POWER AND CHARGE DISSIPATION FROM
AN ELECTRODYNAMIC TETHER

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The Plasma Motor-Generator project utilizes the influence of the geomagnetic field on a conductive tether attached to a LEO spacecraft to provide a reversible conversion of orbital energy into electrical energy. The behavior of the current into the ionospheric plasma under the influence of the geomagnetic field is of significant experimental and theoretical interest. Theoretical calculations are reviewed which start from Maxwell's equations and treat the ionospheric plasma as a linear dielectric medium. These calculations show a charge emitting tether moving in a magnetic field will generate electromagetic waves in the plasma which carry the charge in the direction of the magnetic field. The ratio of the tether's speed to the ion cyclotron frequency which is about 25 m for a LEO is a characteristic length for the phenomena. Whereas for the dimensions of the contact plasma much larger than this value the waves are the conventional Alfvén waves, when the dimensions are comparable or smaller, diffraction effects occur similar to those associated with Fresnal diffraction in optics.

The power required to excite these waves for a given tether current is used to estimate the impedance associated with this mode of charge dissipation. The result, on the order of an ohm, is encouraging.

I.   INTRODUCTION

According to Faraday's law of induction, a voltage will be induced in a wire moving across magnetic field lines. Provided there is a stationary return path for current, electrical power can be extracted (an IxB force will act to reduce the relative speed of the wire with respect to the magnetic field), or with the application of a reversed voltage greater than the induced voltage, electrical power will be expended and propulsion will result. In low Earth orbit when the orbital speed, v_c is about $8 \cdot 10^3$ m/s, a 10 kilometer long wire would have an induced voltage of slightly more than 2 kV. Neglecting losses and assuming good electrical contact with the ionosphere, 20 kw of power would be equivalent to a propulsion thrust of about 2.5 N.[1]

The essential elements in the proposed system is the establishment of a good electrical contact with the ionosphere. The most convenient way to establish this contact is with a plasma generator capable of producing plasma in such quantities that it will exceed the ionosphere plasma out to a distance of ten meters or so. Hollow cathodes[2-5] will supply both the plasma need for electrical contact with the ionosphere and electrons needed for the electrical current. Electrical power/propulsion systems using plasma contactors are generally called Plasma Motor/Generators (PMG).

In the literature, most authors[6-10] assume Alfvén waves are responsible for carrying charge from the ends of electrodynamic tethers along magnetic field lines into the ionosphere. The theoretical calculations are extremely long and the discrepancies of their results,

e.g. impedance of the ionosphere vearying from 1 to 10^5 ohms, require

that careful study of such calculations be undertaken to identify

results common to all calculation and those effected by model dependent

assumptions. Consequently, the next section will consider the general

solutions of Maxwell's equations under the common assumption that the

ionosphere can be considered linear dielectric material. Subsequent

sections will consider the effects of tether configuration and

frequency bands on the predictions.

II. THE WAVE EQUATION AND ITS SOLUTION

Maxwell's equations for the rext frame of the ionosphere are

$$\vec{\nabla}\cdot\vec{E} = 4\pi\rho \qquad , \qquad \vec{\nabla}\cdot\vec{B} = 0$$

$$c\,\vec{\nabla}\times\vec{E} = -\dot{\vec{B}} \qquad , \qquad c\,\vec{\nabla}\times\vec{B} = \dot{\vec{E}} + 4\pi\vec{j} \tag{1}$$

when $\vec{E}$ and $\vec{B}$ are the electric and magnetic fields, and ρ and $\vec{j}$ are

the total charge and current densities.

Using the Fourier transform notation

$$F(\vec{k},\omega) = \left(\tfrac{1}{2\pi}\right)^2 \int e^{-i(\vec{k}\cdot\vec{r}-\omega t)}\, F(\vec{x},t)\, d^3x\, dt \tag{2}$$

$$F(\vec{x},t) = \left(\tfrac{1}{2\pi}\right)^2 \int e^{i(\vec{k}\cdot\vec{r}-\omega t)}\, F(\vec{k},\omega)\, d^3k\, d\omega$$

and combining the latter two Maxwell equations yields the wave equation

$$c^2 k^2\, \vec{E} - c^2 (\vec{k}\cdot\vec{E})\,\vec{k} = \omega^2 \vec{E} + 4\pi i \vec{j} \tag{3}$$

Treating the ionospheric plasma as medium described by a dielectric

tensor $\overleftrightarrow{\varepsilon}$:

$$\vec{D} = \overleftrightarrow{\varepsilon}\cdot\vec{E} \tag{4}$$

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and introducing the current density within the plasma by

$$\vec{J_P} = \overleftrightarrow{\sigma} \cdot \vec{E} = -\frac{i\omega}{4\pi}(\vec{D} - \vec{E}) \qquad (5)$$

where $\overleftrightarrow{\sigma} = -\frac{i\omega}{4\pi}(\overleftrightarrow{\varepsilon} - I)$ is the plasma conductivity tensor, the wave equation can be written as

$$c^2 k^2 \vec{E} - c^2 (\vec{k} \cdot \vec{E}) \vec{k} = \omega^2 \vec{D} + 4\pi i\omega \vec{J}c \qquad (6)$$

where $\vec{J_c}$ is the current density within the conducting tether. Choosing a coordinate system such that $\hat{z}$ is parallel to the geomagnetic field and $\hat{x}$ is the direction of tether motion which is assumed for simplicity to be perpendicular the geomagnetic field, the dialectric tensor of the form[11]

$$\overleftrightarrow{\varepsilon} = \begin{pmatrix} \varepsilon_\perp & +ig & 0 \\ -ig & \varepsilon_\perp & 0 \\ 0 & 0 & \varepsilon_\parallel \end{pmatrix} \qquad (7)$$

For frequencies in the region of interest, i.e. much less than the plasma frequency, ωpe, $\varepsilon_\parallel$ is very large and E_z can be neglected compared to D_z in the wave equation.

Without specifying further the frequency dependence of the dialectric tensor or of the tether current density, the wave equation can be solved for the electric field:

$$E_x^y = \left(\frac{4\pi i\omega}{c^2}\right)\left[\frac{\vec{k} \cdot \vec{J}(k^2 - k_a^2) k_y / k_\perp^2 \mp (k \cdot j)_z (k_z^2 - k_a^2) / k_\perp^2 \pm ig\omega^2 j_y/c^2}{(k^2 - k_a^2)(k_z^2 - k_a^2) - g^2 \omega^4/c^4}\right] \qquad (8)$$

where $k_\perp^2 = k_x^2 + k_y^2$ and $k_A^2 = \omega^2 \varepsilon_\perp/c^2$.

As will be explained later, the g^2- term in the denominator is neglible. Consequently the E-field involves Cauchy poles in the

complex k-plane at $k^2 = k_A^2$, $k_z^2 = k_A^2$ and $k_\perp^2 = 0$.

The power needed for the establishment and radiation of electromagnetic fields and the associated charge and current densities is given by Poynting's theorem

$$P = W - \dot{U} \tag{10}$$

where

$$W = -\int \vec{j} \cdot \vec{E} \, d^3x$$

$$U = \frac{1}{8\pi} \int (E^2 + B^2) d^3x$$

W is the rate of conversion of mechanical energy to electromagnetic energy and U is the field energy. Since the fields do not change with time in the rest frame of the tether, the time derivative of U will be zero. Further, with the assumption of a cold, collisionless plasma, there is no conversion of electromagnetic energy into mechanical and the power dissipation takes the form

$$P = -\int \vec{J}_c \cdot \vec{E} \, dx^3 \tag{11a}$$

As a result of the tether having a constant speed, v_c, and a constant current, I, both $\vec{j}$ and $\vec{E}$ have Fourier transforms of the form

$$F(\vec{k}, \omega) = F(\vec{k}) \, \delta(\omega - k_x v_c)$$

which allows the power dissipation to be recast as the real part of a Fourier integral:

$$P = -\frac{Re}{2\pi} \int \vec{j}_c^*(\vec{k}) \cdot \vec{E}(\vec{k}) \, d^3k \tag{11b}$$

Since the current density in the tether, $\vec{j}_c$, is proportional to the tether current I, and the electric field is linear in $\vec{j}_c$, the power is proportional to I^2. Consequently the impedance, Z, of the ionosphere required for the maintenance of these charge carrying, electromagnetic waves is given by

$$Z = - \frac{Re}{2\pi I^2} \int \vec{j}_c^*(\vec{k}) \cdot \vec{E}(\vec{k}) d^3k \tag{12}$$

The charge density in these waves can be calculated from the continuity equation

$$\dot{\rho}(\vec{x},t) = -\vec{\nabla} \cdot \vec{j}(\vec{x},t) \quad or \quad \rho(\vec{k},\omega) = + \vec{k} \cdot \vec{j}(\vec{k},\omega)/\omega \tag{13}$$

where $\vec{j}(\vec{k},\omega)$ is the total current density $\vec{j} = \vec{j}_c + \vec{j}_p = \vec{j}_c - \frac{i\omega}{4\pi} (\vec{D} - \vec{E})$
From the wave equation, it follows that $- \frac{i\omega}{4\pi} \vec{k} \cdot \vec{D} = - \vec{k} \cdot \vec{j}_c$ and thus

$$\rho(\vec{k},\omega) = \frac{i}{4\pi} \vec{k} \cdot \vec{E}(\vec{k},\omega) \tag{14}$$

As a result of equations (8), (12) and (14) the electric field, the impedance, and the charge density are determined once the frequency dependence of the dialectric tensor and the model used for the current density of tether are specified.

III. MATHEMATICAL ORIGIN OF ALFVEN WAVES

In the frequency regions were the g^2 - term in the denominator of equation (8) can be neglected the E-field is seen to have Cauchy-poles at $k_z^2 = k_A^2$, $k^2 = k_A^2$ and $k_x^2 + k_y^2 = 0$

where $k_A^2 = \omega^2 \varepsilon_\perp/c^2 = k_x^2 (\frac{v_c}{c}) \cdot \varepsilon_\perp(k_x v_c)$

Only the first two of these poles involve k_z and thus are candidates for the generation of waves moving along the direction of the

geomagnetic field. As a result, in performing the inverses Fourier

transform, the most general k_z-integral will be of the form

$$\int e^{ik_z z} \left[\frac{A}{k_z^2 - k_A^2} + \frac{B}{k^2 - k_A^2} \right] dk_z \tag{15}$$

where A and B are functions of k which are non-singular in k_z.

The $k_z^2 = k_A^2$ pole is really two poles

$$k_z = \pm k_x \left(\frac{vc}{x}\right) \epsilon_\perp \tag{16}$$

which would result in z-dependencies of the form $exp\left[\pm i k_x \left(\frac{v_c}{c}\right) \epsilon_\perp^{1/2} z\right]$

When combined with the k_x integration this factor would result in

$exp\left[i k_x \left(x \pm \left(\frac{v_c}{c}\right) \epsilon_\perp^{1/2} z\right)\right].$

If there is to be radiation into the wake of the tether then two

conditions must be satisfied. First, the appropriate pole is the +

one of z>o and - om for z<o. The path of integration along the k_z axis

should be appropriately chosen to retain only the physical pole. Second,

in order that the waves not be damped, i.e. that they are able to carry

the charge deep into the ionosphere, requires that $\epsilon_\perp$>o.

This latter condition will be discussed again in the next section where

it will restrict the frequencies leading to wave generation to three

regions or bands.

If the above two conditions are satisfied, then each k_x- wave

will have a wake angle (see figure 1) given by

$$\alpha(k_x) = \tan^{-1}\left(\frac{v_x}{v_z}\right) = \tan^{-1}\left(\frac{v_c}{v_A}\right) \tag{17}$$

where $v_A = c/\epsilon^{1/2}$, is their speed in the z-direction. The k_z poles

remaining to be considered come from $k_z^2 = k_A^2 - k_x^2 - k_y^2$. The condition

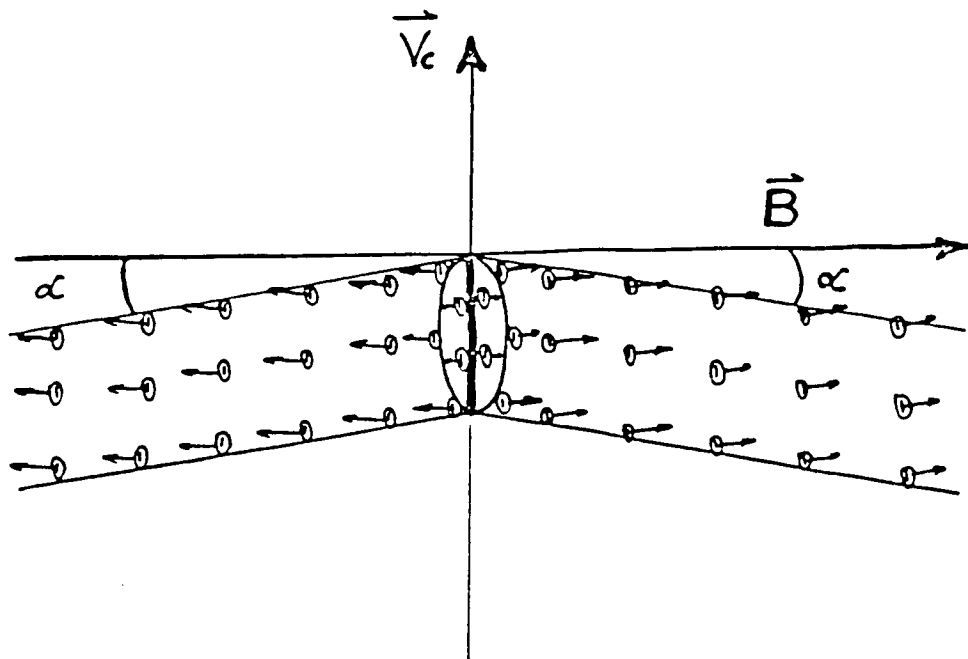that the waves propagate and carry charge along the z-axis, requires

Figure 1.   Bottom Tether Wire As Viewed From Earth

that $k_z^2 > 0$, or $k_x^2 \left(\left(\frac{v_c}{c}\right)^2 \varepsilon_\perp - 1 \right) > k_y^2$

Since $(v_c/c)^2 \cong 7 \cdot 10^{-10}$, only frequency regions where $\varepsilon_\perp > 10^9$ contribute, a severe restriction. Even then, the contributions from the k_x-k_y plane will be very restricted.

Consequenty, only the pole at the $k_z^2 = k_A^2$ will contribute to waves capable of carrying charge into the ionosphere. It should be pointed out that contributions to the integral in equation (12) in which k_z becomes imaginary are themselves imaginary and do not contribute to the impedance.

IV. THE DIELECTRIC TENSOR FOR THE IONOSPHERE

The resonance frequencies for electrons and ions in the ionosphere are due to plasma oscillations and cyclotron motion about the B-field. The dielectric tensor has the form given in equation (7) with

$$\varepsilon_\perp = \tfrac{1}{2}(L+R) \ , \quad g = \tfrac{1}{2}(L-R) \ , \quad \varepsilon_{ii} = 1 - \sum_k \left(\frac{\omega_{pk}}{\omega}\right)^2 \qquad (18a)$$

when

$$R = 1 - \sum_k \left(\frac{\omega_{pk}}{\omega}\right)^2 / (1 + Z_k \, \Omega_k/\omega)$$

$$L = 1 - \sum_k \left(\frac{\omega_{pk}}{\omega}\right)^2 / (1 - Z_k \, \Omega_k/\omega)$$

and $\quad \omega_{pk}^2 = 4\pi n e^2/m_k \ , \quad \Omega_k = \frac{eB}{m_k c} \ , \quad Z_k = \begin{cases} + \text{ ions} \\ - \text{ electrons} \end{cases}$

The sums are to be taken over electrons and all ion species. For a two component plasma,

$$\varepsilon_\perp = \frac{(\omega^2 - \omega_\ell^2)\cdot(\omega^2 - \omega_u^2)}{(\omega^2 - \Omega_i^2)\cdot(\omega^2 - \Omega_e^2)} \ , \quad g = \frac{\omega \, \omega_{pe}^2 \, \Omega_e \left(1 - \left(\frac{\Omega_i}{\Omega_e}\right)^2\right)}{(\omega^2 - \Omega_i^2)(\omega^2 - \Omega_e^2)} \qquad (18b)$$

where $\quad 2\omega_{\ell}^2 = \Omega_i^2 + \Omega_e^2 + \omega_{pi}^2 + \omega_{pe}^2 \mp \sqrt{(\Omega_e^2 - \Omega_i^2 + \omega_{pe}^2 - \omega_{pi}^2)^2 + 4\omega_{pe}^2 \omega_{pi}^2}$

are the lower and upper hybrid frequencies. A sketch of $\varepsilon_\perp$ as a

function of ω is shown in figure 2. Clearly the requirement that the waves not be damped, i.e. $\varepsilon_\perp > 0$, is satisfied in only three frequency regions:

Region I $\qquad 0 \leq \omega < \Omega_i$ $\hspace{4cm}$ (19)

Region II $\qquad \omega_L < \omega < \Omega_e$

Region III $\qquad \omega_u < \omega$

According to reference 11, numerical values appropriate to an oxygen-electron plasma are $\Omega_i = 2.0 \cdot 10^6 Hz, \omega_L = 3.5 \cdot 10^4 Hz, \omega_{pi} = 2 \cdot 10^5 Hz, \Omega_e = 5.9 \cdot 10^6 Hz$ $\omega_{pe} = 3.4 \cdot 10^7 Hz, \omega_u = 3.5 \cdot 10^7 Hz$. In the first region $\varepsilon_\perp \simeq \left(\frac{c}{v_{Ao}}\right)^2 / \left(1 - \left|\frac{\omega}{\Omega_i}\right|^2\right)$, where $v_{Ao} = c(\Omega_i / \omega_{pi})$ is the Alfvén velocity, and $g \simeq \frac{\omega}{\Omega_i} \varepsilon_\perp$.

V. MODELS FOR THE TETHER CURRENT DENSITY

The simplest model for the current density of the tether is that of Estes[6]. In this model the vertical tether extends from $-L/2 \leq y \leq L/2$ and is infinitely thin:

$$j_y(\bar{x},t) = I \, \delta(x') \, \delta(z) \left[\theta(y_+) - \theta(y_-) \right]$$

where $x' = x - v_c t, y_\pm = y \pm L/2$ and the θ function is given by

$$\theta(y) = \begin{cases} 1 & y > 0 \\ 0 & y < 0 \end{cases} .$$

There is an infinitely thin horizontal tether at each end of the vertical tether extending from $-L_x/2 \leq x' \leq L_x/2$ such that the currents split at $x' = 0$ into equal parts, one in the positive x-direction, and another in the negative x-direction:

where $x'_\pm = x' \pm L_x/2 = x - v_c t \pm L_x/2$. Using equation (2) the corresponding Fourier transformed currents are
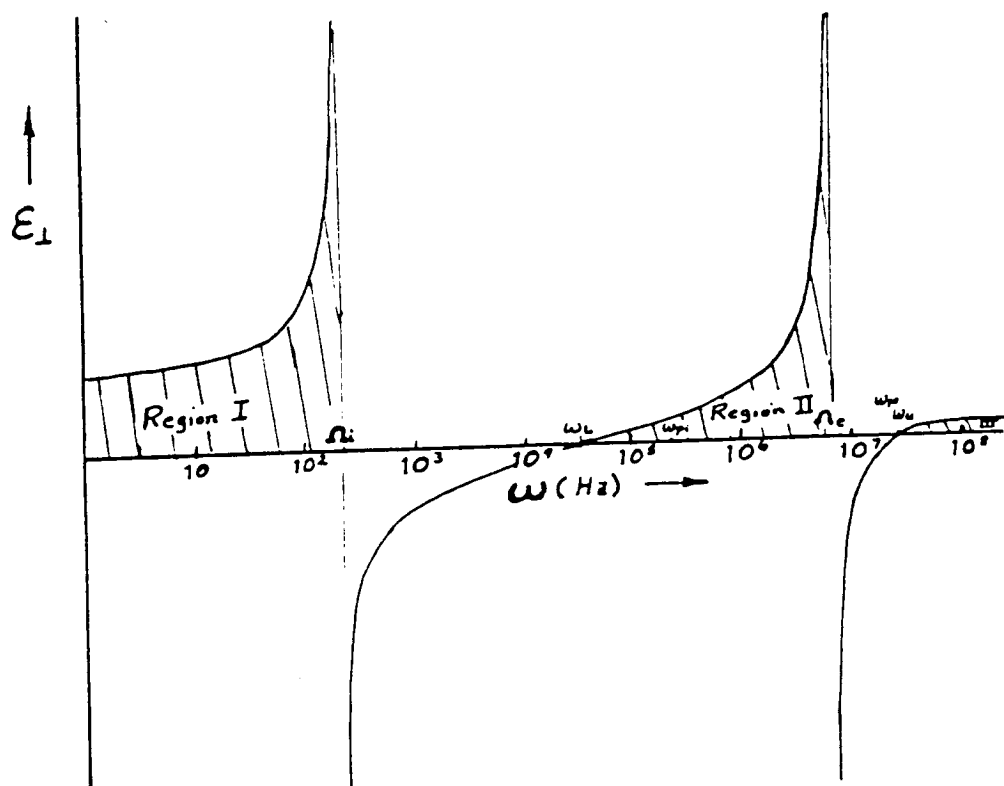
Figure 2. Sketch of $\varepsilon_\perp$ as a Function of Frequency

$$j_x = \frac{I}{\pi} \delta(\omega - k_x v_c) \left[\frac{\sin k_x L_x/2}{k_x L_x/2} - \frac{1}{k_x} \right] \sin k_y L/2 \qquad (20)$$

$$j_y = \frac{I}{\pi} \delta(\omega - k_x v_c) \frac{\sin k_y L/2}{k_y}$$

Thus the Fourier transform of $\vec{\nabla} \cdot \vec{J}$ is given by

$$\vec{k} \cdot \vec{J} = \frac{I}{\pi} \delta(\omega - k_x v_c) \frac{\sin k_x L_x/2}{k_x L_x/2} \sin k_y L/2 \qquad (21a)$$

If, as done in reference (7), the horizontal wires are made into sheets, the result is

$$\vec{k} \cdot \vec{j} = \frac{I}{\pi} \delta(\omega - k_x v_c) \frac{\sin k_x L_x/2}{k_x L_x/2} \cdot \sin k_y L/2 \cdot \frac{\sin k_z L_z/2}{k_z L_z/2} \qquad (21b)$$

It is not possible to tell from the above expressions for $\vec{k} \cdot \vec{J}$ whether the vertical wire is infinitely thin or not. Since there is no divergence of $\vec{J}$ coming from the vertical wire, its cross section does not effect $\vec{k} \cdot \vec{J}$. It is of course possible to turn the horizontal sheets into parallelepipeds. To describe the charge carried into the ionosphere by the plasma cloud generated by a hollow cathode, it might suffice to use a spherical or elliptical surface of appropriate size over which the divergence of the current density is constant.

As emphasized by Estes[6] the factor $(\frac{\sin k_x L_x/2}{k_x L_x/2})$ in the expressions for $k \cdot j$ acts as a frequency cut-off for finite L_x, i.e. the dominate frequencies are those for which

$$\omega = k_x v_c < \pi v_c/L_x \qquad (22)$$

According to Estes[6], this cut-off for a minimal tether extent ($L_x \cong$ m)

is just below the lower hybrid frequency for the F-layer of the ionosphere. Consequently, in calculating the impedance where the square of this cut-off factor occurs, there will be no significant contributions from any but the lowest frequency band for the PMG project.

VI. RESULTS FOR THE IMPEDANCE AND CHARGE DENSITY

The impedance, Z, of the ionosphere required for the maintenance of the charge carrying electromagnetic waves is given by a k-space integral of $\vec{j}_c{}^* \cdot \vec{E}$. Using the electric field found in equation (8) yields

$$\vec{j}(\vec{k}) \cdot \vec{E}(\vec{k}) = \frac{4\pi i \omega}{c^2} \left[\frac{|\vec{k} \cdot \vec{j}|^2}{(k_z^2 - k_A^2) k_\perp^2} + \frac{|(\vec{k} \times \vec{j})_z|^2}{(k^2 - k_A^2) k_\perp^2} + O(q^2 \omega^4) \right] \quad (23)$$

As shown in section III, the pole at $k^2 = k_A^2$ gives no significant contribution to the generation of waves and to the impedance. The cut-off factor discussed in the last section, limits the frequencies to those of region I, i.e. $\omega \leq \Omega_i^*$ and where $g(\frac{\omega}{\Omega_i}) \epsilon_\perp$ Consequently, using the simplest model given by equation (21a) yields

$$Z = - \frac{Re}{\pi^2 c^2/2} \int_{-\kappa}^{\kappa} dk_x \int_{-\infty}^{\infty} dk_y \int_{-\infty}^{\infty} dk_z \frac{i k_z v_c \left(\frac{\sin k_z L/2}{k_z L/2} \right)^2 \sin^2 k_y L/2}{(k_z^2 - k_A^2) k_\perp^2} \quad (24)$$

where $k_A = k_x (\frac{v_c}{v_A}) \cdot (1 - (\frac{k_x v_c}{\Omega_i})^2)^{-\frac{1}{2}}$ and $\kappa = \frac{\Omega_i}{v_c}$. With the appropriate contour for the k_z - integration, the expression for the impedance becomes

$$Z = \frac{Re}{\pi c^2/2 v_A} \int_{-\kappa}^{\kappa} dk_x \int_{-\infty}^{\infty} dk_y (1 - (\frac{k_x v_c}{c})^2)^{\frac{1}{2}} \frac{\left(\frac{\sin k_x L/2}{k_x L/2} \right)^2 \sin^2 k_y L/2}{k_x^2 + k_y^2}$$

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The integration over dk_y is easily done to give

$$Z = 2 \frac{V_A}{c^2} \int_0^1 \left(\frac{\sin k \bar{L}_x/2}{k \bar{L}_x/2} \right)^2 (1 - e^{-k\bar{L}}) \sqrt{1-k^2} \frac{dk}{k} \qquad (25)$$

where $\bar{L} = L/d$, $\bar{L}_x = L_x/d$ with $d = v_c/\Omega_i \cong 25$ meters.

The difference of this final expression for Z and that given in reference (6) is just the contribution for $j_x E_x$ which that author neglected in his determination of Z.

If the limit of $L_x << d << L$ is taken, then

$$Z \longrightarrow 2 \frac{V_A}{c^2} \left(\ln 2L + \gamma - 1 \right) \qquad (26)$$

where γ is the Euler constant. In this limit the horizontal extent of tether is no longer important and agreement is obtained with reference 7, for their contribution from the first frequency region. Clearly in this limit, the physically important frequency cut-off factor coming from the finite horizontal extend of the tether system is turned off. Consequently, frequencies are no longer limited to the first band and the higher bands could give the significant contributions found by reference 7 for a horizontal extend of a few tenths of a millimeter, i.e. $Z_I \cong .35\Omega$, $Z_{II} \cong 10^5\Omega$ and $Z_{III} \cong 10^4\Omega$. Such extents are unphysical and one should use the results of this reference best as a mathematical check.

A second limit in which $L_x >> L >> d$ is perhaps more physical. In this limit

$$Z \longrightarrow \frac{2\pi V_A}{c^2} \left(\frac{L}{L_x} \right) \qquad (27)$$

first obtained by reference 10. However this limit could not describe the PMG project since a plasma having such a large extend in the direction of motion would similarly extend in the vertical direction and envelope both ends of the tether and thus short-circuit the system.

The inclusion of the factor $\frac{\sin k_z L_z/2}{k_z L_z/2}$ of equation (21b), due to the extent in the z-direction, brings in a factor of

$$\left(\frac{\sin k_A L_z/2}{k_A L_z/2}\right)^2$$ into the impedance calculation. Since

$k_A = k_x (\frac{v_c}{c})\epsilon_{\perp}^{\frac{1}{2}}$ this factor will further strengthen the frequency cut-off.

The plasma generated by a hollow cathode will itself be distorted by the geomagnetic field. Due to the large value of the conductivity in the direction of the geomagnetic field compared to that perpendicular to the field, one could expect the effective extend of the plasma cloud in the field direction, L_z, to be very much larger than that in the perpendicular direction, L_x. It might well be that in the PMG project the spatial extend of the plasma cloud along the field lines will be the critical parameter.

Turning to the predictions of the models for charge density in the Alfvén wings, equations (8) and (14) yield

$$\rho(\vec{k},\omega) = \frac{\omega}{c^2}\left[\frac{k\cdot j}{k_z^2 - k_A^2} + \frac{i(k\cdot j)_z g\omega^2/c^2}{(k^2 - k_A^2)(k_z^2 - k_A^2)} + O(g^2\omega^4/c^4)\right] \quad (28)$$

Rewriting the denominator of the second term as

$$\frac{1}{(k^2 - k_A^2)(k_z^2 - k_A^2)} = \left[\frac{1}{k_z^2 - k_A^2} - \frac{1}{k^2 - k_A^2} \right] \frac{1}{k_\perp^2}$$

the pole structure remaining, after the non-contributiong pole at $k^2 = k_A$ is discarded, is just that encountered in equation (24). The result of the k_z and k_y integration is a contribution similar to that resulting from the first term in the equation (28) but reduced by a factor of $(v_c/c)^2$.

The contribution of the first term gives

$$\rho(\vec{x},t) = \frac{I v_A}{2\pi c^2} \left(\delta(y - \tfrac{1}{2}) - \delta(y + \tfrac{1}{2}) \right) \int_0^{'} dk \left(\frac{\sin k \, L_x/2}{k \, L_x/2} \right) \cos k \, (\bar{x} + 1 \bar{z}'1) \cdot \sqrt{1 - k^2}$$

where $\bar{x} = x/d$, $\bar{z}' = \bar{z} \, (v_c/v_A) \, (1-k^2)^{-\frac{1}{2}}$.

Only for $L_x >> d$ does the upper limit of the integration become unimportant and diffraction effects due to the $(1-k^2)^{\frac{1}{2}}$ factor relating z' and z result in the normal sharp Alfvén wings. Figure 3 shows the situation for the case of $L_x = 10$ m and $L_x = 800$m.
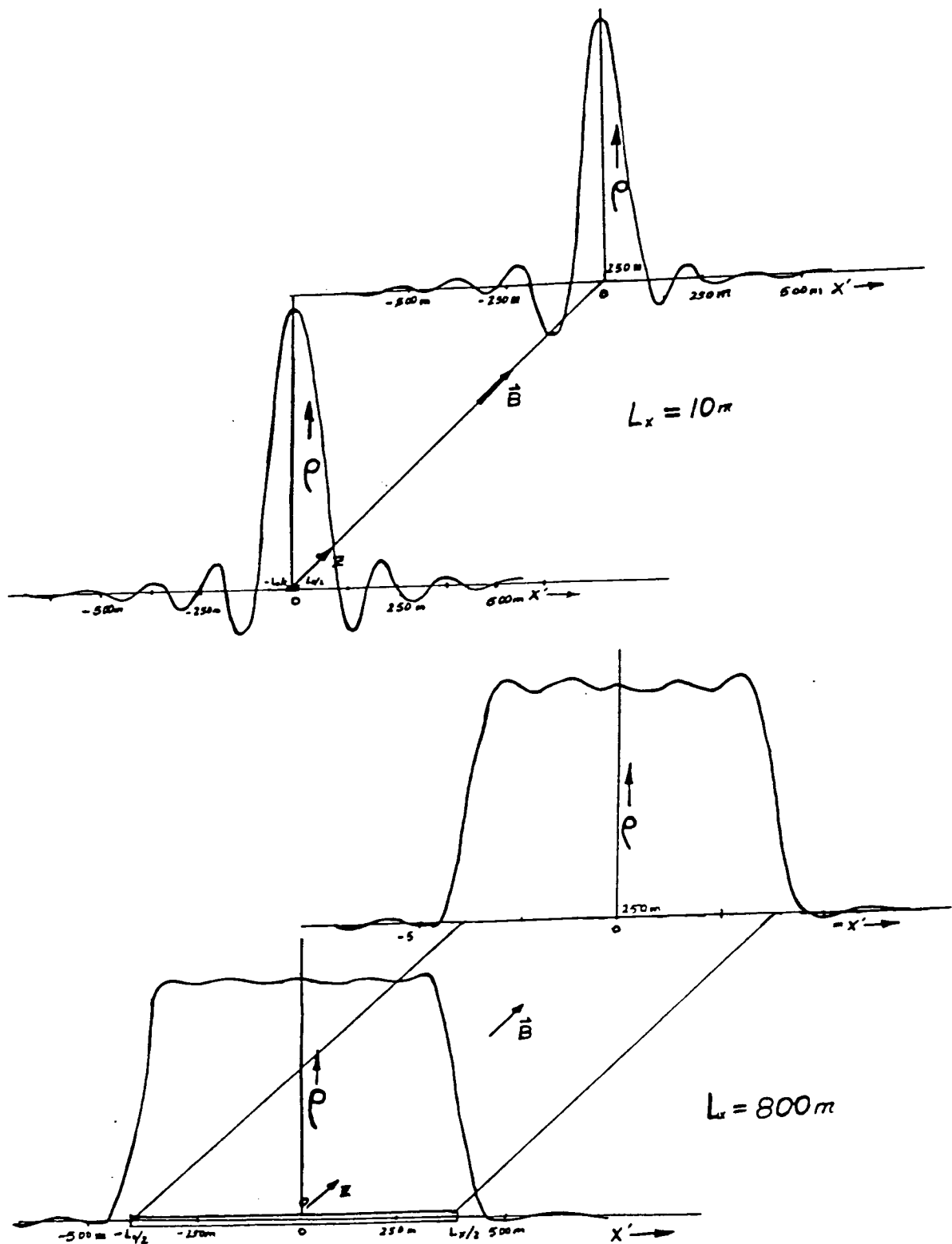
Figure 3.   Charge Density in Upper Wing for L_x = 10 m and 800 m.

VII. CONCLUSION

This report has attempted to clearly point out the assumptions made and common results found in theoretical models put forth to explain the dissipation of charge into the ionosphere from the ends of a conductive tether. The most dubious of the assumption is that of treating the ionosphere in a "cold plasma" dialectric medium. Once the assumption is made the spatial extend of the tether need be modeled in a reasonable manner. The dimensions of the region over which the charge is released into the ionosphere, is shown to play a crucial role in restricting the phenomena to low frequencies. If a physically reasonable size is used, calculations show that the models predict an impedance on the order of a few ohms.

The adaption of this model to describe the PMG project is not easy since the plasma clouds generated at the ends of the tether will themselves be strongly affected by the geomagnetic field. Still it seems reasonable to accept the impedance value mentioned above as a good first approximation.

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