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METEOR VELOCITY DISTRIBUTION AND AN OPTIMUM MONITORING MATHEMATICAL MODEL

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There are not many papers devoted to an optimization of meteor observations. For example, the optimization of the azimuth of antenna orientation of a 2 or 3-station network is given in MALYNJAK (1982). Optimization methods also have been used to determine the main parameters of a meteor body and the atmosphere in the meteor zone in KOSTYLEV (1982). At present, there are a great number of radio meteor, ionosphere and rocket observation data for the altitude range of 80-100 km which indicate the existence of large scale circulation systems in the mesopause-low thermosphere range which change regularly with season and latitude. But the existing observation network and observation programs are not optimal for revealing the main factors forming the circulation mode at these altitudes.

This paper offers for consideration a generalized optimum monitoring mathematical model. The model input data are distribution density, response function, individual measurement root mean square uncertainty and detection effectiveness function. The minimization of the prediction variance due to upper atmosphere ionization and impacts by sporadic meteors may also serve as the effectiveness function (SALIMOV et al., 1979). In the latter case, the results of a previously developed meteor thermal fragmentation model and laboratory data may be used (KOSTYLEV, 1982; LEBEDINETS et al., 1967; LEBEDINETS et al., 1968; MALYNJAK, 1982; SALIMOV, et al., 1979). The model makes it possible to obtain the observation distribution density, the minimal possible dispersion and optimized system effectiveness.

Suppose that $n(X)dX$ is the number of observed value (OV) magnitudes in the interval OV $(X, X + dX)(\int n(X)\, dX = N)$ that is connected with some effect by the relation $h = f(X)$. The purpose of monitoring is to achieve a certain value of the function $h(X)$ (denoted H)

$$H = \frac{1}{N} \int f(X)\, n\,(X)\, dX$$

by means of OV detection with sampling of the fixed size M. Let $m(X)\, dX$ be the monitoring sampling size $(\int m(X)\, dX = M)$. The problem of monitoring optimization is to find the $m(X)$ function which achieves the least dispersion in the assessment of the value H.

It can be easily proved that dispersion of the value H with fixed $m(X)$, M and $f(X)$ is

$$D_H = \sigma_H^2 = \frac{1}{N^2} \int \left(\frac{df}{dX}\right)^2 \sigma_x^2 \frac{n^2(X)}{m(X)}\, dX \quad ,$$

where σ_x^2 is the dispersion of the OV measurement. The minimization of this expression on condition that $\int m(X)\, dX = M)$ leads to an $m(X)$ function of the following form

$$m(X) = \frac{M}{\int \left|\frac{df}{dX}\right| n(X)\ \sigma_x dX} \left|\frac{df}{dX}\right| n(X)\ \sigma_x$$

When the f(X) function has some inherent uncertainty then this formula is valid. Only the minimum value and optimization effectiveness are changed.

The relation for the minimum dispersion of the H value assessment of optimized system is

$$D_{Hmin} = \frac{1}{M} \left(\int_0^\infty \left|\frac{df}{dX}\right| w\ \sigma_x\ dX\right)^2$$

where $w = w(X)$ is the distribution density (i.e., weights) of OV. The parameter for characterizing optimization effectiveness is

$$E = \frac{D_H}{D_{Hmin}} = \frac{\int \left(\frac{df}{dX}\right)^2 \sigma_x^2 \frac{w^2}{m_n} dX}{\left(\int \left|\frac{df}{dX}\right| w\sigma_x dX\right)^2}$$

Two cases were considered: m_n = *const and* $m_n = w$ where m_n is the observation distribution for the unoptimized system.

For the case of a power law response function $f = \alpha X^\beta$ and a constant relative error of measurement (not depending on OV) δ we have

$$m_n = \begin{cases} 1/(X_M - X_m) & X_m < X < X_M \\ 0 & X < X_m,\ X > X_M \end{cases}$$

$$m \sim X^\beta w, \qquad E = a_\beta (X_M - X_m)/m_\beta^2$$

where $a_\beta = \int_{X_m}^{X_M} (X^\beta w)^2 dX\ ,\quad m_\beta = \int_{X_m}^{X_M} X^\beta\ w dX$

X_m and X_M are the minimum and maximum magnitudes of OV respectively.

For the case $m_n = w$ and a log normal OV distribution density

$$\ell n\ E = \beta^2\sigma^2 \quad , \quad E = \exp(\beta^2\sigma^2)$$

The relative error of effectiveness determination in this case is calculated as follows:

$$\delta_E = 2(\beta^2\delta_\beta + \sigma^2\delta_\sigma),$$

$$\delta_E = \frac{dE}{E}, \quad \delta_\beta = \frac{d\beta}{\beta}, \quad \delta_\sigma = \frac{d\sigma}{\sigma} .$$

So $$E = \exp(\beta^2\sigma^2)[1 \pm 2(\beta^2\delta_\beta + \sigma^2\delta_\sigma)] .$$

To implement the derived relations for real meteor observations, the radio meteor data obtained from Obninsk in 1967 - 1968 and which were published by the World Data Center B in 1981 - 1982 were analyzed (LEBEDINETS et al., 1967; LEBEDINETS et al., 1982). The results of observations during one full year were processed. The number of detected meteors in different months varied from 10^2 in September to $3{,}3 \cdot 10^3$ in December of that year. The distribution density of meteor body extra-atmospheric velocities was recorded.

Suppose that the OV is the meteor body velocity and the predicted effect is the atmospheric ionization at a certain altitude. To simplify the problem for the sake of demonstration, we shall consider $\beta = 6$ and assume that the effect does not depend on other variables except velocity. The distribution density of the number of measurements was calculated using the following formulae:

$$\bar{v}_i = \frac{v_i + v_{i+1}}{2}, \quad N = \Sigma N_i, \quad m_\sigma = \Sigma \bar{v}_i^{\sigma} w_i; \quad m_i = \frac{\bar{v}_i^{-\sigma} w_i}{m_\sigma},$$

$$M = N/E, \quad E = \frac{ka_\sigma}{M_\sigma}, \quad a_\sigma = \Sigma(\bar{v}_i^{-\sigma} w_i)^2, \quad k = \frac{v_m - v_m}{\Delta v}, \quad M_i = Mm_i .$$

The results of the calculations are given in the Table.

Table 1

Observation distribution density.

NN	v_i-v_{i+I} kms	$\bar{v}_i$ kms	w_i	$m_i\Delta v$	N_i	M_i	M_i/N_i
1	12-17	14.5	0.014	0.0000	131	0	0.00
2	17-22	19.5	0.064	0.0002	599	1	0.00
3	22-27	24.5	0.068	0.0006	636	2	0.00
4	27-32	29.5	0.072	0.002	674	7	0.01
5	32-37	34.5	0.125	0.009	1170	31	0.03
6	37-42	39.5	0.129	0.022	1207	76	0.06
7	42-47	44.5	0.087	0.030	814	103	0.13
8	47-52	49.5	0.069	0.045	646	155	0.24
9	52-57	54.5	0.108	0.125	1011	430	0.42
10	57-62	59.5	0.127	0.248	1188	853	0.72
11	62-67	64.5	0.096	0.304	898	1046	1.16
12	67-72	69.5	0.035	0.174	328	599	1.83
13	72-77	74.5	0.004	0.030	37	103	2.78
14	77-82	79.5	0.001	0.011	9	38	4.22

k=14 $\Sigma w_i = 0.999$ $\Sigma m_i\Delta v = 1.001$, $\Sigma N_i = 9348$, $\Sigma M_i = 3444$

$N = 9358$ $E = 2.72$, $M = 3440$

The effectiveness in this case is 2.7. This means that the number of observations can be reduced 2.7 fold without reducing the accuracy of the predicted results. When v > 52 kms the optimum observation distribution density becomes greater than the OV distribution density. When v > 62 kms, the number of meteors needed for each velocity range exceeds the number of detected meteors (the boundary of sampling and complete monitoring). The $N_i = N_i(v)$ function is bimodal and had two maxima (with 40 and 60 kms). The $M_i = M_i(v)$ function has one maximum when v = 64 kms. The M_i/N_i ratio is the monotonically increasing function of velocity.

If we consider the dependence of velocity on space coordinates and time we can derive recommendations concerning the observational distribution in space and time. For the cases when the predicted effect depends not on one but two (velocity and mass, for example), a multiple variable calculation is carried out in a similar way for such 2-dimensional or multi-dimensional problems. The described approach may be implemented for other effectiveness functions, such as wind speed determination at an altitude of 80-100 km through meteor observations.

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