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Analysis of Photopole Data Reduction Models

James B. Cheek

An estimate of the total impulse obtained from a buried explosive charge can be calculated from displacement versus time points taken from successive film frames of high-speed motion pictures of the explosive event. The indicator of that motion is a pole and baseplate ("photopole"), which is placed on or within the soil overburden. This paper is concerned with the precision of the impulse calculation and ways to improve that precision. Typically, a general cubic power series in time is fitted to the deflection versus time data points that describe the explosive-induced motion of photopoles. The resulting equation fails to meet the initial conditions of the actual test. This paper examines the effect of each initial condition on the curve-fitting process and shows that the zero initial velocity criteria should not be applied due to the linear acceleration versus time character of the cubic power series. It points out the role of the nonzero initial velocity in helping the linear model deal with the effects of the highly nonlinear pressure versus time conditions in the explosive test bed. Last, this paper illustrates the applicability of the new method to photopole data records whose early-time motions are obscured. It describes how and why the early-time data serve to degrade the data fit in the region of the maximum velocity as does any constraint on initial conditions. It concludes that future photopole data processing must not include the early-time data points, that constraints should not be applied to the initial conditions modeled by the data fit, and that the photopole data should include points well beyond the explosive cavity venting time.

INTRODUCTION

This report is about processing the data obtained from high-speed photographs of "photopoles." For those unfamiliar with the use of photopoles to estimate the

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total impulse produced by a buried explosive, the following overview is provided. Photopoles typically consist of a length of pipe (several feet long) attached to a circular baseplate. The pole is placed on or within the soil (overburden) that covers the explosive charge and the specimen being tested. Each pole is painted a background color. A contrasting colored horizontal band is painted around the vertical pole. This band serves as a witness mark for subsequent measurements. When the explosive charge is initiated (zero time), the photopole and the mass of soil beneath its baseplate are assumed to be lifted as a unit by the force of the blast. Prior to the test event, several 1,000 frames per second motion-picture cameras are sited so as to have the photopoles positioned near the bottom of the cameras' field of view. The high-speed photographs taken by each camera provide a position versus time history for each pole. That record covers the time period from before zero time and continuing until the pole is out of the cameras' field of view or is obscured by flash, smoke and/or dust.

Using a photographic film reader, measurements are made on each film frame of the vertical position of each pole with respect to a single reference point whose elevation remains essentially constant during test event. These data, together with a scale factor to convert the film measurements to engineering units, are processed to produce a deflection (S) versus time (t) history for each pole. Those data are further processed to obtain the maximum velocity of the photopole (V_m). With that value, the impulse is calculated as the product of the sum of the soil and photopole mass times V_m.

The best way to estimate the soil mass is by no means agreed upon by all researchers. Nevertheless, once the mass value is established, the impulse calculation depends entirely on the value calculated for V_m. For that reason, the dependability and the accuracy of the data-processing procedure are very important issues in the overall testing process. Those issues are the focus of the remainder of this paper.

CALCULATING THE MAXIMUM VELOCITY, CURRENT PROCEDURE

The first step in calculating V_m is a curve-fitting process. In order to fit a curve to data, a function form must first be chosen. Past experience indicates that the displacement versus time history of photopoles is satisfactorily modeled by the function:

$$S = At^3 + Bt^2 + Ct + D \tag{1}$$

Using Equation 1, the maximum velocity is at the time when $\frac{d^2S}{dt^2}$ is zero. Thus:

$$V_m = C - \frac{B^2}{3A} \tag{2}$$

Current practice is to estimate the time at which the pole first begins to move (lift-off time, T) by examining the plot of the S versus t as shown on Figure 1. Points prior to T are deleted and the values for the constants A, B, C, and D are then determined by the least squares curve-fitting procedure. The points and fitted curve are shown in Figure 2.

The procedure just described has been used for several years. The calculated impulse values agree within the limits of experimental error with the results obtained by other means. However, there are a few bothersome details relating to

the photopole's initial conditions. Specifically, the nature of a least squares fit to a data set assures that the coefficients obtained result in the best fit of the chosen function to the data. Since the graph of that function may not pass through some of the data points, this leads to the potential for violations of known initial conditions. Thus, the equation may have a nonzero deflection at zero time, wrongly indicating that the photopole moved before the test began. The slope of the equation at zero time may be nonzero, again, wrongly indicating that the pole was moving when the test began. Finally, when the lift-off is not at zero time, the velocity may also be nonzero. This is also incorrect. While all those errors in the initial condition specification are part of the current procedure, it is important to note that they appear not to degrade the process of finding V_m. How can this be so? How can one ignore known initial conditions?

One answer to those questions is that the actual task is not to fit a curve to all of the data. Instead the task is to get a good fit to the data on each side of the point in time at which V_m is developed. Then V_m is calculated from the curve fitted to that region alone. One might conclude that the early-time data should be deleted from the curve fit since only the maximum velocity region is important. The need to examine the merit of that conclusion presented itself when a test failed to produce any early-time photopole data. Efforts to obtain a good fit to those incomplete data sets required a detailed analysis of the role of initial conditions in the curve-fitting process.

MODELING THE INITIAL CONDITIONS

A study of the high-speed photographs shows that the lift-off time appears to occur several milliseconds after zero time, see Figure 1. As noted previously, this forces the user to choose T and delete points prior to that time. This choice is not always an easy one to make. Consequently, it was decided to let the program select the best value for T. In order to meet the S = 0 at t = T condition, Equation 1 was transformed from S as a function of t, to S as a function of U, where U is zero at lift-off time. This results in U = t-T, dU = dt, and $V = \frac{dS}{dt} = \frac{dS}{dU}$, which when substituted into Equation 1 gives:

$$S = AU^3 + BU^2 + CU \tag{3}$$

That expression forces S to zero when U is zero, thereby meeting one of the two initial conditions. The second condition is met by differentiating Equation 3 with respect to U, which gives:

$$V = 3AU^2 + 2\,BU + C \tag{4}$$

Since V is zero when U is zero, C is zero and that allows Equation 3 to be written as:

$$S = AU^3 + BU^2 \tag{5}$$

This displacement versus shifted time expression meets both initial conditions.

Unfortunately, Equations 5 and 3 are in terms of three and four unknowns, respectively. This is apparent when they are expressed in terms of t as:

$$S = A(t-T)^3 + B(t-T)^2 + C(t-T) \tag{6}$$

$$S = A(t-T)^3 + B(t-T)^2 \tag{7}$$

The problem with those equations is that the coefficients and T parameter cannot be obtained directly by the least squares procedure. In order to overcome that difficulty, a minimum search program was used to find the T value that yields the best of the "best fits." However, T can be calculated directly from a fit with Equation 1 by setting S to zero and solving the resulting cubic equation for t (which is T since t = T when S = 0).

INITIAL CONDITIONS FOR MISSING EARLY-TIME DATA CALCULATIONS

In order to test the effects of the initial conditions on the curve-fitting process, all combinations of constraints on initial velocity and T were tried. However, the requirement that deflection be zero at T was applied to all calculations. During this study, quite a few runs were made using the four initial conditions with data sets having various combinations of total number of points and time of first point. The results presented in Figures 3 through 6 are typical of those obtained for each initial condition during the study. Data for the four examples presented are taken from the original data set, Figure 1, with the time of the first point set to ten milliseconds (data prior to that time are ignored).

The results shown in Figure 3 are with lift-off at zero time and lift-off velocity constrained to zero. It was expected that those results would be the best because the specified initial conditions agree with the known conditions in the test. They are not. The curve fit to the data points both in the early- and the late-time regions is unsatisfactory. From this we conclude that either the lift-off at zero time or the zero velocity at lift-off constraint is improper.

With that in mind we look at calculations wherein the velocity at lift-off is unconstrained while lift-off is at zero time. Those results, shown in Figure 4, show a slightly better fit of the curve to the data.

The third example, Figure 5, presents the results of forcing lift-off velocity to zero and allowing the program to find the T value that best fits the data. We expected the lift-off to take place a few milliseconds after zero time, thereby agreeing with the observation of the photopole performance. Such is not the case; T is negative.

Since constraining either the velocity at lift-off or the lift-off time produces undesirable effects, the results of the unconstrained case are presented in Figure 6. We find, much to our liking, that T is positive and close to the time indicated by the original data, Figure 1.

DELETING EARLY-TIME DATA

Since the curve fit shown in Figure 6 is better than the fit to the time shifted data shown in Figure 2, we wonder just how many points should be deleted in order to get the best results. For this test, the criteria for goodness of fit is the smallest error mean square. This value (labeled EMS on each plot) is computed as:

$$EMS = \frac{1}{N} \sum_{i=1}^{N} [S_i - f(t_i)]^2 \qquad (8)$$

where t_i, S_i are the coordinates of the i^{th} data point, $f(t_i)$ is the value of the equation at the time coordinate of the data point, and N is the number of data

points. Note that the number of points need not be the same for each data set in order to use these criteria.

Also in question is how does V_m change as the early-time data points are deleted. Those points are addressed in the two plots shown in Figure 7. The curves were produced by making a series of curve fits. First, to all of the data points, then all but the first point, then all but the first two points and so on until only a few data points remain. Plotting the EMS and V_m as a function of the time of the first point produces the curves shown. In all cases tested, the curves show the slight decline in V_m and a rapid decline in EMS out to 5 to 10 milliseconds. From that point on, both V_m and EMS hold almost constant values until the EMS and/or V_m begin to show sensitivity to removing a single point (the points in this data set are at intervals of 1 millisecond). This sensitivity is due to the decrease in the total number of points in the data set and the fact that points are being removed from the region that defines V_m.

LATE-TIME POINTS

The need for points well beyond the time (T_m) at which V_m occurs is illustrated in Figure 8. The original data set was used for this test, except all points beyond 55 milliseconds were deleted. Here we see the degrading in the fit because of the poor definition of the region beyond T_m. As mentioned previously, dust and the pole's motion limit the total number of points that can be obtained. Nevertheless, this series of calculations well illustrates the importance of those late-time points and justifies the extra effort expended in obtaining them.

ANALYSIS AND CONCLUSIONS

It may seem trite to observe, "When one chooses a model, one also takes the first step in reducing the accuracy and precision with which we model the effects in question." However, grasping the implications of that observation is critical to understanding the seemingly strange results produced during this series of computational experiments. At this point, a close examination of strongly held beliefs regarding the role and importance of initial conditions is in order. As the results demonstrate, forcing the fit to meet the known initial conditions produces the least satisfactory results. Why?

Responding to that question, we look to the general form of the displacement versus time model, Equation 1. From that model we extract the underlying acceleration (a) model:

$$a = 6At + 2B \tag{9}$$

That model describes the photopole motion as being the result of an acceleration that is a linear function of time. On the other hand, our knowledge of the explosive test environment tells us that the pressure on the soil mass beneath the photopole's baseplate is highly nonlinear. From its initial peak value, the pressure declines exponentially and falls to zero shortly after venting. Assuming constant pole-soil mass and base area, this means that the actual acceleration of the photopole is highly nonlinear in the early time portion of the test. However, as the pressure declines during the late time portion of the test, the exponential decay curve has only a slight nonlinearity. Thus, the acceleration experienced by the pole is more or less linear as the blast-induced acceleration matches the gravitational acceleration at t_M. This is why the model produces a good fit in the maximum velocity region. But why a negative T value as shown in Figure 5?

The model is doing what we told it to do, and in so doing, it is "talking" to us. It says, "If you force acceleration to be linear and the initial velocity and displacement to be zero, then the photopole must get an early start in order to gain enough momentum to best fit the data in the maximum velocity region." As first thought, this seems too much to ask of us. How can we accept initial conditions and a model that produce pole movement before the blast is initiated?

We can and we must accept the conditions if we accept the model. While it is silly to say that the photopole actually moved before the blast was initiated, it is equally silly to say that the initial velocity was not zero (as is done in the current curve-fitting procedure). In either situation, we are in effect helping our linear model deal with the nonlinear part of its forcing function by relaxing the constraints on the initial conditions. Purely from the standpoint of modeling the data to obtain the maximum velocity in the (almost) linear acceleration region, one must view the negative lift-off time as reasonable, provided the initial velocity must be zero. In like manner, the nonzero initial velocity must be accepted. For the complete relaxation of all constraints, the nonzero displacement at zero time must be allowed. Thus, the concept of valid initial conditions at zero time has no meaning in this sense. Only when the task is to accurately describe the entire displacement versus time function must we focus on initial conditions.

In the light of the above discussion, one is pressed to conclude that the early-time data points serve no useful purpose. Instead of improving the data-fitting process, they poison it.

As mentioned previously, the pressure versus time relationship is almost linear at t_M, which is to imply a low pressure level. But how low? Since the blast pressure is a decreasing function of time, the photopole will reach its maximum velocity when the air drag force and the force induced on the pole-soil mass by the gravitational acceleration are equal to the force induced by the effective pressure on the pole's base area. For a typical pole-soil mass, the force balance pressure is around 1.5 pounds per square inch. One may therefore conclude that t_M will not be 1 or 2 milliseconds after cavity venting, but quite a few more milliseconds later. This further substantiates the need for late-time data points.

RECOMMENDATIONS

Using a linear-in-time acceleration model on photopole data forces us to restrict its application to deflection versus time data taken somewhat before and after the time at which the maximum velocity is anticipated. The specification of the pole's initial conditions at $t = 0$ has no meaning in this application. If those conditions are forced, the fit tends to degrade the maximum velocity result. Consequently, this study recommends that we ignore the initial conditions without having bad feelings about the failure to model every aspect of the actual situation.

Beyond being comforted, this study recommends that the best maximum velocity results are produced from photopole data having no definition of the early-time deflections, provided enough late-time data points are included to model the photopole's decrease in velocity from its maximum.

CAUTIONS ON MODELING THE COMPLETE DEFLECTION VERSUS TIME HISTORY

There is a growing interest in being able to model the entire deflection versus time history of photopole data. Here, the early-time points become important as do some, if not all, of the initial conditions. However, using a cubic power series to

model all of the initial conditions and subsequent motion in the early-time data region is nothing less than wrong. Efforts are under way to apply proper models to the several regions of the photopole data that exhibit unique effects and properly link those models in the transition zones.

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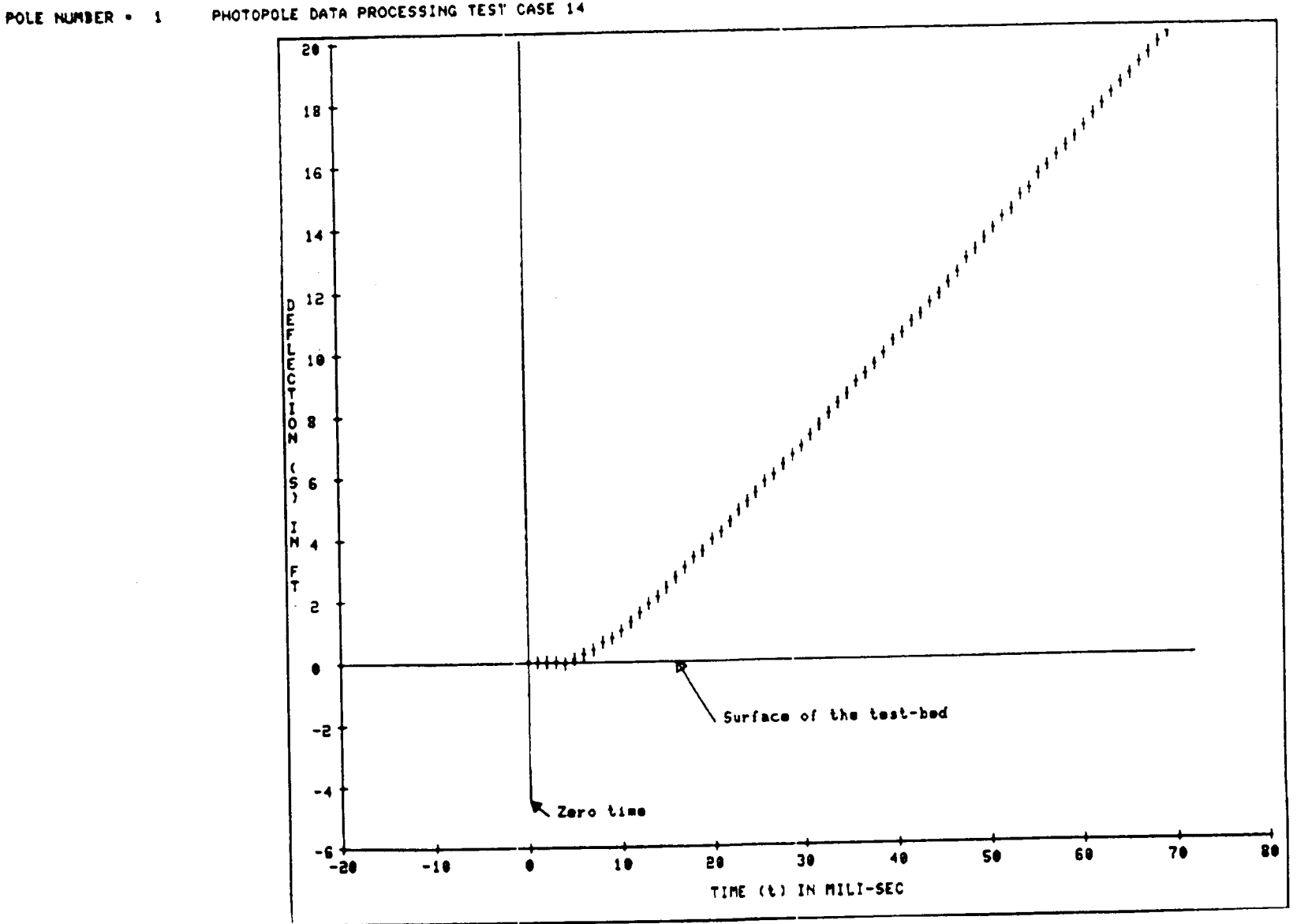

Figure 1. Original data.

PROG JPP3Z: S = A * U**3 + B * U**2 + C * U; U = t - T

POLE NUMBER = 1 PHOTOPOLE DATA PROCESSING TEST CASE 14

Lift-off time = T = 0.

EMS= 1.229888E-02

A= -2.480953E-05
B= 3.221560E-03
C= 1.973052E-01

INITIAL CONDITIONS:
T = 0.0
S(T) = 0.0
V(T) IS UNCONSTRAINED

TIME AT VMAX = 43.28
VMAX = 336.75

USER SHIFTED DATA

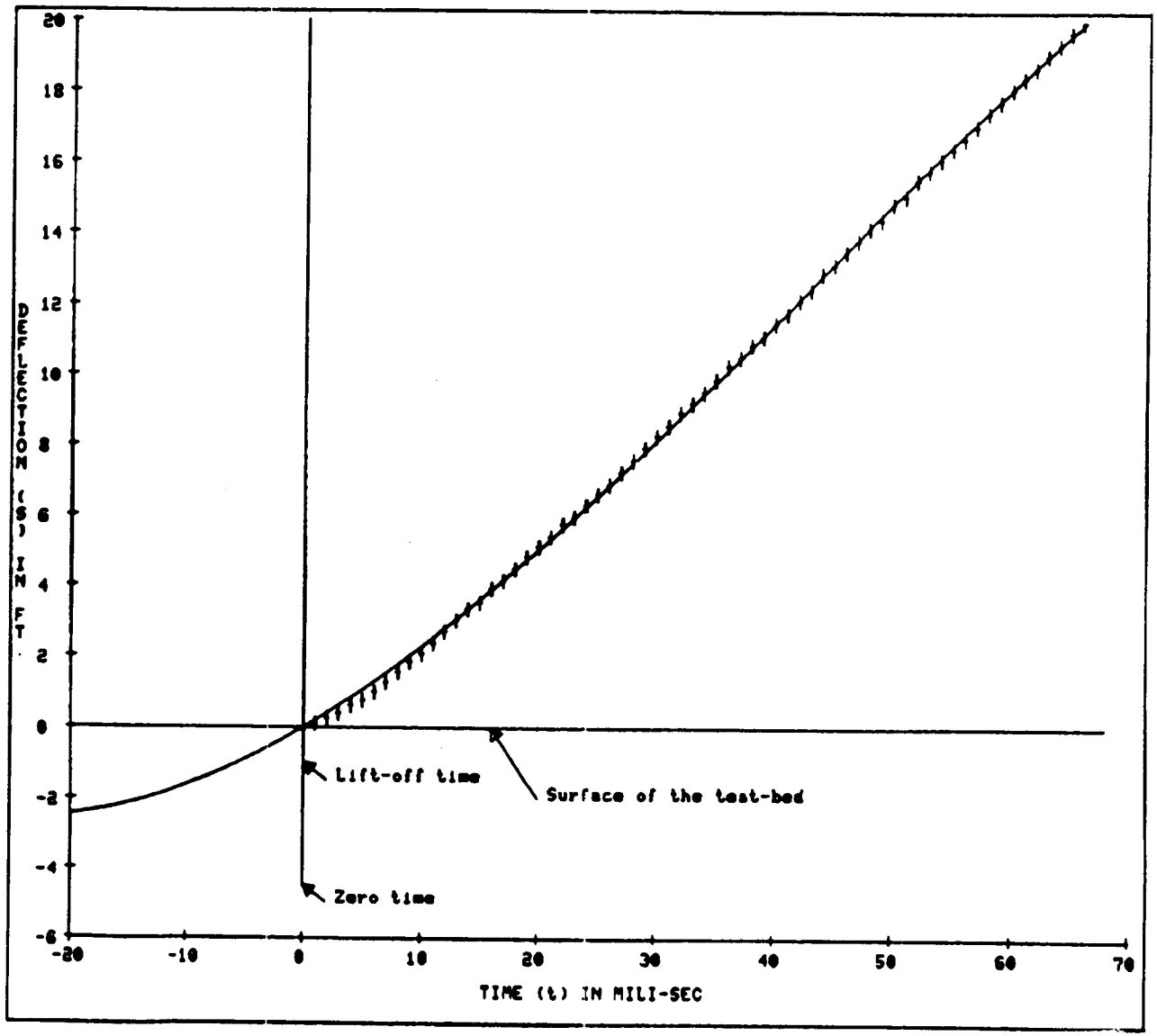

Figure 2. Time shifted data.

PROG JPPFZ, S = A * U**3 + B * U**2, U = t - T

POLE NUMBER = 1 PHOTOPOLE DATA PROCESSING TEST CASE 14

Lift-off time = T = 0

EMS= 2.257489E-01

A= -7.943992E-05
B= 9.525614E-03

INITIAL CONDITIONS
T = 0.0
S(T) = 0.0
V(T) = 0.0

TIME AT VMAX = 39.97
VMAX = 380.74

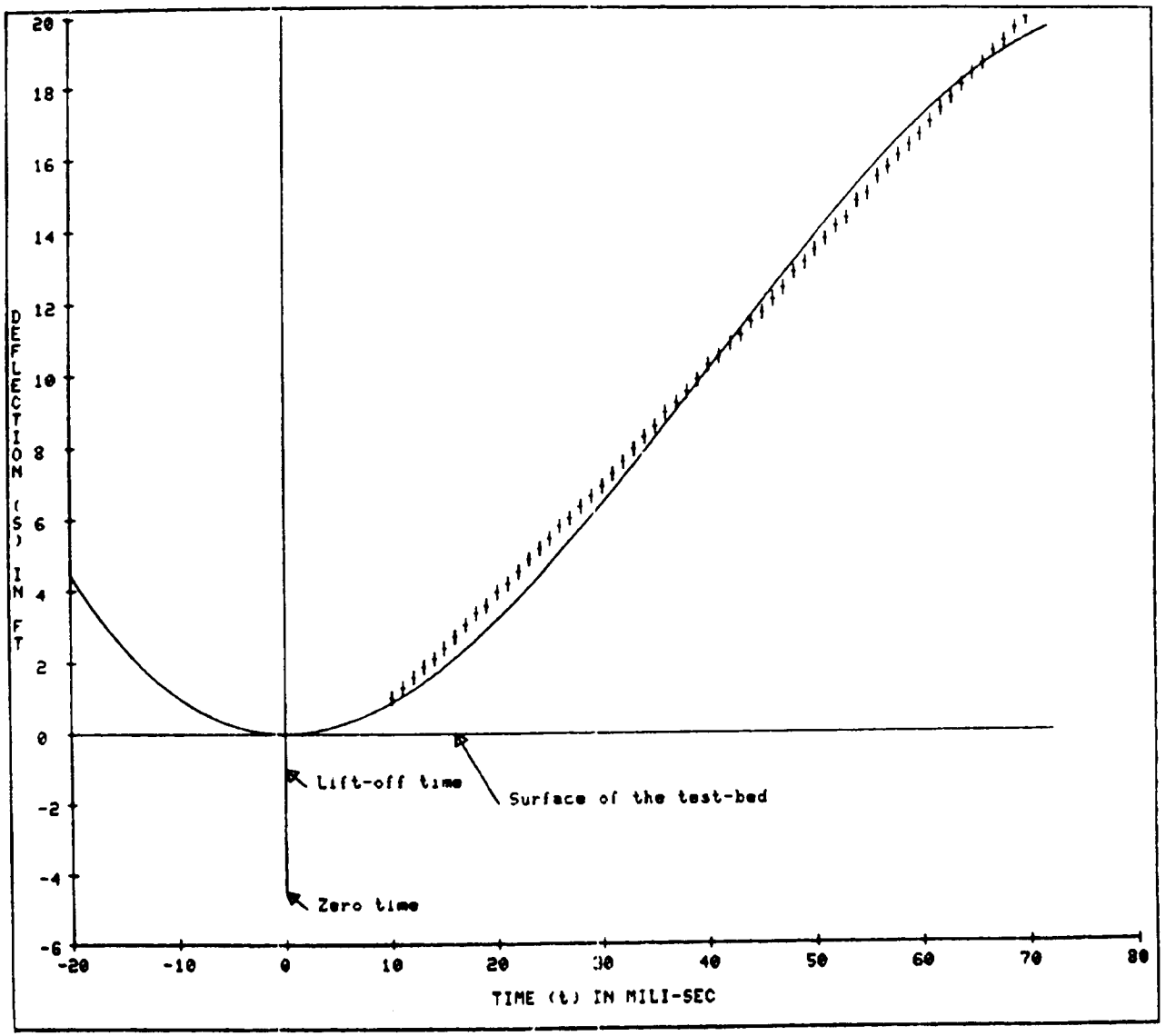

Figure 3. Initial conditions satisfied.

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PROG  JPP3Z1    S = A * U**3 + B * U**2 + C * U,    U = t - T
POLE NUMBER =  1      PHOTOPOLE DATA PROCESSING TEST CASE 14
Lift-off time = T =  0
EMS=    2 705044E-02
  A=   -3 732499E-05
  B=    5 157079E-03
  C=    1 058439E-01
INITIAL CONDITIONS
 T = 0.0
 S(T) = 0 0
 V(T) IS UNCONSTRAINED
TIME AT VMAX =    46 06
       VMAX =   343.36
```

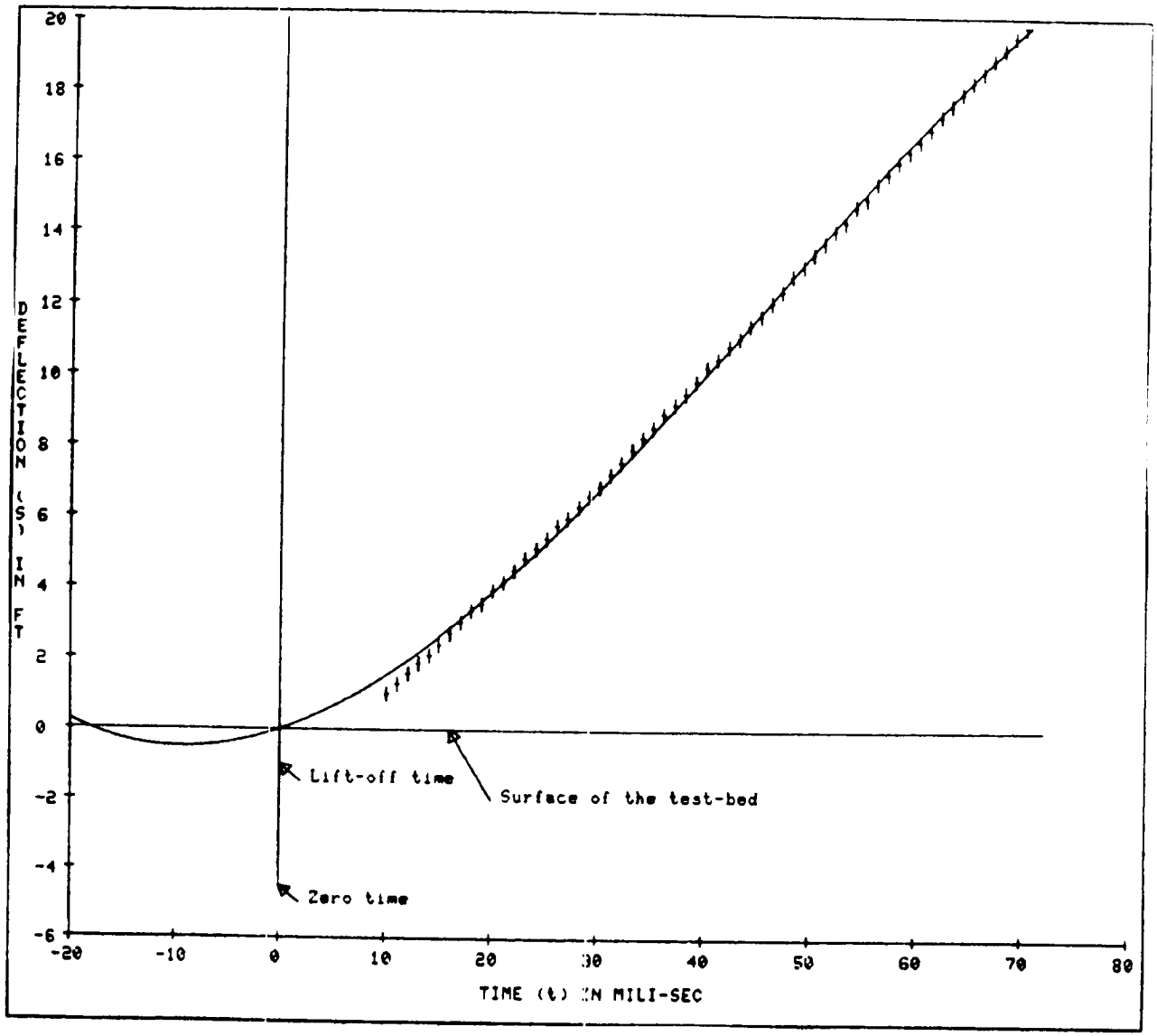

Figure 4. Nonzero initial velocity.

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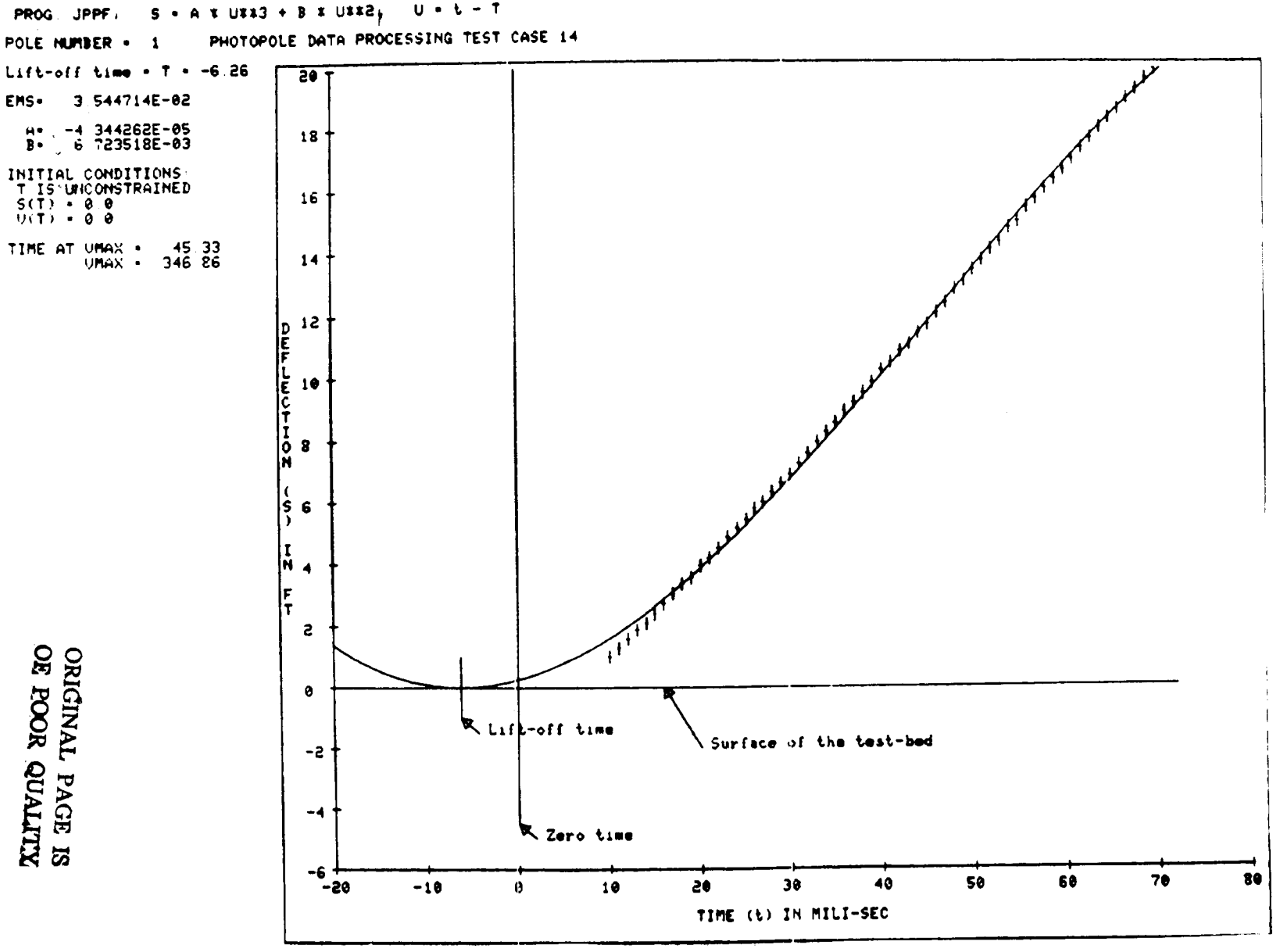

Figure 5. Volocity zero at lift-off, best lift-off time.

PROG JPP3, S = A * U**3 + B * U**2 + C * U, U = t - T

POLE NUMBER = 1 PHOTOPOLE DATA PROCESSING TEST CASE 14

Lift-off time = T = 6.45

EMS= 1.454138E-03

A= -7.704468E-06
B= 1.135611E-03
C= 2.737727E-01

INITIAL CONDITIONS:
T IS UNCONSTRAINED
S(T) = 0.0
V(T) IS UNCONSTRAINED

TIME AT VMAX = 55.58
VMAX = 329.57

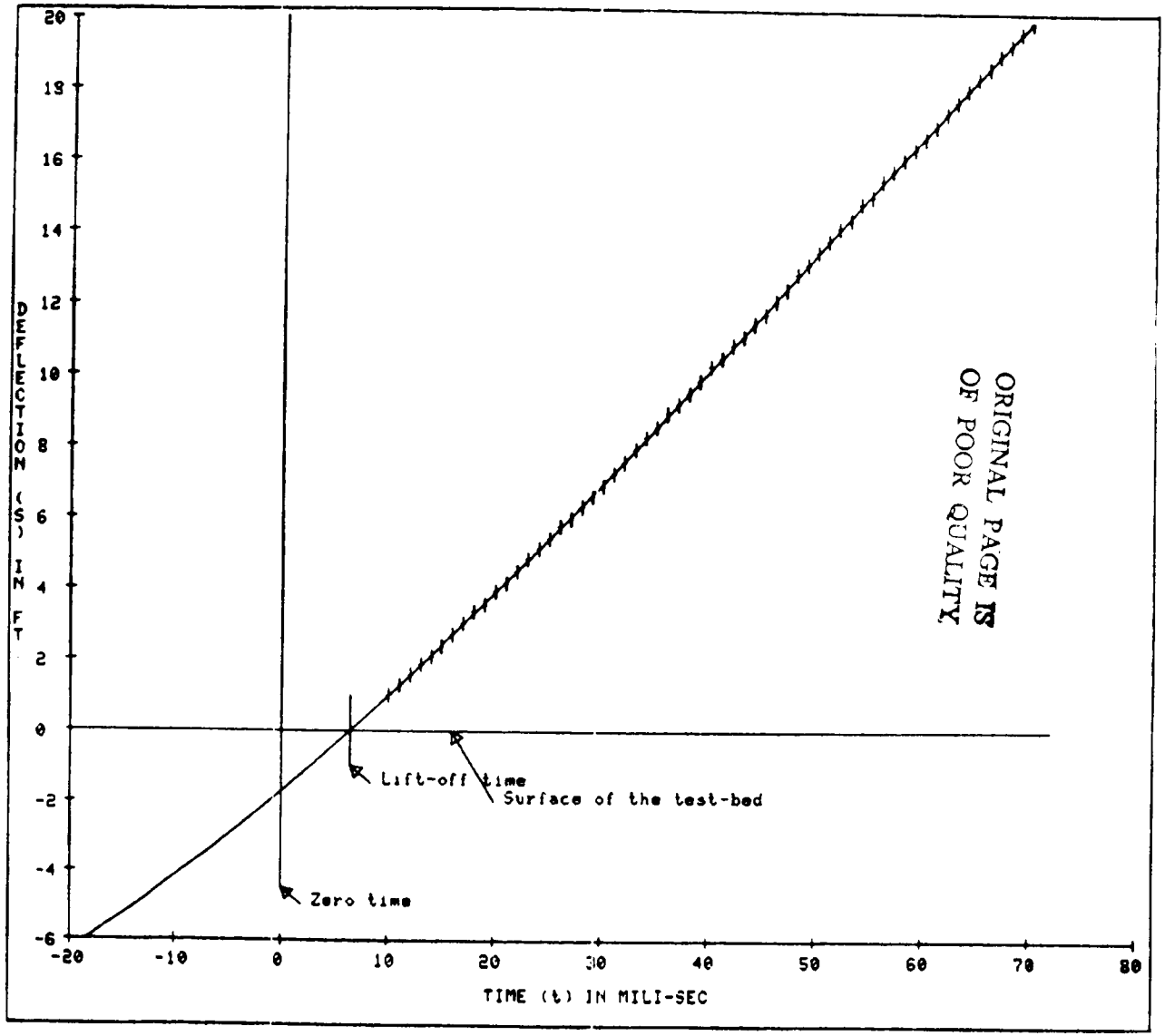

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Figure 6. Best fit to data, no time or velocity constraints.

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PROGRAM JPEMS-S, 29 JUNE 1987
PHOTOPOLE DATA PROCESSING TEST CASE 14 (PPTC-D)
POLE NUMBER 1

Time	EMS	VMX
0.0	0.025036	339.46
1.0	0.018892	337.54
2.0	0.013450	335.75
3.0	0.008803	334.01
4.0	0.005409	332.49
5.0	0.004222	331.80
6.0	0.003147	331.05
7.0	0.002298	330.43
8.0	0.001919	329.88
9.0	0.001488	329.56
10.0	0.001454	329.57
11.0	0.001473	329.63
12.0	0.001488	329.49
13.0	0.001497	329.47
14.0	0.001497	329.44
15.0	0.001508	329.36
16.0	0.001512	329.47
17.0	0.001538	329.61
18.0	0.001558	329.61
19.0	0.001514	329.21
20.0	0.001518	329.62
21.0	0.001508	329.41
22.0	0.001526	329.41
23.0	0.001570	329.42
24.0	0.001535	329.17
25.0	0.001527	328.97
26.0	0.001546	330.28
27.0	0.001401	324.77
28.0	0.001382	312.32
29.0	0.001450	329.15
30.0	0.001450	325.21
31.0	0.001500	324.52
32.0	0.001424	329.38
33.0	0.001541	330.12
34.0	0.001676	330.47
35.0	0.001552	332.30
36.0	0.001652	329.63
37.0	0.001575	326.59
38.0	0.001732	330.39
39.0	0.003419	328.52
40.0	0.002422	331.14
41.0	0.002412	327.43
42.0	0.004751	328.81
43.0	0.005086	329.73
44.0	0.002078	330.28
45.0	0.007424	331.07
46.0	0.005267	328.59
47.0	0.027574	333.84
48.0	0.062165	340.07
49.0	0.076508	335.16
50.0	0.001874	319.21

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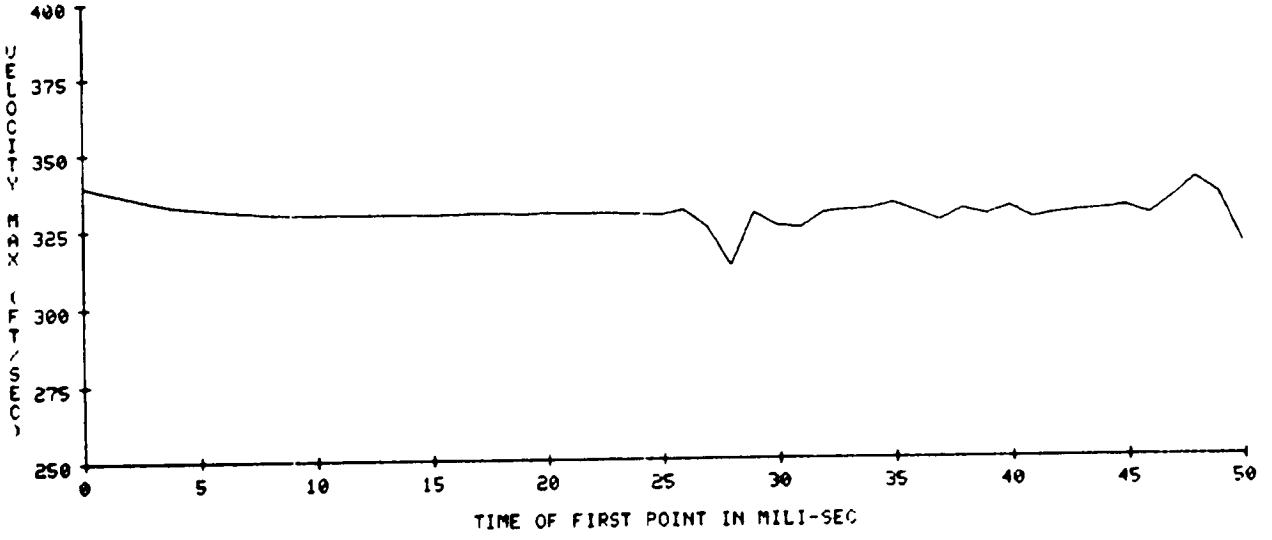

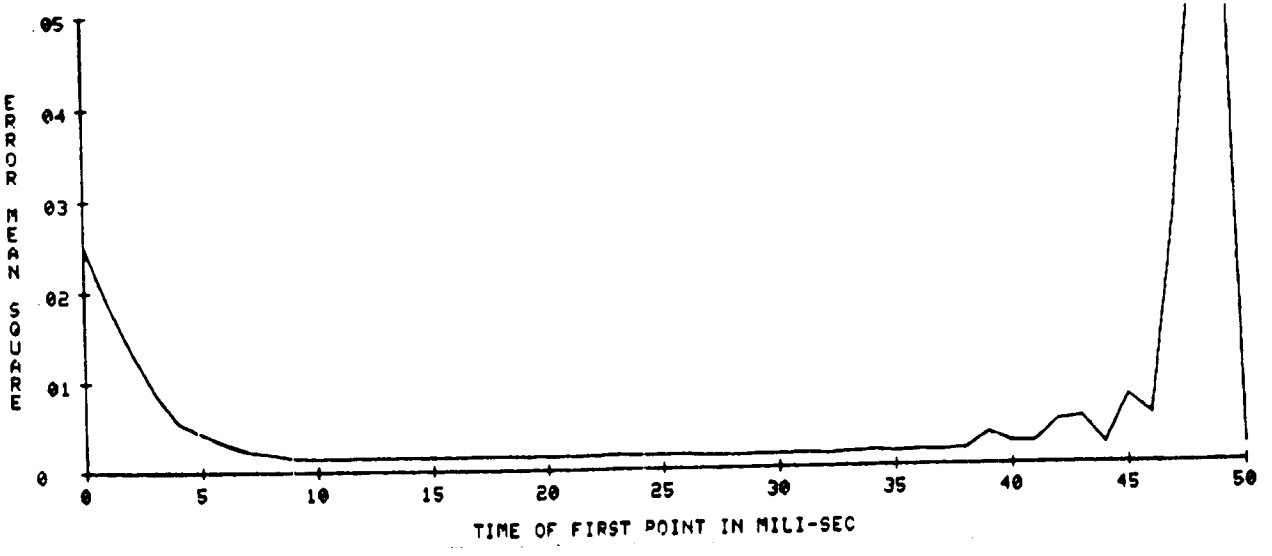

Figure 7. Sensitivity to time of first point, 72 point data set.

PROGRAM JPEMS-S. 29 JUNE 1987
PHOTOPOLE DATA PROCESSING TEST CASE 14 (PPTCS-D)
POLE NUMBER 1

Time	EMS	VMX
0	0.017809	336.99
1.0	0.014130	334.88
2.0	0.010523	332.72
3.0	0.007124	330.62
4.0	0.004473	328.85
5.0	0.003738	328.12
6.0	0.002968	327.25
7.0	0.002295	326.59
8.0	0.002019	326.60
9.0	0.001614	326.63
10.0	0.001615	326.87
11.0	0.001651	326.73
12.0	0.001688	326.89
13.0	0.001721	326.99
14.0	0.001746	328.15
15.0	0.001741	327.21
16.0	0.001719	326.78
17.0	0.001749	326.90
18.0	0.001795	326.73
19.0	0.001793	327.07
20.0	0.001760	327.07
21.0	0.001784	326.86
22.0	0.001796	326.71
23.0	0.001822	327.81
24.0	0.001860	327.65
25.0	0.001870	326.85
26.0	0.001925	326.59
27.0	0.001793	325.87
28.0	0.001717	323.01
29.0	0.001732	322.67
30.0	0.002905	329.70
31.0	0.001795	322.08
32.0	0.003791	328.20
33.0	0.001971	320.33
34.0	0.002647	322.55
35.0	0.032592	319.74
36.0	0.003067	319.03
37.0	0.019408	330.61
38.0	0.004309	317.35
39.0	0.011062	327.15
40.0	0.186084	1019.82
41.0	0.017527	364.12
42.0	0.123292	340.30
43.0	4.362924	289.99
44.0	0.565563	333.34
45.0	4.168564	340.10
46.0	**********	**********
47.0	0407.405880	-946.99
48.0	9.470748	463.26
49.0	**********	801.75

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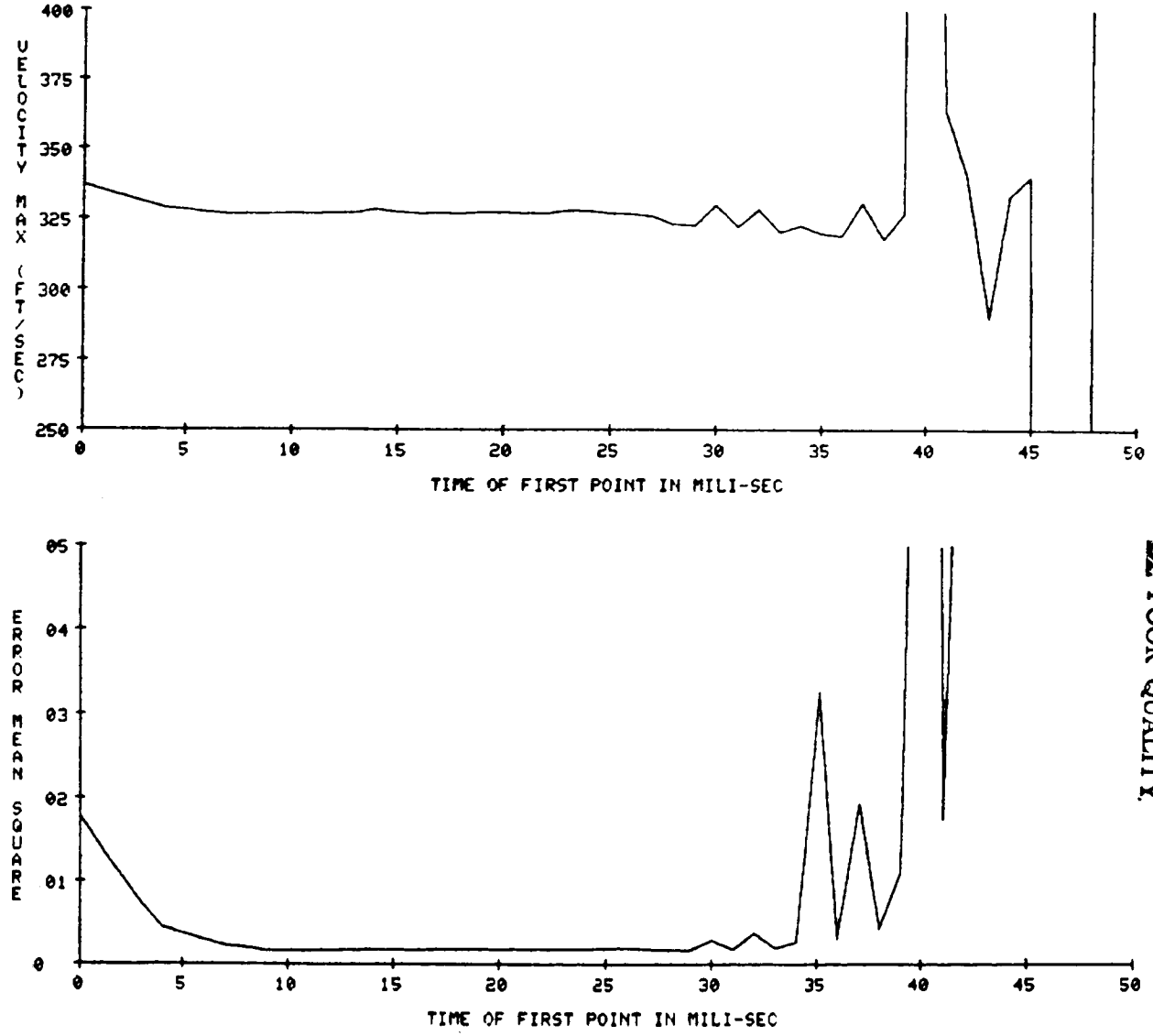

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Figure 8. Sensitivity to time of first point, 55 point data set.