The Analysis of Nonstationary Vibration Data

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Procedures for analyzing the random vibration environments of transportation vehicles and other machinery are well defined and relatively easy to accomplish, as long as the vibration data are stationary in character; i.e., the average properties of the vibration do not vary with time. There are cases, however, where a random vibration environment of interest is naturally nonstationary in character, for example, the vibrations produced during the launch of a space vehicle. A well developed methodology exists for the analysis of nonstationary random data, but the resulting analysis procedures require measurements from repeated experiments that often cannot be obtained in practice. The alternative then is to employ a parametric analysis procedure that can be applied to individual sample records of data, under the assumption that the data have a specific nonstationary character. This paper reviews the general methodology for the analysis of arbitrary nonstationary random data, and then discusses a specific parametric model, called the product model, that has applications to space vehicle launch vibration data analysis. Illustrations are presented using nonstationary launch vibration data measured on the Space Shuttle orbiter vehicle.

INTRODUCTION

The launch vibration environment of space vehicles is highly nonstationary in character due to a sequence of time-varying aeroacoustic events that govern the dynamic loads on the vehicle. The most important of these events and the excitations they produce are (a) the acoustic noise excitation from the rocket motors during lift-off, (b) the excitation due to shock wave/boundary layer interactions during transonic flight, and (c) the turbulent aerodynamic boundary layer excitation

during flight through the region of maximum dynamic pressure (max "q").
These three events are clearly apparent in the time history of the
typical Space Shuttle launch vibration measurement shown in Figure 1.
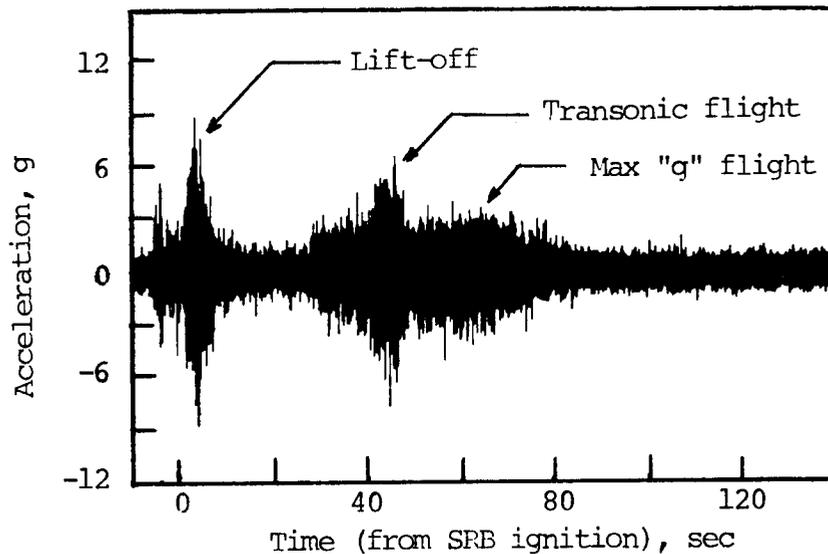
Figure 1. Typical Vibration Time History During A Space Shuttle Launch.

Traditionally, the vibration data measured on space vehicles during
launch are analyzed by selecting short time slices of data at those
times when the overall value of each measurement reaches a maximum
during each of the above noted events. Auto (power) spectral density
functions are then computed for each time slice, and are used to
describe the vibration spectra for those events. However, because space
vehicle launch vibration data are basically random in character, this
analysis procedure poses a serious problem. On the one hand, it is
clearly desirable to make the spectral analysis with a small frequency
resolution bandwidth B to properly extract the spectral variations in
the data, and also with a small averaging time T to properly define the
time variations in the data. On the other hand, because the data repre-
sent a random process, the spectral estimates will involve a statistical
sampling error [2, p. 283], which can be approximated in terms of a
normalized random error (coefficient of variation) by $\epsilon = [BT]^{-1/2}$.
Hence, as the bandwidth B is made smaller to obtain a better spectral
resolution, and the averaging time T is made smaller to obtain a better
time resolution, the random error in the estimate increases, often to
levels in excess of the bias errors that would have occurred if a wider
resolution bandwidth B and/or averaging time T had been used.
    Several studies of this nonstationary data analysis problem have
been performed dating back to the 1960's [3,4], and including a recent
study directed specifically at the analysis of the Space Shuttle launch
aeroacoustic and structural vibration environment [5]. This paper sum-
marizes the theoretical background for improved vibration data analysis
procedures recommended in [5], and illustrates applications using a
Space Shuttle launch vibration measurement. The suggested procedures
should also be applicable to expendable launch vehicle vibration data.

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ANALYTICAL BACKGROUND

Many analytical methods have been proposed over the years for describing the spectra of nonstationary random data. From the viewpoint of describing mathematically rigorous input-output relationships for physical systems, including time-varying systems, the double frequency (generalized) spectral density function is broadly accepted as the most useful spectral description for nonstationary data. A full development and discussion of various forms of the double frequency spectrum are presented in [2, pp. 448-456]. For the purposes of applied data analysis, the instantaneous spectral density function (sometimes called the frequency-time spectrum or the Wigner distribution) is usually considered a more useful spectral description for nonstationary data. The instantaneous spectrum is detailed in [2, pp. 456-465], and will be used here as the starting point for the analytical discussions of improved spectral analysis procedures for launch vehicle vibration data.

The Instantaneous Spectrum

Consider a random process defined by an ensemble of sample functions {x(t)}, where individual measurements of the random process produce time history records $x_i(t)$; i = 1,2,3,....,N, as illustrated in Figure 2. The instantaneous autocorrelation function of the random process is given by [2, p. 445]

$$R_{xx}(\tau, t) = E[x(t-\tau/2) x(t+\tau/2)] \qquad (1)$$
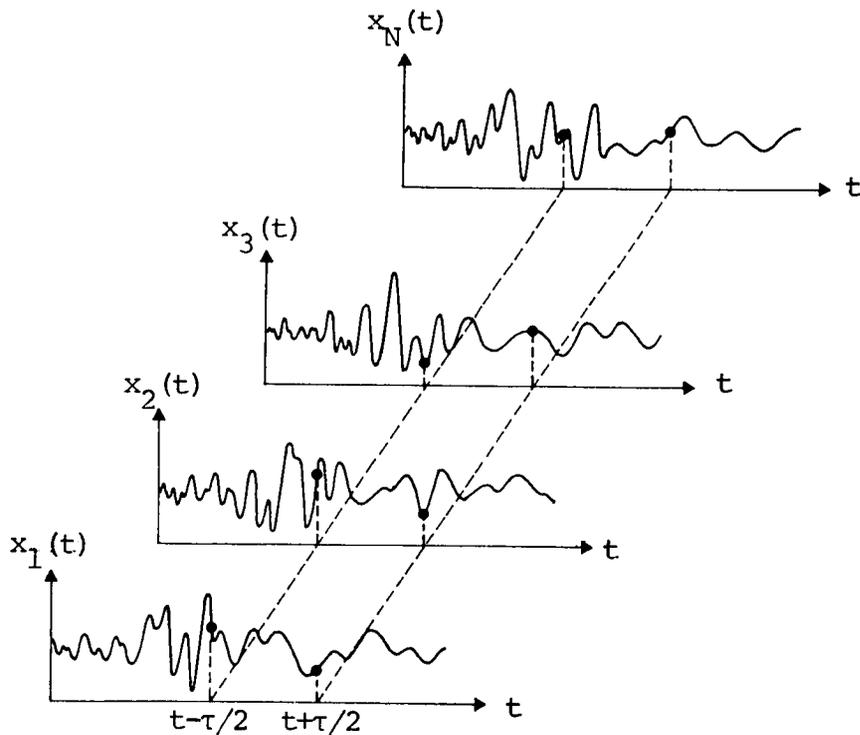
Figure 2. Sample Records Forming Nonstationary Random Process.

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where E denotes an "expected value", which in practice would be approximated by an ensemble average over i = 1 to N sample records. The instantaneous autospectral density function is then defined by

$$W_{xx}(f,t) = \int_{-\infty}^{\infty} R_{xx}(\tau,t)\cos 2\pi f\tau\, d\tau \qquad (2)$$

In words, the instantaneous autospectral density function is the Fourier transform of the instantaneous autocorrelation function computed over τ. Since the instantaneous autocorrelation function is always real valued, the Fourier transform involves only the cosine term. This fact also leads to the following properties of the instantaneous autospectrum:

$$W_{xx}(f,t) = W_{xx}(-f,t) \qquad\qquad W_{xx}(f,t) = W_{xx}^{*}(f,t) \qquad (3)$$

$$\int_{-\infty}^{\infty} W_{xx}(f,t)\, df = E[x^2(t)] \qquad\qquad \int_{-\infty}^{\infty} W_{xx}(f,t)\, dt = E[|X(f)|^2] \qquad (4)$$

In Equation (3), the asterisk (*) denotes complex conjugate, and in Equation (4), X(f) is the Fourier transform of x(t) given by

$$X(f) = \int_{-\infty}^{\infty} x(t) e^{-j2\pi ft}\, dt \qquad (5)$$

where the integral in Equation (5) is assumed to exist. In words, Equation (4) says that the integral of the instantaneous autospectrum over frequency at any time yields the mean square value of the data at that time, while the integral over time at any frequency yields the "energy" spectral density of the data at that frequency. These two properties closely relate the instantaneous autospectrum to the ordinary autospectrum (also called the "power" spectrum) that is commonly computed for random data in practice, including launch vehicle vibration data.

Practical Measurement Considerations

The instantaneous autospectrum, W(f,t) defined in Equation (2), provides a rigorous description of nonstationary vibration data that lends itself well to the formulation of design criteria and test specifications. However, the estimation of an instantaneous autospectrum theoretically requires an ensemble average over a collection of sample records to first obtain the instantaneous autocorrelation function defined in Equation (1). For the case of spacecraft launch vibration data, this means that measurements would have to be made at identical locations on numerous launches of the same spacecraft under identical conditions. For most space vehicles, this clearly is not feasible. For the special case of Space Shuttle, where the same launch vehicle is used many times and the outputs of certain vibration transducers have been recorded several times on at least one of the orbiter vehicles, an ensemble averaging analysis approach might be considered. Even here, however, the launch conditions have not been identical on the various Space Shuttle launches due to differences in payload weights and SSME thrust. Furthermore, there have not been a sufficient number of launches to date (with repeated measurements at identical locations) to provide

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ensemble averaged estimates with acceptable sampling (random) errors. See [2, p. 285] for a discussion of the sampling errors in spectral estimates produced by ensemble averaging procedures.

The alternative to ensemble averaging is the short time averaging analysis procedure discussed in the Introduction. However, this approach involves an inherent conflict between the frequency resolution bandwidth B and the averaging time T needed to achieve spectral estimates with acceptable bias and random estimation errors. To elaborate on this problem, from [2], the bias error due to the finite frequency resolution bandwidth B used to compute the spectral density estimate at any time t is approximated by

$$b[\hat{W}(f,t)] \approx (B^2/24) \, d^2[W(f,t)]/df^2 \qquad (6)$$

where the hat ($\wedge$) denotes "estimate of". The approximation for the bias error due to the finite averaging time T used to compute the spectral density estimate at any frequency f has a similar form, namely,

$$b[\hat{W}(f,t)] \approx (T^2/24) \, d^2[W(f,t)]/dt^2 \qquad (7)$$

In Equations (6) and (7), the second derivatives essentially represent the sharpness of peaks and notches in the variations of $W(f,t)$ with both frequency and time. Finally, as noted in the introduction, the random error due to the finite sample size used to compute the spectral density estimate is approximated in terms of the normalized standard deviation of the estimate (the coefficient of variation) by

$$\epsilon = \sigma[\hat{W}(f,t)]/W(f,t) \approx 1/[BT]^{1/2} \qquad (8)$$

Since B and T appear in the denominator of Equation (8), but in the numerator of Equations (6) and (7), respectively, it is clear that there will be at least some frequencies and times when it is not possible to estimate a time varying spectrum by short time averaging procedures that will have an acceptable combination of bias and random errors. This problem can be circumvented only by assuming a specific model for the nonstationary character of the data, which can then be exploited for analysis purposes (a parametric procedure).

The Product Model

A special model for nonstationary data that has received considerable attention for applications to space vehicle launch vibration data [2-5], as well as nonhomogeneous atmospheric turbulence data [6], is the product model, which is defined as a nonstationary random process {x(t)} producing sample records of the form

$$x(t) = a(t)u(t) \qquad (9)$$

where a(t) is a deterministic function, and u(t) is a sample record from a stationary random process {u(t)} with zero mean and unit variance; i.e., $\bar{u} = 0$ and $\overline{u^2} = 1$. For the special case where a(t) is restricted to positive values, it can be interpreted as the time varying standard deviation of {x(t)}. It follows from Equations (1) and (2) that the

instantaneous autocorrelation and autospectral density functions for the product model are given by

$$R_{xx}(\tau, t) = R_{aa}(\tau, t) R_{uu}(\tau) \qquad W_{xx}(f, t) = \int_{-\infty}^{\infty} S_{aa}(\beta, t) S_{uu}(f-\beta) \, d\beta \qquad (10)$$

where the S terms in the spectral result are two-sided spectral density functions defined as

$$S_{aa}(f, t) = \int_{-\infty}^{\infty} R_{aa}(\tau, t) e^{-j2\pi f \tau} \, d\tau \qquad S_{uu}(f) = \int_{-\infty}^{\infty} R_{uu}(\tau) e^{-j2\pi f \tau} \, d\tau \qquad (11)$$

Note in Equations (10) and (11) that the autocorrelation and auto-spectral density functions are nonstationary for a(t), but are stationary for {u(t)} since this is a stationary random process. Also note that it is common practice to work with one-sided spectra (defined for positive frequencies only), as opposed to two-sided spectra. The one-sided spectra are related to the two-sided spectra as follows:

$$W(f, t) = 2W(f, t); 0 \leq f \qquad G(f) = 2S(f); 0 \leq f \qquad (12)$$
$$\quad\quad = 0 \quad\quad ; f < 0 \qquad\quad\quad = 0 \quad\quad ; f < 0$$

The Locally Stationary Model

An important special class of nonstationary random processes that fit the product model of Equation (9) are locally stationary data [2,7] (sometimes called uniformly modulated data [8]). Data are said to be locally stationary if they fit Equation (9) where a(t) varies slowly relative to u(t); i.e., the highest frequency component in $S_{aa}(f,t)$ is at a much lower frequency than the lowest frequency component in $S_{uu}(f)$. For this case, the instantaneous autospectral density function in Equation (10) can be approximated by

$$W_{xx}(f, t) \approx a^2(t) G_{uu}(f) \qquad (13)$$

where W(f,t) and G(f) are one-sided spectra, as defined in Equation (12), and $a^2(t)$ is the instantaneous mean square value of the data (the instantaneous variance if the mean value is zero). Hence, the instantaneous autospectrum for locally stationary data becomes a product of independent time and frequency functions that can be measured separately. Specifically, one can estimate the instantaneous autospectrum from a sample record of length T_r by two operations, as follows:

(1) Compute the autospectrum of the record by averaging over the entire record length T_r using a narrow spectral resolution bandwidth B.

(2) Compute the time varying mean square value over the entire record bandwidth B_r using a short averaging time T.

The above operations permit the estimation of the instantaneous spectrum with a good resolution in frequency and time, which suppresses the

frequency and time interval bias errors in the estimate, while still achieving a large bandwidth-averaging time product to suppress the random errors in the estimate.

Hard-Clipped Analysis

The locally stationary model in Equation (13) is valid only if a(t) varies slowly relative to u(t) in Equation (9). If this is not the case, a(t) will act as a modulating function on u(t), and cause the spectrum of x(t) to spread [2]. It follows that the computed average autospectrum of x(t) will not be proportional to the autospectrum of the fictitious stationary component u(t) in the product model; i.e., $G_{xx}(f) \neq c\, G_{uu}(f)$ where c is a constant. However, the autospectrum of the stationary component u(t) can still be estimated, even though u(t) cannot be directly measured, by a special procedure suggested and illustrated in [6]. Specifically, since a(t) represents a standard deviation which never takes on negative values, and assuming the mean values of x(t) and u(t) in Equation (9) are zero, it follows that the zero crossings of x(t) will be identical to those of u(t). Under the further assumption that the random process {u(t)} has a normal (Gaussian) probability density function, the stationary autospectrum of {u(t)} can be computed from a hard-clipped version of a sample record x(t) by applying the "arc-sine" rule [9], as follows:

(1) Hard-clip the record x(t) to obtain a new record y(t) defined by

$$y(t) = \ 1 \ \text{for} \ x(t) \geq 0 \qquad\qquad (14)$$
$$= -1 \ \text{for} \ x(t) < 0$$

(2) Compute the autocorrelation function of y(t) to obtain $R_{yy}(\tau)$.

(3) Compute the normalized autocorrelation function of u(t) from

$$R_{uu}(\tau) = \sin[(\pi/2)R_{yy}(\tau)] \qquad\qquad (15)$$

(4) Compute the autospectrum of u(t) by Fourier transforming the autocorrelation function of u(t) over a delay time (1/B), where B is the desired spectral resolution, as follows:

$$G_{uu}(f) = 4\int_0^{(1/B)} R_{uu}(\tau)\cos(2\pi f\tau)\ d\tau \qquad\qquad (16)$$

The computed spectrum of u(t) in Equation (16) does not, of course, represent the actual autospectrum of the nonstationary vibration environment that was measured. It is simply the autospectrum of the fictitious u(t) in the theoretical product model given by Equation (9). However, with this term, and an estimate for the time-varying term a(t) in Equation (9), one could simulate the nonstationary spectrum $W_{xx}(f,t)$ in the laboratory by applying a nonstationary excitation produced by multiplying a signal with a stationary spectrum $G_{uu}(f)$ by a time-varying signal a(t).

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APPLICATIONS TO LAUNCH VIBRATION DATA

It is generally agreed that the desired spectral representation for the nonstationary vibration measurements made on space vehicles during launch is a spectrum that defines the maximum mean square value in each frequency resolution bandwidth during the nonstationary event, independent of the times when the maximum values in the various bandwidths occur. Such a spectral representation, referred to hereafter as the maximax spectrum, generally will not represent the instantaneous spectrum of the data at any specific instant of time, unless the data are stationary or locally stationary. However, since vibration induced malfunctions and failures of space vehicle structures and equipment tend to be frequency dependent, the maximax spectrum does provide a conservative measure of the environment from the viewpoint of damage potential and, hence, constitutes a rational basis for the derivation of test specifications and design criteria.

The issue at hand is whether the desired maximax spectrum of the space vehicle vibration response during a nonstationary launch event can be adequately approximated by the maximum of the instantaneous spectrum computed assuming the data are locally stationary during the event. This matter was empirically evaluated for Space Shuttle launch vibration and aeroacoustic data in [5]. The general conclusion from that reference is that, even though the launch vibration data do not always make a rigorous fit to the locally stationary model, the discrepancies of the maximum instantaneous spectrum (computed assuming local stationarity) from the maximax spectrum for each launch event are negligible compared to the errors that occur in a short time averaged spectral calculation. (It should be mentioned that this conclusion does not necessarily apply to aeroacoustic data). An illustration of the results from [5] is now presented for a vibration measurement on a Space Shuttle payload during the STS-2 launch.

Data Evaluation Procedures

To illustrate the data evaluation procedures, consider a vibration measurement made on the OSTA-1 payload during the second Space Shuttle launch (STS-2). The exact location of the measurement in terms of orbiter coordinates was $x = 920$, $y = -73$, and $z = 413$. The measurement is identified in [1] as V08D9248A (hereafter referred to as Accel 248). The short time averaged ($T = 1$ sec) overall rms value for this measurement is shown in Figure 3. Note that the three primary nonstationary launch events are clearly apparent in the overall level versus time. The maximum overall values during these events occur at about T+4 for lift-off, T+45 sec for transonic flight, and T+60 sec for max "q" flight.

To calculate an approximate maximax spectrum and facilitate other studies, the short time averaged spectra of this vibration measurement through the three primary nonstationary launch events were computed every sec (every 3 sec for max "q" data) from the beginning to the end of each event, considered to be as follows:

(a) Lift-off: T+2 to T+8 sec (6 sec duration).
(b) Transonic flight: T+40 to T+48 sec (8 sec duration).
(c) Max "q" flight: T+52 to T+70 sec (18 sec duration).
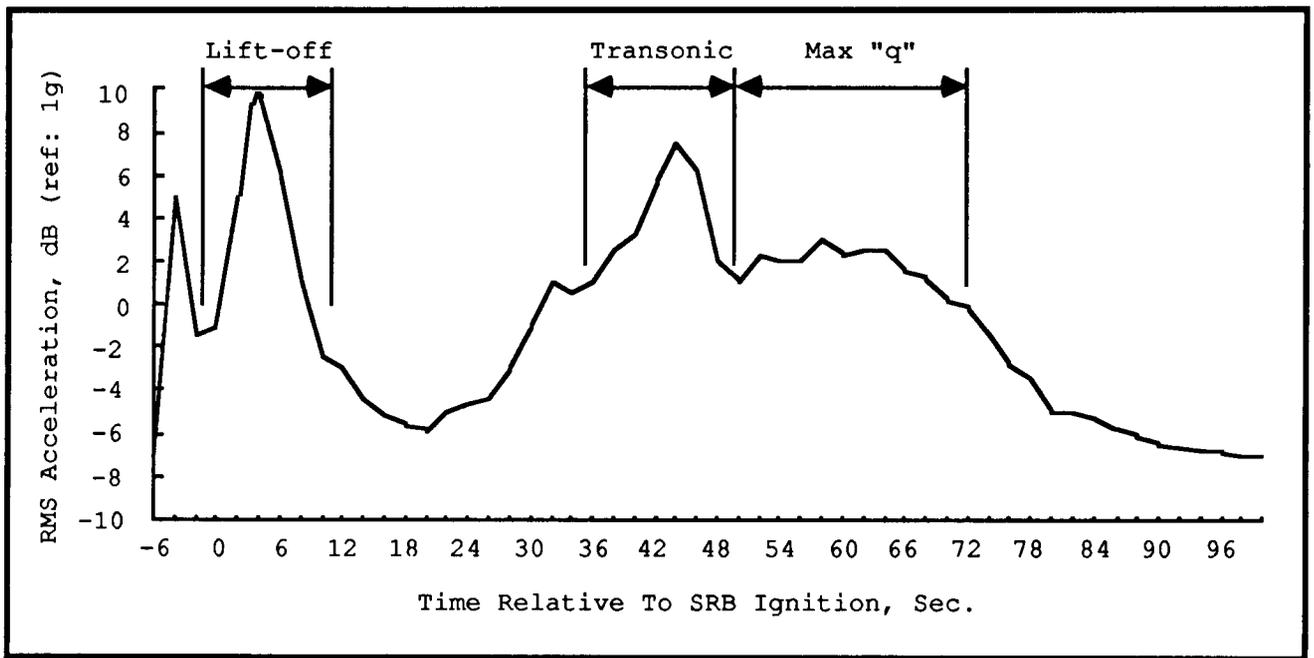
Figure 3. Overall Vibration level Versus Time Measured By Accel 248 During STS-2 Launch.

These durations were selected to avoid contamination of the data for each event by other events. For example, the transonic and max "q" events clearly overlap, but the selected 4 sec separation should be adequate to avoid serious contamination of the data for one event by the other. Also, the first 2 sec of lift-off were omitted to avoid contamination by the lift-off transient. The spectra were computed in 1/3 octave bands to enhance the BT product of the calculations and suppress random errors. The frequency range for the 1/3 octave band calculations was 16 to 1000 Hz. The results are presented in the appendix (Figures A1 through A3). To further facilitate data interpretations, all 1/3 octave band spectra were normalized to an overall mean square value of unity. These normalized results are also detailed in the appendix (Figures A4 through A6). If the data were locally stationary and there were no random sampling errors in the spectral estimates, these normalized spectral plots for each event would be identical. From Equation (8), the random error in the 1/3 octave band estimates is $\epsilon \approx 2.1/[f_o]^{1/2}$ ($\epsilon \approx 1.2/[f_o]^{1/2}$ for the max "q" data), where f_o is the center frequency of the octave band.

Average Spectra

There are two basic ways to calculate the spectral portion of locally stationary data, as given by $G_{uu}(f)$ in Equation (13). The first and most common way is simply to compute the average spectrum over the entire nonstationary event, or a sufficiently long portion of the event (providing a total record length T_r), to yield a BT_r product that will adequately suppress the random errors in the estimate. The values of the resulting spectrum can then be divided by the area under the spectrum to obtain a result with a mean square value of unity. This approach is hereafter referred to as the direct average procedure. It is clear that

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the direct average procedure will give greatest weight to the spectral values at those times when the magnitudes are large.

The direct average spectra of the 1/3 octave band values in the appendix (Figures A1 through A3) were computed as follows. Let MS_{ij} denote the mean square value of the data measured during the $\underline{ith}$ time slice ($i = 1,2,\ldots,k$) and in the $\underline{jth}$ frequency band ($j = 1,2,\ldots,r$) during a given event. The average mean square value in each 1/3 octave band over the nonstationary event of interest is given by

$$MS_j = \sum_{i=1}^{k} MS_{ij}/k \qquad (20)$$

The direct average spectrum (without a normalization on bandwidth) is then given by

$$G_{uu}(j) = MS_j / \sum_{j=1}^{r} MS_j \qquad (21)$$

The second way to estimate $G_{uu}(f)$ in Equation (13) is to first normalize the instantaneous spectrum of the data to a mean square value of unity at all instances of time, and then compute the average of the normalized spectrum over the entire nonstationary event. This approach is hereafter referred to as the normalized average procedure. Unlike the spectra produced by the direct average approach, the normalized average procedure weights the spectral values at all times during the nonstationary event equally, independent of their magnitudes.

The easiest way to normalize the instantaneous spectrum of data is by the hard clipping procedure detailed in Equations (14) through (16). This permits the normalized average spectrum to be computed directly by Equation (16). However, for the Space Shuttle data, the normalized average spectra were approximated by averaging the normalized 1/3 octave band values given in the appendix (Figures A4b through A6b). Specifically, for the $\underline{ith}$ time slice and the $\underline{jth}$ frequency interval,

$$NMS_{ij} = MS_{ij} / \sum_{j=1}^{r} MS_{ij} \qquad (22)$$

The normalized average spectrum (without a normalization on bandwidth) is then given by

$$G_{uu}(j) = NMS_j = \sum_{i=1}^{k} NMS_{ij}/k \qquad (23)$$

Errors In Average Spectral Computations

The most direct way to assess the errors that would occur if the Space Shuttle vibration measurement were analyzed using the locally stationary assumption is first to compute its average spectrum by both the direct and normalized average procedures, and then to compare these results with the maximax spectrum after adjustments to make the mean square values of the different spectral computations equal. This was accomplished for each of the three primary nonstationary launch events. The discrepancies between each of the average spectra, $G_{ave}(f)$, and the

corresponding maximax spectrum, $G_{max}(f)$, were computed in dB using the formula

$$dB \ error(f) \ = \ 10 \ log_{10}[G_{ave}(f)/G_{max}(f)] \hspace{2cm} (24)$$

The dB errors between the average and maximax spectra determined using Equation (24) are plotted in Figure 4. The standard deviations for these errors are detailed in Table 1.

Table 1. Summary of Errors Between Average And Maximax Spectra.

Type of Average	Standard Deviation of Errors, dB		
	Lift-off	Transonic	Max "q"
Direct average	0.38	0.98	0.44
Normalized average	0.81	1.09	0.48

It is clear from Figure 4 and Table 1 that the direct average produces a smaller net discrepancy from the maximax spectrum than the normalized average. From [5], this was true for all the Space Shuttle vibration and aeroacoustic measurements considered. Hence, there is no question that the direct average provides the superior approach. This is a gratifying result, since the direct average is the easier of the two calculations to perform using current data analysis equipment and software. Specifically, one simply computes the autospectrum of the record as if it represented stationary data, except the ordinate scale of the resulting spectrum must be independently established.

Applications To Narrowband Autospectra

The previous results and conclusions are based upon the spectra of data measured in 1/3 octave bands under the assumption that the nonstationary trends observed in the 1/3 octave band data should be representative of the trends in narrowband auto (power) spectra data (PSD's). To check this assumption, the maximax and direct average autospectra for the data measured by Accel 248 during lift-off were computed using a 10 Hz frequency resolution bandwidth with the results shown in Figure 5. The analysis was conducted to only 500 Hz because, in terms of constant bandwidth autospectra, the spectral levels of the data from Accel 248 during lift-off were very small above this frequency. The errors between the average and maximax spectrum, as defined in Equation (24), are detailed in Figure 6. The standard deviation of the errors for the narrow bandwidth (10 Hz) analysis is 0.6 dB, somewhat larger than the net error of about 0.4 dB calculated from the 1/3 octave band data in Table 1 because of the larger random errors in the narrowband spectral estimates (the random error between the maximax and average spectra should be small since they are computed from the same data record, but there is some random error because the maximax spectra are computed over a shorter time interval).

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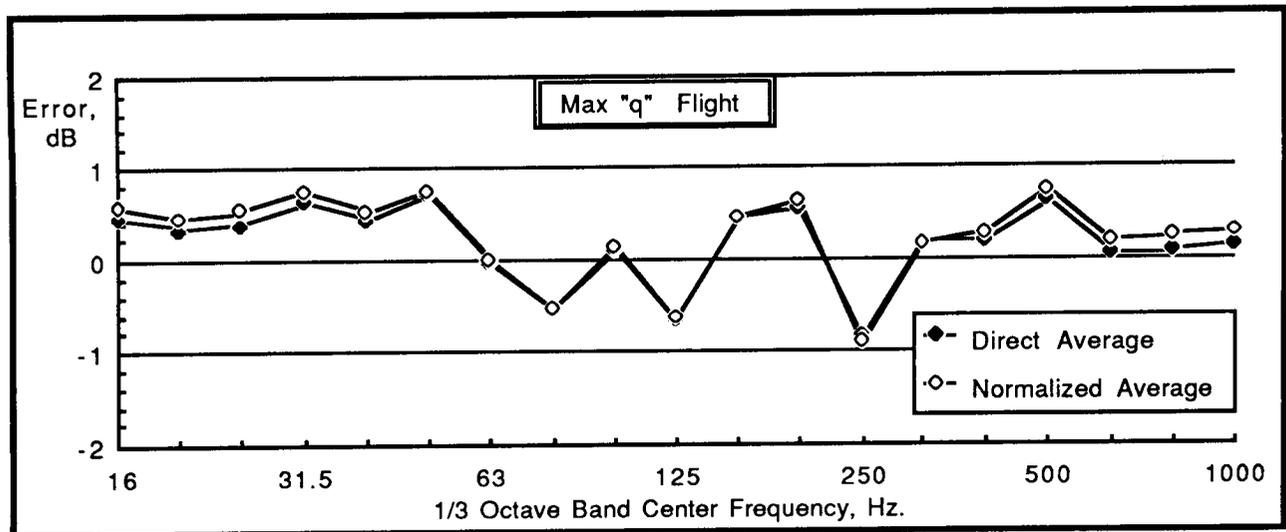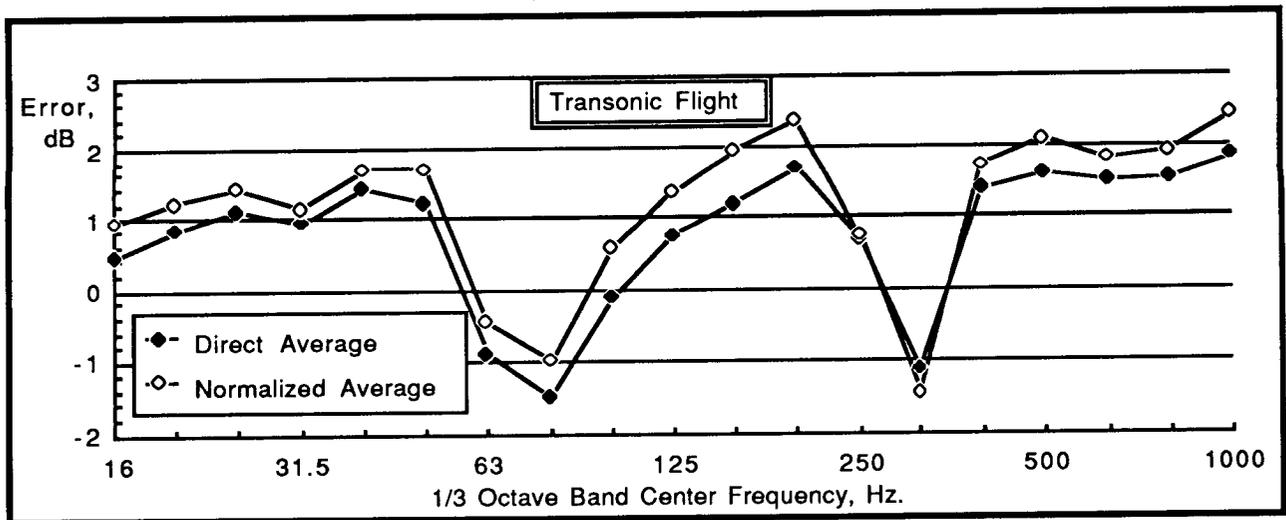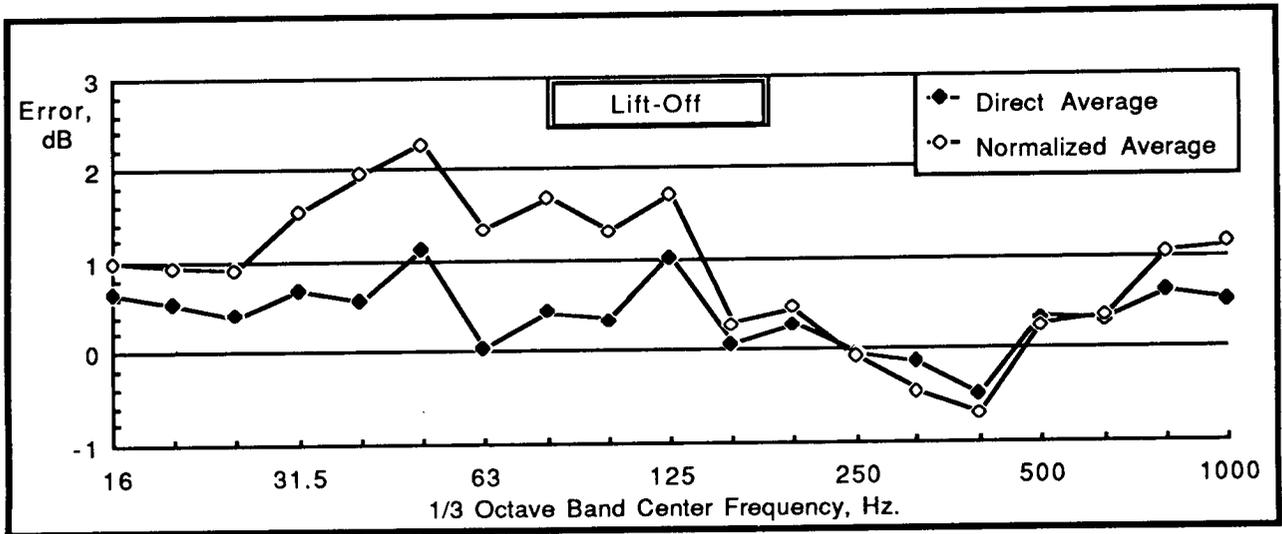
Figure 4. Errors Between Average And Maximax Spectra for Accel 248.
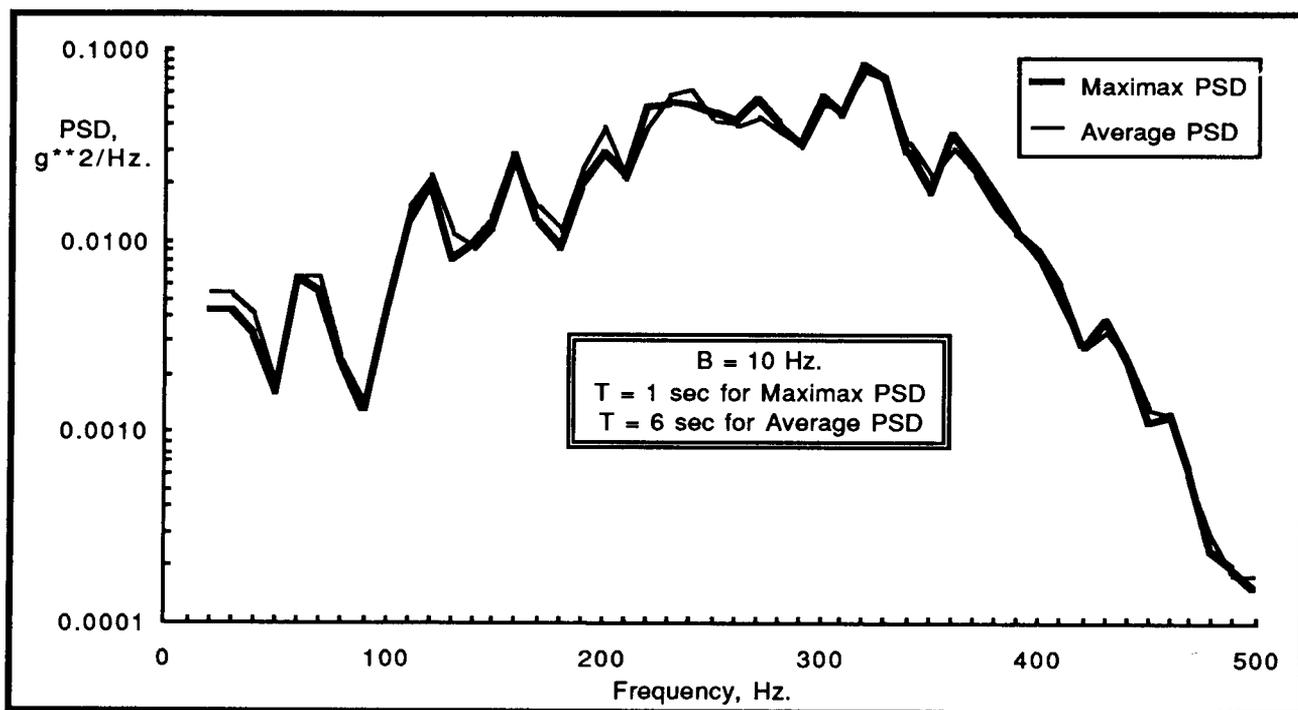
Figure 5. Average And Maximax Autospectra For Accel 248 During STS-2 Lift-Off.
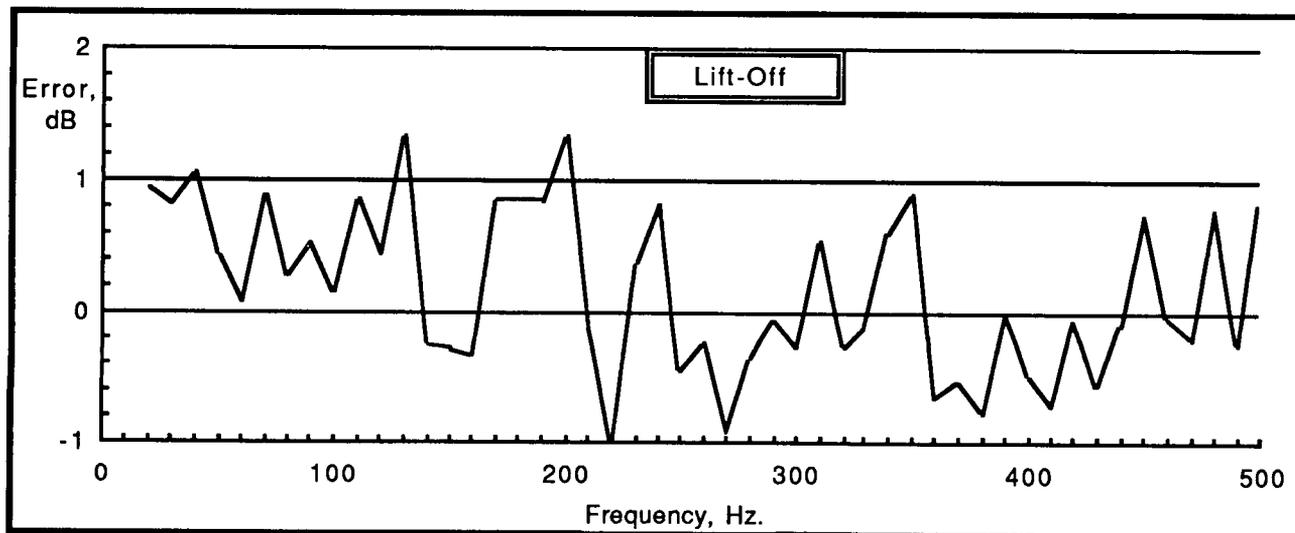
Figure 6. Errors Between Average And Maximax Autospectra Of Vibration Measured By Accel 248 During STS-2 Lift-Off.

Estimation of Maximax Overall Value

The computation of an average spectrum constitutes only half the analysis for data assumed to be of the locally stationary form. The second half of the required analysis involves the computation of the maximum overall value of the data during each nonstationary event of interest; i.e., the maximum value of a(t) in Equation (13). As discussed

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earlier, the desired overall value here is the overall for the maximax spectrum, and not the maximum instantaneous spectrum. Of course, for data which rigorously fit the locally stationary model, the maximax and maximum instantaneous overalls would be equal since the mean square values of locally stationary data reach their maximum values in all frequency bands at exactly the same time. However, from [5], much of the Space Shuttle launch vibration data do not rigorously fit the locally stationary model; the locally stationary assumption is being used here only as an approximation to be exploited for data analysis purposes. This fact poses a practical analysis problem since the maximum instantaneous overall value of nonstationary data is relatively easy to estimate, but a determination of the maximax overall value requires a knowledge of the maximum value in each individual frequency band independent of when that maximum value occurs.

The easiest way to approximate the overall value of the maximax spectrum is to use the maximum instantaneous overall value as an approximation. To assess the potential errors of such an approximation, the overall values of the maximax spectrum and the maximum instantaneous spectrum for Accel 248 during the three nonstationary launch events were computed with the results shown in Table 2. Also shown in Table 2 are the differences between the two overall values in percent and dB. The maximax overall values were determined by selecting the highest spectral value for each event in each 1/3 octave band in Figures A1 through A3, independent of the time it occurred, and summing these 1/3 octave values. The maximum instantaneous overall values were determined by short time averaging procedures with an averaging time of 1 sec for the lift-off and transonic data and 3 sec for the max "q" data, as shown in Figures A4a through A6a (the error associated with this calculation is discussed later).

Table 2. Summary of Errors Between Maximum Instantaneous and Maximax Overall Values.

Calculation	Lift-off	Transonic	Max "q"
Maximax overall, g	3.06	2.78	1.46
Max. instant. overall, g	2.99	2.61	1.37
Percent difference (rms), %	2.3	6.5	6.6
Percent difference (ms), %	4.7	13.4	13.6
Decibel difference, dB	0.2	0.6	0.6

It is seen from Table 2 that the maximax overall value always exceeds the maximum instantaneous overall value as expected, but generally by less than 15% of the mean square value (from [5], this appears to be a reasonable error bound for both vibration and aeroacoustic data, at least for Space Shuttle launches). Hence, to be conservative, it would be wise to multiply the computed maximum instantaneous overall mean square value of the vibration measured during each nonstationary event by a factor of 1.15 to estimate the maximax overall mean square value. Of course, there is still the problem of making an accurate estimate for the maximum instantaneous mean square value during each of the primary nonstationary launch events, which is discussed next.

Estimation Of Maximum Instantaneous Overall Values

The maximum instantaneous overall value of vibration data during a nonstationary event can be estimated in two general ways. The first and easiest way is to compute the time varying mean square value of the record during the nonstationary event using a short averaging time. The maximum instantaneous overall value is then given by the square root of the maximum mean square value calculated during the event. The short time average may be computed using either linear or exponentially weighted averaging procedures. The only problem is to select an appropriate averaging time.

The optimum averaging time to estimate the time-varying overall value of nonstationary data is the longest averaging time that can be used without smoothing the nonstationary trend in the overall value. In more quantitative terms, it is the longest averaging time that will not cause a significant bias error as defined by Equation (7). From the time-varying overall values presented for the Space Shuttle launch vibration data in [1], it appears that the most rapid variations in mean square value with time (using an averaging time of T = 1 sec) occur for the lift-off and transonic data, and resemble a half sine wave with a period of at least 5 sec; that is,

$$a^2(t) \approx \sin(\pi t/5) \qquad (25)$$

Using this criterion as the worst case for lift-off and transonic data, it follows from Equation (7) that the bias error in the estimate of $a^2(t)$ due to the finite averaging time T is given by

$$b[\hat{a}^2(t)] \approx -[(\pi T)^2/600] \sin(\pi t/5) \qquad (26)$$

The largest bias error occurs where t = 2.5 sec (the peak mean square value) and is approximated by

$$b_{max}[\hat{a}^2(t)] \approx -(\pi T)^2/600 \approx -0.0165 \ T^2 \qquad (27)$$

where the minus sign means the finite averaging time always causes an underestimate of the instantaneous overall value. Hence, an averaging time of T = 1 sec, as used in this study, produces maximum mean square value estimates that are biased on the low side by up to 1.7% or 0.07 dB (rms value estimates that are low by less than 0.9%). From [10], a linear averaging time of T = 1 sec is broadly equivalent to an exponentially weighted average with a time constant of about TC = 0.5 sec.

For the max "q" data in [1], the variations in the mean square value with time are slower, more closely fitting a half sine wave with a period of at least 12 sec. The maximum bias error due to the finite averaging time in this case is approximated by

$$b_{max}[\hat{a}^2(t)] \approx -0.00286 \ T^2 \qquad (28)$$

Plots of the finite averaging time bias errors defined in Equations (27) and (28) are shown in Figure 7.

It is seen in Figure 7 that the error in estimating the time-varying mean square value for Space Shuttle launch vibration data by short time averaging procedures will be about -0.1 dB with linear averaging times of T = 1 sec for the lift-off and transonic regions, and T = 3 sec for the max "q" region. These linear averaging times are statistically equivalent [10] to exponentially weighted averaging time constants of TC = 0.5 and 1.5 sec, respectively. An error of -0.1 dB corresponds to an underestimate of the maximum mean square value of 2.3%, which is considered an acceptable error.
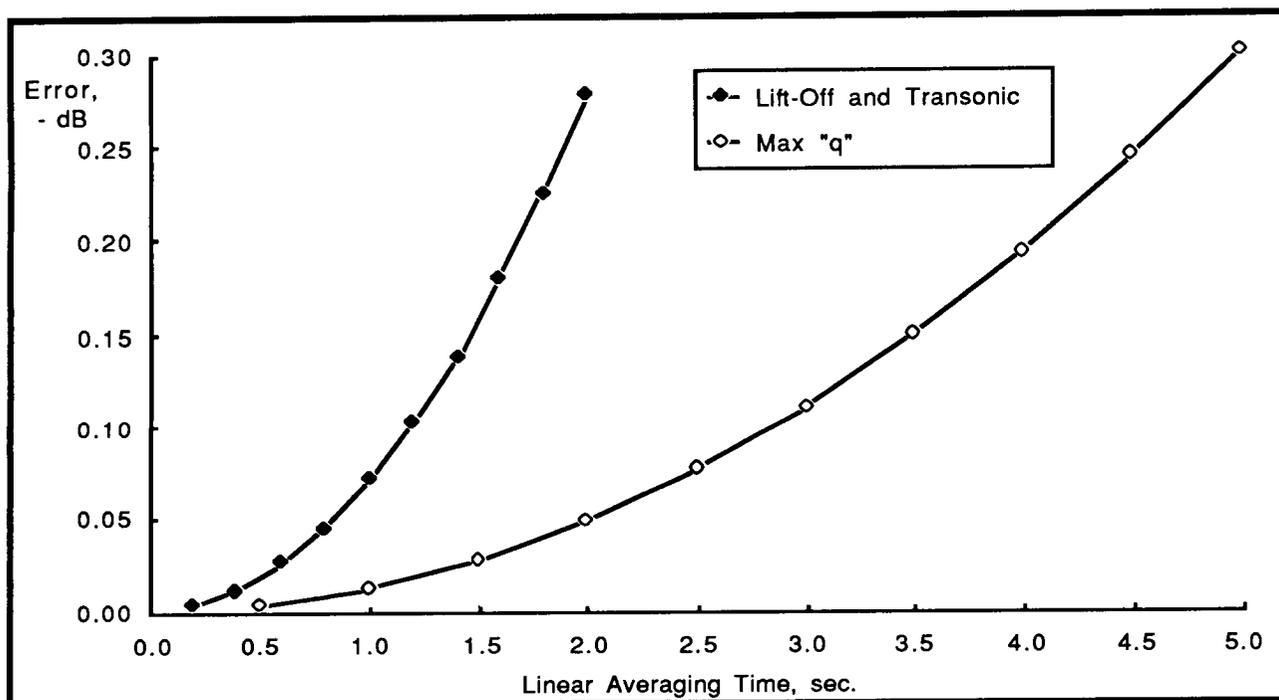
Figure 7. Maximum Bias Errors In Space Shuttle Overall Vibration Level Estimates Due Finite Averaging Time.

In closing on this subject, it must be emphasized that the bias errors in Figure 7 apply only to the Space Shuttle launch environment. Because of differences in the early launch acceleration of various space vehicles, the time durations for the primary nonstationary launch events will be different, meaning the averaging time required to suppress the bias errors in short time averaged mean square value estimates will also be different.

Now concerning the random errors in short time averaged mean square value estimates, Equation (8) applies where $B = B_r$, the equivalent total bandwidth of the data. The equivalent bandwidth B_r will equal the actual bandwidth of the data only for the case of "white noise"; i.e., data with a constant autospectrum. As a rule of thumb, B_r will usually be at least one-quarter of the actual bandwidth of random vibration data. Hence, even with the T = 1 sec averaging time, the normalized random error of the overall mean square value estimate at any instant for a 1 kHz bandwidth vibration record is given by Equation (8) as $\epsilon[\hat{a}^2(t)] \approx$ 0.063 or about ±0.3 dB, which is considered an acceptable random error.

The second way to estimate the overall value of vibration data during a nonstationary event is by fitting an appropriate series function to the squared values of the individual data points using conventional regression analysis procedures. For relatively simple mean square value/time variations, where there is a single maximum with values falling monotonically on both sides of the maximum, a trigometric set will often provide a good fit with only a few terms (see [6] for an illustration). However, a more common approach is to fit the individual squared values of the data, $w_n = x^2(n\Delta t)$; $n = 1,2,...,N$, with a Kth order polynomial,

$$\hat{a}^2(n\Delta t) = \sum_{k=0}^{K} b_k(n\Delta t)^k \; ; \; n = 1,2,...,N \qquad (29)$$

where $3 \leq K \leq 5$ is usually adequate, as long as the time variation of the mean square value is of the relatively simple form described above. A least squares fit of the function in Equation (29) to the individual squared data values yields a set of equations of the form [2, p. 363]

$$\sum_{k=0}^{K} b_k \sum_{n=1}^{K} (n\Delta t)^{k+m} = \sum_{n=1}^{N} w_n(n\Delta t)^m \; ; \; m = 1,2,...,K \qquad (30)$$

This set of K+1 simultaneous equations are solved for the regression coefficients, b_k, which are then substituted into Equation (29) to obtain the mean square value estimate versus time.

The regression analysis approach offers the advantage of potentially lower random errors than are achievable by the short time averaging analysis procedure described earlier, if the order of the fitted polynomial is low. This is true because, with a low order fit, more data are used to define the mean square value estimate at each instant of time. However, if the time variations of the mean square value are not relatively smooth through the maximum value, a significant bias error can occur in the calculated maximum mean square value with a low order polynomial fit. Increasing the order K of the fit to suppress this possible bias error will increase the random error of the estimate, exactly as reducing the averaging time T in the short time averaging approach will increase the random error of the estimate.

CONCLUSIONS

The maximax auto (power) spectral density functions for the non-stationary vibration data produced during a space vehicle launch can be closely approximated by separate time and frequency averaging procedures. This approach to the analysis of such data allows the estimation of spectral density functions with a much smaller combination of bias and random errors. Although developed for Space Shuttle applications, this same procedure, with appropriately modified averaging times, should apply to the analysis of the launch vibration data for expendable launch vehicles as well.

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APPENDIX

Figures A1-A3. 1/3 octave band spectra for OSTA-1 payload vibration mea-
surement (Accel 248) during Space Shuttle launch (STS-2).

Units: Mean square acceleration (MS Accel) in g^2 (g**2).

Averaging Time: 1 sec for lift-off and transonic flight; 3 sec for
max "q" flight (T+x is end of averaging interval).

Figures A4-A6. Overall values and normalized 1/3 octave band spectra for
OSTA-1 payload vibration measurement (Accel 248).

Units: Overall values - Mean square acceleration (MS Accel) in g^2
(g**2).
Normalized spectra - normalized mean square acceleration
(NMS Accel) relative to the overall
(OA) in linear units and dB.

Averaging Time: 1 sec for lift-off and transonic flight; 3 sec for
max "q" flight (T+x is end of averaging interval).
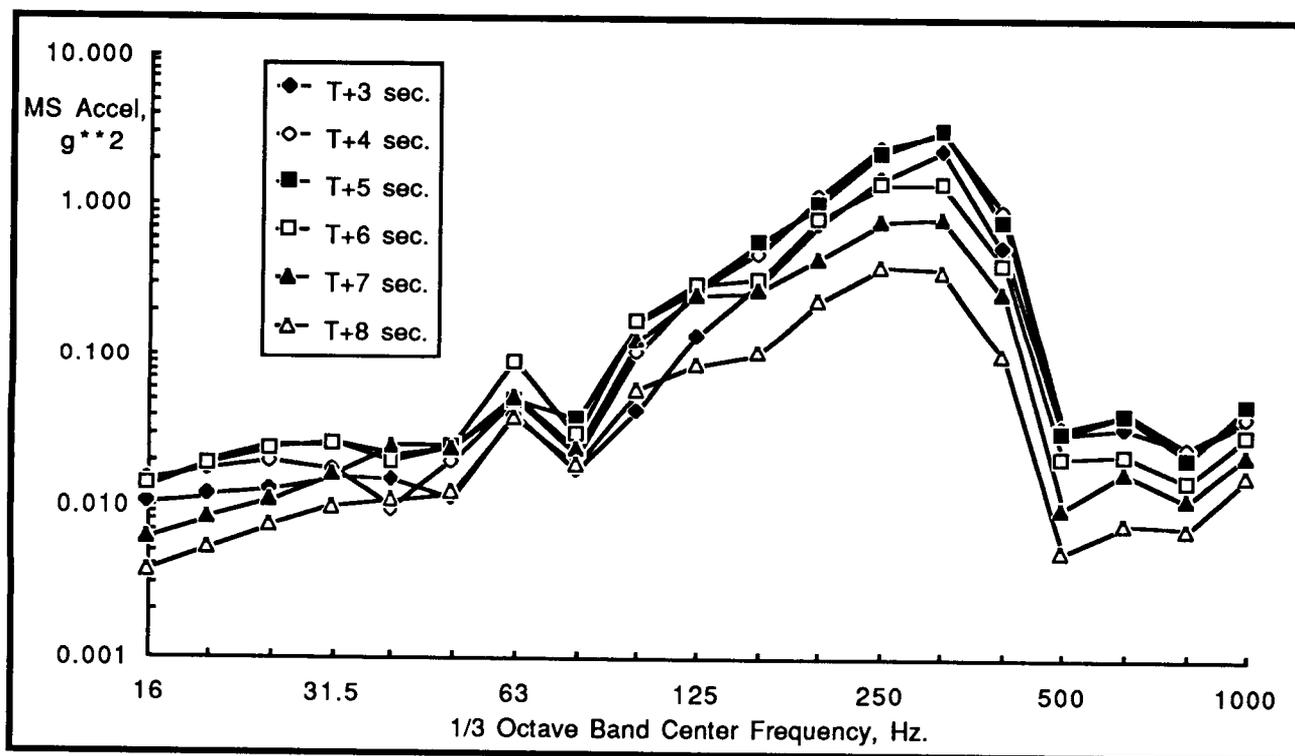
Figure A1. 1/3 Octave Band Vibration Levels During Space Shuttle
Lift-Off; STS-2 Accel 248.
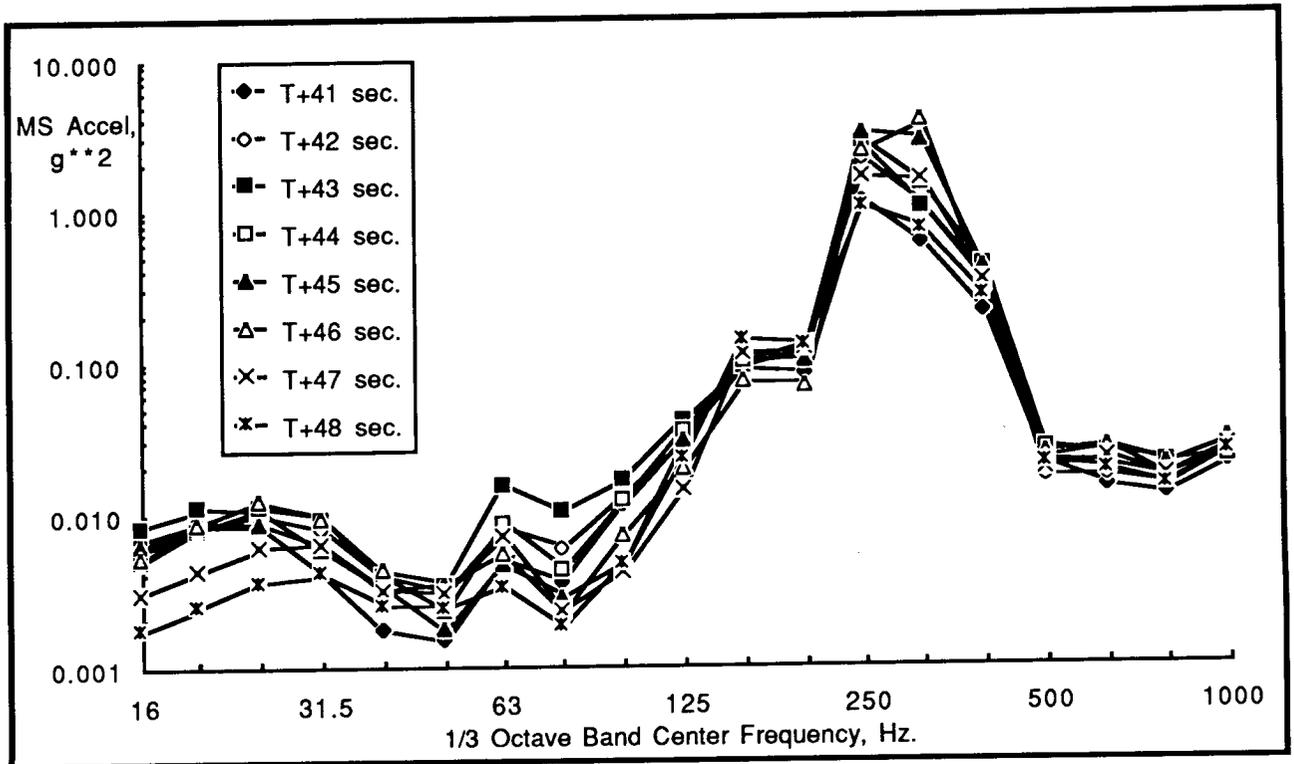
Figure A2. 1/3 Octave Band Vibration Levels During Space Shuttle Transonic Flight; STS-2 Accel 248.
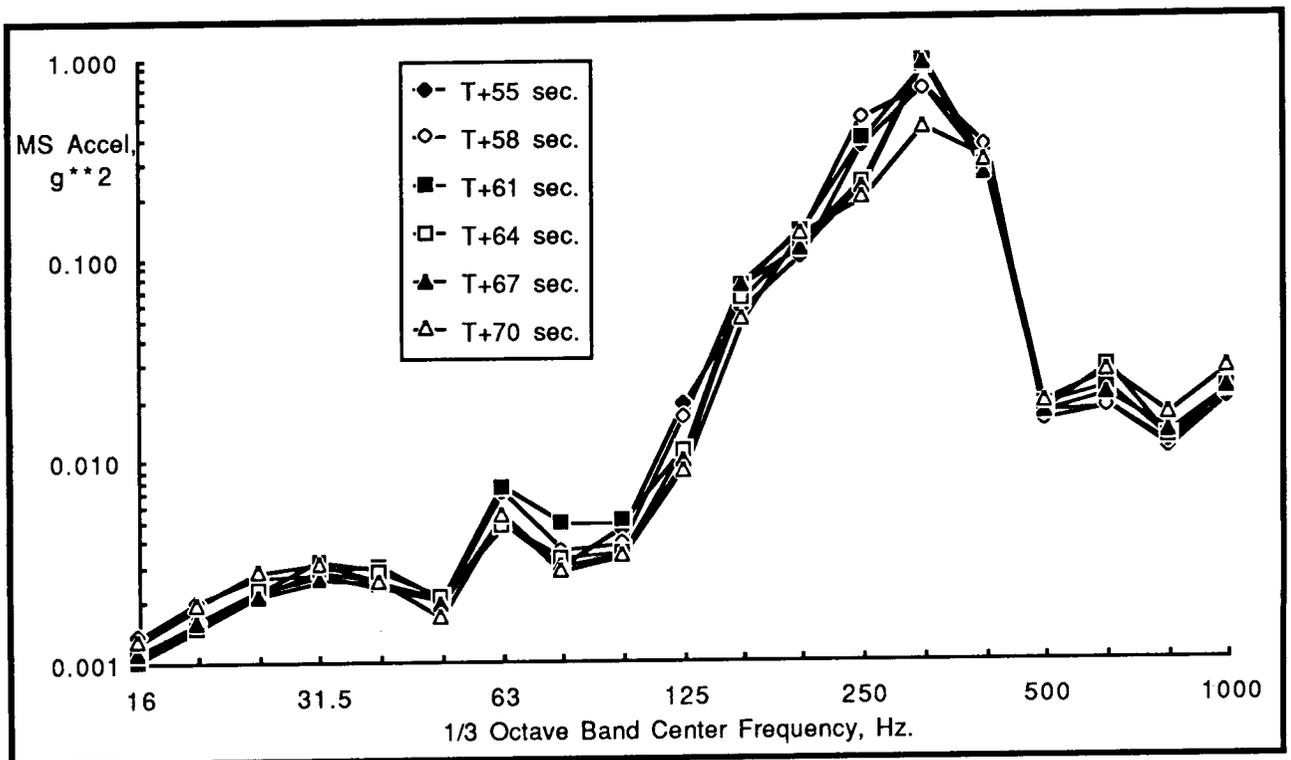
Figure A3. 1/3 Octave Band Vibration Levels During Space Shuttle Max "q" Flight; STS-2 Accel 248.

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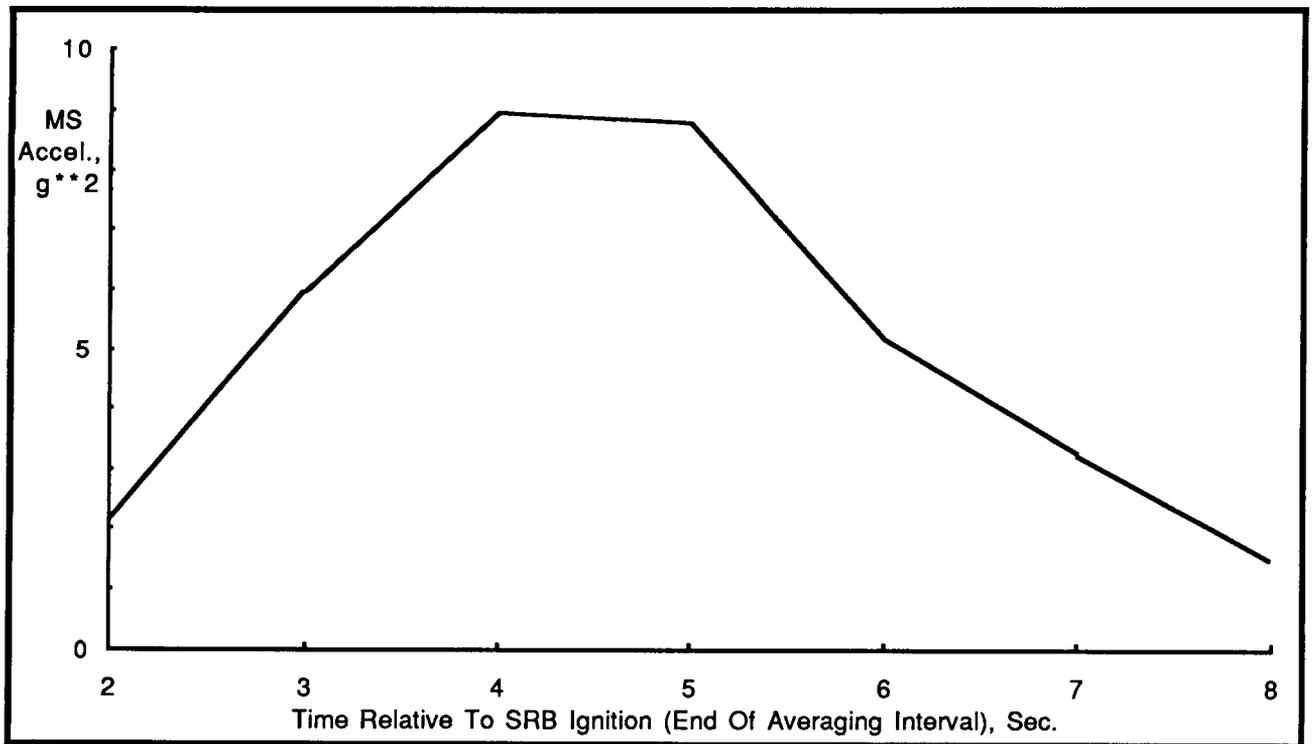
Figure A4a. Overall MS Vibration Level During Space Shuttle Lift-Off; STS-2 Accel 248.
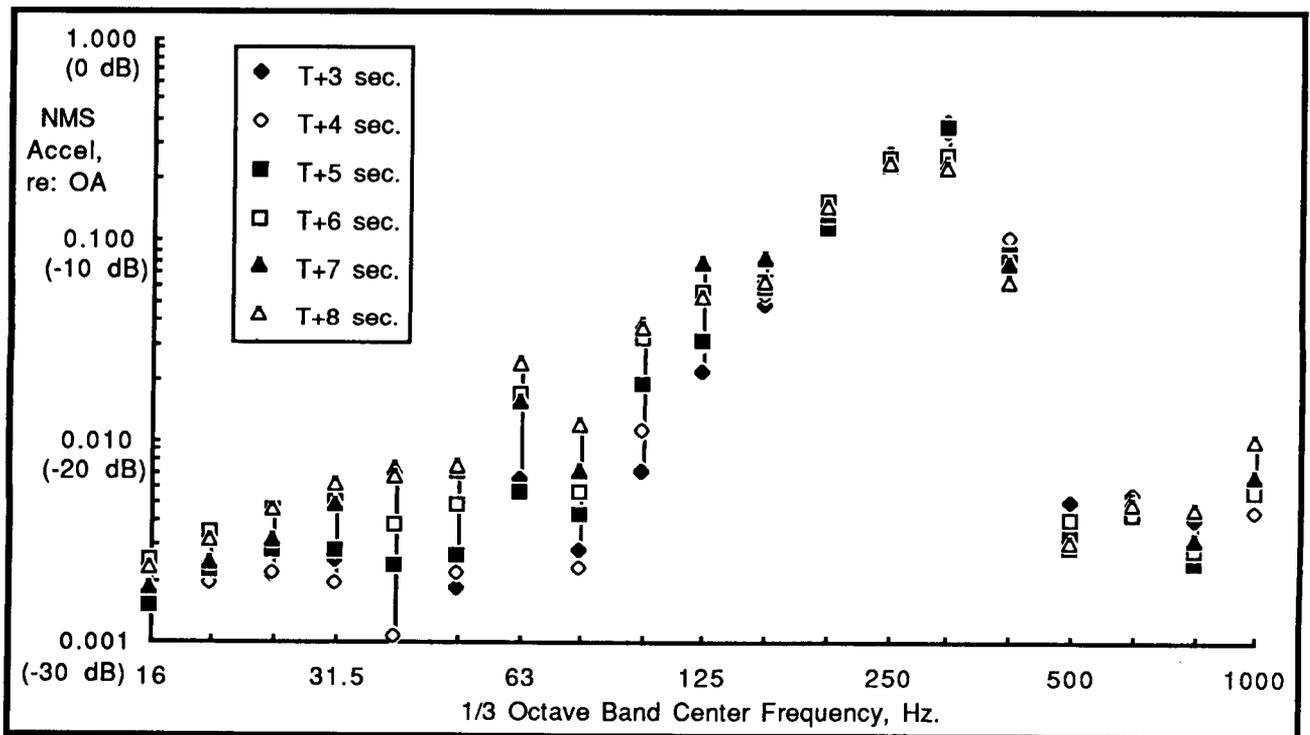
Figure A4b. 1/3 Octave Band Spectra Of Normalized MS Vibration Levels During Space Shuttle Lift-Off; STS-2 Accel 248.
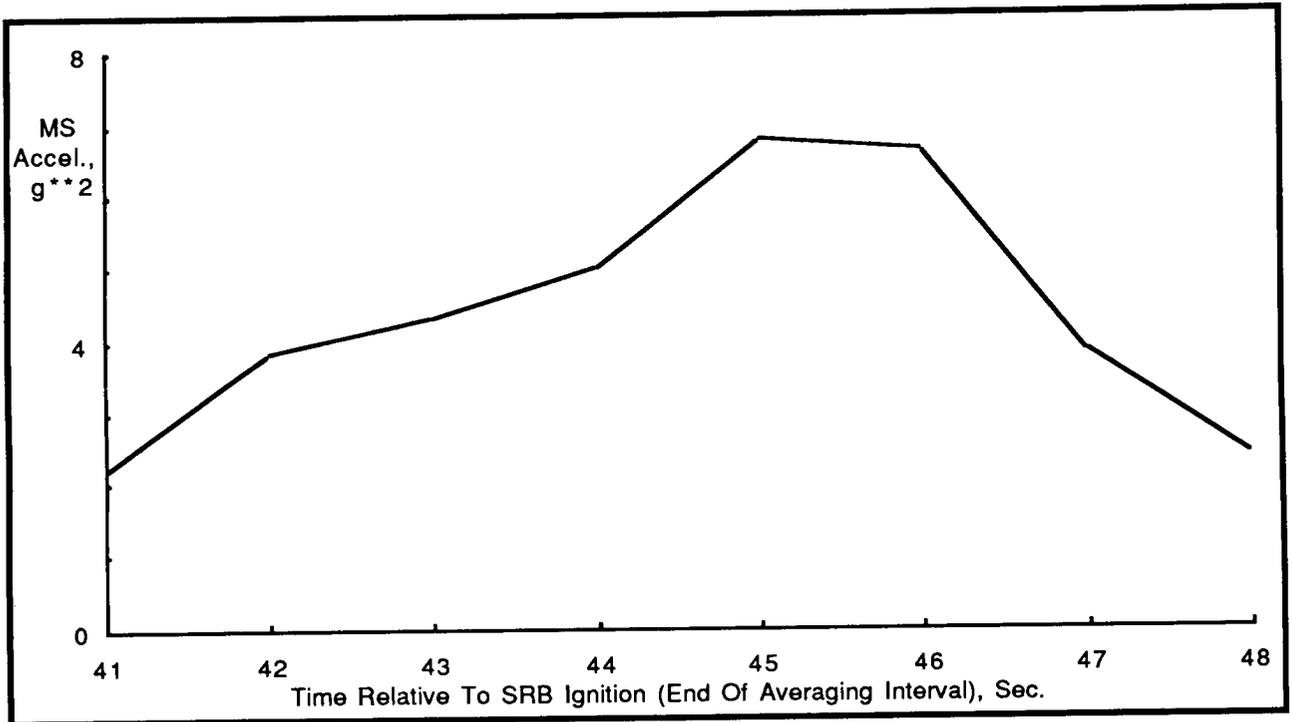
Figure A5a. Overall MS Vibration Level During Space Shuttle Transonic Flight; STS-2 Accel 248.
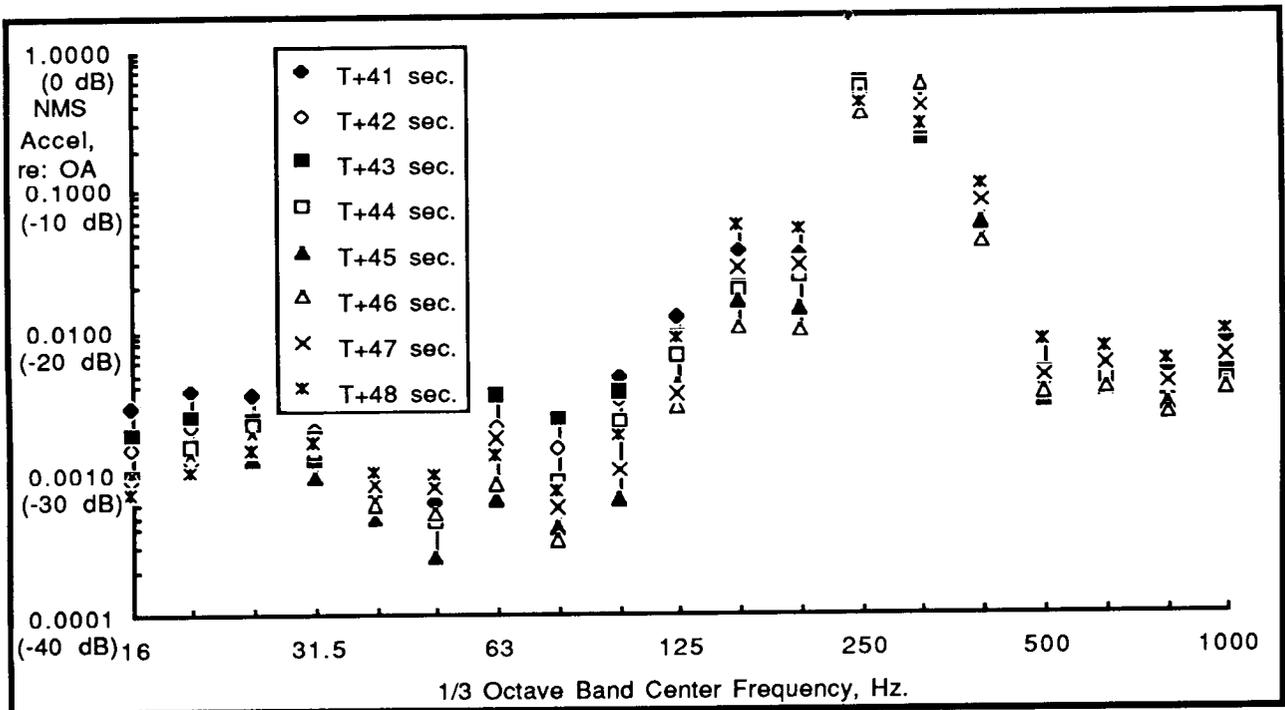
Figure A5b. 1/3 Octave Band Spectra Of Normalized MS Vibration Levels During Space Shuttle Transonic Flight; STS-2 Accel 248.
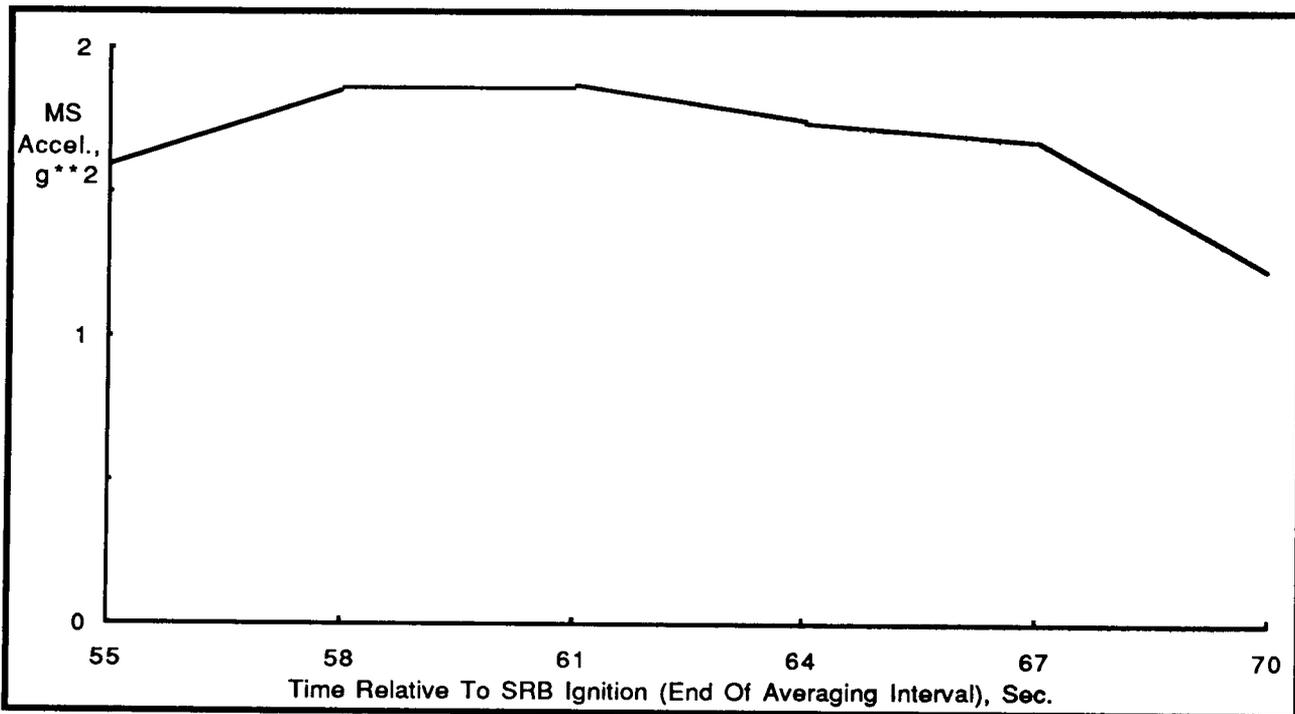
Figure A6a. Overall MS Vibration Level During Space Shuttle Max "q" Flight; STS-2 Accel 248.
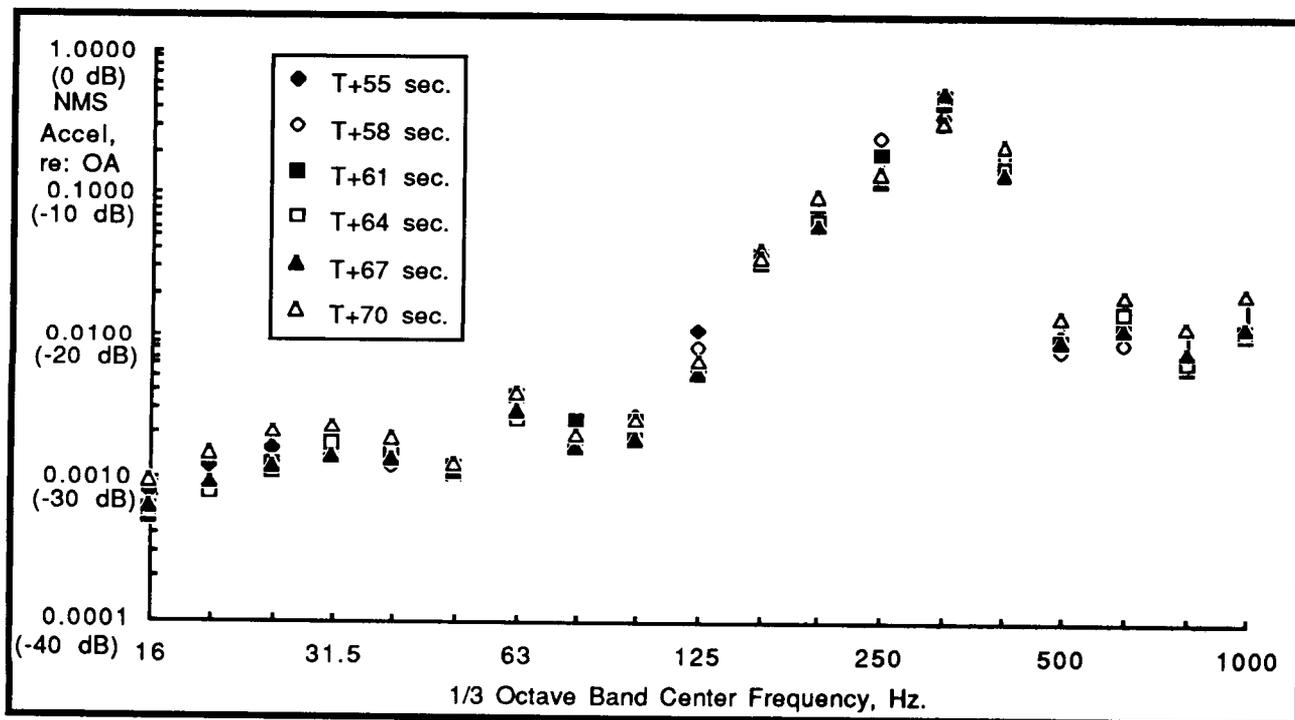
Figure A6b. 1/3 Octave Band Spectra Of Normalized MS Vibration Levels During Space Shuttle Max "q" Flight; STS-2 Accel 248.