A Passive Vibration-Cancelling Isolation Mount

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Machines operating at constant rpm are found in many industrial, domestic, and military applications. For those machines which generate single-frequency vibration, a conceptual alternative to a system of conventional (KR) or compound (CD) mounts is a system of passive vibration-cancelling (VC) mounts.

This paper presents an analysis of an idealized passive vibration-cancelling two-terminal mount with one degree-of-freedom at each mechanical terminal isolating a nonrigid machine from a nonrigid foundation. To evaluate the VC mount, its effectiveness as a function of frequency is compared with the effectivenesses of both KR and CD mounts isolating a rigid machine from a nonrigid foundation. The comparisons indicate that a carefully designed and manufactured VC mount should provide substantially greater vibration reduction at its cancellation frequency than either a KR or CD mount having the same low frequency stiffness, i.e. stiffness at the natural frequency of the machine-mount system.

Although there are a number of practical problems to be solved before VC mounts can become a reality, and some additional analytical work should be done to "fine tune" their design, there appears to be nothing of a technical nature to preclude their successful development.

INTRODUCTION

Common engineering practice for reducing machine-excited structural vibration is to interpose vibration isolation mounts between the machine and the supporting structure. For many vibration problems, conventional vibration mounts provide adequate vibration reduction; however, if the reduction provided by conventional mounts is insufficient, more elaborate isolation mounts or mount systems are required. A compound mounting system may be employed in which the machine is attached to a stiff massive platform by one set of isolation mounts, the platform to the supporting structure by a second. Alternately, the machine may be supported by a system of compound mounts, each of which consists conceptually of two damped springs connected together by a rigid mass [1].

For single frequency single degree-of-freedom vibration reduction, or for vibration reduction when the vibratory output of a machyine is dominated by vibration at a single frequency in a single degree-of-freedom, an alternative to either a set of conventional(KR) or compound (CD) mounts is a set of vibration-cancelling (VC) mounts.

A passive-vibration cancelling mount consists of a compound mount paralleled by a third resilient element, the stiffness and loss factor of which are chosen such that the forces transmitted from the machine through the two mechanically paralleled paths are equal in magnitude and opposite in phase, so that when superposed, they cancel.

VC MOUNT ANALYSIS AND ITS SPECIALIZATION TO INCLUDE KR AND CD MOUNTS

The analysis of the VC mount will be carried out using the simple model in Fig.1 in which the CD mount consisting of z_1, z_2, and M_1, is paralleled by a third mechanical element z_3.

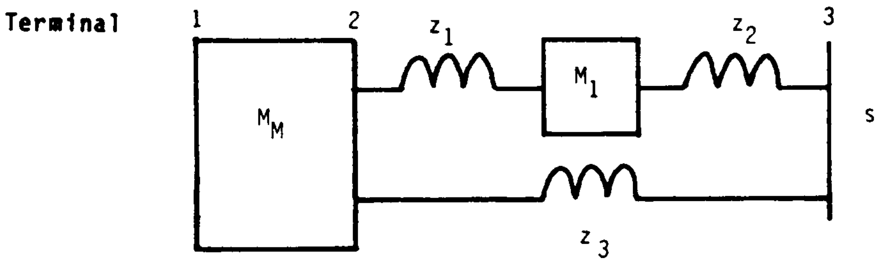


Fig. 1 Model of a Nonrigid Machine M_M Isolated from a Nonrigid Supporting Structure s by a Vibration-Cancelling Mount Consisting of Four Elements z_1, z_2, z_3, and M_1.

The analysis will assume steady-state sinusoidal vibration, that the vibratory input at Terminal 1 of the machine can be characterized as a force rather than a motion, and that the structure of the mount is such that the left terminals of z_1 and z_3 undergo the same motion, and the right terminals of z_2 and z_3 undergo the same motion*. The phasor of the vibratory velocity of the supporting structure will be obtained by two applications of a mechanical version of the Thevenin Electrical Network Theorem [2]. (A different statement of the Thevenin Theorem is required if motion rather than force is specified at Terminal 1 of the machine.) The analysis will make use of a two-terminal version of multiterminal network theory derived in [2], in which each two-terminal mechanical element is characterized by two point and two point-to-point (transfer) free admittances or blocked impedances.

Vibratory Velocity of the Supporting Structure

Let $f_{2,2\beta}$ be the phasor of the force at Terminal 2 of the machine with Terminal 2 blocked. By Thevenin's Theorem [2], the phasor $\dot{x}_{2,3\beta}$ of the velocity at Terminal 2 of the VC mount when Terminal 3 is blocked is given by

$$\dot{x}_{2,3\beta} = \frac{f_{2,2\beta}}{z_M + z_{CD22} + z_{(3)22}}, \qquad (1)$$

where:

z_M is the impedance of the machine at Terminal 2 with Terminal 1 free;

z_{CD22} is the impedance at Terminal 2 of the CD section of the mount with Terminal 3 blocked; and

*Different fonts are used for text and equations; however, the meaning of literal symbols is the same in both fonts.

$z_{(3)22}$ is the impedance at terminal 2 of the mechanical element z_3 with Terminal 3 blocked.

To calculate the velocity at Terminal 3 when the mount is connected to the supporting structure by Thevenin's Theorem, it is first necessary to calculate the phasor $f_{3,3\beta}$ of the blocked force at Terminal 3. By superposition, the blocked force at Terminal 3 is the sum of the forces transmitted through the CD and paralleled sections of the mount and applied to the blocking structure. The phasors of these forces can be obtained by calculating the phasors of the forces applied to the two sections of the mount at Terminal 2, and by multiplying these forces by the negatives of their force transmissibilities T_{CDf32} and $T_{(3)32}$.

Let $f_{2CD,3\beta}$ be the phasor of the force applied at Terminal 2 of the CD section of the mount, and $f_{2(3),3\beta}$ be the phasor of the force applied at Terminal 2 of the paralleled section, both with Terminal 3 blocked.

By superposition,

$$f_{3,3\beta} = T_{CDf32}\, f_{2CD,3\beta} + T_{(3)f32}\, f_{2(3),3\beta} \, . \tag{2}$$

Since

$$f_{2CD,3\beta} = z_{CD22}\, x_{2,3\beta} \, , \quad \text{and} \tag{3}$$

$$f_{2(3),3\beta} = z_{(3)22}\, x_{2,3\beta} \, , \tag{4}$$

then

$$f_{3.3\beta} = \left[T_{CDf32}\, z_{CD22} + T_{(3)f32}\, z_{(3)22}\right] x_{2,3\beta} \, . \tag{5}$$

Substituting for $x_{2,3\beta}$ from Eq.(1), Eq.(5) becomes

$$f_{3,3\beta} = T_{CDf32}\, z_{CD22} + T_{(3)f32}\, z_{(3)22} \frac{f_{2,2\beta}}{z_M + z_{CD22} + z_{(3)22}} \, . \tag{6}$$

By Thevenin's Theorem, the phasor x_{3VC} of the velocity of the supporting structure when machine, mount, and supporting structure are connected is given by

$$x_{3VC} = \frac{f_{3,3\beta}}{z_{VCM} + z_S} = \frac{\left[T_{CDf32}\, z_{CD22} + T_{(3)f32}\, z_{(3)22}\right]}{z_M + z_{CD22} + z_{(3)22}} \frac{f_{2,2\beta}}{z_{VCM} + z_S} \, , \tag{7}$$

where

z_{VCM} is the impedance at Terminal 3 of the VC mount with Terminal 2 of the mount attached to the machine, with Terminal 1 of the machine free.

From Appendix 1, z_{VCM} is given by

$$z_{VCM} = (z_{CD33} + z_{(3)33}) \frac{[(1/y_{(3)22}) + (1/y_{CD22})] + z_M}{z_{CD22} + z_{(3)22} + z_M} \, , \tag{8}$$

where:

$z_{(3)33}$ is the impedance at Terminal 3 of the mechanical element z_3 with Terminal 2 blocked;

$y_{(3)22}$ is the admittance of z_3 at Terminal 2 with Terminal 3 free; and

y_{CD22} is the admittance of the CD section of the mount at Terminal 2 with Terminal 3 free.

Substituting for z_{VCM} from Eq.(8) into Eq.(7), the phasor x_{3VC} of the velocity of the supporting structure when machine, mount, and supporting structure are connected is given by

$$x_{3VC} = \left[\frac{(T_{CDf32}\, z_{CD22} + T_{(3)f32}\, z_{(3)22})}{z_M + z_{CD22} + z_{(3)22}}\right] \frac{f_{2,2B}}{(z_{CD33} + z_{(3)33})\left[\frac{(1/y_{(3)22}) + (1/y_{CD22}) + z_M}{z_{CD22} + z_{(3)22} + z_M}\right] + z_S} . \tag{9}$$

The equation for the phasor x_{3CD} of the velocity at Terminal 3 with z_3 removed and the CD section remaining can be obtained from Eq.(9) by making the substitutions $T_{(3)f32} = z_{(3)22} = 1/[y_{(3)22}] = z_{(3)22} = 0$.

$$x_{3CD} = \frac{T_{CDf32}}{z_M + z_{CD22}} \frac{z_{CD22}}{z_{CD33}\, \frac{(1/y_{CD22}) + z_M}{z_{CD22} + z_M} + z_S} f_{2,2B} . \tag{10}$$

The equation for the phasor x_{3KR} of the velocity at Terminal 3 with the CD section of the mount removed and only z_3 remaining can be obtained from Eq.(9) by making the substitutions $T_{CDf32} = z_{CD22} = 1/(y_{CD22}) = z_{CD33} = 0$.

$$x_{3KR} = \frac{T_{CDf32}}{z_M + z_{(3)22}} \frac{z_{(3)22}}{z_{(3)33}\, \frac{(1/y_{(3)22}) + z_M}{z_{(3)22} + z_M} + z_S} f_{2,2B} . \tag{11}$$

When z_3 is a massless spring-dashpot mount, so that $1/[y_{(3)22}] = 0$, Eq. (11) can be further simplified to

$$x_{3KR} = \frac{f_{2,2B}}{z_M + z_S + \frac{z_M\, z_S}{z_{(3)22}}} . \tag{12}$$

Requirements for Vibration Cancellation

It can be seen from Eq.(9) that the condition for vibration cancellation, namely that the forces transmitted to the supporting structure via the two paths in the VC mount be equal in magnitude and opposite in phase, is equivalent to requiring that the phasor x_{3vc} of the velocity of the supporting structure be zero, i.e. that

$$T_{CDf32}\, z_{CD22} + T_{(3)f32}\, z_{(3)22} = 0 \; . \tag{13}$$

This section of the paper will determine the relationships that must be established between the properties of the components of the CD and paralleled paths to achieve cancellation when z_1, z_2,and z_3 are massless spring-dashpots, and M_1 is a rigid mass.

Given that:

$$z_1 = R_1 - j(K_1/w) \; ; \tag{14}$$

$$z_2 = R_2 - j(K_2/w) \; ; \tag{15}$$

$$z_3 = R_3 - j(K_3/w) \; ; \tag{16}$$

$$z_M = jwM_1 \; ; \tag{17}$$

then:

$$T_{CDf32} = \frac{R_2 - j\frac{K_2}{w}}{R_2 + j(wM_1 - \frac{K_2}{w})} \; ; \tag{18}$$

$$z_{CD22} = \frac{[R_1 - j\frac{K_1}{w}] \quad [R_2 + j\,(wM_1 - \frac{K_2}{w})]}{R_1 + R_2 + j\,[wM_1 - \frac{(K_1 + K_2)}{w}]} \; ; \tag{19}$$

$$T_{(3)f32} = 1 \; ; \tag{20}$$

$$z_{(3)22} = R_3 - j\frac{K_3}{w} \; ; \tag{21}$$

where:

M_1 is the mass of the inertial element in th CD section of the mount;
R_1, R_2, R_3 are the mechanical resistances of z_1,z_2,z_3, respectively;
K_1, K_2, K_3 are the stiffnesses of z_1,z_2,z_3, respectively;

$$w = 2\pi f \; , \tag{22}$$

f being the frequency in Hertz.

Substituting from Eqs.(18) - (21) into Eq. (13), the cancellation condition for a VC mount constructed from ideal mechanical elements (massless spring-dashpots and rigid lossless masses) becomes

$$\frac{(R_1 - j\frac{K_1}{w})\ (R_2 - j\frac{K_2}{w})}{R_1 + R_2 + j\ [wM_1 - \frac{(K_1 + K_2)}{w}]} + R_3 - j\frac{K_3}{w} = 0\ . \tag{23}$$

Eq.(23) is complex, and for it to be zero, both real and imaginary parts must be zero. Collecting real and imaginary parts, equating both to zero, defining the loss factors of r_1, r_2, r_3,

$$r_1 = \frac{wR_1}{K_1}\ , \tag{24}$$

$$r_2 = \frac{wR_2}{K_2}\ , \tag{25}$$

$$r_3 = \frac{wR_3}{K_3}\ , \tag{26}$$

and solving the resulting pair of equations simultaneously, one can show that to achieve cancellation at the circular frequency w_c

$$w_c = 2\,\pi\, f_c\ , \tag{27}$$

where f_c is the cancellation frequency, r_3 and M_1 must have the magnitudes:

$$r_3 = \frac{r_1K_1\ (1+\frac{K_2}{K_3})\ + r_2K_2\ (1+\frac{K_1}{K_3})}{\frac{K_1K_2}{K_3}\,\frac{(1-r_1r_2)}{2} + \left[[\ \frac{K_1K_2}{K_3}\frac{(1-r_1r_2)}{2}\ ]^2 - [r_1K_1+r_2K_2][r_1K_1(1+\frac{K_2}{K_3})+r_2K_2(1+\frac{K_1}{K_3})]\right]^{1/2}}\ ; \tag{28}$$

$$M_1 = \frac{1}{w_c^2}\left[K_1+K_2+\frac{K_1K_2}{K_3}\,\frac{(1-r_1r_2)}{2} + \left[\frac{K_1K_2}{K_3}\frac{(1-r_1r_2)}{2}^2 - [r_1K_1+r_2K_2][r_1K_1(1+\frac{K_2}{K_3})+r_2K_2(1+\frac{K_1}{K_3})]\right]^{1/2}\right]. \tag{29}$$

From Eqs. (28) and (29), r_3 and M_1 depend on the stiffnesses of the resilient elements in both the CD and paralleled (KR) sections of the mount, and on the loss factors in the CD section.

In the next section of the paper, formulae will be developed for calculating the effectivenesses of VC, CD, and KR mounts.

MOUNT EFFECTIVENESS

Definition

For steady-state sinusoidal machine-excited supporting structure vibration, mount effectiveness is defined as the ratio of the phasor of the velocity of the supporting structure when the machine is directly attached to it, to the phasor of the velocity of the supporting structure when the machine is attached to it by an isolation mount.

Velocity of the Supporting Structure with the Machine Directly Attached

Let $f_{2,2B}$ be the phasor of the force at Terminal 2 of the machine with Terminal 2 blocked. Let x_2 be the phasor of the supporting structure velocity with the machine directly attached to it.

By Thevenin's Theorem, x_2 is given by

$$x_2 = \frac{f_{2,2B}}{z_M + z_S} . \tag{30}$$

Effectiveness Equations for VC, CD, and KR Mounts

By the definition above, the effectiveness, E_{VC}, of the VC mount is given by the ratio

$$E_{VC} = \frac{x_2}{x_{3VC}} ; \tag{31}$$

the effectiveness, E_{CD}, of the CD mount by the ratio

$$E_{CD} = \frac{x_2}{x_{3CD}} ; \tag{32}$$

and the effectiveness, E_{KR}, of the KR mount by the ratio

$$E_{KR} = \frac{x_2}{x_{3KR}} . \tag{33}$$

From Eqs. (9) and (30),

$$E_{VC} = \frac{[z_{CD33} + z_{(3)33}]\,[(1/y_{CD22}) + (1/y_{(3)22}) + z_M] + z_S\,[z_M + z_{CD22} + z_{(3)22}]}{[z_M + z_S]\,[T_{CDf32}\, z_{CD22} + T_{(3)f32}\, z_{(3)22}]} . \tag{34}$$

From Eqs. (10) and (30),

$$E_{CD} = \frac{z_{CD33}\,[(1/y_{CD22}) + z_M] + z_S\,[z_M + z_{CD22}]}{[z_M + z_S]\,T_{CDf32}\,z_{CD22}} \,. \tag{35}$$

From Eqs. (12) and (30),

$$E_{VC} = \frac{z_M + z_S + \dfrac{z_M\,z_S}{z_3}}{[z_M + z_S]} \,. \tag{36}$$

Eqs. (34) and (35) are quite broad in applicability. Eq. (34) gives the effectiveness of two arbitrary mechanical elements in parallel in isolating a nonrigid machine from a nonrigid supporting structure; Eq. (35), the effectiveness of a single arbitrary mechanical element in isolating a nonrigid machine from a nonrigid supporting structure.*

Note that zeros for undamped systems, and minima for damped systems occur in the denominators of E_{VC}, E_{CD}, and E_{KR} at the frequency at which the reactive components of z_M and z_S are equal in magnitude and opposite in phase, a condition under which, in the absence of an isolation mount, the machine resonates with the supporting structure.

In the next section of the paper, effectiveness equations will be obtained for CD and VC mounts constructed from idealized mechanical elements.

Effectiveness Equations for CD and VC Mounts Constructed from Ideal Mechanical Elements

From Eqs. (14) - (17):

$$z_{CD33} = \frac{[R_2 - j\frac{K_2}{w}]\quad [R + j\,(wM_1 - \frac{K_1}{w})]}{R_1 + R_2 + j\,[wM_1 - \frac{(K_1 + K_2)}{w}]} \,; \tag{37}$$

$$y_{CD22} = \frac{R_1 + j(wM_1 - \frac{K_1}{w})}{(R_1 - j\frac{K_1}{w})\; j\,wM_1} \,; \tag{38}$$

*Consider two arbitrary mechanical elements constrained so that their input terminals both experience the same motion $x_1(t)$, and their output terminals the same motion $x_2(t)$. By definition, elements constrained in this manner are said to be mechanically in parallel.

$$z_{(3)33} = R_3 - j\frac{K_3}{w} \; ; \tag{39}$$

$$y_{(3)22} = \frac{1}{U} = \infty \; ; \tag{40}$$

$$z_M = jwM_M \; ; \text{ and} \tag{41}$$

$$z_S = R_S + jX_S \; ; \tag{42}$$

where:

M_M is the mass of the machine;

R_s and X_s are the mechanical resistance and reactance of the supporting structure, respectively.

From Eqs. (14) - (21), (24) - (26), and (37) - (43), it can be shown that E_{CD} and E_{VC} can be written in nondimensional form as:

$$E_{CD} = \frac{N_1 + N_2 + N_3}{D_1} \; ; \tag{43}$$

$$E_{VC} = \frac{N_1 + N_2 + N_3 + N_4 + N_5}{D_1 D_2} \; ; \tag{44}$$

where:

$$N_1 = \left[\frac{M_1}{M_M} + \frac{r_1 + j\,[(\frac{w}{w_1})^2 - 1]}{r_1 - j} \right] j \left(\frac{w}{w_{OCD}}\right)^2 ; \tag{45}$$

$$N_2 = \left[\frac{r_2 + j\,[(\frac{w}{w_2})^2 - 1]}{r_2 - j} \right] \frac{w}{w_{OCD}} \quad z_s' ; \tag{46}$$

$$N_3 = \left[\frac{\frac{b}{b+1} r_1 + \frac{r_2}{b+1} + j[(\frac{w}{w_{12}})^2 - 1]}{(r_1-j)\ (r_2-j)} \right] j\left(\frac{w}{w_{OCD}}\right)^2 \frac{w}{w_{OCD}} \quad z_s' ; \tag{47}$$

$$N_4 = \frac{b+1}{bk_2}\left[\frac{(r_3-j)\left(\frac{b}{b+1} r_1 + \frac{r_2}{b+1} + j[(\frac{w}{w_{12}})^2 - 1]\right)}{(r_2 - j)}\right]\left[\frac{1}{r_1 + j[(\frac{w}{w_1})^2 - 1]}\ \frac{M_1}{M_M} + \frac{1}{r_1-j}\right] j \left(\frac{w}{w_{OCD}}\right)^2 ; \tag{48}$$

$$N_5 = \frac{b+1}{bk_2}\ \frac{r_3-j}{r_1-j} \left[\frac{\frac{b}{b+1} r_1 + \frac{r_2}{b+1} + j[(\frac{w}{w_{12}})^2 - 1]}{(r_2-j)} \right] \frac{w}{w_{OCD}} \quad z_s' ; \tag{49}$$

$$D_1 = j \left(\frac{w}{w_{OCD}}\right)^2 + \frac{w}{w_{OCD}} \quad z_s' ; \tag{50}$$

$$D_2 = 1 + \frac{b+1}{bk_2} (r_3-j) \left[\frac{\frac{b}{b+1} r_1 + \frac{r_2}{b+1} + j[(\frac{w}{w_{12}})^2 - 1]}{(r_1 - j)\ (r_2 - j)} \right] ; \tag{51}$$

$$b = K_1/K_2 \; ; \quad (52) \qquad j = (-1)^{1/2} \; ; \quad (53)$$

$$k_2 = K_2/K_3 \; ; \quad (54) \qquad r_1 = wR_1/K_1 \; ; \quad (55)$$

$$r_2 = wR_2/K_2 \; ; \quad (56) \qquad r_3 = wR_3/K_3 \; ; \quad (57)$$

$$w = 2\pi f = 2\pi \text{ (frequency in Hertz)} \; ; \quad (58) \qquad w_1^2 = K_1/M_1 \; ; \quad (59)$$

$$w_2^2 = K_2/M_1 \; ; \quad (60) \qquad w_{12}^2 = (K_1 + K_2)/M_1 \; ; \quad (61)$$

$$w_{OCD}^2 = [K_1K_2/(K_1+K_2)]/M_M \; ; \quad (62) \qquad z_s' = w_{OCD}\, z_s/[K_1K_2/(K_1+K_2)] \; . \quad (63)$$

In these equations:

w_{OCD} is the natural frequency of the machine M_M on the CD mount -- or CD section of the VC mount -- neglecting the reactance of M_1;

z_s' is the ratio of the complex supporting structure impedance z_s to $w_{OCD}K_1K_2/(K_1+K_2)$, the magnitude of the stiffness reactance of the CD mount (or the CD section of the VC mount), neglecting the reactance of M_1.

The loss factor r_3 and the mass ratio M_1/M_M in Eq. (43) and (44) -- See Eqs. (45) - (63)-- can be obtained from Eqs. (28) and (29), or in terms of the parameters b and k_2 from the equations:

$$r_3 = \frac{r_1b+r_2 + bk_2(r_1+r_2)}{bk_2\,\frac{(1-r_1r_2)}{2}+\left[[bk_2\frac{(1-r_1r_2)}{2}]^2-[r_1b+r_2][r_1b+r_2+bk_2(r_1+r_2)]\right]^{1/2}} \; ; \quad (64)$$

$$\frac{M_1}{M_M} = (b+1)k_2\left(\frac{w_{OVC}}{w_c}\right)^2\left[\frac{b+1 + bk_2\frac{(1-r_1r_2)}{2}+\left[[bk_2\frac{(1-r_1r_2)}{2}]^2- [r_1b+r_2][r_1b+r_2+bk_2(r_1+r_2)]\right]^{1/2}}{1+b(1+k_2)}\right]; \quad (65)$$

where

$$w_{OVC} = 2\pi f_{OVC} \; , \quad (66)$$

f_{ovc} being the natural frequency of the machine M_M on the VC mount, neglecting the reactance of M_1.

$$w_{OVC}^2 = [K_3 + K_1K_2/(K_1+K_2)]/M_M \; ; \quad (67)$$

and

$$w_C = 2\pi f_C \, , \tag{68}$$

f_c being the frequency at which cancellation occurs.

Effectiveness Calculations

Mount effectiveness is a complex quantity having a magnitude and phase. Its magnitude is a measure of the isolation provided by a vibration mount.

Curves giving the magnitudes in dB of the effectivenesses, E_{KR}(dB), E_{CD}(dB), and E_{VC}(dB), for spring-dashpot (KR), compound (CD), and vibration-cancelling (VC) mounts vs. frequency ratio w/w_{OKR} (or w/w_{OCD}) for resistive, masslike, and springlike supporting structures are presented in Figs. (2) - (10)*. In these figures, the ratio of the supporting structure impedance (whether resistive or reactive) to the mount impedance at the natural frequency of the machine-mount system on an infinite impedance supporting structure is taken as a parameter.

Figs. (2) - (4) compare KR and CD mounts; Figs. (5) - (7) compare KR and CD mounts; and Figs. (8) - (10) compare CD and VC mounts.

The calculations were performed for CD-VC mount pairs for which the CD mount and the CD section of the VC mount were identical. A result of this procedure is that the natural resonant frequency of a rigid machine on the VC mount is always somewhat higher than that of the machine on the CD mount. Such a procedure is justified if the the VC mount is to be constructed from commercially-available isolation mounts, but is less than ideal for CD and VC mount comparison in that the improvement in the performance of the VC mount over that of the CD mount in the frequency band about the cancellation frequency is less than if the two natural frequencies were the same. However, since the difference in natural frequencies is small, the difference in the relative performance of the CD and VC mounts near the cancellation frequency is also small.

For the KR effectiveness curves,

$$w_{OKR}^2 = K_3/M_M \, , \tag{69}$$

where:

K_3 is the stiffness (spring constant) of the mount;
M_M is the mass of the machine; and
z_s' is the nondimensional supporting strucure impedance defined as

$$z_s' = w_{OKR} z_s / K_3 , \tag{70}$$

*The effectiveness of a mount in dB is defined by the equation

$$E(\text{dB}) = 20 \log |E| \, . \tag{71}$$

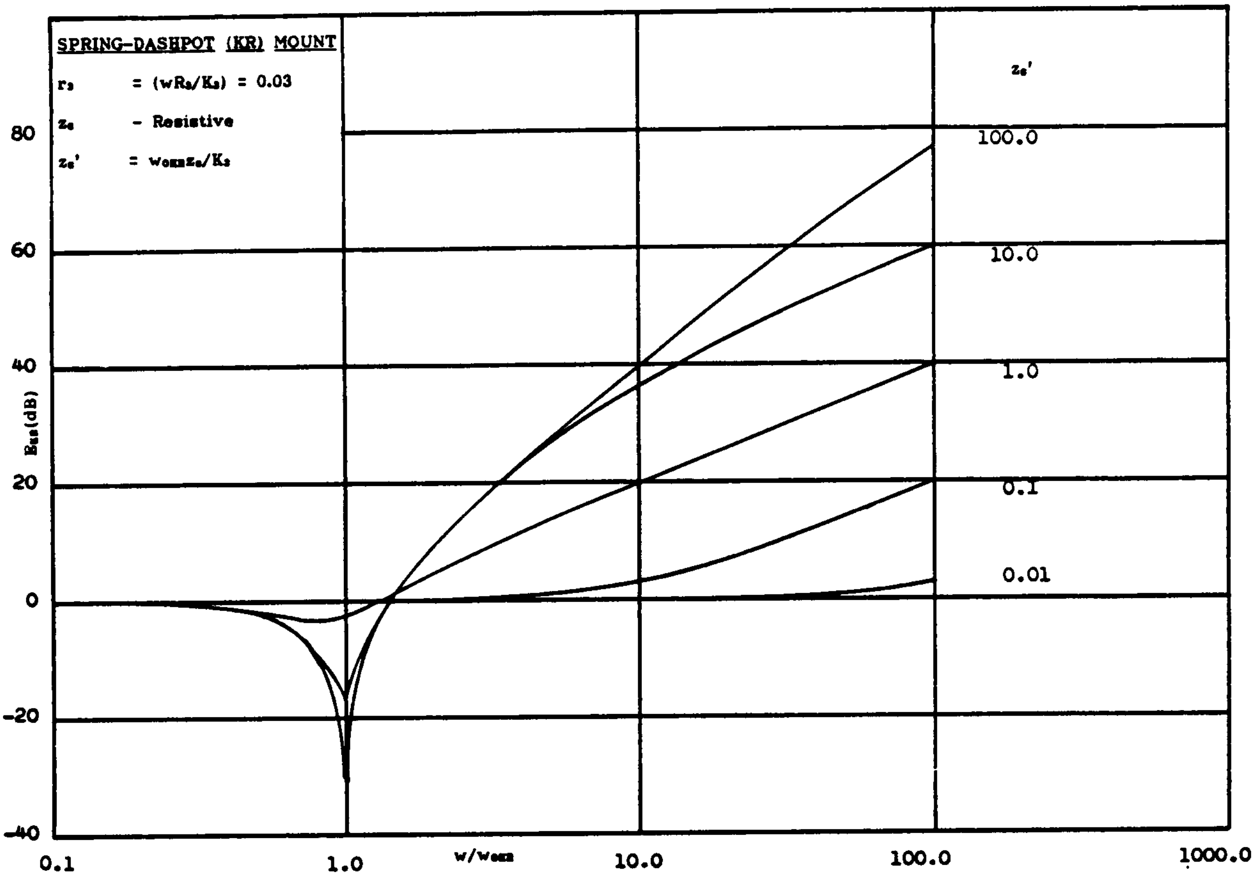
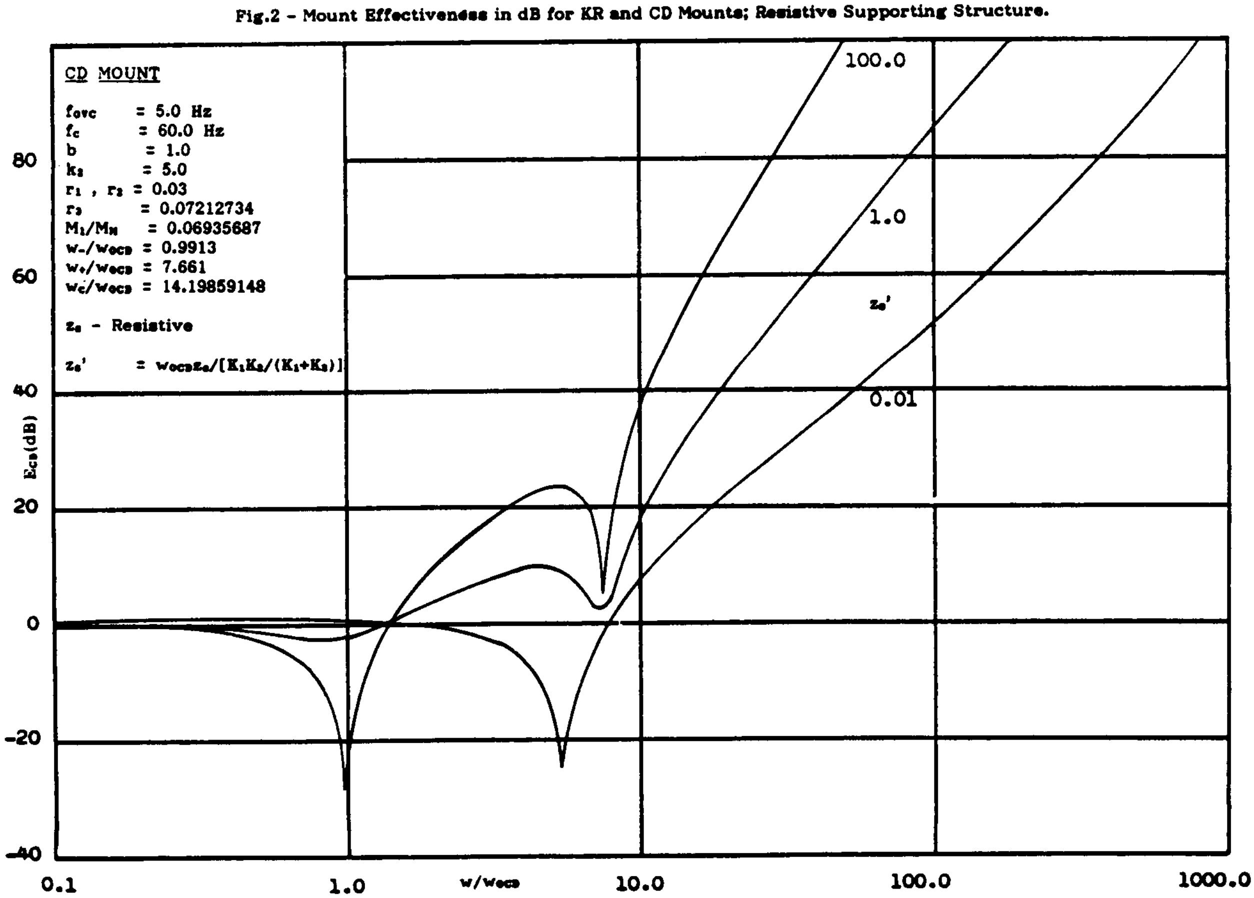


Fig.2 - Mount Effectiveness in dB for KR and CD Mounts; Resistive Supporting Structure.



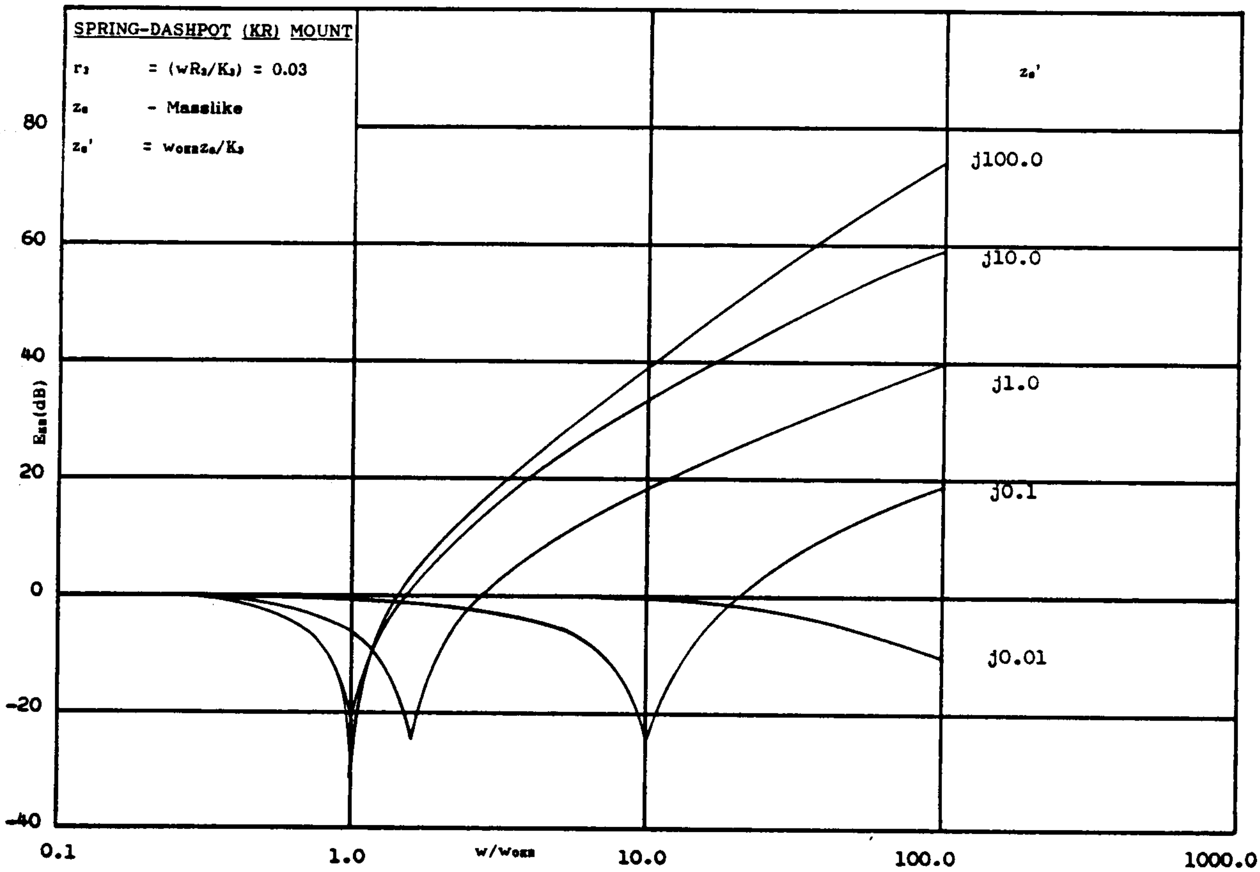
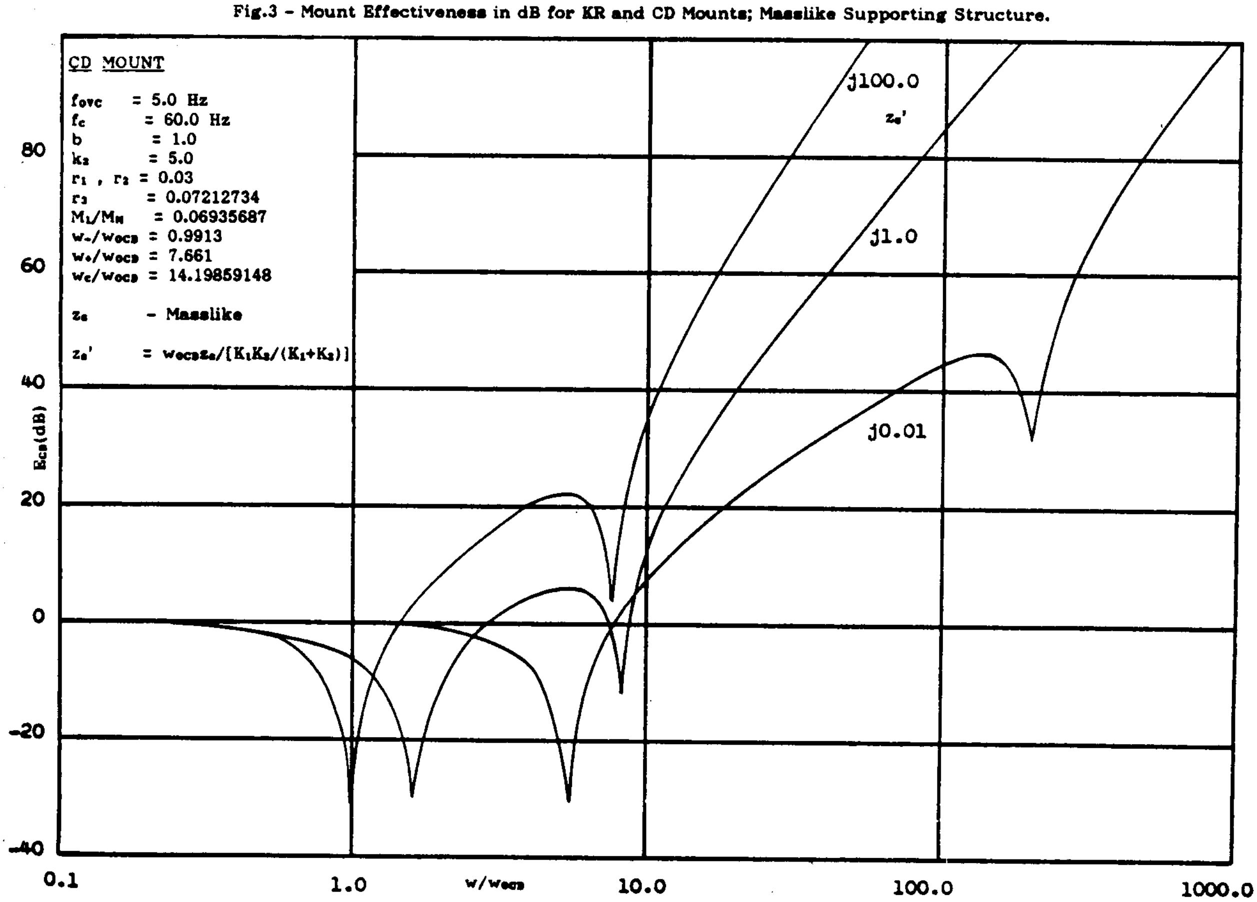


Fig.3 - Mount Effectiveness in dB for KR and CD Mounts; Masslike Supporting Structure.



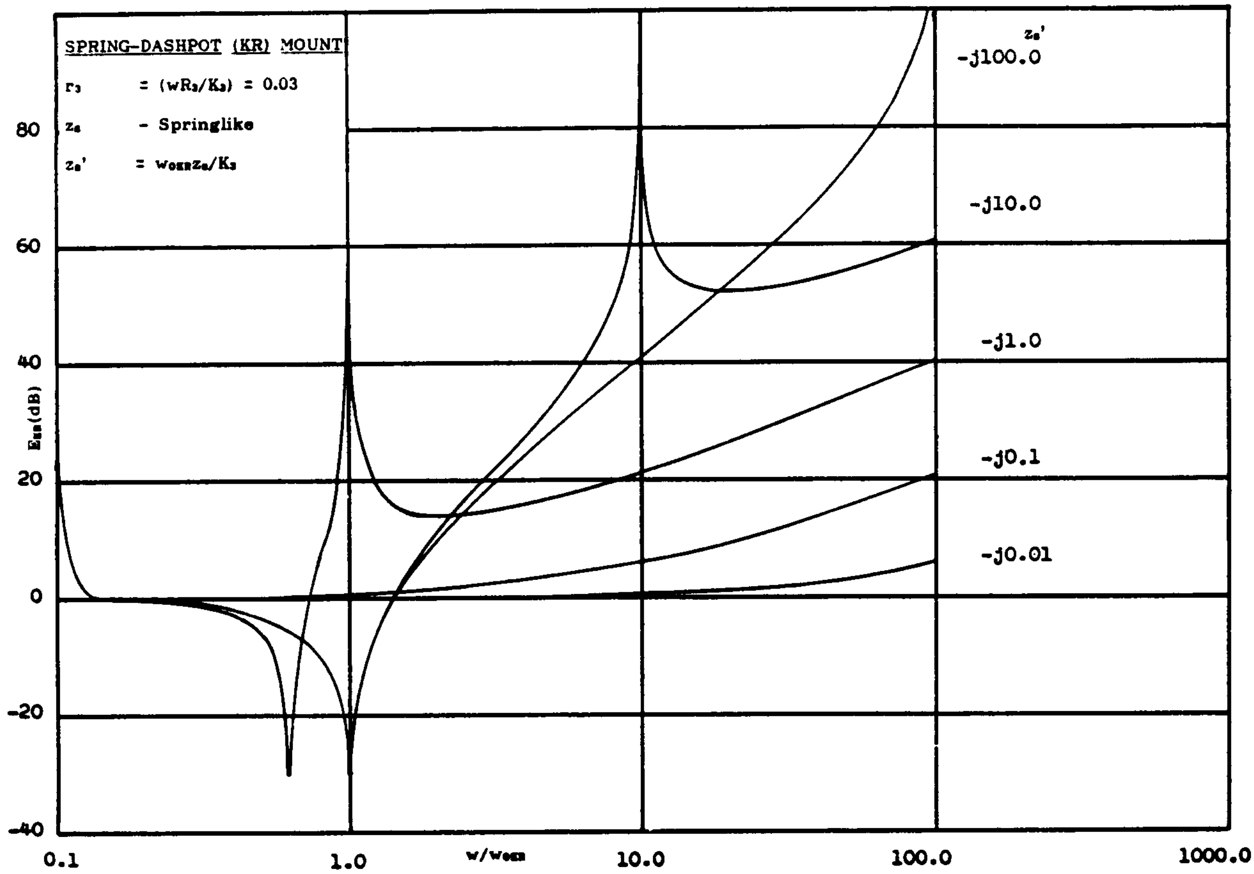
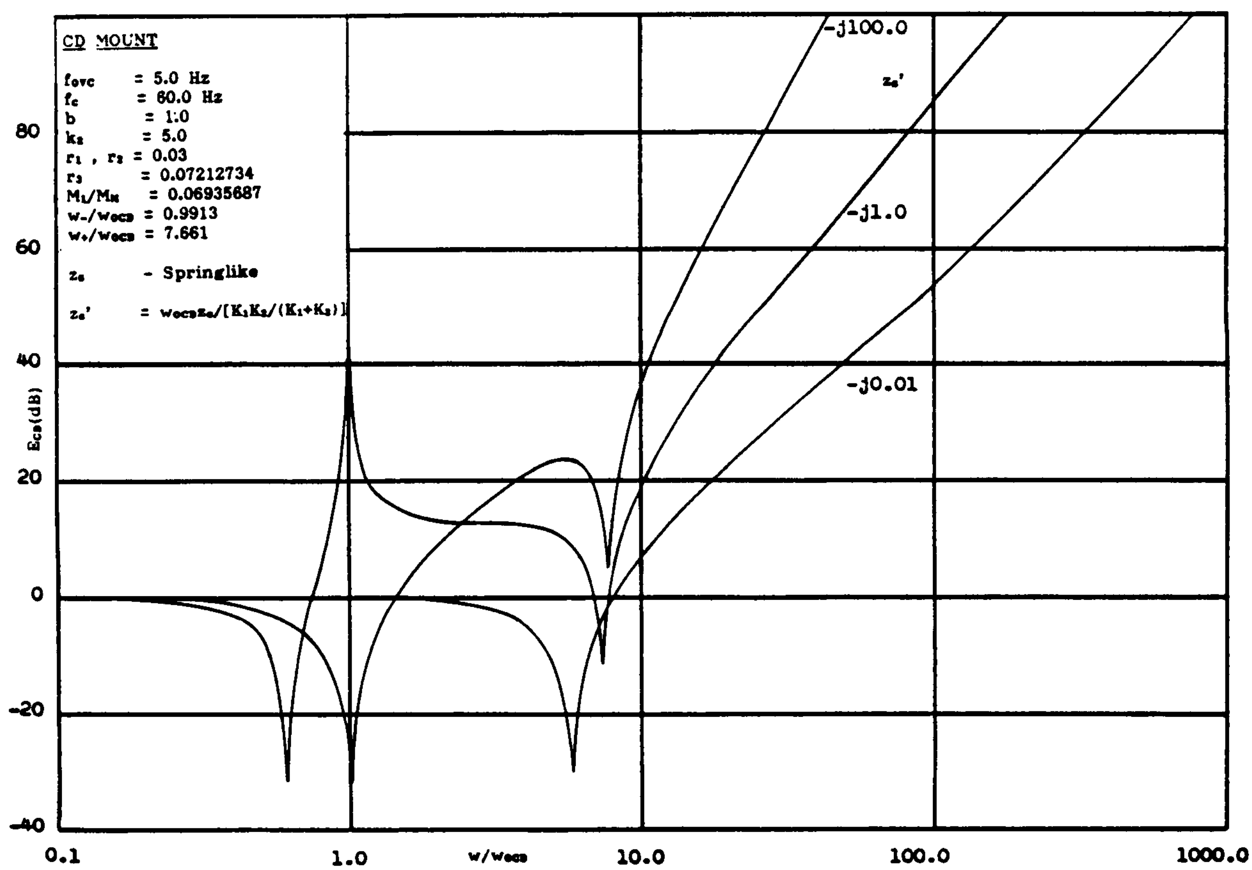


Fig.4 - Mount Effectiveness in dB for KR and CD Mounts; Springlike Supporting Structure.



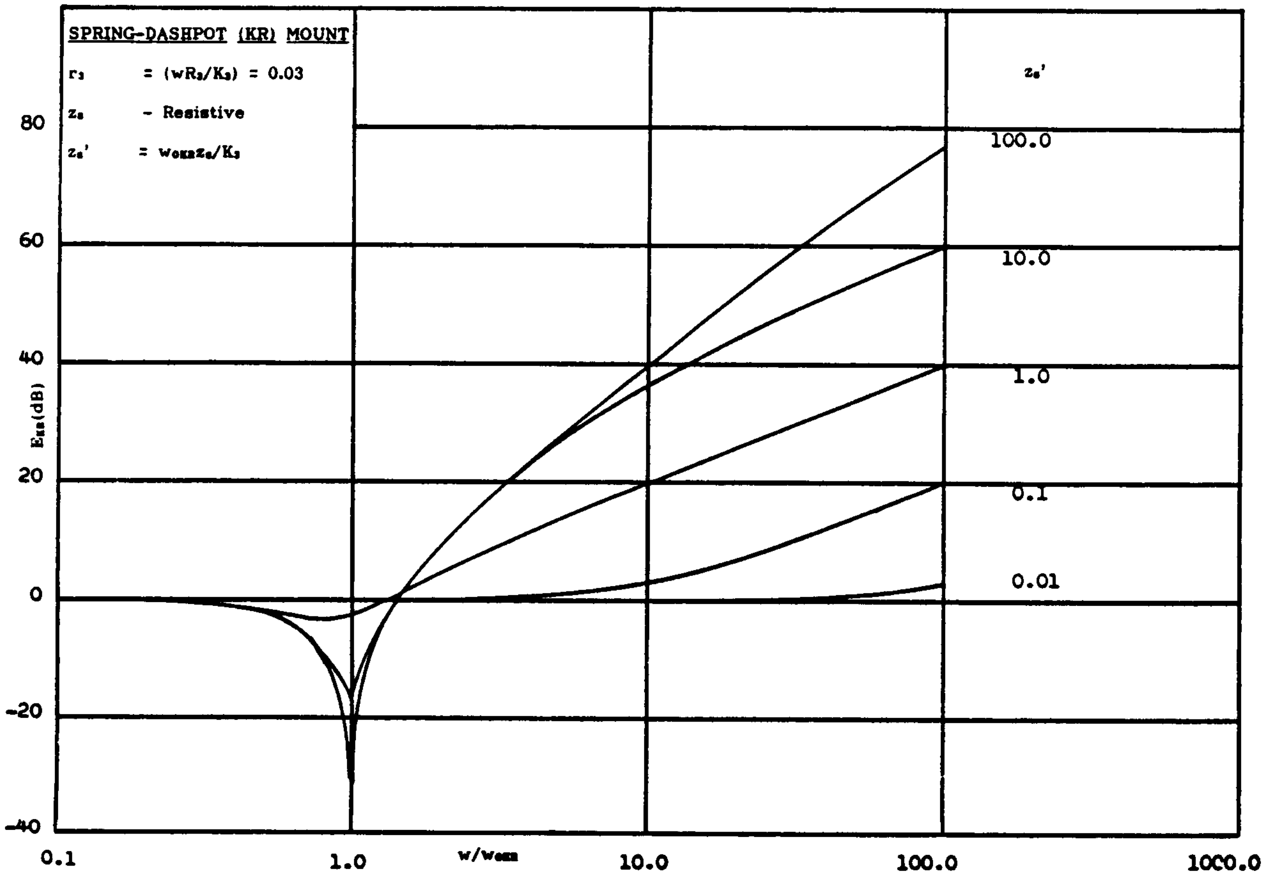
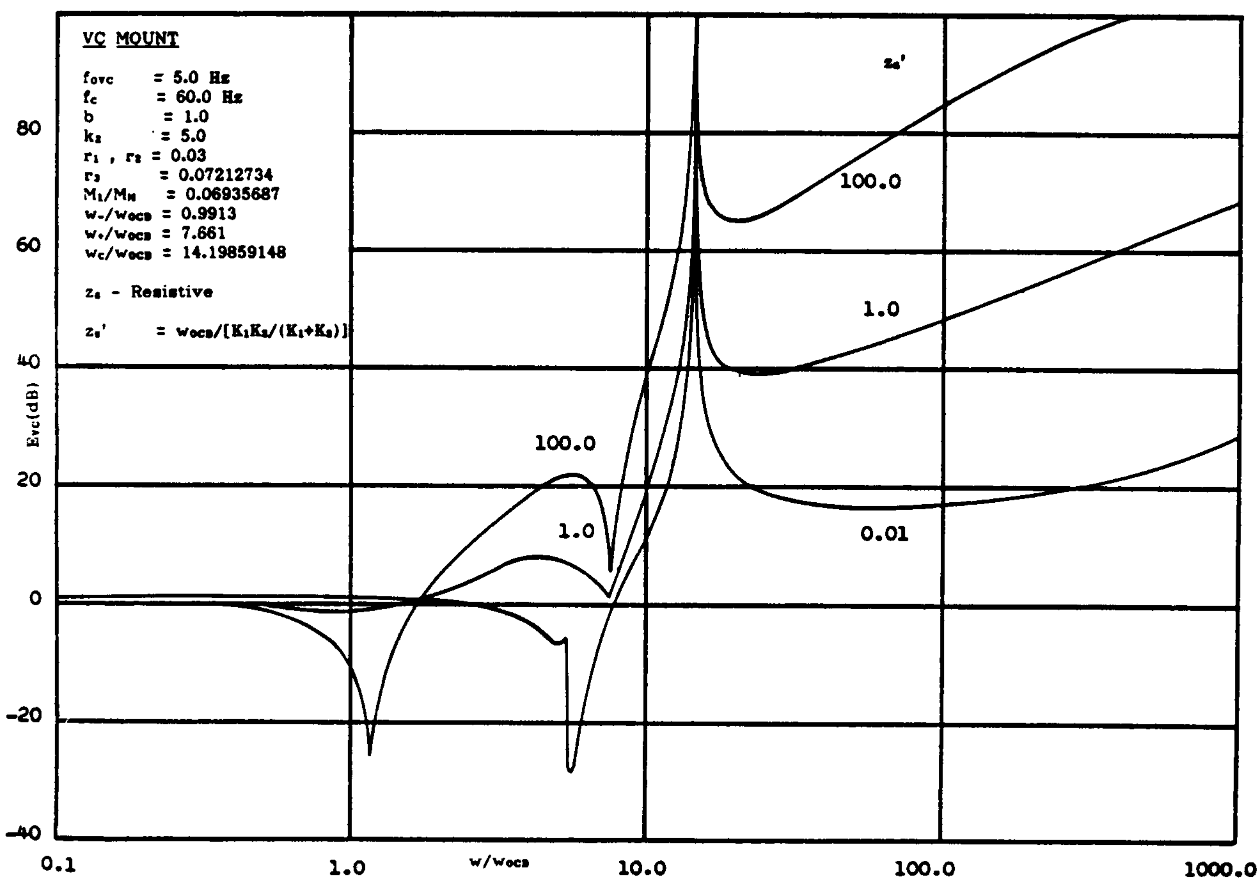


Fig.5 - Mount Effectiveness in dB for KR and VC Mounts; Resistive Supporting Structure.



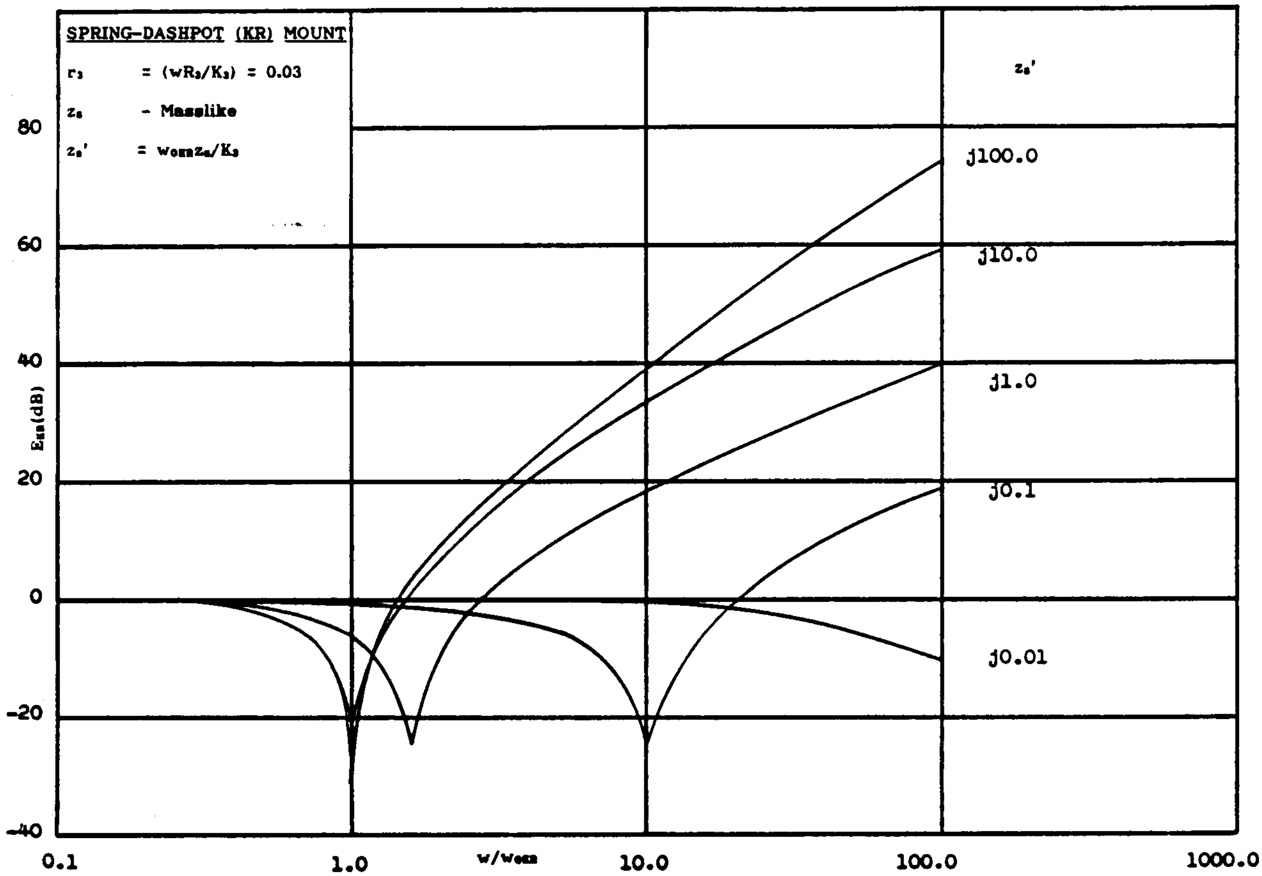
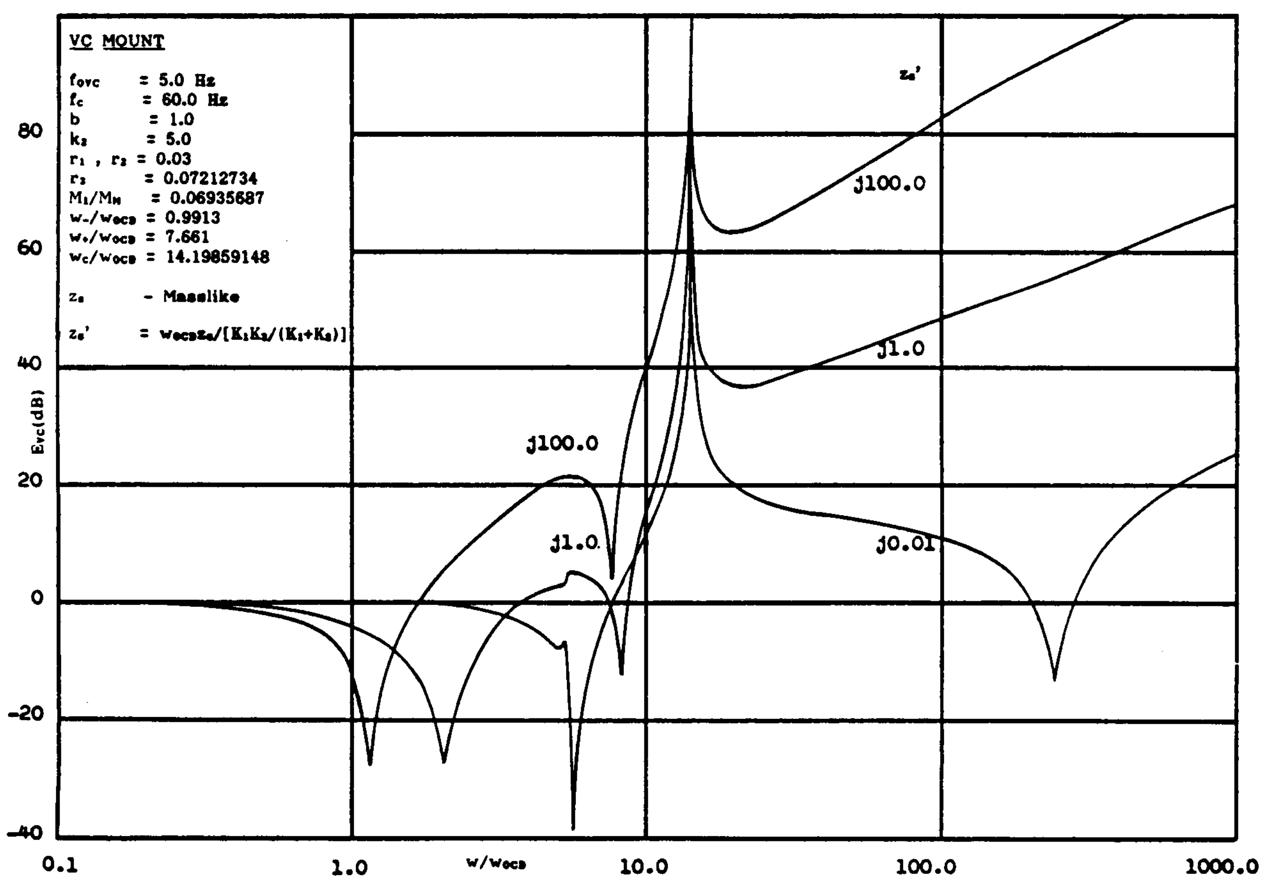


Fig.6 - Mount Effectiveness in dB for KR and VC Mounts; Masslike Supporting Structure.



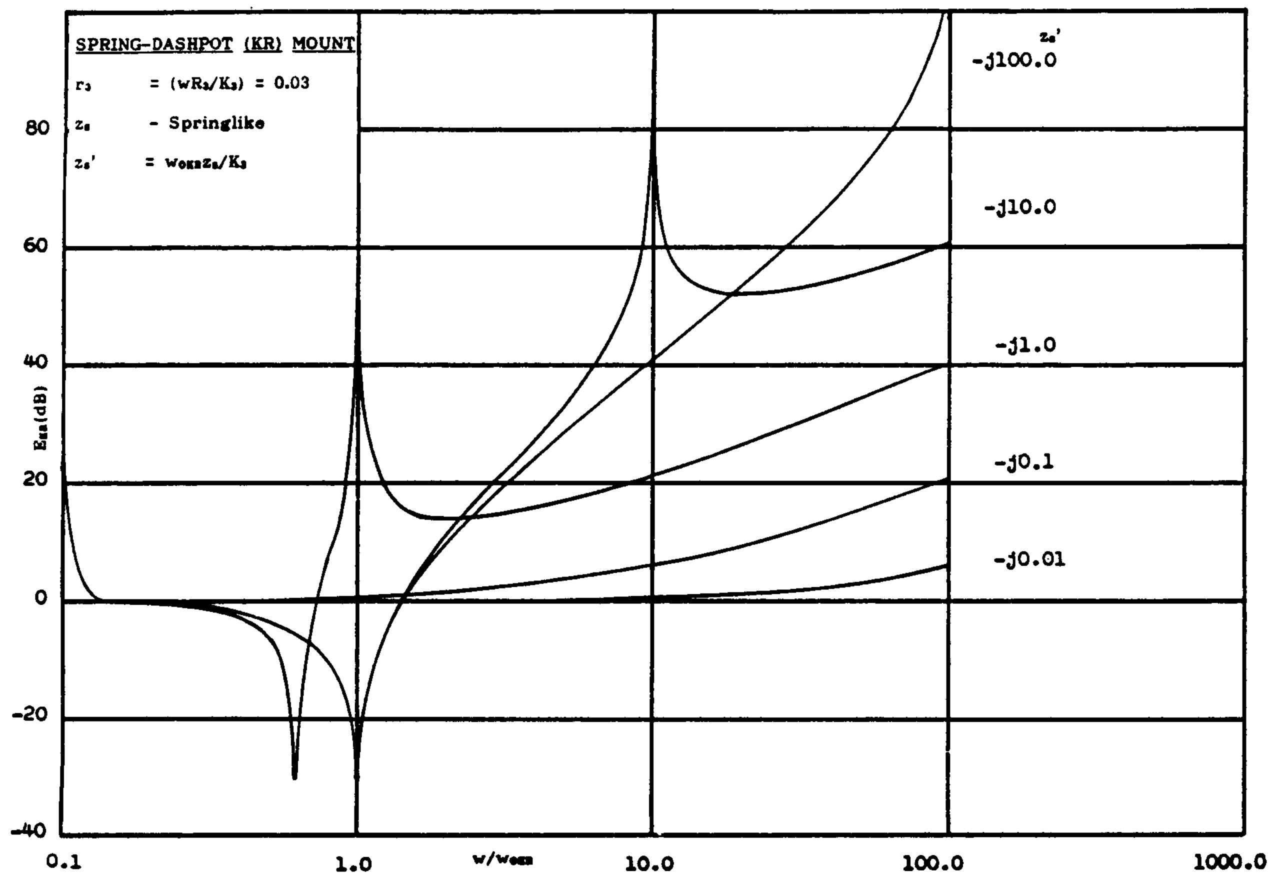
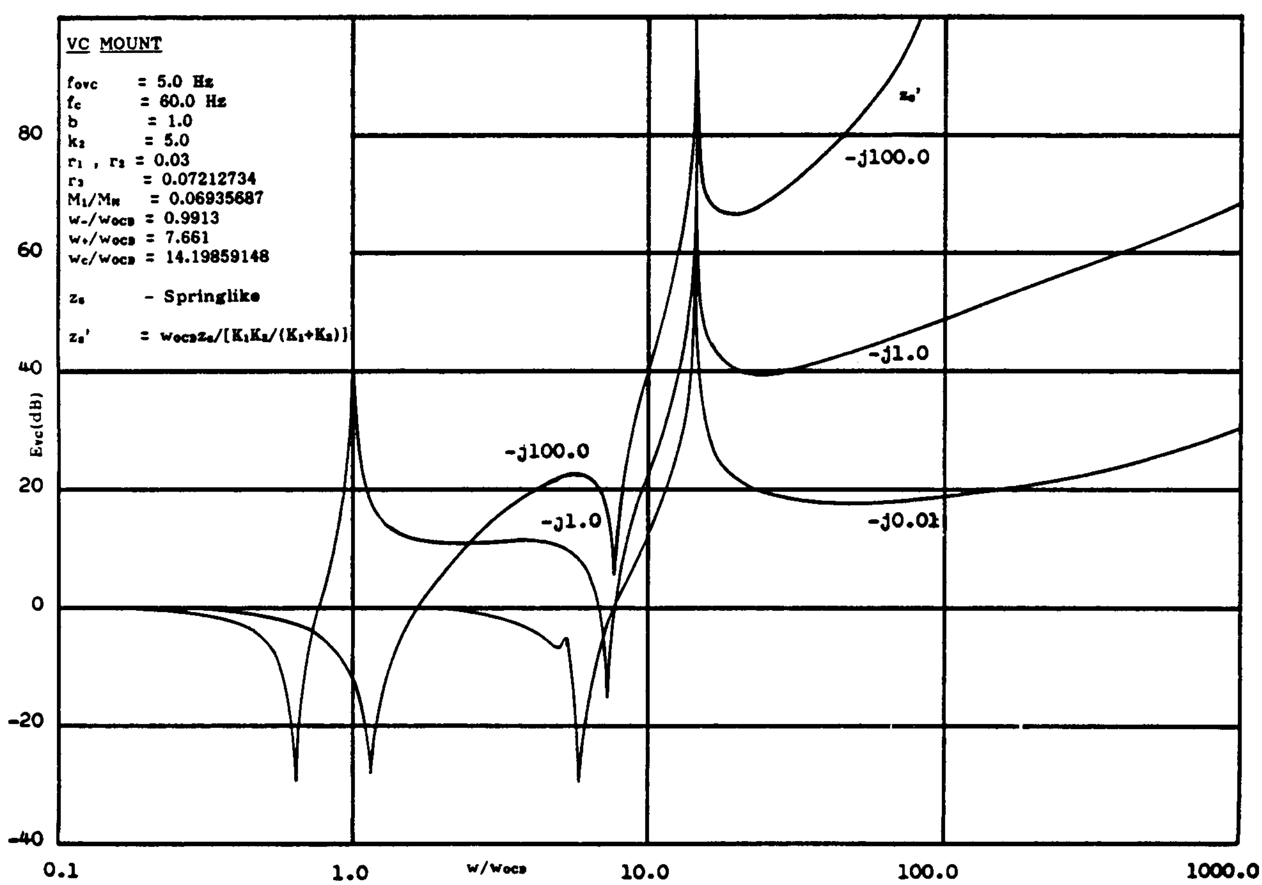


Fig.7 - Mount Effectiveness in dB for KR and VC Mounts; Springlike Supporting Structure.



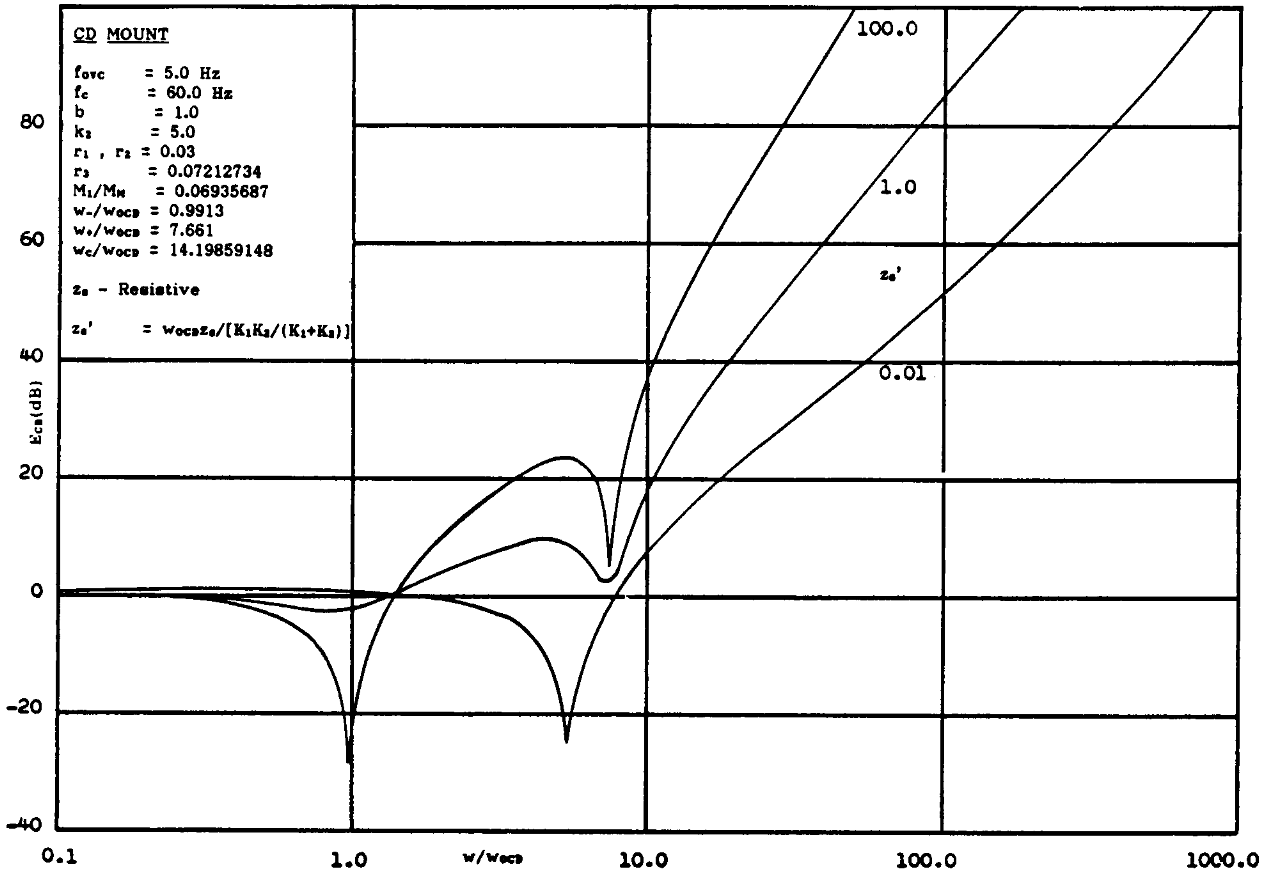
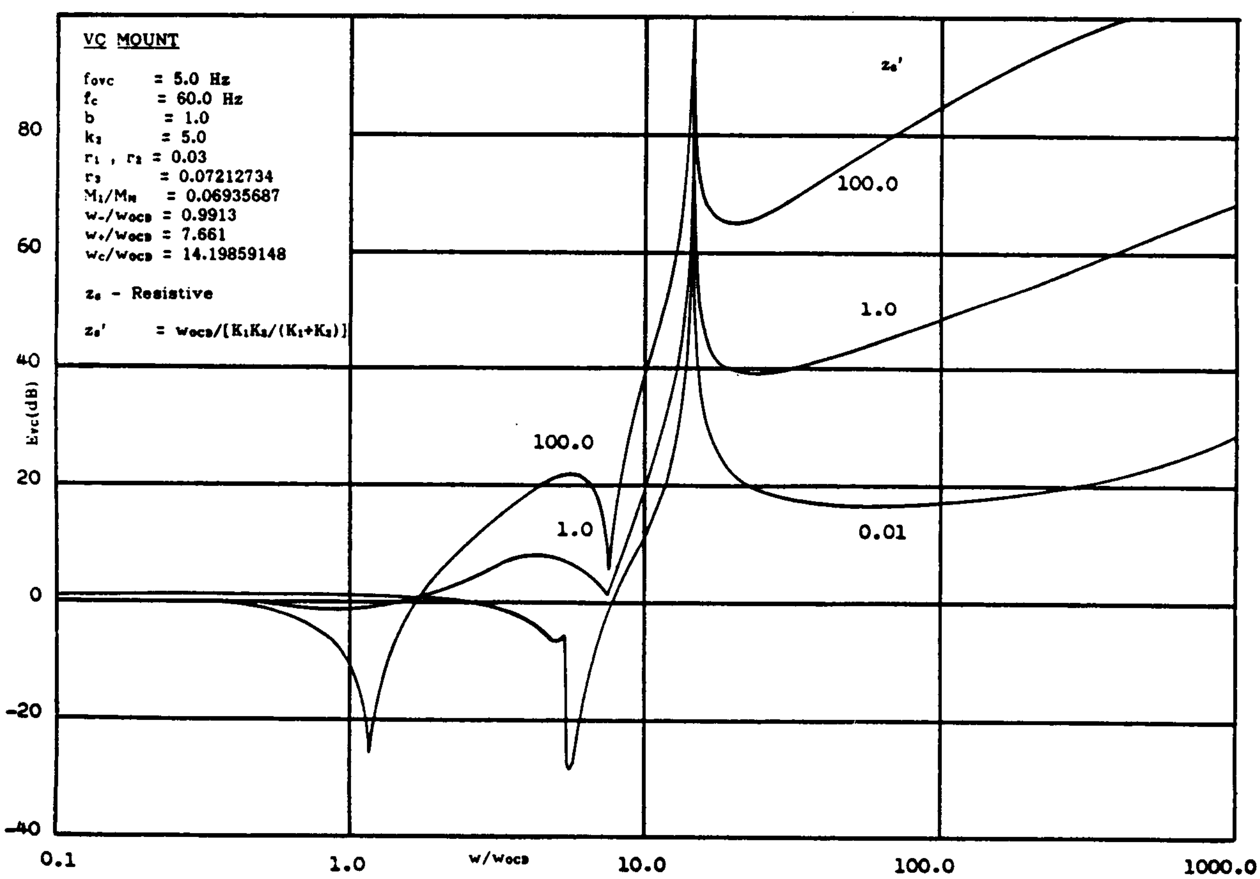


Fig.8 - Mount Effectiveness in dB for CD and VC Mounts; Resistive Supporting Structure.



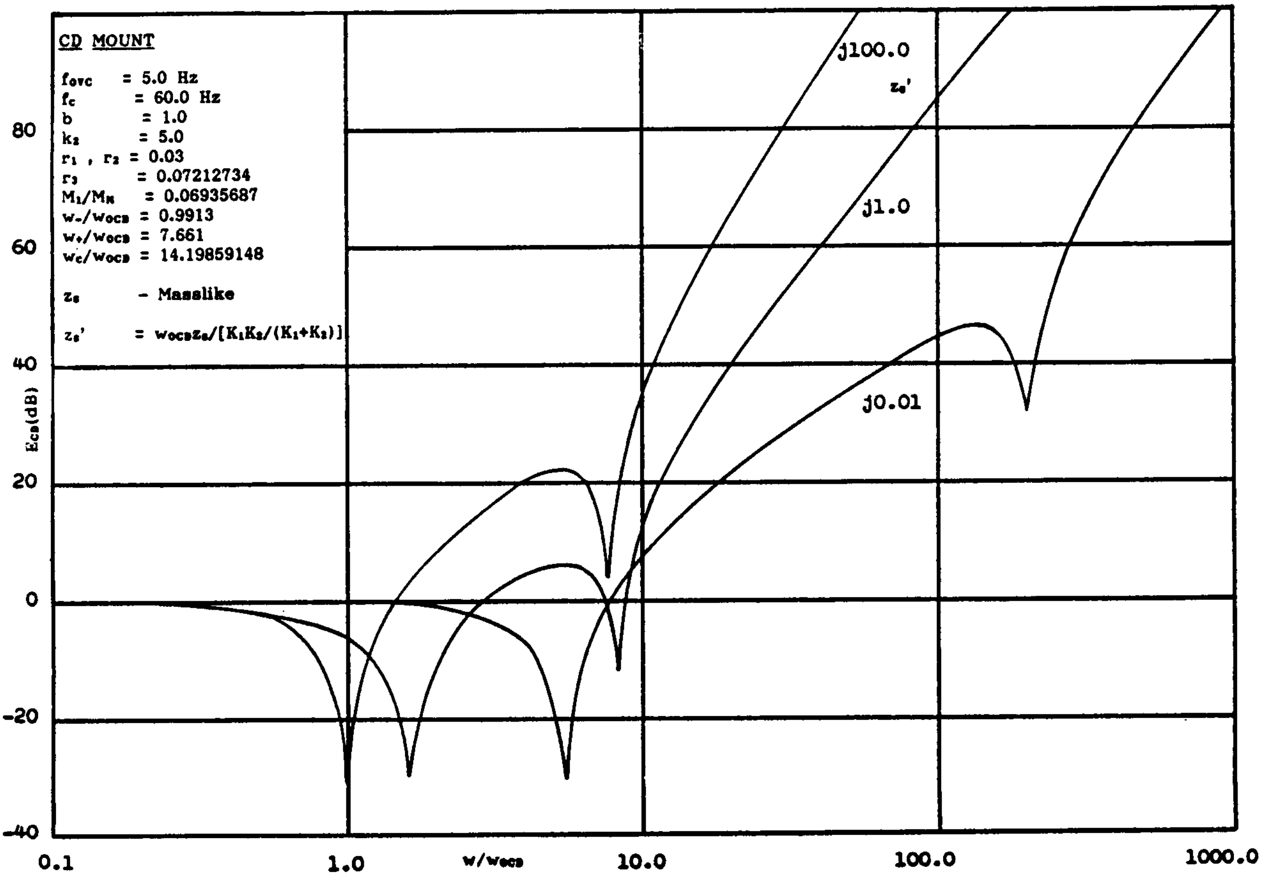
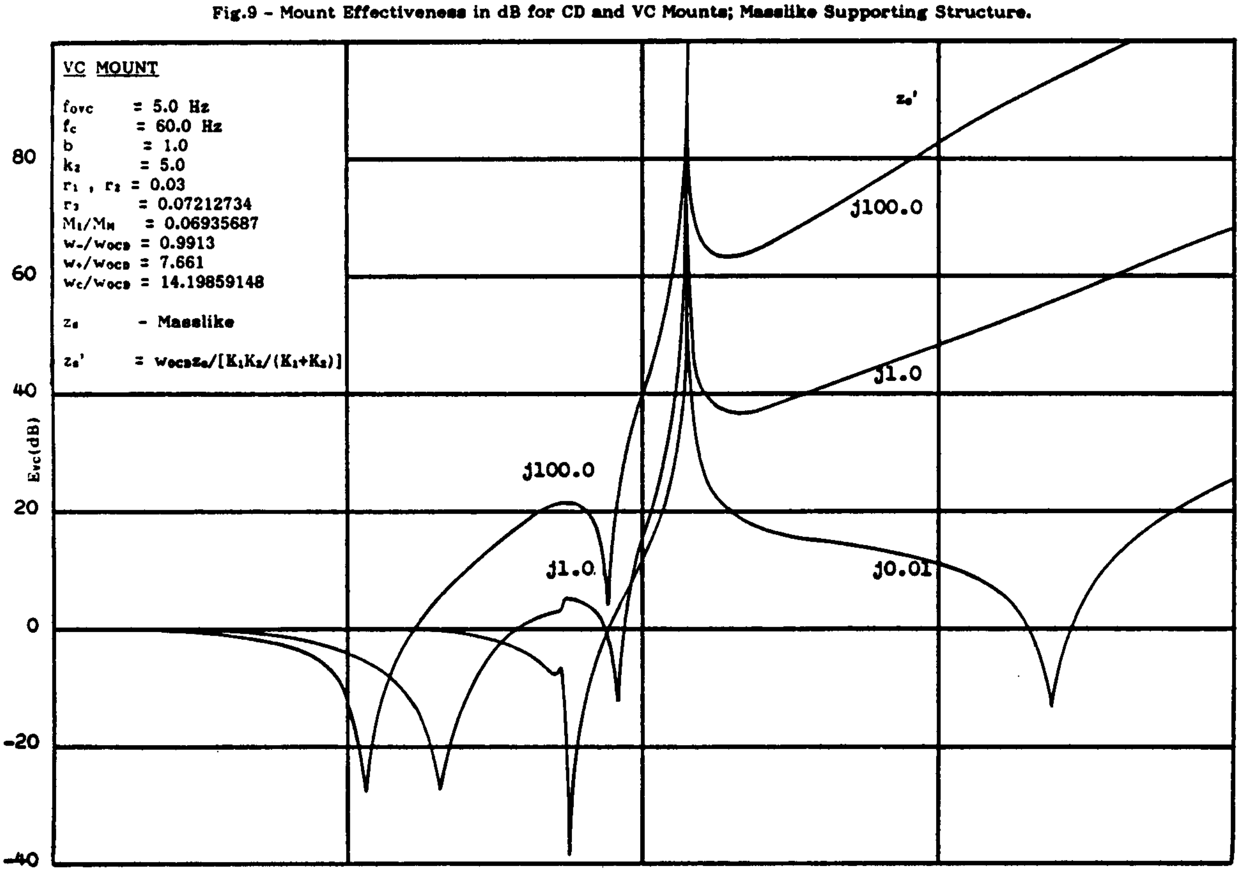


Fig.9 - Mount Effectiveness in dB for CD and VC Mounts; Masslike Supporting Structure.



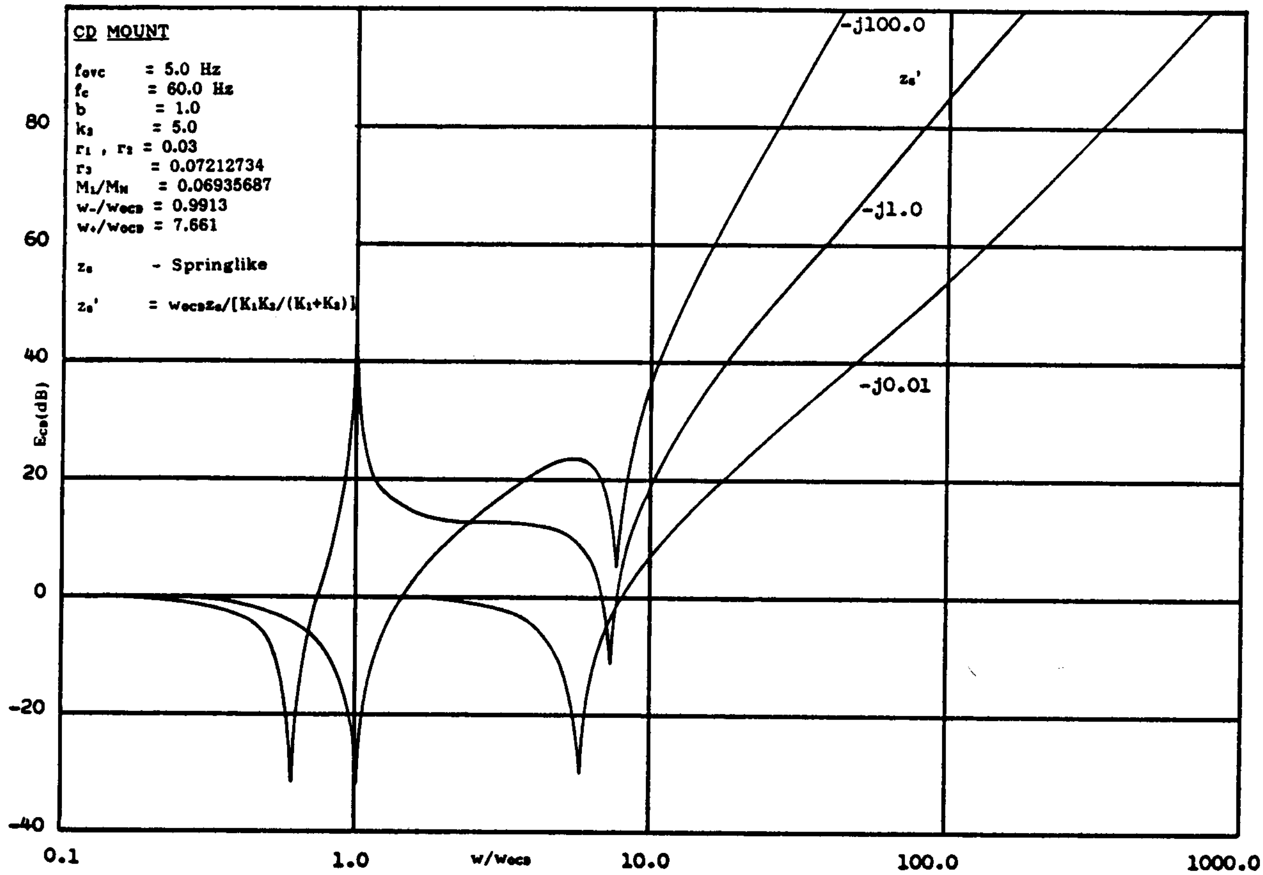
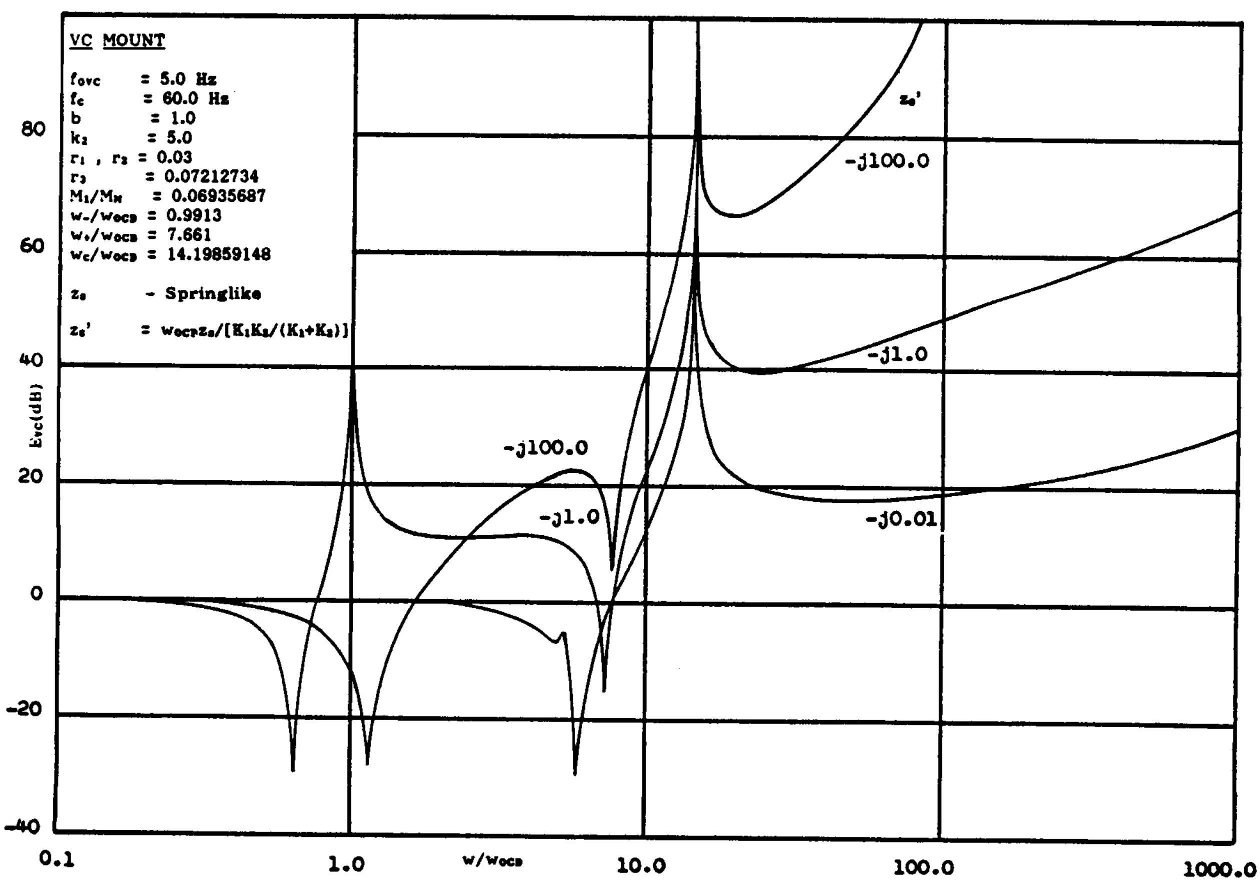


Fig.10 - Mount Effectiveness in dB for CD and VC Mounts; Springlike Supporting Structure.



where z_s is the supporting structure impedance.

For the CD and VC mount effectiveness curves,

$$w_{OCD}^2 = [K_1K_2/(K_1+K_2)]/M_M = [bK_2/(b+1)]/M_M \ , \tag{72}$$

where:

K_1 is the stiffness of the resilient element in the CD mount (CD section of the VC mount) attached to the machine;
K_2 is the stiffness of the resilient element in the CD mount (CD section of the VC mount) attached to the supporting structure;
M_M is th mass of the machine;

$$b = K_1/K_2; \tag{73}$$

$$z_s' = w_{OCD}\ [(K_1+K_2)/K_1K_2]\ z_s = w_{OCD}[(b+1)/bK_2]\ z_s\ . \tag{74}$$

The example chosen for the calculations is the isolation of a machine by a VC mount, the natural frequency f_{ovc} of the machine on the mount being 5.0 Hz, the cancellation frequency f_c being 60.0 Hz. The mount parameters chosen for the calculation were: $b = 1.0$; $k_2 = 5.0$; and $r_1 = r_2 = 0.03$.

Given these data, r_3 and M_1/M_M were calculated from Eqs. (64) and (65). From these equations: $r_3 = 0.07212734$; $M_1/M_M = 0.06935687$.

The effectiveness calculations for the CD and VC mounts were made by substituting Eqs. (45)-(51) into Eqs. (43) and (44) for E_{CD} and E_{VC}, and then expressing $(w/w_1)^2$, $(w/w_2)^2$, $(w/w_{12})^2$ in terms of $(w/w_{OCD})^2$ by means of the relationships:

$$(w/w_1)^2 = (M_1/M_M)[1/(b+1)](w/w_{OCD})^2\ ; \tag{75}$$

$$(w/w_2)^2 = (M_1/M_M)[b/(b+1)](w/w_{OCD})^2\ ; \tag{76}$$

$$(w/w_{12})^2 = (M_1/M_M)[b/(b+1)^2](w/w_{OCD})^2\ . \tag{77}$$

The compound section of the VC mount is treated as the compound mount, the effectiveness of which is to be compared with that of th KR and VC mounts. Using the relationships

$$w_{OVC}^2 = (2\pi f_{OVC})^2 = [K_3+K_1K_2/(K_1+K_2)]/M_M \ , \tag{78}$$

where w_{OVC}^2 is the square of the natural circular frequency of the machine on the VC mount, and Eq. (72) where w_{OCD}^2 is the square of the natural circular frequency of the machine on the CD section of the mount, one can find the relationship between w_{OCD} and w_{OVC}.

$$w_{OCD} = [bk_2/(b+1+bk_2)]^{1/2} \ w_{OVC} \ . \tag{79}$$

Given that b = 1.0 , k_2 = 5.0 , and f_{ovc} = 5.0 Hz, from Eq. 79, f_{OCD} = 4.22 Hz.

The frequency f_{12} at which M_1 resonates with K_1 and K_2, with the input terminal of K_1 and the output terminal of K_2 blocked, can be calculated from Eqs. (61) and (78):

$$w_{12} = 2\pi f_{12} = [(b+1)^2/(b+1+bk_2)]^{1/2} \ [M_M/M_1]^{1/2} \ w_{CVC} \ ; \tag{80}$$

hence, f_{12} = 32.09 Hz.

Comparison of the Effectivenesses of KR, CD, and VC Mounts as a Function of Frequency, and Supporting Structure Resistance or Masslike or Springlike Reactance

From Figs. 2 - 10, the effectivenesses of KR, CD, and VC mounts are strongly dependent on supporting structure impedance. At frequency ratios less than about 10, for nondimensional impedance ratios z_s' = 0.01 , 1.0 , 100.0 , the simple KR mount is more effective than either the CD or VC mount; however, at frequency ratios greater than 10, for z_s' = 1.0 or greater, both CD and VC mounts have greater effectivenesses than the KR mount. In the frequency range from about 10 - 20, the VC mount has significantly higher effectiveness than the CD mount, and this increase in effectiveness would be even greater were the VC and CD mounts to have the same stiffness, i.e., were the natural frequencies of the machine on the two mounts to be the same. The effectiveness of the VC mount at the frequency ratio 14.2, which corresponds to the cancellation frequency, is infinite and independent of the supporting structure and machine impedances. Variations in source frequency of as much as + 5% can be tolerated without reducing the effectiveness of the VC mount below that of KR or CD mounts of the same stiffness because of the relatively broad frequency ratio band over which appreciable cancellation occurs.

For masslike supporting structures, for which $z_s' \ll j1.0$, a third minimum in effectiveness occurs for both CD and VC mounts at a frequency at which the impedance "looking back" into the mount at Terminal 3 is springlike and of equal magnitude to the supporting structure impedance, e.g., at (w/w_{OCD}) = 200 for z_s' = j 0.01 in Figs. 7 and 8. (Whether such minima will occur at high frequency ratios is dependent on the losses -- the mechanical resistance -- of the supporting structure.)

For springlike supporting structures, a maximum in effectiveness occurs at a frequency ratio corresponding to the frequency at which the masslike reactance "looking back" into the machine is equal in magnitude and opposite in sign to the springlke reactance of the supporting structure,e.g., at (w/w_{OCD}) =1.0 for z_s' = -j 1.0 , and for (w/w_{OCD}) = 100 for

$z_s' = -j\ 100.0$ in Figs. 9 and 10. The maximum in effectiveness occurs because introducing a mount between machine and supporting structure eliminates the resonance that occurs in its absence.

VC MOUNT DESIGN

A straightforward procedure has been developed for the design of VC mounts. It is assumed that:

1. The natural frequency f_{ovc} of the machine of mass M_M is specified on the basis of static loading, shock, or other considerations;

2. M_M is known;

3. Appropriate materials and techniques are available for constructing resilient elements z_1, z_2, z_3 having specified stiffnesses K_1, K_2, K_3 and loss factors r_1, r_2, r_3;

4. The stiffness ratio $b = K_1/K_2$ is specified;

5. The cancellation-frequency f_c is specified.

The stiffness K_T of a VC mount with M_1 having negligibly small impedance compared with that of M_M at w_{ovc} is given by -- see Fig. 1 --

$$K_T = K_3 + \frac{K_1K_2}{K_1+K_2}\ . \tag{81}$$

The stiffnesses K_1, K_2, K_3 of resilient elements z_1, z_2, z_3 can be calculated from b, k_2, and K_T which can be determined from $w_{ovc} = 2\pi f_{ovc}$ and M_M.

$$K_T = w_{OVC}^2\ M_M. \tag{82}$$

From Eq. (81) :

$$K_1/K_T = [b+1]k_2/[b+1+bk_2]\ ; \tag{83}$$

$$K_2/K_T = [b+1]k_2/[b+1+bk_2]\ ; \tag{84}$$

$$K_3/K_T = [b+1]/[b+1+bk_2]\ . \tag{85}$$

The loss factor r_3 for resilient element z_3, and the mass M_1 required in the compound section of the mount for cancellation at the frequency $f_c = w_c/2\pi$ can be calculated from w_{ovc}, w_c, b, k_2, r_1, and r_2 from Eqs. (64) and (84) below:

$$r_3 = \frac{r_1b+r_2 + bk_2(r_1+r_2)}{bk_2\,\frac{(1-r_1r_2)}{2}+\left[\left[bk_2\frac{(1-r_1r_2)}{2}\right]^2-[r_1b+r_2][r_1b+r_2+bk_2(r_1+r_2)]\right]^{1/2}}\ ; \tag{64}$$

$$M_1 = (b+1)k_2\left(\frac{w_{OVC}}{w_c}\right)^2\left[\frac{b+1 + bk_2\frac{(1-r_1r_2)}{2}+\left[\left[bk_2\frac{(1-r_1r_2)}{2}\right]^2- [r_1b+r_2][r_1b+r_2+bk_2(r_1+r_2)]\right]^{1/2}}{1+b(1+k_2)}\right]M_M\ . \tag{86}$$

Eqs. (64) and (83) - (86) can be used to calculate VC mount design curves which show the variation of K_1/K_T , K_2/K_T , M_1/M_M and r_3 as a function of k_2 with f_{ovc} , f_c , b , r_1 and r_2 as parameters. A sample set of curves for f_{ovc} = 5.0 Hz, f_c =60.0 Hz, b = 1.0, $r_1 = r_2 = 0.03$ is presented in Fig. 11. Such curves are useful, not so much for detailed design, but for selecting ranges of acceptable values of M_1/M_M , r_3 , K_1/K_T , $K_2//K_T$, and K_3/K_T for specified values of f_{ovc} , b , r_1 , r_2 , and f_c .

From Fig. (11), for the specified values of f_{ovc} , b , r_1 , r_2 , and f_c , resilient material considerations suggest that acceptable mount designs might have stiffness ratios k_2 in the range 0.1 - 2.0 , mass ratios M_1/MM in the range 0.015 - 0.25 , and loss factors r_3 in the range 0.06 - 0.12.

As an example of the use of the curves, for k_2 = 1.0 : (M_1/M_M) = 0.014 ; r_3 = 0.12 ; K_1/K_T , K_2/K_T , K_3/K_T = 0.67 .

CONCLUSION

An analysis of a single-frequency vibration-cancelling (VC) isolation mount, and calculations of its effectiveness as well as those of conventional (KR) and compound (CD) mounts have been presented.

The VC mount, at and near its cancellation frequency, provides substantially greater vibration reduction than either a KR mount, or a CD mount having the same intermediate mass as the CD section of the VC mount, both the KR and CD mounts having the same low frequency stiffness as the VC mount. At its cancellation frequency, its performance is independent of the natural frequency of the machine /mount system, which suggests that single-frequency vibration reduction should be possible with relatvely stiff VC mounts, an advantage for many applications. Its performance at its cancellation frequency is also independent of both machine and supporting structure impedance.

Conceptually, whether a VC , CD , or KR mount should be selected for a particular application depends on the vibration spectrum of the source, and on the machine and supporting structure impedances at the source frequencies. For a complex source spectrum, a careful analysis is required before a selection can be made, but for certain simple source spectra, the advantage of one or another of the three mounts is clear.

For a stable single-frequency source, or a source the spectrum of which is dominated by a single-frequency component, the VC mount is the best choice since its effectiveness at its cancellation frequency, if perfectly designed and manufactured, is infinite.

For a source with a broadband output spectrum, a compound mount is the best choice if the intermediate mass in the mount can be large enough that the higher normal mode

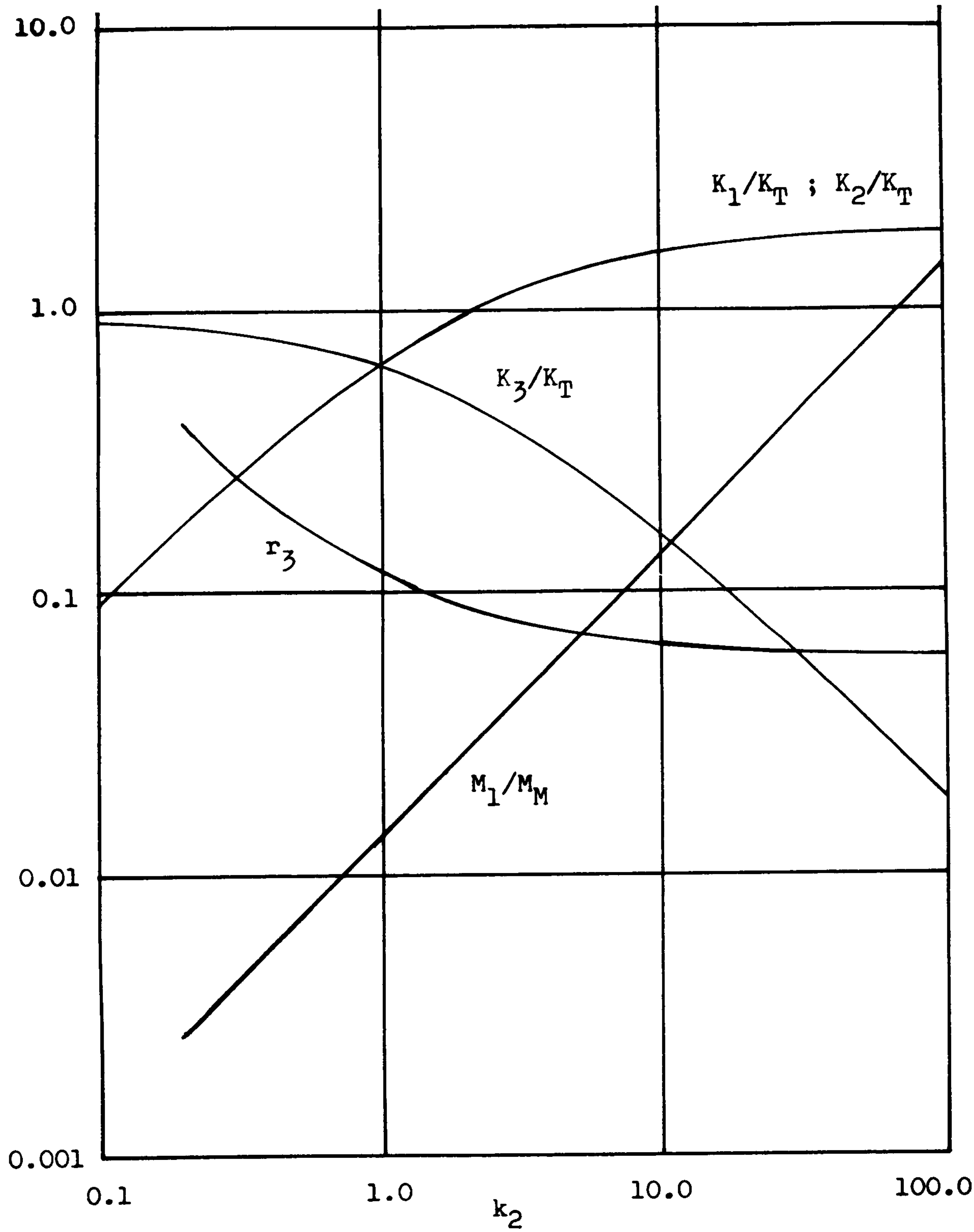


Fig.11 - VC Mount Design Curves. K_1/K_T , K_2/K_T , K_3/K_T , r_3 , and M_1/M_M vs. k_2. f_{ovc} = 5.0 Hz ; f_c = 60.0 Hz ; $b = (K_1/K_2) = 1.0$; $r_1 = r_2 = 0.03$; $k_2 = K2/K_3$; $K_T = [K_1K_2/(K_1+K_2)] + K_3$.

frequency of the machine/mount system, when attached to an infinite impedance supporting structure, is below the frequencies of any important components in the source output spectrum. If restrictions on the size of the intermediate mass place the higher normal mode frequency above the frequencies of important components in the source spectrum, a KR mount may be the best choice.

REQUIREMENTS FOR ADDITIONAL WORK

Although the analysis and calculations presented here establish the feasibility of the VC mount concept for reducing single-frequency vibration, additional analytical and experimental studies are required to facilitate its development for practical applications.

Analytical studies are required to determine:

1. How errors in mount parameters influence mount cancellation-frequency, and effectiveness at and near cancellation frequency;

2. Whether the ideal-element VC mount model is adequate for enginneering purposes, or a more sophisticated model taking account of resilient element mass and intermediate mass compliance is required;

3. The feasibility of multifrequency VC mounts;

4. How the ratio of the stiffnesses of the resilient elements in the CD section of the mount, their loss factors, the natural frequency of the machine/mount system, and the cancellation frequency influence the mass ratio required for cancellation, the normal mode frequencies of the machine/mount system both for low and high supporting structure impedances, and the loss factor for the paralleled section of the mount.

Experimental studies are required to:

1. Test and validate the single-frequency single-degree-of-freedom VC mount concept;

2. Evaluate various concepts for two and three degree-of-freedom VC mounts;

3. Determine how closely the stiffness and loss factors of resilient elements can be predicted and controlled, what variations in the properties of resilient elements can be expected as functions of time, temperature, temperature gradient, and from element-to-element and batch-to-batch. (These studies will determine whether VC mounts can mass-produced, or will require a mount-by-mount "tuning" process.

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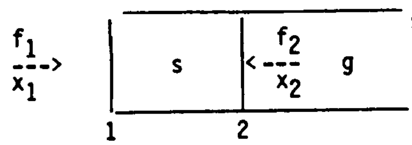
Appendix 1 - The Admittance and Impedance of Two Arbitrary Two-Terminal Mechanical Elements in Tandem



The equation of motion for small amplitude vibration for the arbitrary two-terminal s can be written in phasor form either as:

$$x_1 = y_{s11}f_1 + y_{s12}f_2 \quad ; \tag{A-1}$$

$$x_2 = y_{s21}f_1 + y_{s22}f_2 \quad ; \tag{A-2}$$

or as

$$f_1 = z_{s11}f_1 + z_{s12}f_2 \quad ; \tag{A-3}$$

$$f_2 = z_{s21}f_1 + z_{s22}f_2 \quad . \tag{A-4}$$

Noting from Newton's law of action and reaction that

$$-f_2 = z_g x_2 \ , \tag{A-5}$$

and substituting from Eq. (A-5) into Eq. (A-4),

$$x_2 = - \frac{z_{s21}}{z_{s22} + z_g} x_1 \ . \tag{A-6}$$

Substituting from Eq. (A-6) into Eq. (A-3),

$$f_1 = z_{s11} - \frac{z_{s21}z_{s12}}{z_{s22} + z_g} \; x_1 \; ; \tag{A-7}$$

hence, the drive-point impedance $z_{d.p.}$ at Terminal 1 for s and g in tandem is given by

$$z_{d.p.} = z_{s11} - \frac{z_{s21}z_{s12}}{z_{s22} + z_g} \; . \tag{A-8}$$

Rearranging terms,

$$z_{d.p.} = \frac{z_{s11}\,(z_{s22} + z_g) - z_{s21}z_{s12}}{z_{s22} + z_g} = z_{s11}\,\frac{(z_{s22} - z_{s21}z_{s12}/z_{s11}) + z_g}{z_{s22} + z_g} \; . \tag{A-9}$$

Noting from Eqs. (A-1) – (A-4) that

$$z_{s22} - z_{s21}z_{s12}/z_{s11} = 1/y_{s22} \; , \tag{A-10}$$

Eq. (A-9) can be written

$$z_{d.p.} = z_{s11}\,\frac{(1/y_{s22}) + z_g}{z_{s22} + z_g} \; . \tag{A-11}$$

A similar derivation based on Eqs. (A-1) and (A-2) will show that the drive-point admittance $y_{d.p.}$ at Terminal 1 for s and g in tandem is given by

$$y_{d.p.} = y_{s11}\,\frac{(1/z_{s22}) + y_g}{y_{s22} + y_g} \; . \tag{A-12}$$

Eqs. (A-11) and (A-12) are particularly useful because drive-point impedance and impedance can be calculated without knowledge of point-to-point impedance or admittance. They also provide insight into how the mass of real resilient elements and the stiffness of real masses influence drive-point impedance and admittance.