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An Efficient Frequency Response Solution for Nonproportionally Damped Systems

Paul Conti
William K. Rule

Recent advances in computer technology have made it possible to use large finite element models for shock and vibration analyses. One type of dynamic analysis is the calculation of responses and loads in the frequency domain for steady-state operating conditions. If substantial nonproportional damping levels are generated by energy dissipative components, such as bearings or hydraulic cylinders, then the calculation of accurate frequency domain results can be computationally intensive for large models. To reduce the computational effort, it is common to assume that damping is proportional to the mass and stiffness of the system. This proportional damping approximation can lead to significant errors in the frequency domain results. A method has been developed to produce very accurate results for this type of model without the large computational burden of a traditional nonproportional damping analysis.

INTRODUCTION

Recent advances in computer technology have made it possible, but not always practical, to use large finite element models for shock and vibration analyses. At one time, refined models were used only for linear static stress and deflection analyses which are less computationally demanding. Dynamic analyses were generally limited to smaller lumped parameter or coarse finite element models. However, today's more aggressive design goals are promoting lighter weight structures that must operate effectively in higher performance environments. To help achieve these goals, dynamic analyses of refined finite element models are becoming a more accepted part of the design process.

One type of dynamic analysis is the calculation of vibration responses and loads in the frequency domain for steady-state operating conditions. When damping levels are low and energy dissipation is well distributed throughout the system, proportional or modal damping approximations usually produce sufficiently accurate results with a relatively modest computational effort.

On the other hand, if substantial levels of nonproportional damping are generated by energy dissipative components, such as bearings or hydraulic cylinders, then the calculation of accurate frequency domain results can be computationally intensive for large models. To reduce this effort, it is common to assume that damping is proportional to the mass and stiffness of the system. However, in this case, the proportional damping approximation can produce significant errors in the frequency domain results. A method has been developed to calculate very accurate results for this type of model without the large computational burden of a traditional nonproportional damping analysis.

THEORETICAL BACKGROUND

Finite element models can be used to represent the vibration behavior of structural systems in a steady-state operating condition. In matrix form, the system of N equations of motion can be written as follows:

$$\left(-\omega^2[M] + j\omega[C] + j[H] + [K]\right)_{N\times N} \{u\}_{N\times 1} = \{F\}_{N\times 1} \tag{1}$$

where $[M]$, $[C]$, $[H]$, $[K]$ are the physical mass, viscous damping, hysteretic damping, and stiffness matrices, respectively and $\{u\}$, $\{F\}$ are the physical displacement and force vectors, respectively. The variable ω is the frequency (rad/sec) and $j = \sqrt{-1}$.

When the finite element model represented by Eq. (1) exceeds several hundred degrees of freedom, the system is usually reduced prior to direct frequency domain calculations. A popular reduction method is to use M real normal modes of the system that span the frequency range of interest to create a transformation matrix and constraint relationship as follows:

$$\{u\}_{N\times 1} = [\Psi]_{N\times M} \{\gamma\}_{M\times 1} \tag{2}$$

where $M \ll N$ and the M columns of $[\Psi]$ are the real normal mode shape vectors and $\{\gamma\}$ is the modal displacement vector. By substituting Eq. (2) into Eq. (1) and premultiplying both sides of the resultant equation by $[\Psi]^T$, we have:

$$\left(-\omega^2 \lceil m \rfloor + j\omega[\,c\,] + j[\,h\,] + \lceil k \rfloor\right)_{M\times M} \{\gamma\}_{M\times 1} = \{f\}_{M\times 1} \tag{3}$$

where:

$$\begin{array}{lcll} \lceil m \rfloor & = & [\Psi]^T\ [M]\ [\Psi] & \text{diagonal modal mass matrix} \\ [\,c\,] & = & [\Psi]^T\ [C]\ [\Psi] & \text{modal viscous damping matrix} \\ [\,h\,] & = & [\Psi]^T\ [H]\ [\Psi] & \text{modal hysteretic damping matrix} \\ \lceil k \rfloor & = & [\Psi]^T\ [K]\ [\Psi] & \text{diagonal modal stiffness matrix} \\ \{f\} & = & [\Psi]^T\ \{F\} & \text{modal force vector} \end{array}$$

The reduced system represented by Eq. (3) is an approximation to the original system in Eq. (1). The modal displacement vector $\{\gamma\}$ becomes the new set of independent coordinates and the original physical displacement vector $\{u\}$ is back-calculated through the mode shape matrix in Eq. (2). The modal displacements in Eq. (3) can be calculated at each frequency of interest through a frequency-dependent matrix inversion in this way:

$$\{\gamma\} = \left(-\omega^2 \lceil m \rfloor + j\omega [\,c\,] + j [\,h\,] + \lceil k \rfloor\right)^{-1} \{f\} \tag{4}$$

In the reduced system of Eq. (3) and Eq. (4), the modal mass and stiffness matrices are diagonal, but the damping matrices can have large off-diagonal terms requiring a fully populated complex matrix inversion for each solution frequency. Although the system has been reduced, a large number of retained modes and/or large number of frequency steps can produce a computationally intensive solution. If a proportional damping approximation is made at this stage, the off-diagonal terms in the damping matrices are ignored and the resultant equations of motion are then fully uncoupled. In this case, the frequency domain solution in Eq. (4) becomes very efficient because a scalar, rather than matrix, inversion is required at each frequency of interest. Neglecting the effects of these off-diagonal coupling terms in the damping matrices, however, can generate substantial errors in the frequency response and load calculations.

The presence of discrete damper components in the model does not usually influence all of the system modes to a significant extent. Only those modes that have a substantial amount of relative motion across the dampers will be strongly affected by their energy dissipative properties. When little relative motion exists across the dampers for a given mode, the dampers are not effectively exercised and little energy dissipation is produced for that mode. Therefore, Eq. (3) can be partitioned so that modal coordinates corresponding to the modes that significantly exercise damper components are separated from the other modal coordinates in this way:

$$\begin{aligned}&\left(-\omega^2 \begin{bmatrix} \lceil m_{nn} \rfloor & 0 \\ 0 & \lceil m_{pp} \rfloor \end{bmatrix} + j\omega \begin{bmatrix} c_{nn} & c_{np} \\ c_{pn} & c_{pp} \end{bmatrix}\right. \\ &\quad \left. + j \begin{bmatrix} h_{nn} & h_{np} \\ h_{pn} & h_{pp} \end{bmatrix} + \begin{bmatrix} \lceil k_{nn} \rfloor & 0 \\ 0 & \lceil k_{pp} \rfloor \end{bmatrix}\right) \begin{Bmatrix} \gamma_n \\ \gamma_p \end{Bmatrix} = \begin{Bmatrix} f_n \\ f_p \end{Bmatrix}\end{aligned} \tag{5}$$

where the subscripts n, p denote the 'nonproportionally' and 'proportionally' damped partitions, respectively.

The new analysis technique presented in this paper is a hybrid of the traditional proportional and nonproportional damping solution methods. The off-diagonal terms in the c_{pp} and h_{pp} quadrants of the damping matrices in Eq. (5) should be small compared to the diagonal terms of their respective quadrants. Ignoring these off-diagonal terms and replacing those quadrants with diagonal or proportionally damped matrices is a small approximation. The larger off-diagonal terms in the c_{nn}, c_{np}, c_{pn}, h_{nn}, h_{np}, h_{pn} quadrants are retained to represent the significant nonproportional damping effects. This approximation becomes the basis for a condensation of $\{\gamma_p\}$ onto the $\{\gamma_n\}$ modal coordinates which eventually creates a more efficient method of solving Eq. (3) with a small loss of accuracy. The concept of partitioning and condensing is similar to the Guyan reduction technique [1] although the present application is very different.

The new hybrid formulation begins with the expansion of Eq. (5) for the pn and pp quadrants as follows:

$$\left(-\omega^2 \lceil m_{pp} \rfloor + j\omega \lceil c_{pp} \rfloor + j \lceil h_{pp} \rfloor + \lceil k_{pp} \rfloor\right) \{\gamma_p\} = \{f_p\} - \left(j\omega\, [c_{pn}] + j\, [h_{pn}]\right) \{\gamma_n\} \tag{6}$$

where $\lceil c_{pp} \rfloor$, $\lceil h_{pp} \rfloor$ are the diagonal (proportionally damped) approximations to the original damping matrix quadrants. From Eq. (6), the vector $\{\gamma_p\}$ can be expressed in terms as $\{\gamma_n\}$ as follows:

$$\{\gamma_p\} = \lceil D_{pp} \rfloor^{-1} \left(\{f_p\} - (j\omega\, [c_{pn}] + j\, [h_{pn}])\, \{\gamma_n\} \right) \tag{7}$$

where

$$\lceil D_{pp} \rfloor = \left(-\omega^2 \lceil m_{pp} \rfloor + j\omega \lceil c_{pp} \rfloor + j \lceil h_{pp} \rfloor + \lceil k_{pp} \rfloor \right)$$

By substituting Eq. (7) into the expansion of the nn and np quadrants of Eq. (5), we have:

$$[D_{nn}]\, \{\gamma_n\} + (j\omega\, [c_{np}] + j\, [h_{np}])\, \lceil D_{pp} \rfloor^{-1} \left(\{f_p\} - (j\omega\, [c_{pn}] + j\, [h_{pn}])\, \{\gamma_n\} \right) = \{f_n\} \tag{8}$$

where

$$[D_{nn}] = \left(-\omega^2\, [\, m_{nn}] + j\omega\, [c_{nn}] + j\, [h_{nn}] + [\, k_{nn}\,] \right)$$

Eq. (8) can be simplified and inverted to solve for $\{\gamma_n\}$ as follows:

$$\{\gamma_n\} = \left[\tilde{D}_{nn}\right]^{-1} \left\{\tilde{f}_n\right\} \tag{9}$$

where

$$\begin{aligned}\left[\tilde{D}_{nn}\right] = \Big([D_{nn}] &+ \omega^2\, [c_{np}]\, \lceil D_{pp} \rfloor^{-1} [c_{pn}] \\ &+ \omega\, [c_{np}]\, \lceil D_{pp} \rfloor^{-1} [h_{pn}] + w\, [h_{np}]\, \lceil D_{pp} \rfloor^{-1} [c_{pn}] \\ &+ [h_{np}]\, \lceil D_{pp} \rfloor^{-1} [h_{pn}] \Big)\end{aligned}$$

and

$$\left\{\tilde{f}_n\right\} = \{f_n\} - (j\omega\, [c_{np}] + j\, [h_{np}])\, \lceil D_{pp} \rfloor^{-1} \{f_p\}$$

The calculation of $\{\gamma_n\}$ through the hybrid solution in Eq. (9) provides an accurate, but more efficient, alternative to the traditional nonproportional damping solution in Eq. (4). The hybrid system matrix, $\left[\tilde{D}_{nn}\right]$ although fully populated, is usually significantly smaller than the modal system matrix in Eq. (4). Because the number of calculations required for the inversion of fully populated matrices increases cubically with the matrix order, substantial savings can be gained through order reductions. Additional effort, of course, is required for the calculation of $\left[\tilde{D}_{nn}\right]$ and $\left\{\tilde{f}_n\right\}$ at each solution frequency and offsets some of the computational savings gained from the matrix inversion of a smaller system. However, the small approximation of replacing a nearly-diagonal $[D_{pp}]$ with a diagonal $\lceil D_{pp} \rfloor$ matrix quadrant makes these additional calculations for the generation of $\left[\tilde{D}_{nn}\right]$ quite manageable.

Once $\{\gamma_n\}$ has been determined for each frequency using Eq. (9), the remainder of the modal displacements $\{\gamma_p\}$ can be calculated using Eq. (7). Finally, the physical displacements $\{u\}$ of the original dynamic system can be calculated using Eq. (2) thus completing the frequency domain solution of Eq. (1).

APPLICATION

Recently, the authors performed an evaluation of a rotating equipment design intended for marine application. This evaluation included a frequency response analysis using a very large finite element model of the system where more than 200 resonant modes were needed to span the frequency range of interest. The solution was to be calculated at more than 1500 spectral lines to provide the desired resolution over the frequency range of interest. Because significant levels of nonproportional damping were present in the model and accurate results were required, a traditional nonproportional damping analysis was attempted. Using MSC/NASTRAN software on a CRAY mainframe computer system, a partial solution over a narrow frequency range was performed. The CPU time used for each frequency step of solution was linearly extrapolated to estimate the time for a complete solution. The required CRAY CPU time was estimated to exceed 40,000 seconds and the solution was judged to be impractical using conventional approaches. As a result, the alternative method presented in this paper was developed.

The new hybrid method was first applied to another large finite element model whose size and construction was similar to the model previously discussed. The 71,000 degree of freedom model was divided into six substructures where each substructure was generated using the Craig-Bampton formulation [2]. The residual system of assembled substructures consisted of 4,300 degrees of freedom. It was dynamically reduced prior to frequency response solution using 123 real normal modes that spanned the frequency range of interest as outlined in Eqs. (1), (2), and (3). Viscous damper and hysteretic damper elements were used to represent the energy dissipative effects of fluid-film bearings and elastomeric mounts, respectively. The modal damping matrices were nearly fully populated with large off-diagonal terms indicating that significant levels of nonproportional damping were present in the system model.

The frequency response solution of the reduced system with 123 modal coordinates was calculated at 400 spectral lines for each of the three methods discussed in this paper which include:

- Method 1: Traditional nonproportional damping solution
- Method 2: Traditional proportional damping solution
- Method 3: New hybrid solution

Figure 1 presents a comparison of calculated frequency responses for each of the three solution methods evaluated at the same structural location. Method 1 results are 'exact' and differ substantially from the approximate results of Method 2. The computational time required for the accurate results of Method 1, however, was 52 times greater than the approximate Method 2 solution. On the other hand, Method 3 results from the hybrid solution compare favorably with the 'exact' solution from Method 1, but required only 8% of the computational time needed for that solution.

For this particular hybrid solution, the reduced system matrix in Eq. (5) was partitioned into 31 'nonproportionally-damped' and 92 'proportionally-damped' modal degrees of freedom. In other words, 31 real modes were judged to be significantly influenced by the discrete damper elements in the model. The remaining 92 real modes were not significantly affected and were subsequently approximated with 92 'proportionally-damped' modes, thus producing an efficient solution with little sacrifice in accuracy.

The matrix partitioning process outlined in Eq. (5) was not a straight forward task. As more modes are included in the 'nonproportionally-damped' partition, the accuracy as well as the computational expense will increase and eventually converge to the traditional nonproportional damping solution. Several partitioning schemes were developed and studied based on the relative magnitudes of off-diagonal to diagonal terms in the damping matrices. In the future, more sophisticated partitioning algorithms are expected to generate even more economical solutions.

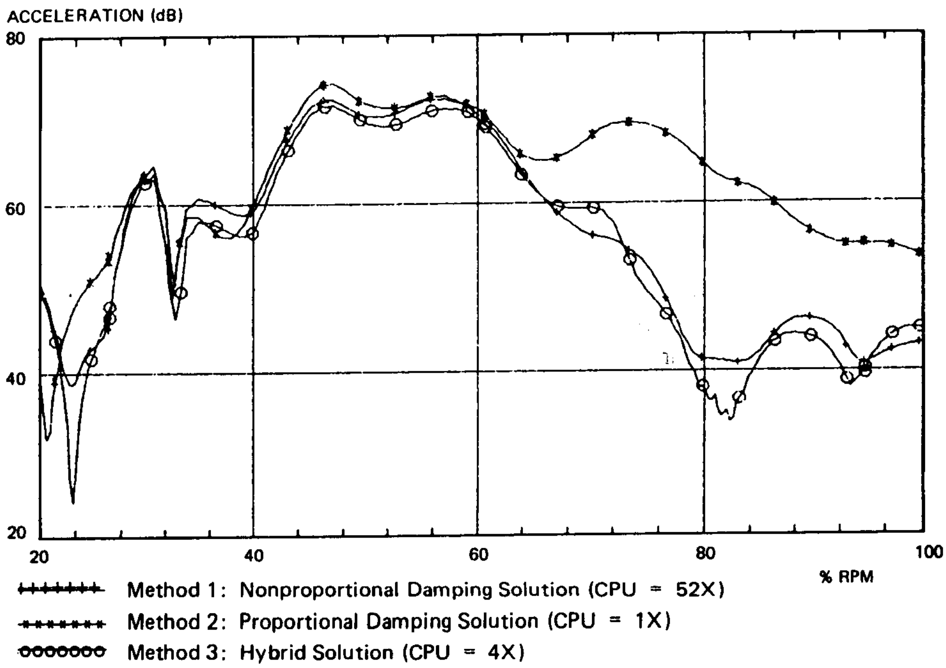

Figure 1
Comparison of Frequency Response Calculations for Different Solution Methods

CONCLUSIONS

A method has been presented to accurately and economically calculate steady-state frequency responses based on the analysis of large finite element models with nonproportional damping effects. The new method is a hybrid of the traditional nonproportional and proportional damping solution methods. It captures the advantages of each conventional approach without the burden of their respective shortcomings, as demonstrated with comparative analyses performed on a large finite element model.

REFERENCES

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