ACTIVE CONTROL OF FLEXURAL VIBRATIONS IN BEAMS

Final Report

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ABSTRACT

The feasibility of using piezoelectric actuators to control the flexural oscillations of large structures in space is investigated. Flexural oscillations are excited by impulsive loads such as Shuttle docking. The vibratory response can degrade the pointing accuracy of cameras and antennae, and can cause high stresses at structural node points. Piezoelectric actuators have the advantage of exerting localized bending moments. In this way, vibration is controlled without exciting rigid body modes in the structure. The actuators are used in collocated sensor/driver pairs to form a feedback control system. The sensor produces a voltage that is proportional to the dynamic stress at the sensor location, and the driver produces a force that is proportional to the voltage applied to it. The analog control system amplifies and phase shifts the sensor signal to produce the voltage signal that is applied to the driver. The feedback control system is demonstrated to increase the first mode damping in a cantilever beam by up to 100 percent, depending on the amplifier gain. An analytical model of the control system has been developed. The estimated and measured vibration control compares favorably. A simulated free-free beam has been fabricated and instrumented with a distribution of piezoelectric sensor/drivers. The purpose is to evaluate the damping efficiency of the control system when the piezoelectrics are not optimally positioned at points of high stress in the beam. The control system is found to reduce the overall vibration response to impact by a factor of two.

INTRODUCTION

Oscillations in the large, flexible structures in space are induced by impact such as docking operations. Flexural vibration transmitted through the structure can degrade the pointing accuracy of devices such as cameras. Transmitted vibrations can degrade the microgravity environment in modules containing vibration sensitive equipment. Large oscillations in the structure can cause high stresses to occur within the structure and at node points. In the extreme case, flexural vibration can cause the structure to go into a rigid body tumbling mode. The oscillatory response of the structure can be controlled through material damping built into the structural elements, or by increasing the mass of the structural elements. However, addition either of mass or damping material imposes a weight penalty, which increases construction cost because of the added weight to be delivered into space.

It is proposed to develop an active control methodology to limit the vibratory response of a large, flexible structure. The purpose of the research conducted in the Summer Faculty Fellowship program is to establish the suitability of piezoelectric actuators to control bending in flexible structures. The piezoelectric actuators are used in pairs, bonded at opposite and equal distances from the neutral axis of the elastic member. When the elastic member is excited by a transient force, the free vibration causes bending moments which produce local stresses along the member's surface. One element senses the local stress and produces a proportional voltage. The voltage signal is amplified and shifted in phase by 180°.

This conditioned signal drives the companion piezoelectric element to produce a localized moment in the elastic member to oppose the dynamic bending. Piezoelectric actuators apply moments locally. Thus, they can control oscillations without exciting rigid body modes in the elastic member, as may be produced by reaction jets.

Piezoelectric actuators have been demonstrated to produce significant flexural control in cantilevered beams, where the sensor/driver pair are bonded near the root of the beam. It is known that the bending stresses are maximum near the root in all modes of vibration. Thus, the control is expected to be optimum for the sensor/driver pair mounted at this known point of high stress. In the current project, a distribution of piezoelectric element pairs is bonded on a free-free beam and the vibration control efficiency is evaluated. The element locations do not necessarily coincide with positions of highest bending stress in the beam. The goal is to determine the efficiency of the piezoelectric actuators when they are not optimally placed at locations of maximum bending stress. This configuration is chosen because it may not be possible to estimate accurately the bending modes of a structure with many substructures and appendages, such as a space structure. This is particularly true since the design is expected to be dynamic with substructure modifications made throughout its mission duration. The travel of the remote manipulator arm trolley also means that the mass distribution of the structure can be changing continuously, even between design changes.

When the sensor/driver pairs are collocated, that is equidistant from the neutral axis, the bending moment exerted by the driver is directly proportional to the stress at the sensor. Thus, the feedback control system is independent of the mode shape of the elastic member. The only expected limitation of the sensor/driver pair to control vibration in the beam is the amplification required between the sensor and driver. Tests have shown that the sensor voltage must be amplified by a factor on the order of 150 to control vibrations in a cantilever beam. Theoretically, any magnitude of amplification is achievable. Higher orders of gain may be required to control oscillations when the sensor/driver pair is not located at the positions of high stress in the beam where the voltage produced by the sensor is high. However, it is also found that the maximum voltage that can be applied to drive the piezoelectric element is approximately 50 volts (A.C.). Above this limit, the ratio of moment produced by the element to applied voltage decreases, and at very high applied voltage, the element depolarizes. Power considerations are important in the space applications. The piezoelectric elements require relatively large voltages up to 50 volts, but the current they draw is on the order of micro-amps. Thus, the power consumed by the vibration control system is not expected to be prohibitive. For example, if each transducer requires 50 volts and draws 10 micro-amps, the power consumed by 2000 sensor/driver pairs is 1 Watt.

BACKGROUND

A perfectly bonded piezoelectric may be modeled as being pinned at either end of the element. A force proportional to the applied voltage is exerted through these pins to the structure. Expressed in terms of modal

parameters, the equation of motion is of the form:

$$M*\ddot{q} + C*\dot{q} + (K+K_p)*q = F \qquad (1)$$

where:
M= the modal mass
C= the modal damping inherent in the structure
K= the modal stiffness
K_p= the stiffness of the piezoelectric element (typically small in comparison to the modal stiffness of the structural member)
q= the modal displacement
F= the force exerted by the piezoelectric element transformed into modal coordinates.

The expression for the force exerted is derived by Crawley and deLuis [1], and is based on the force term shown for a monomorph motor application in the Program Module for Piezo Design Aid [2]. The force has the form:

$$F = K_1*d_{31}*V* \left(\frac{d\phi}{dx}\bigg|_{a+h} - \frac{d\phi}{dx}\bigg|_{a-h}\right) \qquad (2)$$

where:
K_1= a constant incorporating physical parameters of the structural member and of the piezoelectric element, such as size and modulus of elasticity.
d_{31}= the piezoelectric constant.
$V=$ the voltage applied across the piezoelectric element.

$\frac{d\phi}{dx}\bigg|_{a+h} - \frac{d\phi}{dx}\bigg|_{a-h}$ = the spatial derivative of the mode shape at the end points of the piezoelectric element.

The piezoelectric element used is lead zirconate barium titanate ceramic compound, designated G-1195. This material has a relatively high piezoelectric constant. It also has an elastic modulus nearly equal to that of aluminum. All beams in this project are fabricated from aluminum. The equality of moduli of elasticity between the piezoelectric element and the structure to which it is bonded is a recommended design parameter. Each piezoelectric element used is 1.00 inch (25.4 mm) long X 0.25 inch (6.3 mm) wide X 0.01 inch (0.25 mm) thick.

When the piezoelectric is used as a transducer, it produces a voltage which is proportional to the rate of stress at either end of the element. The proportionality is based on the strain gage relationship in the Piezo Design manual [2]; and has the form:

$$V_{out} = K_2 *g_{31}*c* \frac{d}{dt} \left(\frac{d^2y}{dx^2}\bigg|_{a-h} - \frac{d^2y}{dx^2}\bigg|_{a+h}\right) \qquad (3)$$

where:
K_2= a constant incorporating the material properties of the piezoelectric element.
g_{31}= the voltage coefficient.
$c=$ distance to the neutral axis.

It has been assumed that the stress in the piezoelectric element is the same as the stress at the surface of the structural element, and the piezoelectric has no effect on the actual stress at the member surface. While this assumption is not strictly correct, it is expected that the relationship gives an approximation of the voltage produced by the piezoelectric.

The relationships derived above give the expected values both of the voltage produced by the piezoelectric element (as transducer) and of the force produced by the element (as driver).

RESULTS

A series of experiments was performed in order to establish the effect of bonding technique on the performance characteristics of the piezoelectric elements. These tests used a cantilevered beam with the sensor/driver pair mounted near the root. The beam is 6061-T6 aluminum, 0.75 inch (19.1 mm) wide X 0.125 inch (3.2 mm) thick, ranging in length from 15.13 inch (384 mm) to 17.84 inch (453 mm). The bonding parameters varied were the adhesive used to affix the piezoelectric to the beam and the type of electrical lead.

Mounting of the piezoelectric can be achieved with most bonding techniques [2]. However the voltage produced by the sensor or the moment exerted by the driver is affected by the hardness of the bond. Adhesives used for bonding are a 910-type pressure sensitive contact adhesive and a 24-hour cure two part epoxy. The piezoelectric element has a positive and a negative pole. The surface is coated with a nickel electrode to which leads are attached to get voltage out of or into the element. Small tabs of aluminum foil were used in the initial tests. However, a suitable means to affix the wires leading from the tabs could not be found. One mil thick brass tabs were then fabricated. The lead wires could be soldered to the brass tabs. A technique was perfected to solder the lead wires directly to the piezoelectric electrodes. This necessitates drilling a small groove in the beam in order to make room for the solder joint.

It is necessary to isolate the piezoelectric from the beam in order to eliminate ground loop effects. Thus, each piezoelectric is bonded in the following manner. An insulating interlayer is bonded to the beam. The leads are attached to the electrodes of the piezoelectric, and the piezoelectric is bonded to the insulating interlayer. For four of the five beams for which results are reported, the interlayer is Mylar tape. Beam number 4 uses an interlayer of tissue that is soaked with the 910 adhesive.

In the first series of tests, the piezoelectric was driven at steady state at the first resonant frequency of the beam to which it was bonded. The steady state deflection of the beam tip was recorded as a function of drive voltage. Figure 1 shows the test setup. The results of a typical test are shown in figure 2. The tip deflection increases linearly with applied voltage up to approximately 50 volts. Beyond this voltage, the rate of increase of tip deflection flattens. The maximum voltage from the amplifier is 59 volts. The measured results are compared to expected results calculated as follows.

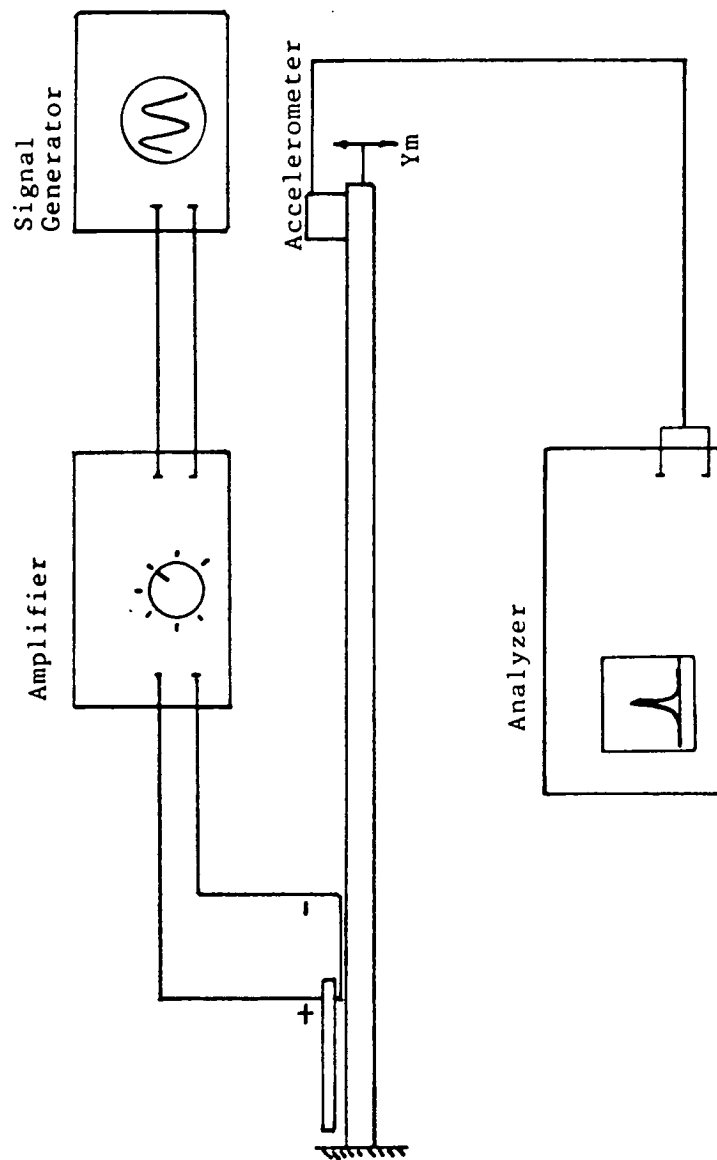The first bending mode of a cantilever beam can be represented by the mode shape function:

Figure 1.   Test Setup to Determine Steady State Tip Deflection for
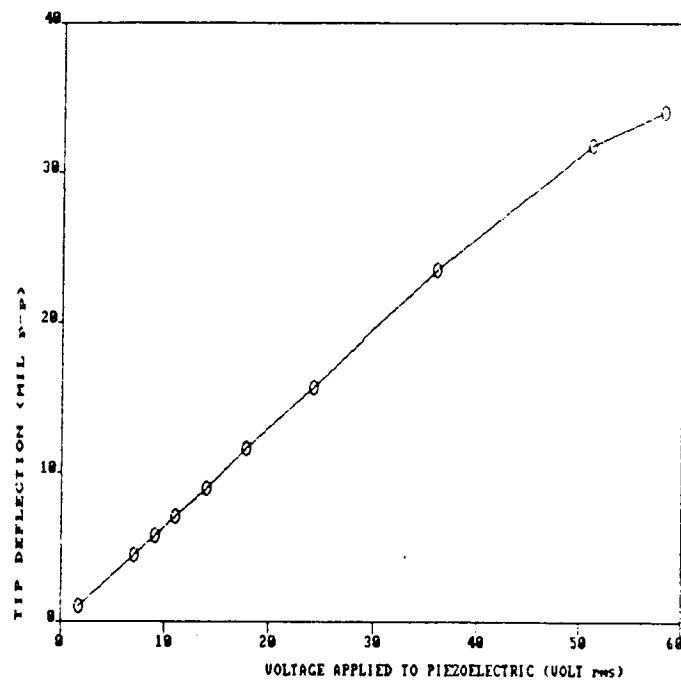Cantilever Beam Driven by Piezolectric

Figure 2. Steady State Tip Deflection of
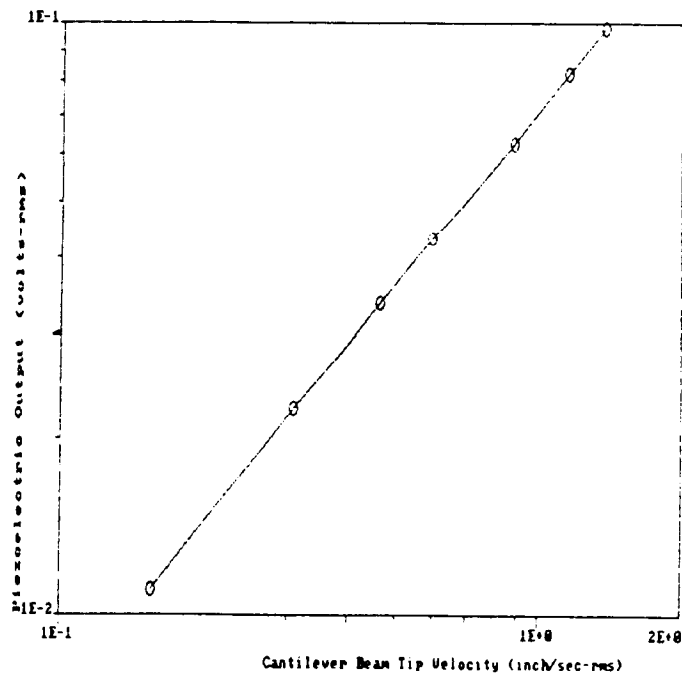Excited Cantilever Beam

Figure 3. Steady State Voltage Signal from
Piezoelectric on Cantilever Beam

Table I. Steady State Response at Cantilever Beam Tip For Excitation at First Resonance.

Aluminum 6061-T6, W = 0.75 in, t = 0.125 in.

BEAM	LENGTH (in)	ELECTRODE	BOND	K (lb/in)	ζ	F_n (Hz)	A_{exp} x10^4 (IN/V)	A_{meas} x10^4 (IN/V)
1	16.844	Aluminum Foil	24-hour Epoxy	3.065	0.0148	14.0	2.92	2.74
2	17.675	Brass	24-hour Epoxy	2.675	.0167	12.75	3.945	1.616
3	15.250	Soldered	Top 24-hour Epoxy Bottom 910	4.130	.00885	16.20	4.864	3.680
4	15.125	Soldered – to beam-top + to beam-bottom	910 adhesive Tissue inter-layer	4.233	.00662	17.00	6.350	12.00
5	15.250	Soldered – to beam-top + to beam-bottom	910 adhesive	4.130	.00716	16.00	6.01	6.370-6.600

$$y(x,t) = q(t) * \phi(x) \qquad (4)$$

where:

q(t)= displacement of the tip of the beam.

and:

$$\phi(x) = \frac{3}{2}\left(\frac{x}{L}\right)^2 - \frac{1}{2}\left(\frac{x}{L}\right)^3 \qquad (5)$$

L= length of the beam.

Equation 5 is derived assuming that the dynamic mode shape is the same as the static mode shape.

When the forcing function in equation 1 is a sinusoid of frequency Ω and magnitude F_o, the temporal response q (t) of the beam tip is also a sinusoid at the same frequency. When the excitation frequency matches the resonant frequency of the beam, the magnitude of the tip deflection is:

$$Y_m = F_o/(2 * \zeta * (K+K_p)) \qquad (6)$$

where:

ζ = the damping coefficient of the beam in the first mode.
For a lightly damped system, this is the maximum tip deflection.

The expected modal force is calculated from equation 2 using the physical parameters of the beam and piezoelectric, and the assumed mode shape, equation 5. Note that the modal force is directly proportional to the applied voltage. The modal damping is measured from free vibration test. The modal stiffness is estimated, using the assumed mode shape, from:

$$K = 3 * E * I / L^3$$

The stiffness of the piezoelectric is evaluated using relationships found in reference [1] and is found to be K =0.091 lb/in. This value is on the order of 1/30 of the range of beam stiffnesses.

These values are combined in equation 10 to estimated the tip deflection as a function of the applied voltage.

The expected and measured values of the tip deflection for the five beams are summarized in table I. The measured tip deflection is within +/- 10% of the expected value for beams 1 (aluminum foil leads) and 5 (soldered leads, 910 adhesive). The brass leads, beam 2, do not permit sufficient voltage to pass the electrodes. Beam 3 has soldered leads. The sensor piezoelectric is affixed with 24-hour epoxy, while the driver is affixed with the 910 adhesive. Except for the difference in adhesive, the mounting technique is essentially the same as for beam 5. It is not clear why beam 3 does not show the expected tip deflection. Beam 4 shows nearly twice the expected tip deflection. This is due to the elimination of Mylar interlayer, which greatly improves the hardness of the bonding.

In the next test, the voltage produced by the piezoelectric was monitored while its companion was driven at steady state. It is expected from equation 3 that the voltage output from the piezoelectric is proportional to the rate of strain. The strain is the second spatial derivative of the beam displacement, which is the tip displacement times the mode shape. The mode shape is constant in time in the first mode.
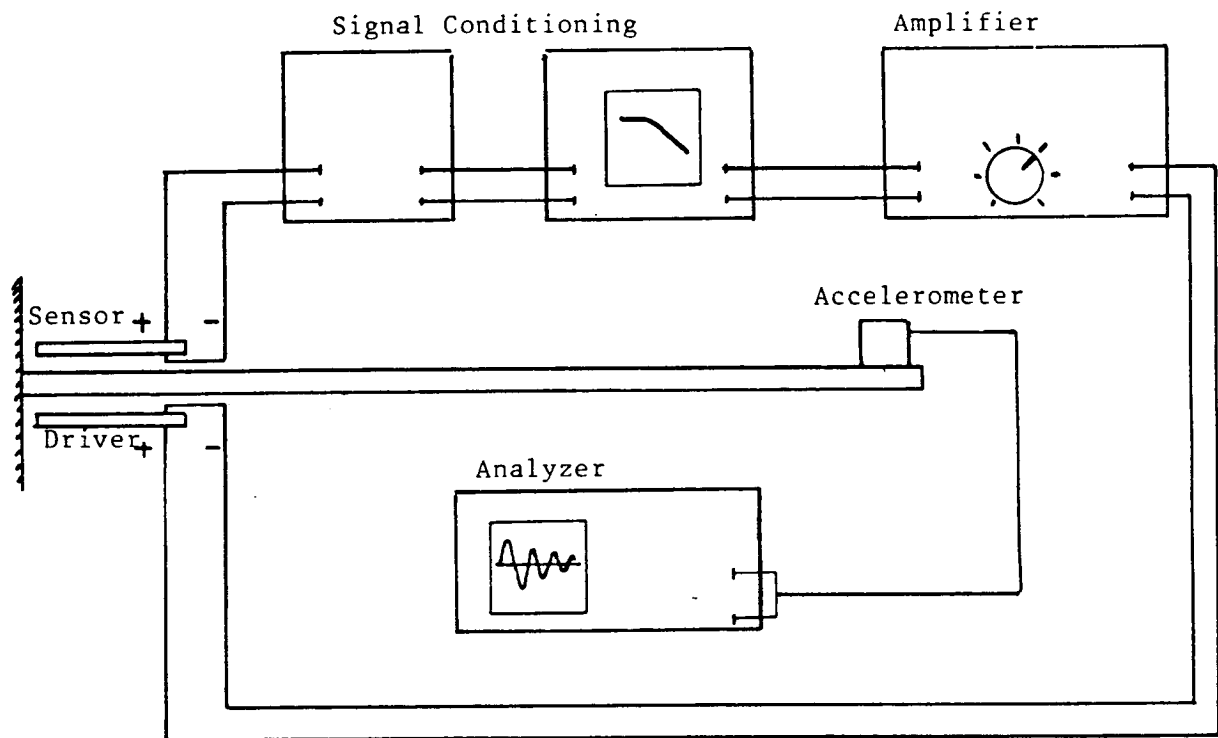
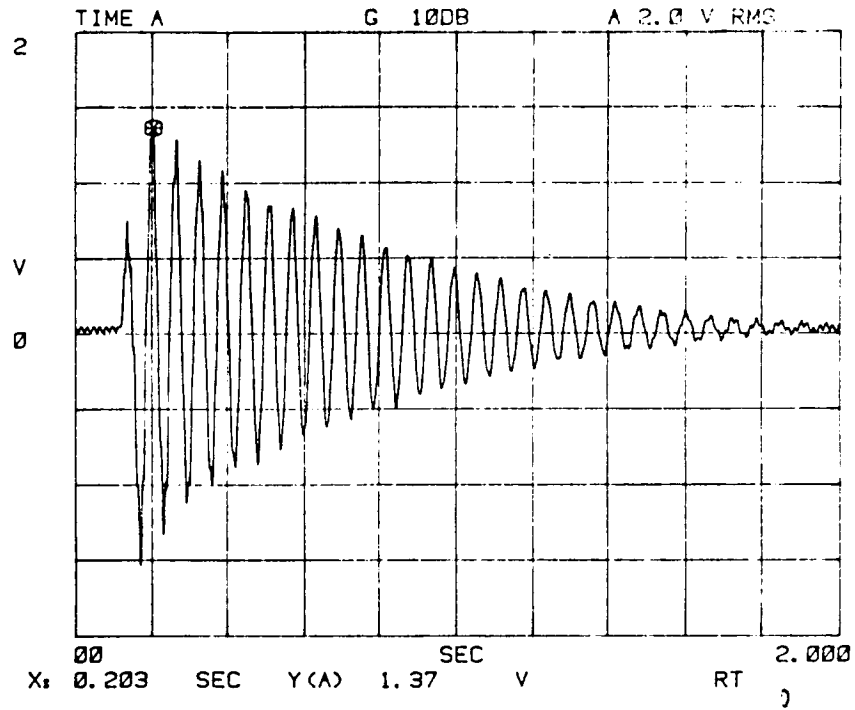Figure 4.   Feedback Control System for Cantilever Beam

TIME A   G 1ØDB   A 2.Ø V RMS

2

V

Ø

ØØ   SEC   2.ØØØ
X: Ø.2Ø3   SEC   Y(A)   1.37   V   RT

Figure 5.   Step Input Response at Cantilever Beam Tip
                -Controller Off

TIME A   G 1ØDB   A 2.Ø V RMS

2

V

Ø

ØØ   SEC   2.ØØØ
X: 1.461   SEC   Y(A)   8.23E-2 V   RT
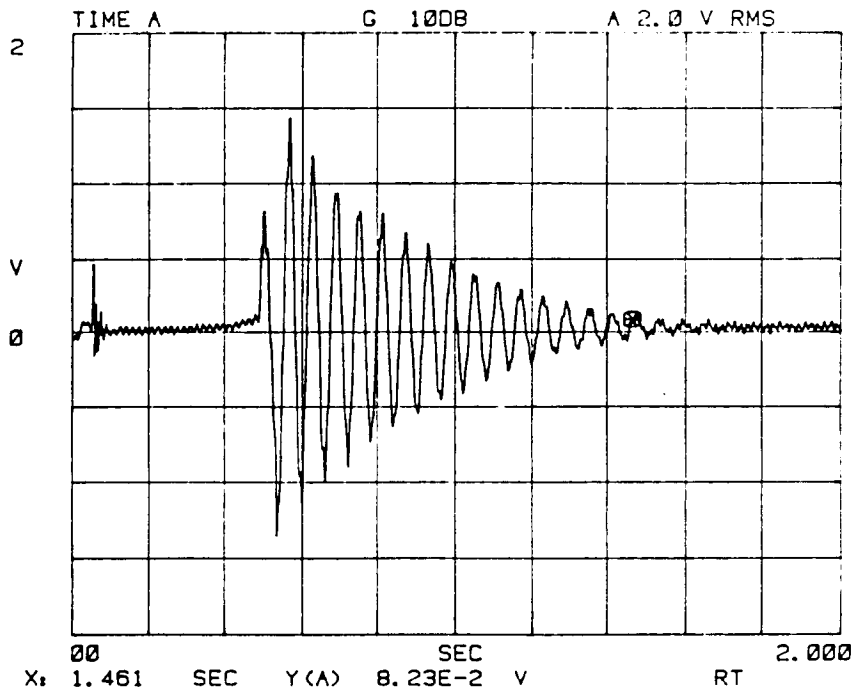
Figure 6.   Step Input Response at Cantilever Beam Tip
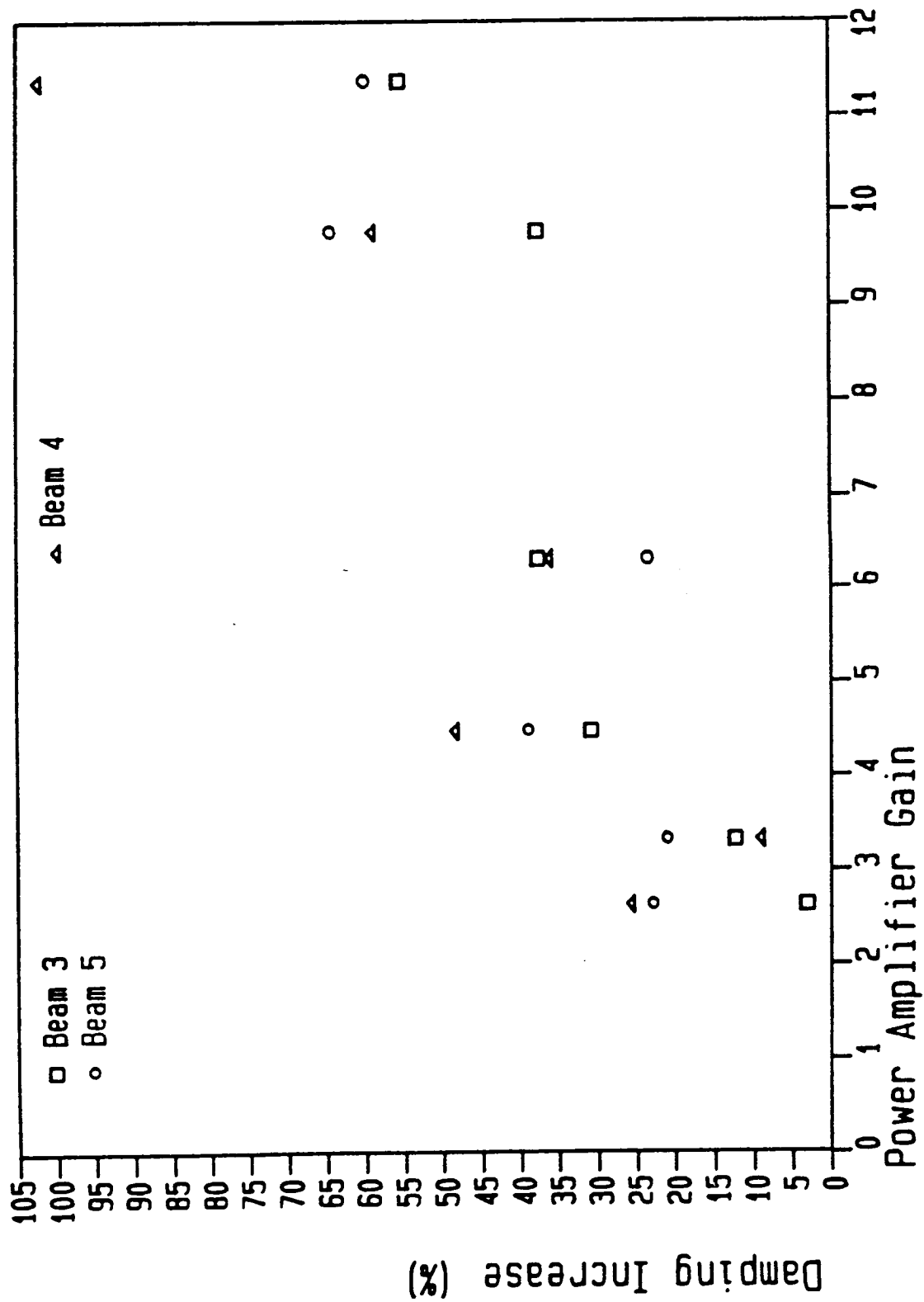   - Controller On

Figure 7. Effect of Bonding Technique on Damping Efficiency

Thus, the rate of strain is proportional to the tip velocity. Figure 3 shows the relationship between voltage output from the piezoelectric and tip velocity. This relationship is linear with a slope of 0.0719 volts/inch/sec. The expected value is obtained from equation 3, where the second derivative of the beam deflection with respect to x is evaluated using the assumed mode shape. The expected slope is 0.0980 volts/inch/sec. The measured value is 25% less than the calculated. This is felt to be due to the fact that the derived value is based on a configuration in which two piezoelectric elements are used to form a strain gage. Thus the model is somewhat different from the experiment. The measured and expected results are of the same order of magnitude. Better correlation is expected when the model is improved.

In the final series of preliminary tests, the piezoelectrics are configured in a sensor/driven pair, as shown in figure 4. The voltage output from the sensor is passed through signal conditioning which shifts the phase by 180°, filters out 60 Hz noise, and amplifies the voltage. The conditioned voltage is passed to the driver. The beam tip is displaced by approximately 0.25 inch, and released from rest. Figure 5 shows a typical free response of the beam from an accelerometer mounted at the tip. The free response is obtained with the feedback control system turned off. When the feedback control system is activated, the response to an initial displacement is as shown in figure 6. It is seen that the feedback control system produces a significant increase in damping.

Beams 3, 4, and 5 were evaluated to determine the increase in damping efficiency as a function of feedback gain factor. These three beams were chosen because beam 2, with the brass leads had already been eliminated, based on the previous series of tests. Beam 1, with the aluminum leads was evaluated, but produced no noticeable increase in damping.

Figure 7 shows the increase in damping with amplifier gain, for factors up to approximately 12. The gain recorded is for the amplifier only. The signal conditioner has a gain of 16.3 at 16.0 Hz. Thus, the total amplification of the signal out of the sensor piezoelectric is as much as 192. The figure shows that the increase in damping generally follows the signal amplification. An anomalous dip in the general trend at the gain factor of 6 is felt to be due to the power amplifier. Beam 4, using the tissue interlayer, appears to have the greatest damping efficiency. However, it was found that at high amplification, around 10, the beam became unstable, with a vibration mode at 104 hertz being self-excited.

Equation 1 is used to evaluate the results of this feedback test. The modal force produced by the drive piezoelectric is proportional to the voltage applied to it. The applied voltage is the amplified voltage produced by the sensor piezoelectric. The sensor voltage is proportional to the time derivative of the strain, which is the modal velocity times the second spatial derivative of the mode shape. Thus, the feedback control signal is proportional to the modal velocity.

The beam is excited into its first resonant mode by an initial displacement of the tip. The equations of motion is·

$$M * \ddot{q} + C * \dot{q} + (K + K_p) * q = -T_1 * T_2 * G * \dot{q} \qquad (7)$$

where:

T_1 = driver proportionality term

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TABLE II. DAMPING EFFICIENCY OF FEEDBACK CONTROL SYSTEM - BEAM 5

POWER AMPLIFIER GAIN	OVERALL SYSTEM	EXPECTED		MEASURED
		ζ EFF	% INCREASE	% INCREASE
2.58	42.05	.00133	19	23
3.30	53.79	.00170	24	22
4.45	72.54	.00229	32	39
6.31	102.9	.00325	45	24
9.75	158.9	.00503	70	65
11.28	183.9	.00582	81	60

T_2 = sensor proportionality term

G = amplified gain

The excitation term is transferred to the left hand side of the equation. Dividing by the modal mass, equation 7 becomes:

$$\ddot{q} + 2 * w_n{}^2 * (\zeta + \zeta_{eff}) * \dot{q} = w_n{}^2 * q = 0 \tag{8}$$

Thus, the feedback signal adds an effective modal damping to the system.

Table II is a summary of the damping increase as a function of power amplifier gain. The signal conditioner circuitry provides a fixed gain of 16.3 at the free vibration frequency of beam 5. The expected values of effective damping are calculated from equation 10. These are used with the inherent damping in beam 5, shown in table I to be 0.00716, to calculate the expected damping increase. The measured values are taken from figure 7. The expected and measured values generally agree to within +/- 7%. The wide divergence at power amplifier gain of 6.3 coincides with a general dip in the curves in figure 7. The measured damping efficiency is 20% less than the expected at the gain of 11.3. This is felt to arise from the fact that, at such high gain, the drive voltage exceeds 50 volts, and the piezoelectric efficiency degrades.

Thus, a simplified algorithm has been developed to estimate the transient response of the beam as a function of feedback amplification. It is expected that this algorithm can be modified for the free-free beam case in order to verify the experimentally obtained results and to aid in the parameter evaluation phase of the project.

TESTS ON FREE-FREE BEAM

The purpose of the preliminary work is to establish the bonding techniques and to aid in the design of feedback circuitry for the free-free beam. Based on the preliminary work, it was decided to mount the piezoelectrics to the beam with the 910 adhesive using a Mylar tape interlayer. The electric leads are soldered to the electrodes of the piezoelectric. The free-free beam experiment is shown schematically in figure 8. The beam is 96 inches long and is suspended vertically from one end. The pendulum length is approximately 25 feet.

The purpose of the long pendulum is to simulate a free end condition with minimum shear and moment. The pendulum frequency is expected to be below 1.0 hertz. This frequency is below the first flexural model of the beam, which is calculated to be at 2.77 hertz. Eight sensor/driver pairs of piezoelectrics are mounted to the beam at intervals of 12 inches. The sensor signal for each piezoelectric pair is amplified and shifted in phase by 180° before being reintroduced to the driver. The signal conditioning also includes a low pass filter to reduce 60 hertz noise. The total gain of each amplifier is 230. In order to ensure that the piezoelectric is not overdriven, the power supply to each amplifier is limited to +/= 70 volts, or approximately 50 volts rms.

A special striker was fabricated to ensure that the beam is excited with the same initial energy each time. Figure 9 shows the free vibration of the beam with the feedback system off. The beam was struck in the center. It was found that the accelerometer signal response was attenuated at the first resonance of 2.77 hertz, and did not give a true
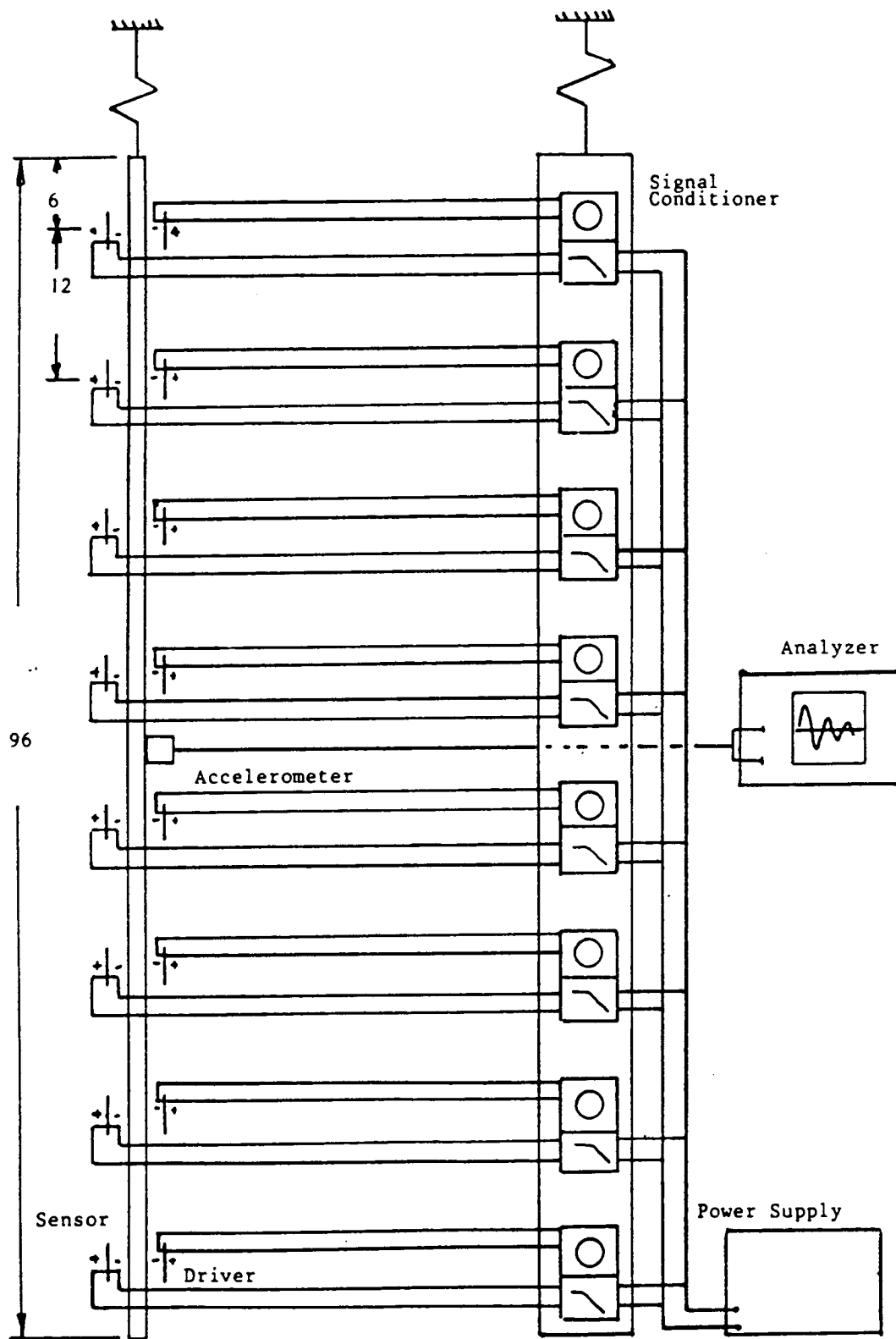
Figure 8. Test Setup for Free-Free Beam Controller

representation of the beam vibration in the first mode. Thus, the signal from a piezoelectric near the center of the beam was used for measurement of vibration. The result obtained with the control system on is shown in figure 10. It can be seen that the vibrations damp out much more quickly than they do with no feedback. The peak displacement of the beam is approximately 1/2 the peak displacement with no control.

The excitation method excites more than one vibration mode in the beam. Figure 11 shows the spectrum of beam oscillation when the beam is excited in the center. The vibration peaks are harmonically related. It is expected that the peaks would occur at odd harmonics of the fundamental (2.77 hertz, 14.95 hertz, 36.96 hertz, and 68.74 hertz) . While this result is generally found, peaks also occur at the odd harmonics of approximately 17.5 hertz (52.5 hertz, and 87.5 hertz). These vibration components arise from transverse vibration in the cable of the pendulum supporting the beam. These vibration components happen to coincide with the beam's flexural components. Since the transverse vibration of the cable is not affected by the piezoelectric control system, this vibration energy adds to the flexural energy in the beam, and degrades the damping efficiency of the piezoelectrics. It is expected that modification of the beam suspension will eliminate the transverse response, and the damping efficient of the feedback control system will be more clearly demonstrated.

Since multiple modes of the beam are excited, evaluation of modal damping is not possible. However, an approximation of the overall energy reduction is made. It is assumed that the area under the response curve is related to the energy. The area is approximated by faring a curve through the peaks of the response. The area under the curve in figure 10 is 47 percent of the area under the curve in figure 9. This indicates that the energy input by the piezoelectric controllers reduces the overall response by a factor of 2.13.

Striking the beam at the end excites both the even and odd harmonics. The results obtained are similar to the case in which the beam was excited at the center. As was the case with excitation in the center of the beam, it is expected that the damping efficient of the feedback control system will be more clearly demonstrated when the transverse cable vibration is eliminated.

CONCLUSIONS

Practical considerations, including techniques for mounting the piezoelectric elements and for attaching electrical leads have been addressed. It is found that bonding the element to the beam using a hard adhesive such a 910 type is suitable because stresses at the beam surface are transferred through a hard bonding layer most efficiently . Mylar tape is used as an insulator to ensure that the element is not grounded to the beam. Soldering the electric leads to the piezoelectric element ensures efficient transfer of the high voltages at low currents either generated by the piezoelectric or used to drive it.

An analytical model of the feedback control system has been developed. This model estimates the voltage generated by the piezoelectric sensor as a function of the dynamic stress at the sensor location, and the force exerted by the driver piezoelectric as a function of signal gain. Both
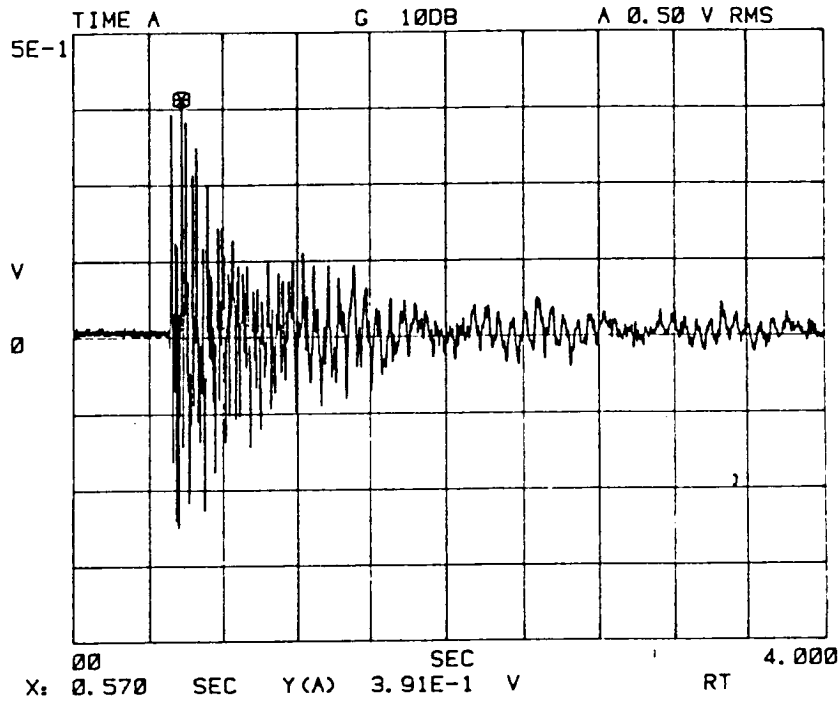
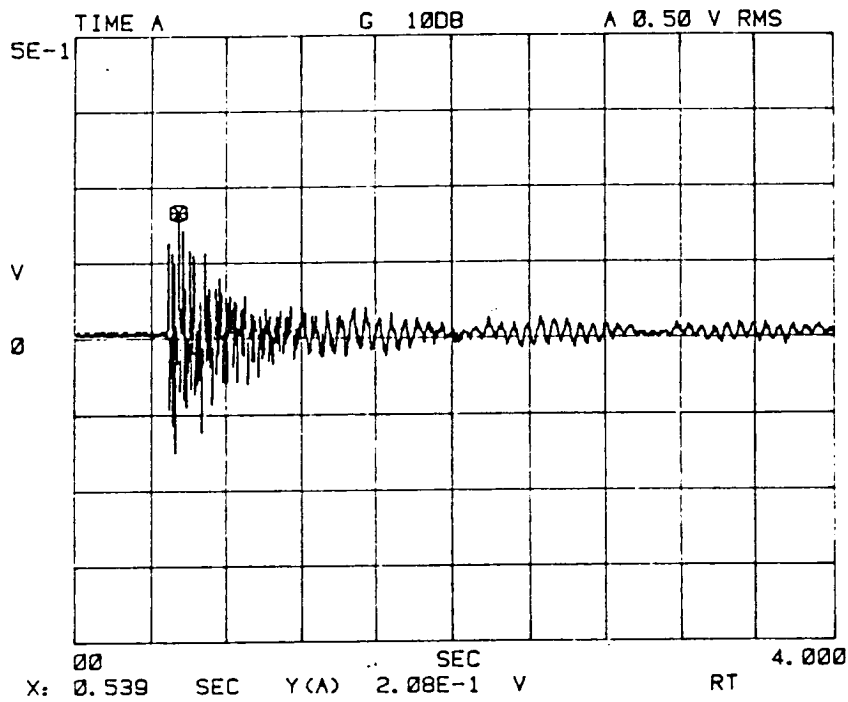Figure 9. Free-Free Beam Excited at Center - Control System Off

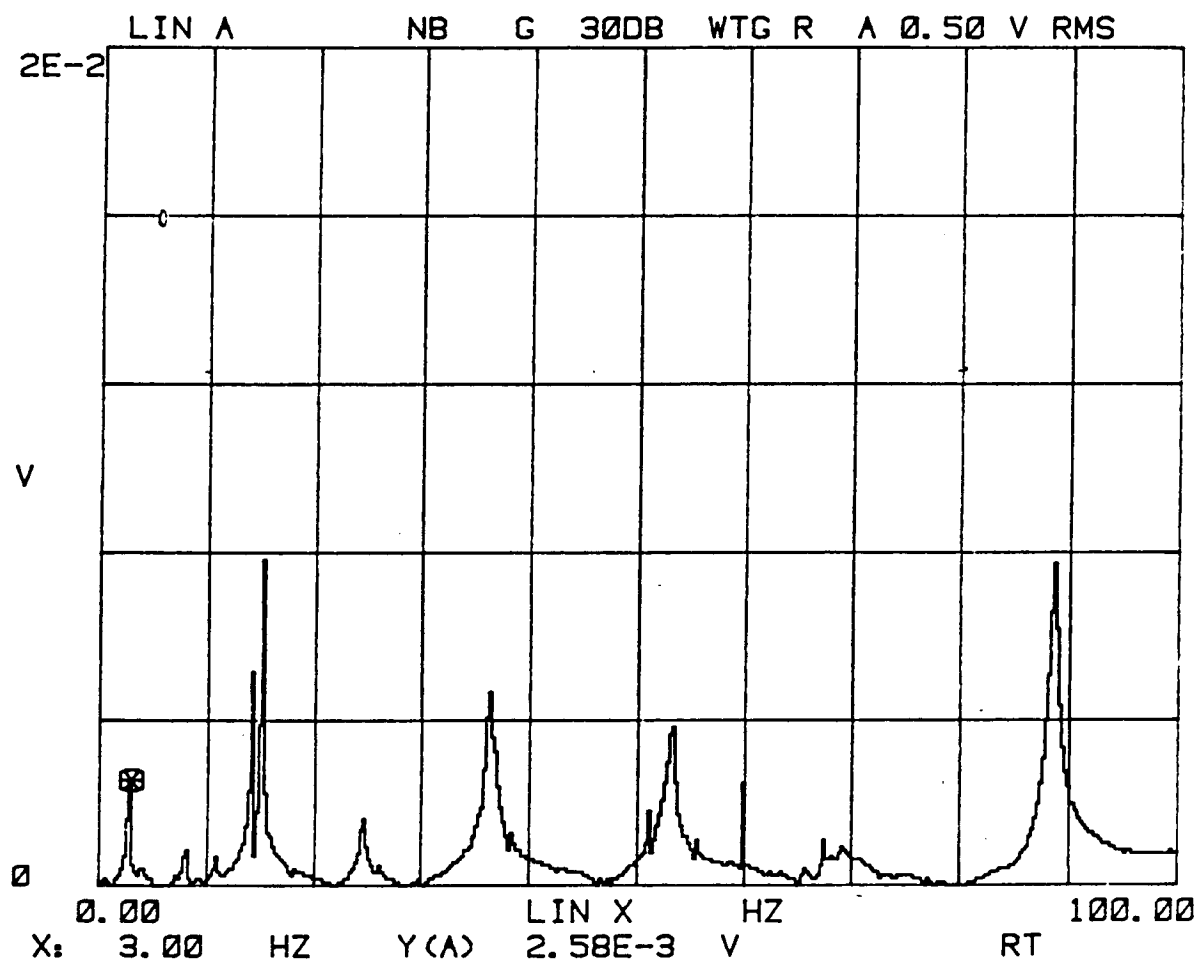Figure 10. Free-Free Beam Excited at Center - Control System On

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11-18

Figure 11.   Spectrum of Free-Free Beam Response

terms are functions of the mode shape of beam vibration. The feedback
control system is expressed in terms of a modal damping factor. The model
has been compared to measured results for a cantilever beam excited to
vibrate in its first natural mode. The estimated increase in first modal
damping factor compares favorable wit the measured results.

A free-free beam configuration has been simulated and instrumented
with a distribution of piezoelectric sensor/driver pairs. The purpose of
the distribution is to test the vibration control efficient of the
piezoelectrics when they are not optimally located at points of high stress
in the beam. The feedback control system is shown to reduce the response
to impulsive excitation by a factor of approximately 2.

ACKNOWLEDGMENTS

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