi time-scale control systems. The closed-loop system must be multi time-scale with bandwidths similar to those of the open-loop system. This is not however a very compelling restriction in the case of a large flexible structure since the amount of control one can get from the actuators is usually limited, and very high gains are not conceivable. The second problem is to evaluate how suboptimal is the design. This is highly dependent on the choice of the fast and the slow modes and the size of the gap between the bandwidths. There must be ways to optimize the choice of the modes in order to obtain a solution as close as

possible to the optimum.

Let's mention for completeness that some results have been presented on the linear filtering of singularly perturbed systems in [16]. Those results are very important since the simplification of the control law requires some simplification of the estimator loop necessary to reconstruct the states.

The multi time-scale idea has also been extended to the frequency domain and some important results are presented in [17]. Those developments led to the derivation of a design procedure in the frequency domain of two time-scale output feedback control laws for two time-scale plants [18].

Decentralized control methods

A more direct approach to controlling large-scale systems consists in considering simpler controllers. One reasonable choice, regarding implementability and simplicity, is to use a nonclassical information pattern, that is, to feed the actuators only a part of the information that can be collected in the system. In the decentralized control case, the control law commanding the actuator of a subsystem is only a function of the subsystem's outputs. If such an architecture appears straightforward and rather appealing, the problem of deriving the control laws is not. As reported in [1,4,32], Wistenhausen showed in 1968 on an example that the solution to the LQ problem with nonclassical information pattern was not linear. Furthermore, the separation principle does not hold even when one restricts the control law to be linear, with specified feedback channels.

Restricting the control law to be linear is a very reasonable approach given the complexity coming from the size of the systems considered. There is no need to increase the complexity by trying to use nonlinear control for which little is known compared to linear control theory. Sandell and Athans introduce in [19] the optimal solution for nonclassical LQG problems for which the optimum is constituted by a linear control law. However, the specificity of the problem makes the solution mostly relevant in the field of decision making processes.

The implementation of a decentralized control law (i.e. each controller

acts based upon the outputs of a subset of sensors) can be the result of different considerations. As seen before, decentralized control laws providing connective stability were found by local optimization of the gains and the remaining structural information was reduced to only provide bounds on the perturbation due to the coupling. This yielded some suboptimal control.

Another possible approach is to get an optimal control law within the admissible set of laws that is fixed by some external considerations which partition the information available to each controller. Such a problem was studied by Chong and Athans in [20]. In their study, the system was considered to have two different sets of inputs and two different sets of outputs. The actions of each set of inputs were constrained to rely only on one set of outputs. The LQG solution for such a constrained controller could then be derived. One very important drawback of the solution is that each control loop uses a compensator of the same order as the plant. Therefore, even if the number of communication channels is reduced, the complexity of the control law remains.

Stabilization and pole placement

Stabilization and pole placement have always been of prime interest in the centralized control of linear systems. The stability of a linear time invariant system is directly related to the location of its closed-loop poles. LQ, and other techniques are however usually preferred as design procedures since it is difficult to relate the input/output properties of a system, its command following properties or noise rejection properties for example, to the location of the closed-loop poles. Anyhow, it is always useful to know if arbitrary dynamics can be obtained, and the following works answer the question for decentralized feedback control.

Wang and Davison introduce in [21] the notion of fixed poles. Those are the uncontrollable or unobservable poles in the context of centralized control. Fixed poles are the ones that cannot be assigned by a given structure of controller. A general result is that a system can be stabilized by a class of controller if the fixed modes found for this class are all contained in the strict left-half plane. The rest of the dynamics

can be placed arbitrarily using dynamic compensation. In [22], Davison uses this result and introduces the notion of robust servomechanism. The robustness is defined as the property for the control system to remain asymptotically stable and regulate with zero steady-state error in the presence of steady disturbances and steady structural error. The centralized robust servomechanism presented in [22] is extended to the decentralized robust servomechanism in [23]. A series of existence theorems stated in [23] give necessary and sufficient conditions for a system to be stabilizable through decentralized control. The generic structure of the decentralized compensator is also characterized. It is shown that there exists a solution to the problem of the robust decentralized servomechanism for "almost all" interconnected systems, provided some controllability and observability properties, where the notion of "almost" is more specifically defined. [24] constitutes a very interesting development of the previous work since it investigates the decentralized robust servomechanism problem for large space structures. The results were derived assuming that position and rate sensors were collocated and were the dual of the actuators -meaning for example that there is a rate sensor collocated with each reaction wheel and that the input axis of the rate sensor coincides with the axis of the reaction wheel. Under those conditions, it is shown that the decentralized robust servomechanism has a solution if and only if the centralized robust servomechanism has one, which equivalently occurs if and only if the rigid body modes are controllable. An other very interesting result is that it is possible to design a controller in the form shown in [24] for which the unmodelled higher order modes will not be destabilized.

The study of [25] gives another complete characterization of the conditions for stability and pole placement using decentralized control. The approach is to determine conditions under which a system made of interconnected subsystems can be made controllable and observable from the inputs and outputs of a particular subsystem using the other controllers to change the coupling between the subsystems. Once the entire system is controllable and observable, dynamic compensation can be employed using the controller singled out to place the poles of the system.

The existence theorems presented in the studies aforementioned are very

powerful theorems, but they are not constructive theorems, in the sense that they cannot be turned into design techniques, or very poor ones in the case of [25]. As mentioned before, pole placement, if it is very useful theoretically, is not a very useful tool as far as specific performances are sought. Consider a MIMO system fully controllable from two different inputs. The same closed-loop poles can be obtained using state feedback and closing the loop on one, or the other input channel. Of course, the properties of the two designs might be very different, even though both have the same closed-loop poles. As in the centralized case, pole placement using decentralized control does not provide enough freedom to the designer.

Simplified centralized control designs

One can opt for a reduced order controller to avoid prohibitively long process time in the control loop. Such an approach yields a centralized control scheme of acceptable complexity. Of course, one is led to look for an optimal design given the order of controller that is chosen. The solution to this problem has been derived in [27]. The problem is the following: given a plant of very high order, some quadratic cost functional on the states and the system's inputs, and some measurements corrupted by noise, find a compensator of given order which will minimize the cost average. When the order of the compensator is equal to the order of the plant, the problem simply reduces to the classic LQG problem. This problem requires the resolution of two uncoupled Riccati equations of order equal to the plant order. When the order of the compensator is smaller than that of the plant, it is shown in [27] that the solution to the optimal problem involves the resolution of two full-order modified Riccati equations coupled by two modified Lyapunov equations via an oblique projection matrix whose rank is equal to the order of the compensator. The projection has to be determined as part of the solution and there exists more than one that will satisfy the necessary conditions stated in [27]. Numerical methods can be developed to solve the optimal projection equations and it is possible, like in the case of model reduction via optimal projection, to make the solution converge toward a local minimum which yields an acceptable value for the

cost. The global minimum might also be reached. There is no theoretical difficulties to derive the problem for discrete-time systems, and the optimal projection equations developed in [28] are for a fixed-order sampled data compensator.

As shown in [29], the optimal fixed order compensator problem can be solved even when the plant is infinite dimensional. The solution is very similar to the finite dimensional case with two modified Riccati equations and two modified Lyapunov equations. In that case however, these equations involve linear operators on an infinite dimensional space. Due to the infinite dimensionality of the state space, one needs to call upon properties of linear operators in Hilbert spaces. The proofs are consequently more involved than when only matrix linear algebra is needed. This result of theoretical importance has little utility for one concerned with finding an actual compensator. It provides however confidence to the designer that by taking a finite dimensional approximation of the plant and solving the optimal projection equations for this model will yield a compensator that should converge to some limit as the dimension of the approximate plant is increased.

The method of optimal projection can be extended to obtain a fixed reduced-order state estimator [30]. One can be tempted to use such an estimator, or to use the reduced order plant model found through optimal projection to design a compensator. Such a design will yield a reduced-order compensator which will be only suboptimal in the class of compensators of the same order. Those suboptimal approaches may on the other hand simplify tremendously the controller synthesis for a tolerable increase in the cost.

It must be emphasized again that the compensator obtained through optimal projection is a reduced-order central compensator: every actuator in the system receives its commands from the same calculator which synthesizes the command inputs using the information of every available sensor. The main benefit of such a method is to reduce the computations to synthesize the commands and not to limit the communication requirements or to make the control robust to components' failure. There is no guarantee of stability and one may have to iterate the design process to find the right order for

the compensator. Finally, little is known about the sensitivity of the design to parameter errors and model uncertainties.

Multilevel techniques

The multilevel, or hierarchical architecture appears to be the next natural step to improve the performance of a decentralized control scheme. At the subsystem level, local controllers operate using local information and information supplied by a global controller. They supply in return the global controller with those informations, as well as their actions, preferably in a condensed manner. The global controller has perfect structural information about the system, and knows in particular how the subsystems interact. Given the information received from the subsystems, the global controller sends directives to each local controller so that more cooperation occurs within the system. Such an architecture is very elegant, but its actual implementation appears to be very difficult.

Periodic coordination

As argued by Chong and Athans in [31], if the global controller is supplied with all the information, the solution to the LQ problem will be for the global controller to cancel out the local actions and superimpose the centralized optimal solution. The approach in [31] is therefore to consider that the global controller operates at a smaller rate than the local controllers. The solution to the LQ problem is called periodic coordination, since the directives arrive at the local level periodically every l time steps, where l is the ratio between the global controller's sampling time and the local controller's sampling time. The system considered in this approach is the interconnection of dynamical systems each of which has its own state variables, sensors and actuators. The overall system command matrix as well as the output matrix are assumed to be block diagonal. This setting is similar to the one considered for connective stability. Under those assumptions, the control law implemented is the following: local controllers drive local actuators based upon local information. The local control law would be LQ optimal if there were no

coupling between the subsystems. At the upper level, the interactions between the subsystem are being estimated based on a priori information and past measurements. The update of the estimate is done only periodically every l steps. Two kinds of periodic control can be distinguished. The first one is qualified as open-loop, meaning that the coordinating parameters are computed based on past information and without expecting future information. Thus, the estimate tries to minimize a mean error for all future times. The second one is qualified as closed-loop, meaning that future measurements are expected: the estimate in that case minimizes a mean error for the next l steps only. The closed-loop scheme is more complex to solve and its resolution does not decouple at the subsystem level. It should however yield a better solution. The method appears to be an elegant design method. Still, even if optimality is reached, little is known about stability.

Goal Coordination and Interaction Prediction methods

Most of the remaining work on hierarchical control does not make any assumption about the information pattern and does not consider different time-scales for the local and global controllers. The problem considered throughout [32] is a LQ type problem for a system constituted of interconnected subsystems. The optimal sequence of controls can be generated on-line for the regulator problem. In order to simplify the computation, additional variables, called the coordination variables, are introduced. They naturally appear as Lagrange multipliers in the cost minimization problem. In the Goal Coordination method ([32,33]) also referred to as the Interaction Balance method (Mesarovic et al., to whom the method is attributed in [32]) the coupling terms are considered as state variables. They are of course related to the system's state variables through linear transformation, and are treated as constraints. The coordination variables are the Lagrange multipliers of those constraints. By use of the coordination variables, the resolution of the problem separates into an upper and a lower level. At the lower, or subsystem level, one is led to solve decoupled optimal regulator problems for the subsystems as if they were isolated. The coordination variables enter as

parameters of the local minimization problems. At the upper level, the coordination variables are updated to reduce the cost. The updating process is truly a minimization algorithm. The gradient of the cost relative to the coordination variables, which is used in the algorithm, depends on the local optimal solutions found at the subsystems' level for the state, costate, coupling and control variables. The optimum is found recursively by first assuming coordination variables, then by computing the gradient of the cost at the lower level, solving only reduced-order minimization problems. A different scheme attributed to Takahara is referred to as the Interaction Prediction method in [32,33]. The method uses both the aforementioned coordination variables as well as the coupling variables to define the coordination vector between the local and the global problems. The solution is found in the same manner as in the Goal Coordination method, by assuming a value for the coordination vector at the upper level and by computing the gradient of the cost at the lower level.

Both methods require the iterative computation of a minimum at each time step. A high convergence rate is reported using either method for fairly complicated systems. The dynamics of the system should however be rather slow so that the time for the algorithm to converge remains small compared to the rate at which one has to sample the system.

Both methods are so-called infeasible methods [32,33] because the constraints are met only at the minimum. The main drawback of such methods is that suboptimal control sequences cannot be obtained by relaxing the accuracy on the determination of the minimum at each time step. Such a control sequence could very well destabilize the plant and does not satisfy any of the problem constraints. Therefore, the expected reduction in computation time due to the breaking down of the large minimization problem to simpler reduced-order problems may very well be over-estimated because of the need to reach accurately a minimum at each time step.

Let's mention for completeness that multilevel methods simplify the computation of centralized control gains [32]. Those methods use coordination variables like the on-line multilevel methods presented above, but gains instead of controls are computed in this case. The methods are also recursive and their main advantage is that they require the resolution

of only reduced-order Riccati equations, whose calculation grows much faster than linearly with the order.

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