BREAKDOWN OF THE CONSERVATIVE POTENTIAL EQUATION

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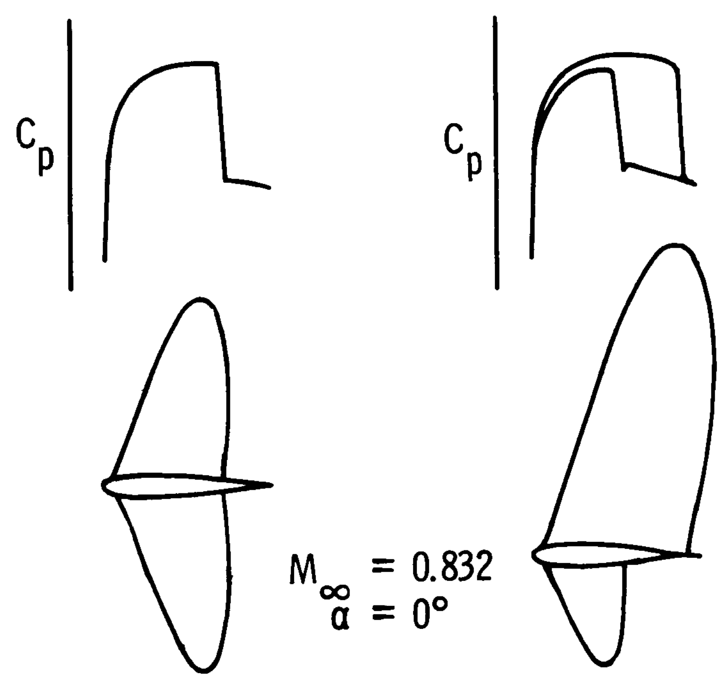
MULTIPLE SOLUTIONS OF THE TRANSONIC POTENTIAL FLOW EQUATION

In 1981, John Steinhoff and Antony Jameson discovered that numerical codes based on the conservative full-potential equation could converge to more than one solution.[1] The case shown corresponds to a symmetrical Joukowski airfoil section at zero angle of attack and 0.832 free-stream Mach number. Two fully converged solutions were obtained by Steinhoff and Jameson, one symmetrical with zero lift and the other asymmetrical with a large positive lift.

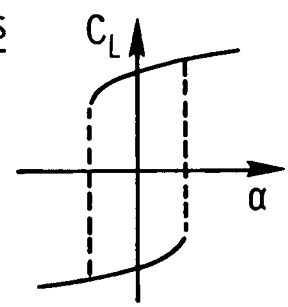


MULTIPLE SOLUTIONS OF THE TRANSONIC POTENTIAL FLOW EQUATION

The major findings reported by Steinhoff and Jameson[1] were the discovery of multiple solutions, usually occurring in narrow bands of M_∞ and α, and a hysteresis loop in the lift behavior. After many tests, they concluded that the problem existed at the differential level (i.e., not a numerical problem); that perhaps it was connected to a physical phenomenon such as buffet; and that, therefore, it could also be observed if the Euler or Navier-Stokes equations were used instead of the conservative potential equation.

Major Findings

- Multiple solutions
- Hysteresis loop
- Narrow bands of M_∞, α



Conclusions

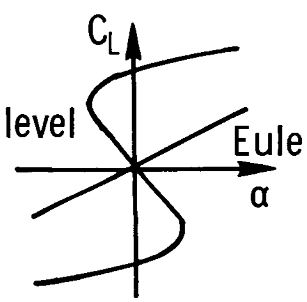
- Problem at differential level
- Possible physical significance
- Possible problem of Euler or Navier Stokes equations

A COMPARATIVE STUDY OF THE NONUNIQUENESS PROBLEM OF THE POTENTIAL EQUATION

To answer some of the issues raised by Steinhoff and Jameson, a study comparing the potential and Euler solutions was conducted in 1983.[2] The results of this work confirmed that the problem existed at the differential level. The hysteresis loop was found to be a result of the "folding back" nature of the lift curve exhibited by the conservative potential equation but not the Euler equations. In addition, it was found that the problem obeyed the transonic similarity law. It was conjectured that the nonuniqueness was associated with the isentropic shock jump conditions.

Major Findings

- Confirmed problem at differential level
- No hysteresis loop
- No problem with Euler equations
- Problem obeyed transonic similarity law



Conclusion

- Problem associated with shock jump conditions

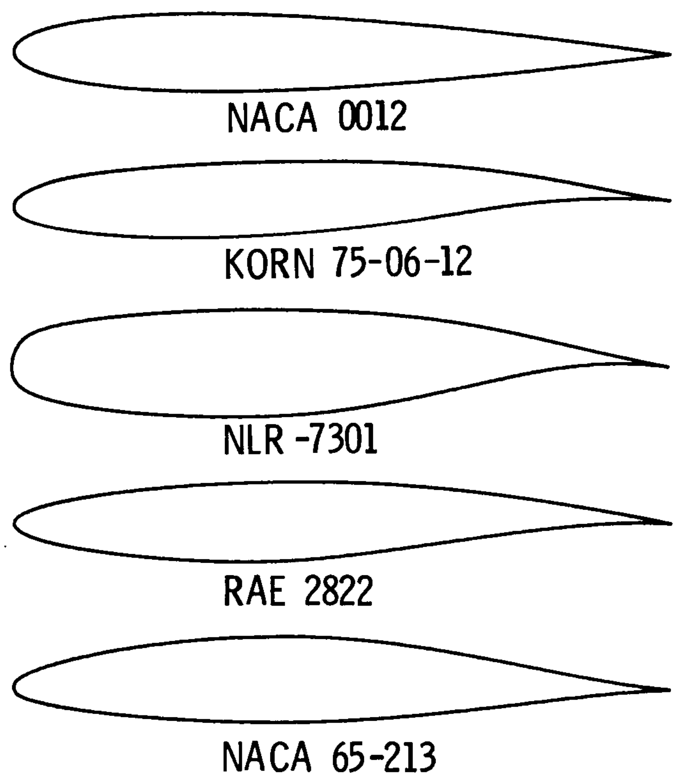
PRESENT INVESTIGATION ADDRESSES THE FOLLOWING QUESTIONS

After these investigations, a number of issues remained to be answered. Since only a few airfoil shapes had been considered, it was not clear if the problem would appear for all airfoil shapes. Was it limited to narrow M_∞, α bands as originally proposed by Steinhoff and Jameson? Or did it occur throughout the M_∞, α plane? What was the cause of the problem?

- Is the problem universal?
- Where does it occur in M_∞, α plane?
- What is its cause?

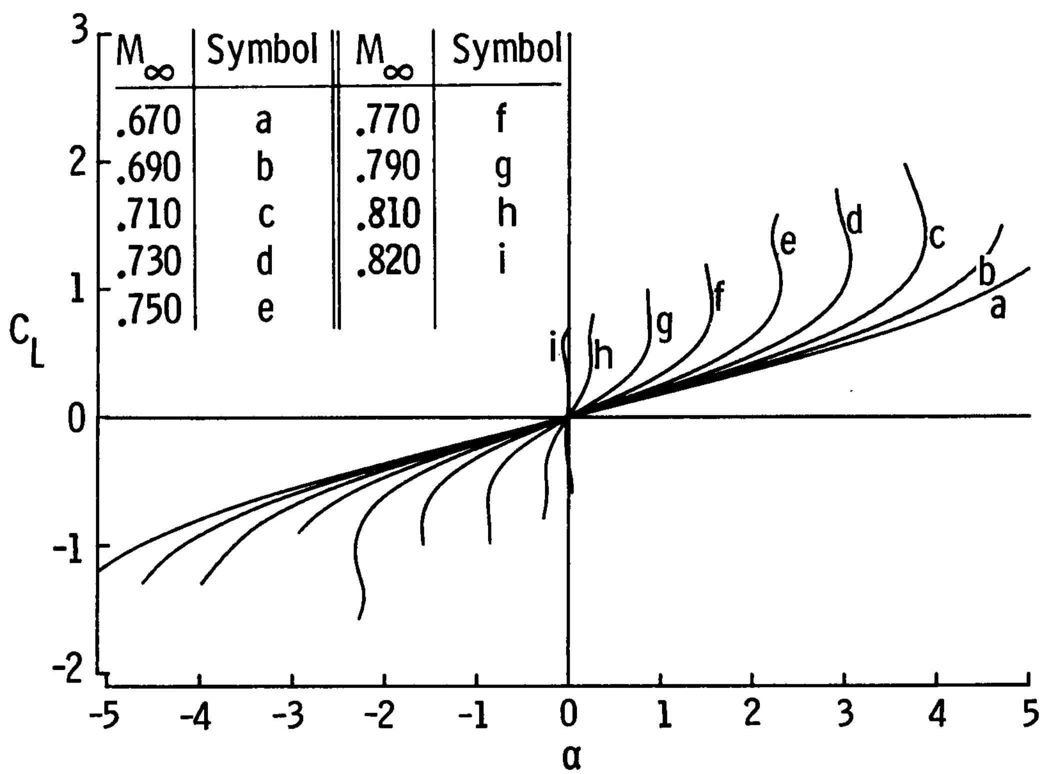
AIRFOIL SECTIONS INVESTIGATED WITH A MULTIGRID CONSERVATIVE POTENTIAL CODE

To answer these questions an investigation was conducted for the five airfoil shapes shown in this figure. Two of these airfoils, the KORN 75-06-12 and the NLR-7301, are shockless supercritical airfoils at design conditions. For each airfoil, approximately 10 different free-stream Mach numbers were tested at approximately 20 different angles of attack using a conservative potential code. Overall, approximately 1,000 calculations were performed.

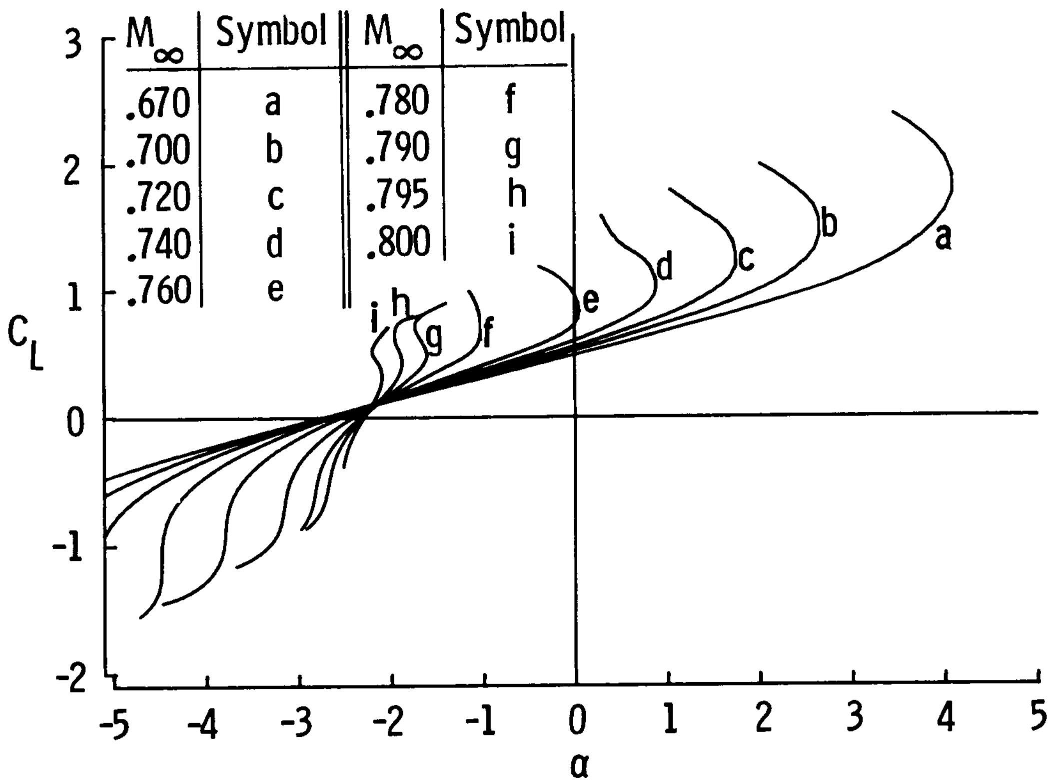


NACA 0012

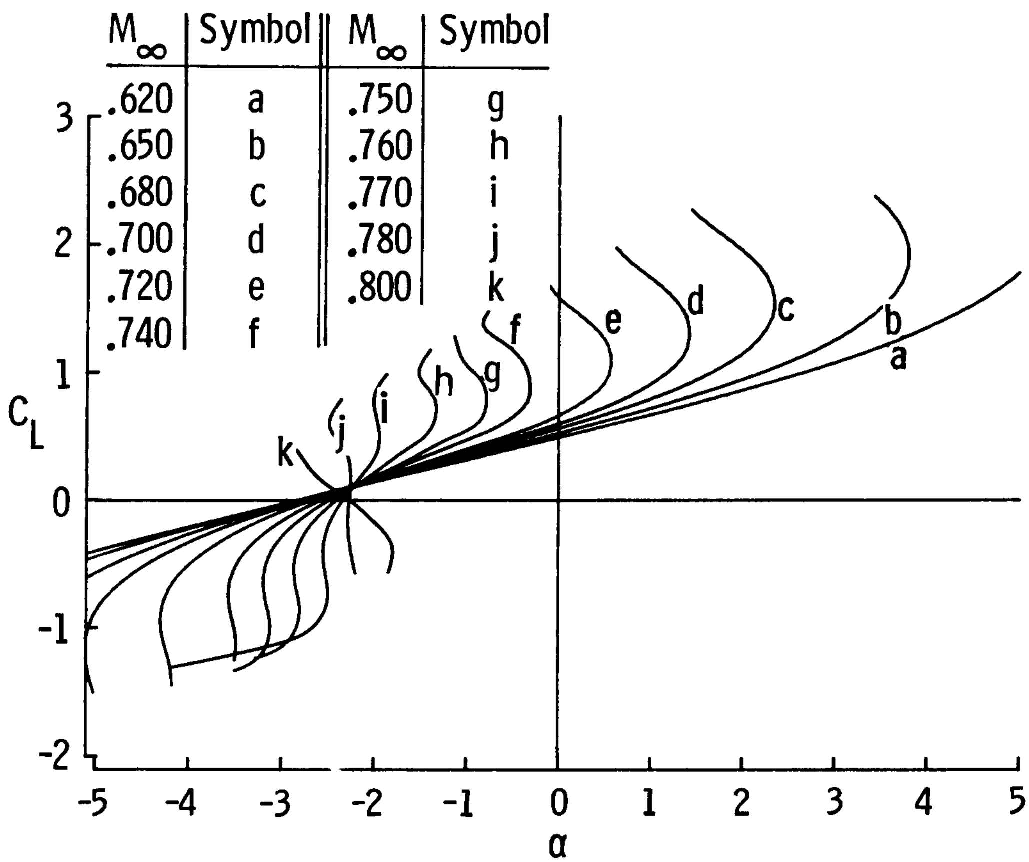
Spline-fitted lift curves for an NACA 0012 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



Spline-fitted lift curves for a KORN 75-06-12 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.

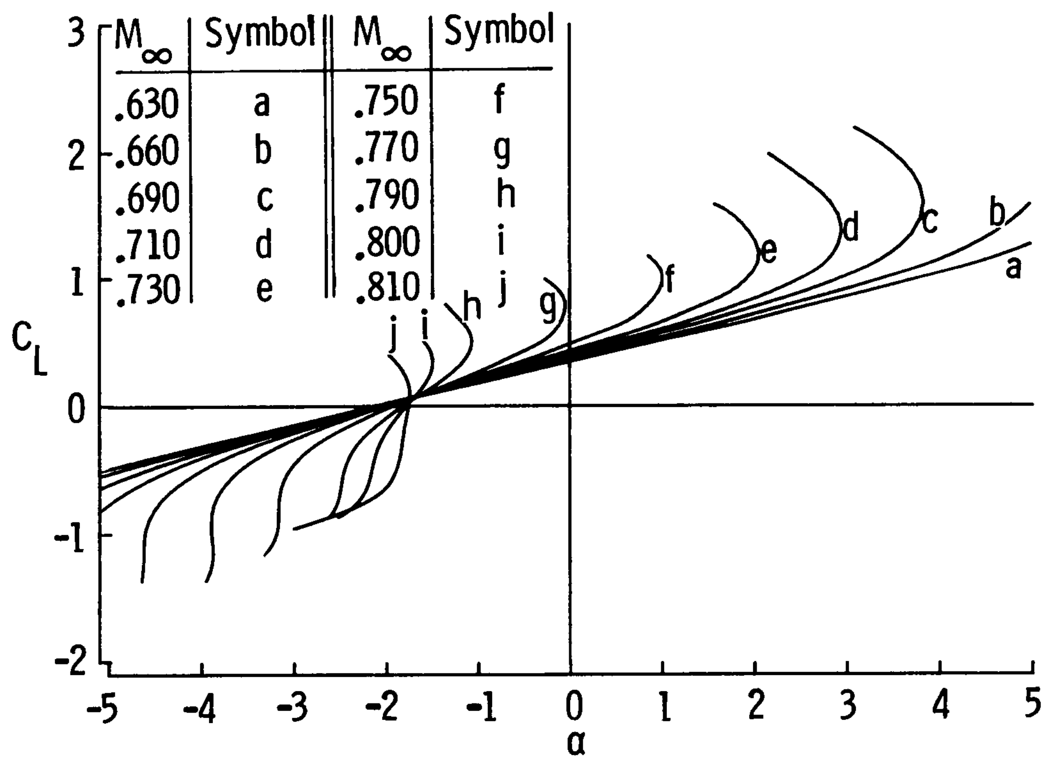


Spline-fitted lift curves for an NLR-7301 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



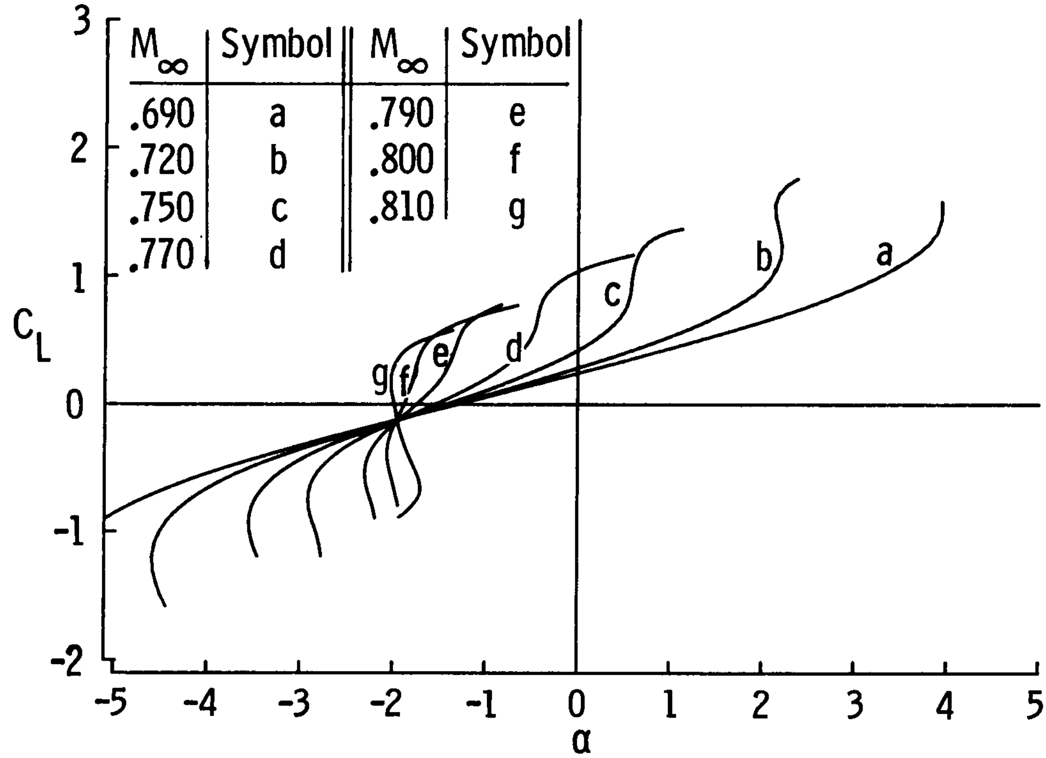
RAE 2822

Spline-fitted lift curves for an RAE 2822 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



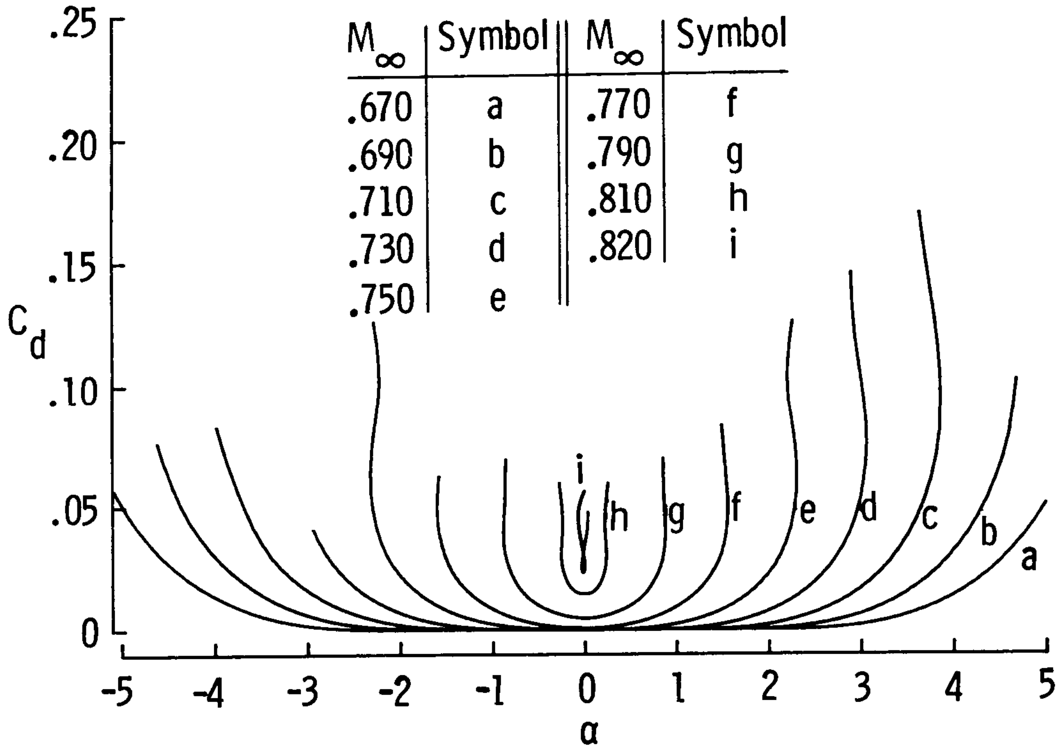
NACA 65-213

Spline-fitted lift curves for an NACA 65-213 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



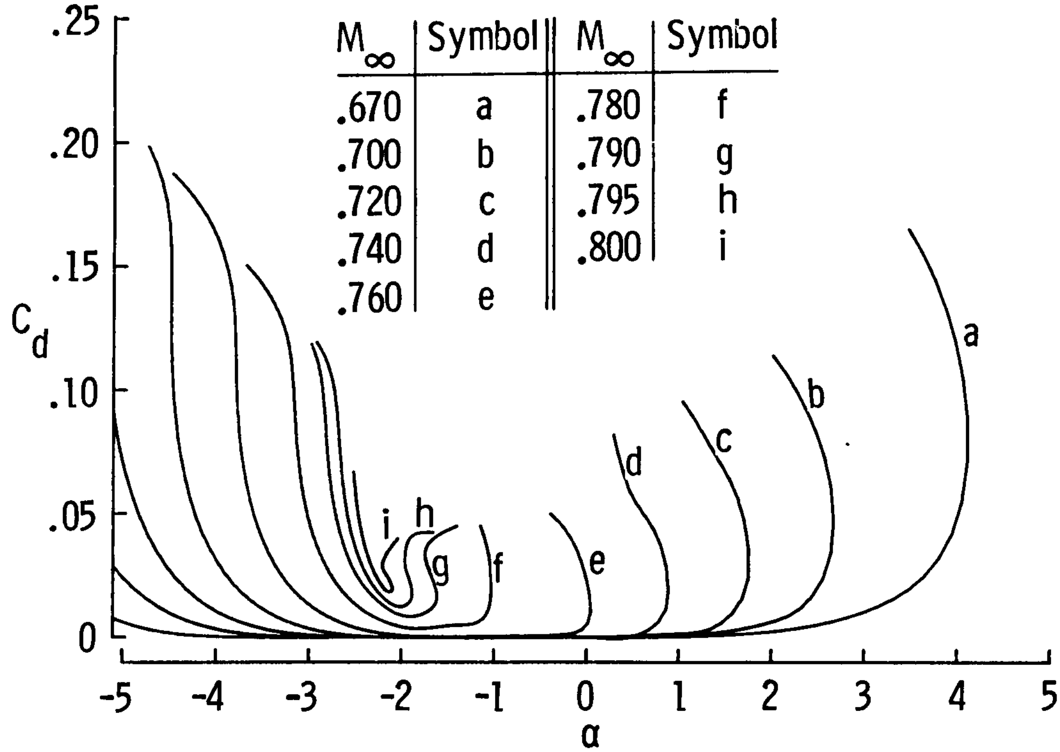
NACA 0012

Spline-fitted drag curves for an NACA 0012 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



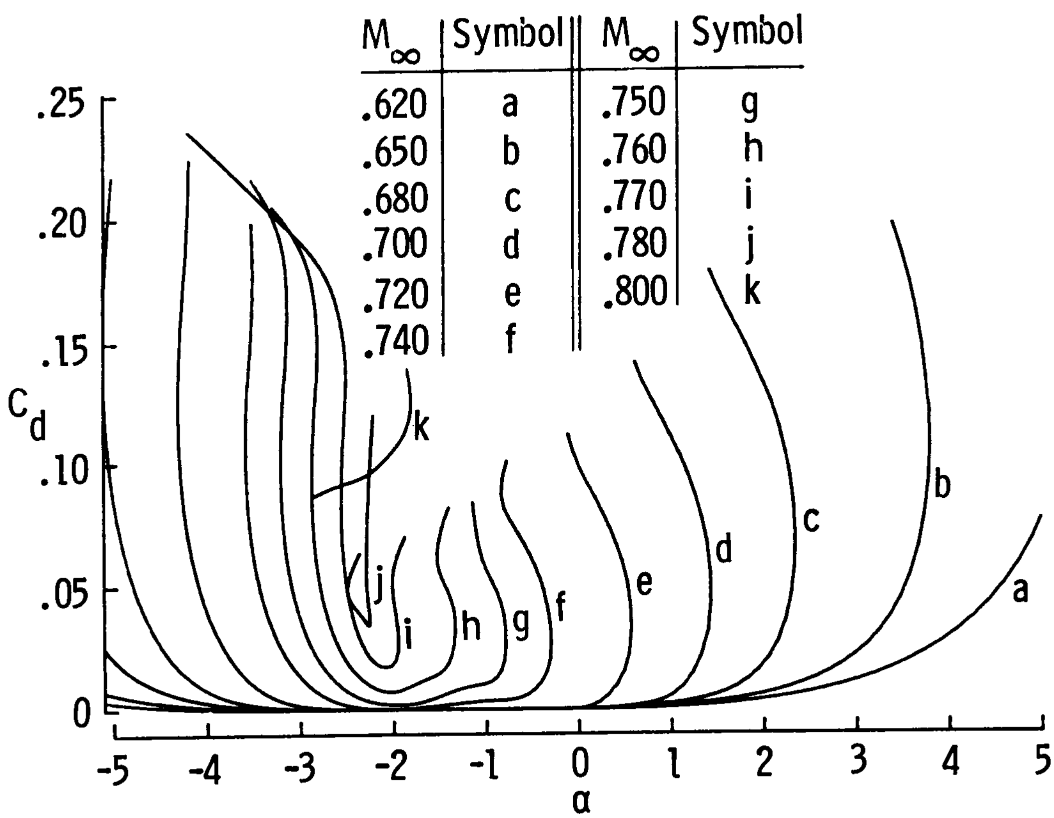
KORN 75-06-12

Spline-fitted drag curves for a KORN 75-06-12 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



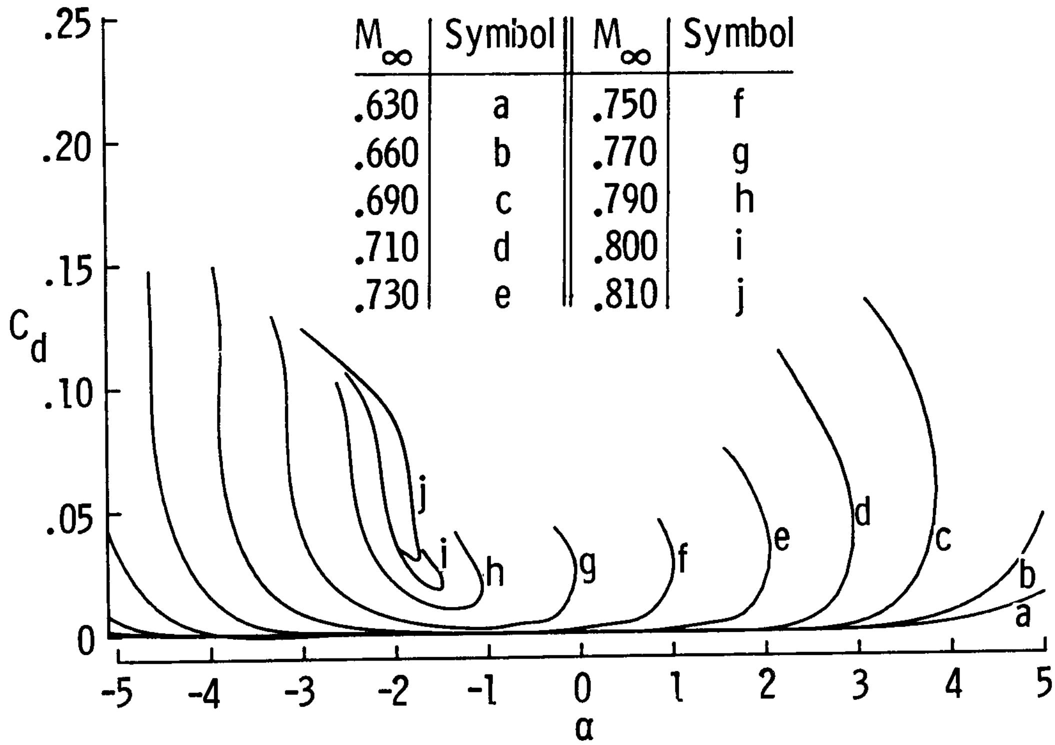
NLR-7301

Spline-fitted drag curves for an NLR-7301 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



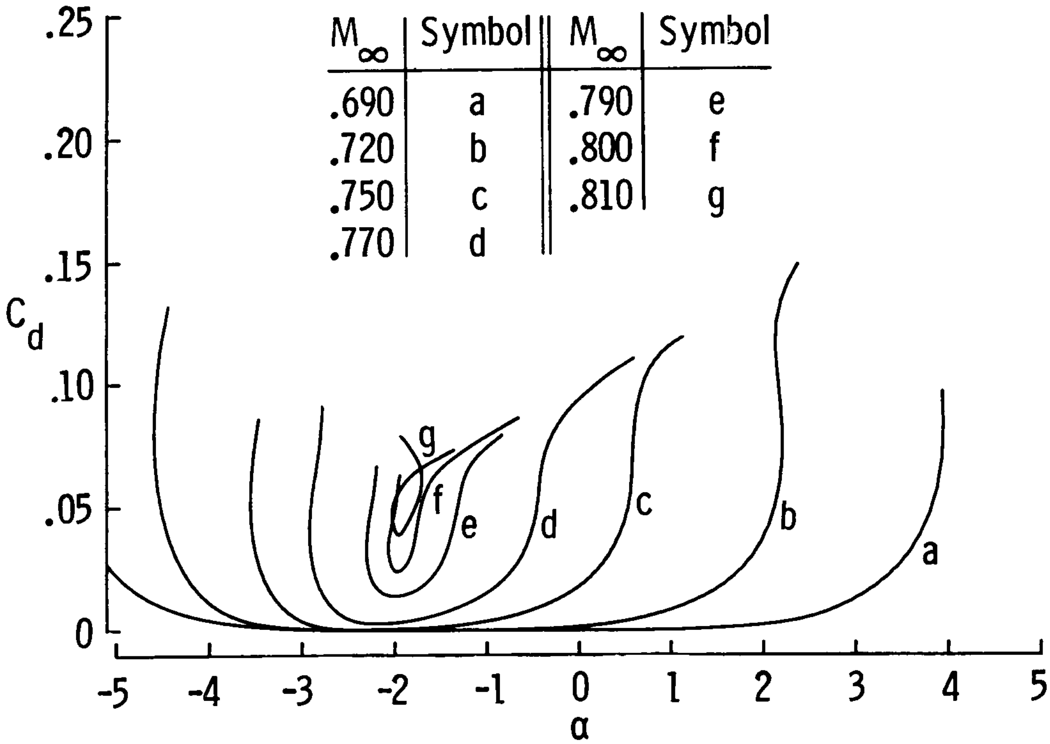
RAE 2822

Spline-fitted drag curves for an RAE 2822 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



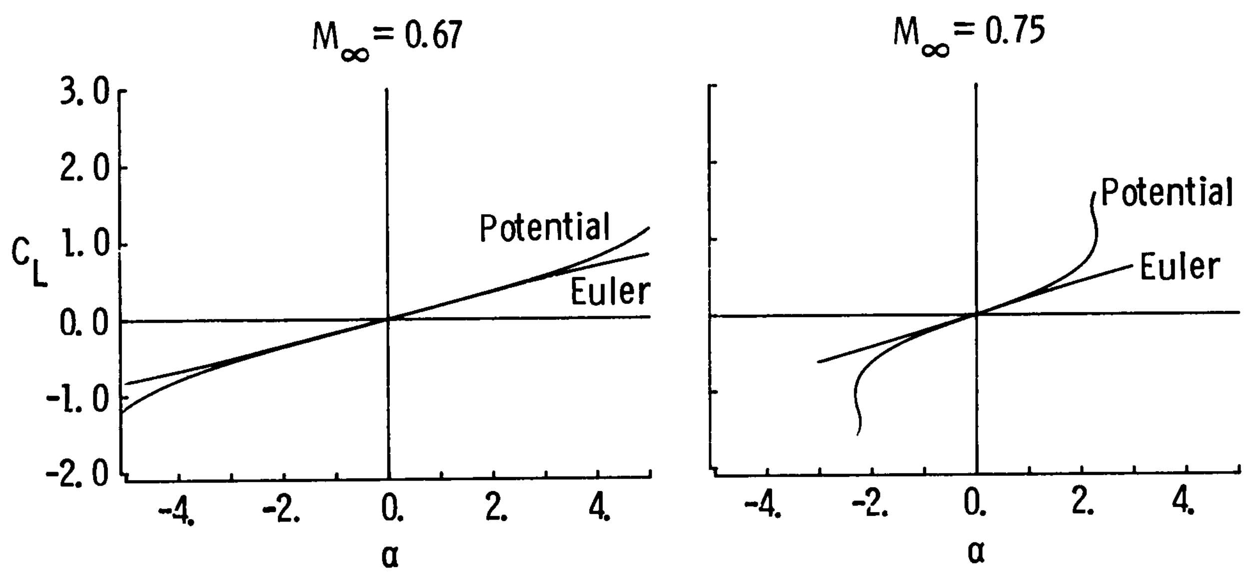
NACA 65-213

Spline-fitted drag curves for an NACA 65-213 airfoil section at various M_∞ obtained from a conservative potential code are shown on this figure.



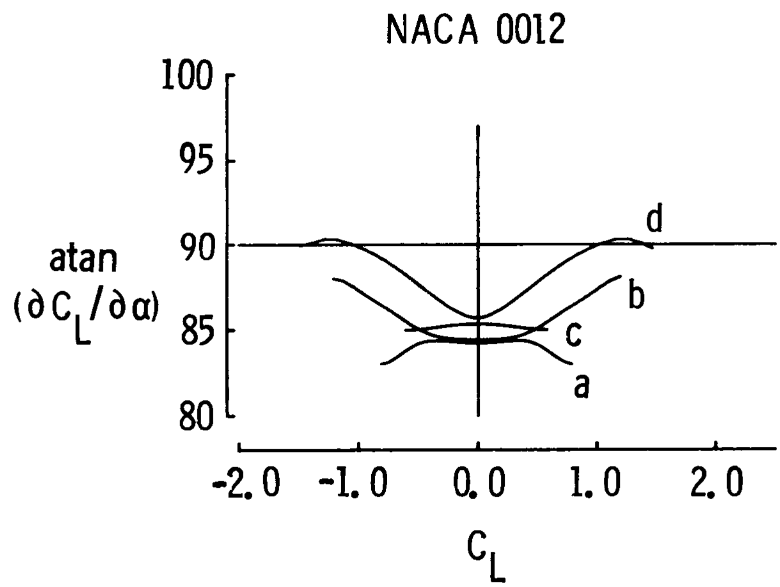
DETAILED LOOK AT NACA 0012

A comparison between the conservative potential results and Euler results shows a remarkable qualitative difference in the lift behavior. Results are shown for an NACA 0012 section, but the same trend is observed for the other airfoils tested.



LIFT-CURVE SLOPE

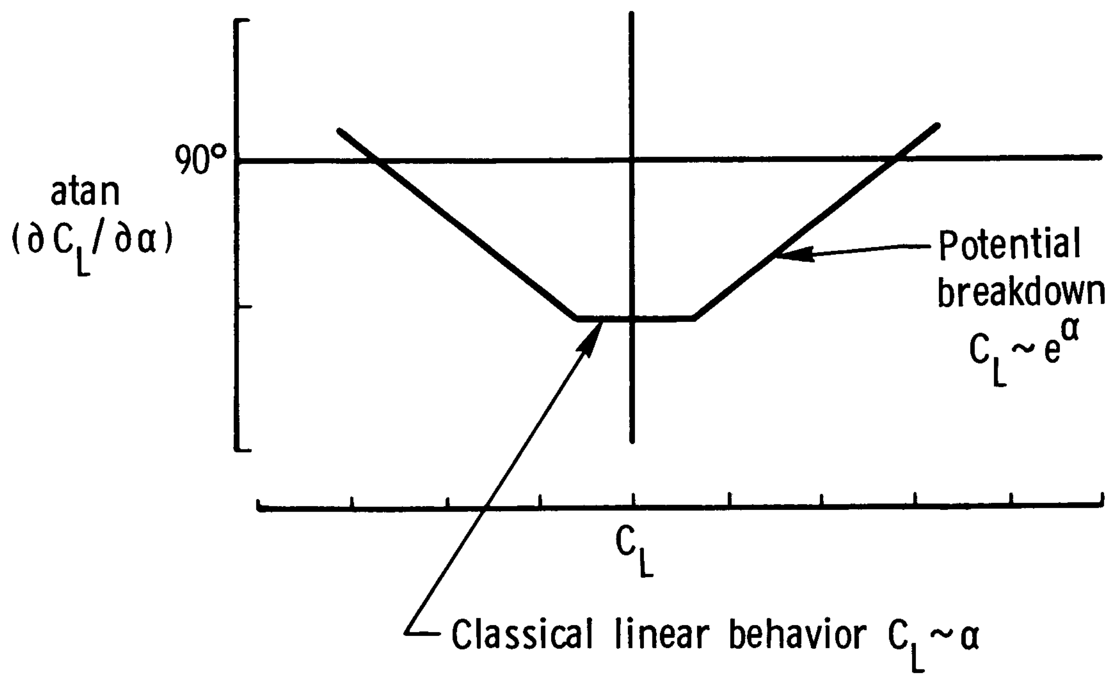
An inspection of the angle made by the lift curve with the horizontal, plotted versus the lift coefficient, shows that outside of the Prandtl-Glauert region (linear) the potential and Euler behaviors follow opposite trends. If the point at which the two depart is examined, we find that it corresponds to the appearance of shock waves in the calculation (i.e., strong nonlinear behavior).



a	Mach .67 Euler
b	Mach .67 Potential
c	Mach .75 Euler
d	Mach .75 Potential

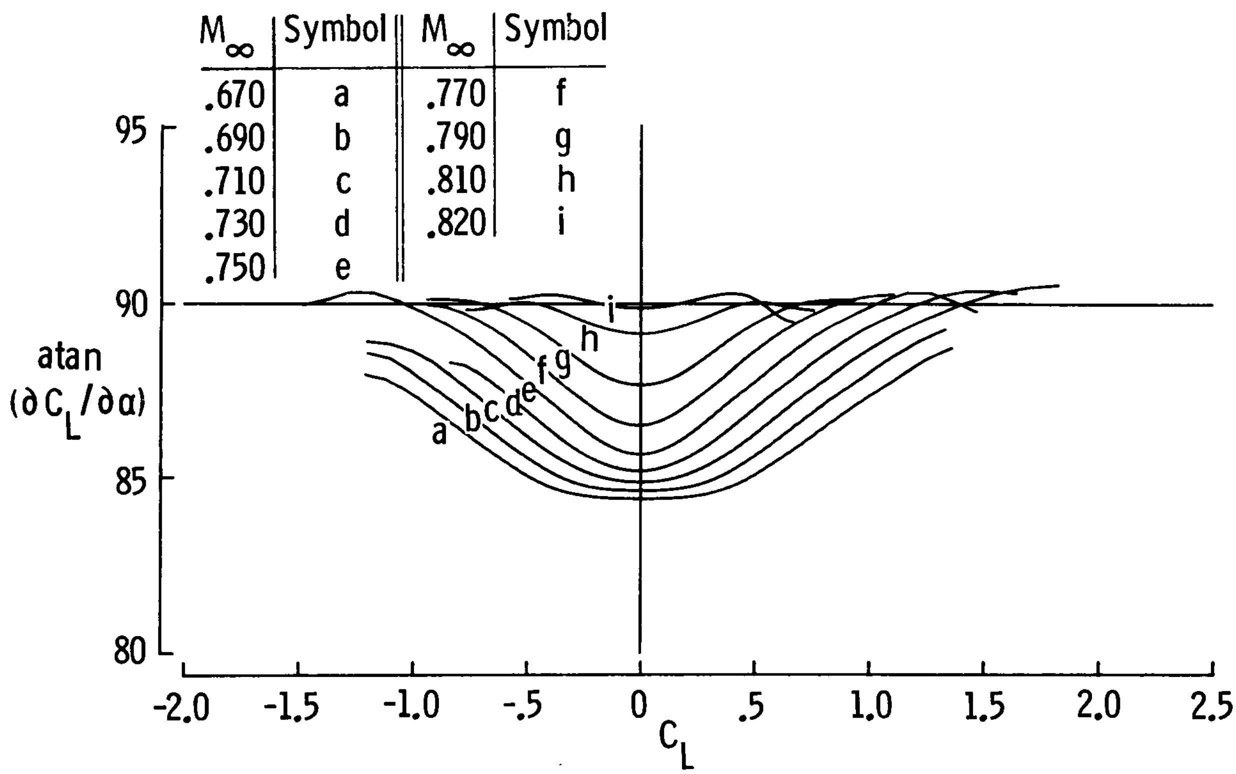
SKETCH OF LIFT-CURVE SLOPE MAIN FEATURES

The typical conservative potential behavior is shown in this figure. The potential and Euler solutions agree in the linear behavior region. Once a shock wave occurs, the lift predicted by the potential approximation increases like e^{α}. It is this behavior of the potential approximation which eventually leads to a "folding back" of the lift curve as its slope increases past 90°.

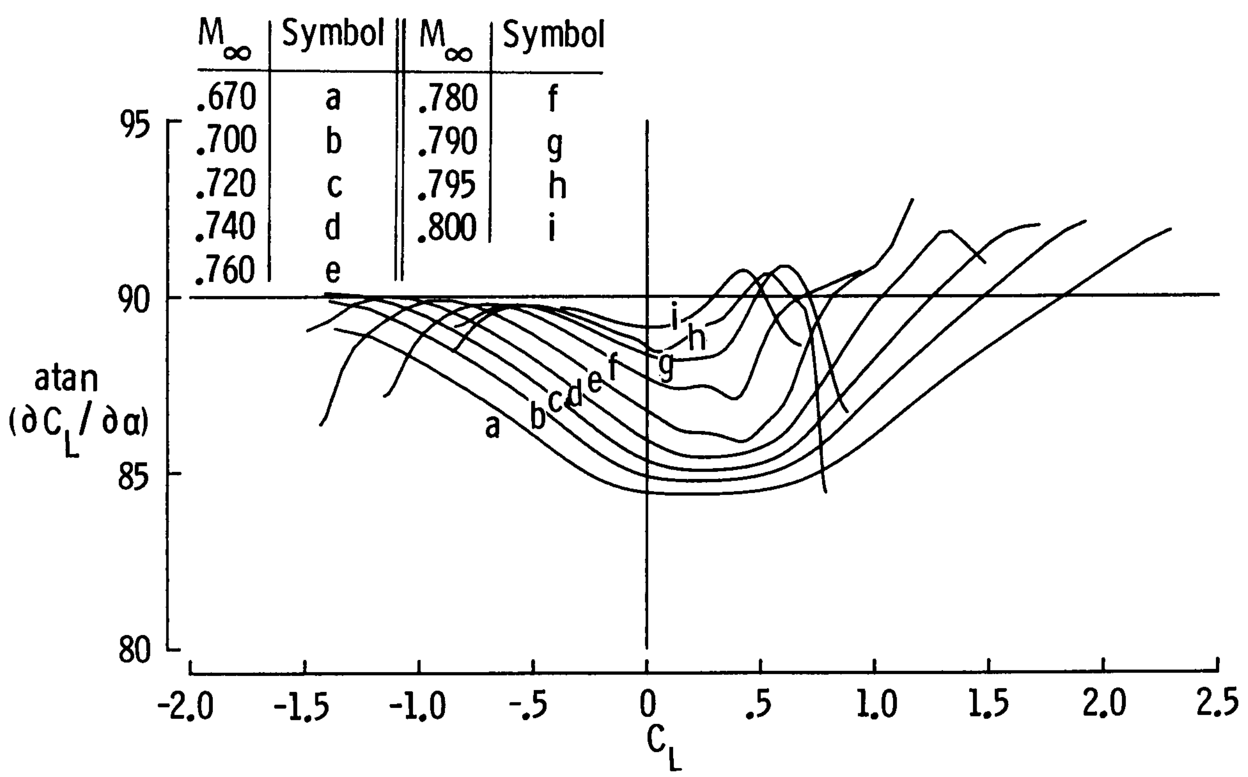


NACA 0012

The angle made by the lift curve plotted as a function of C_L for the NACA 0012 airfoil as predicted by the conservative potential code is shown on this figure.

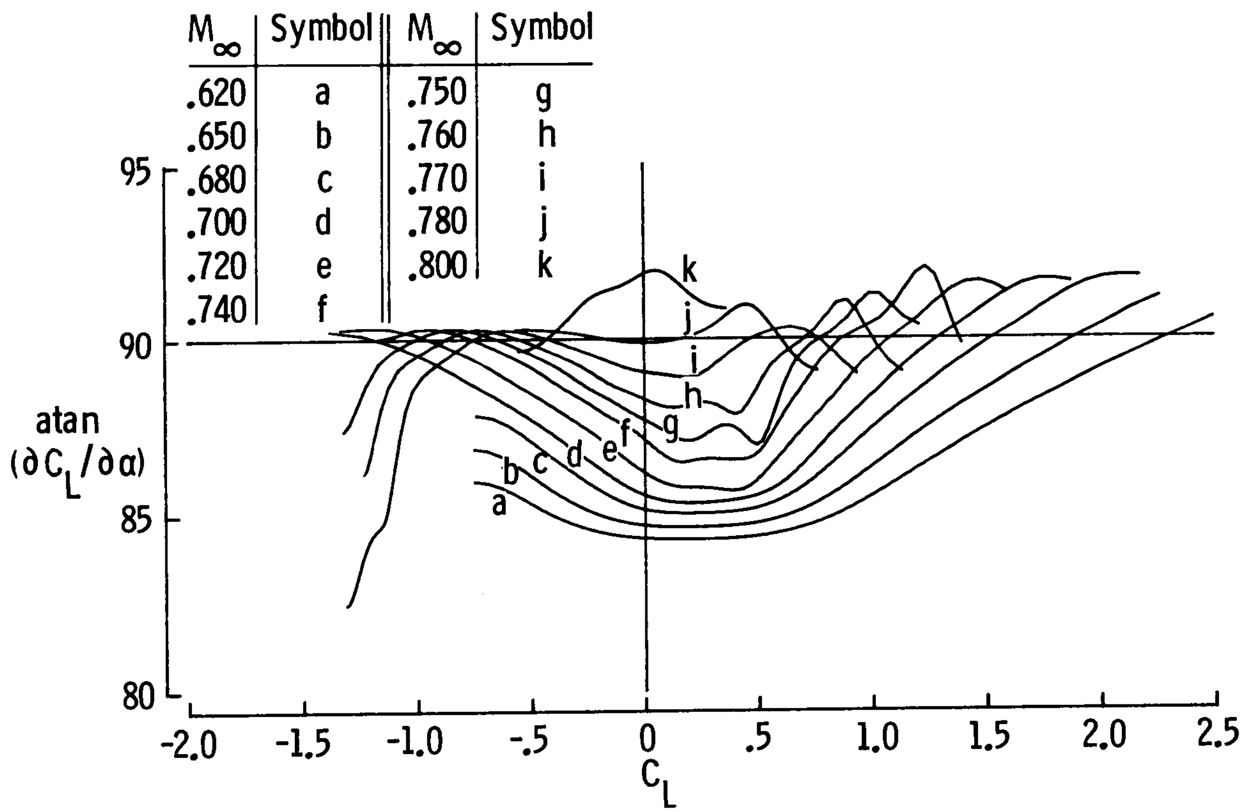


The angle made by the lift curve plotted as a function of C_L for the KORN 75-06-12 airfoil as predicted by the conservative potential code is shown on this figure.



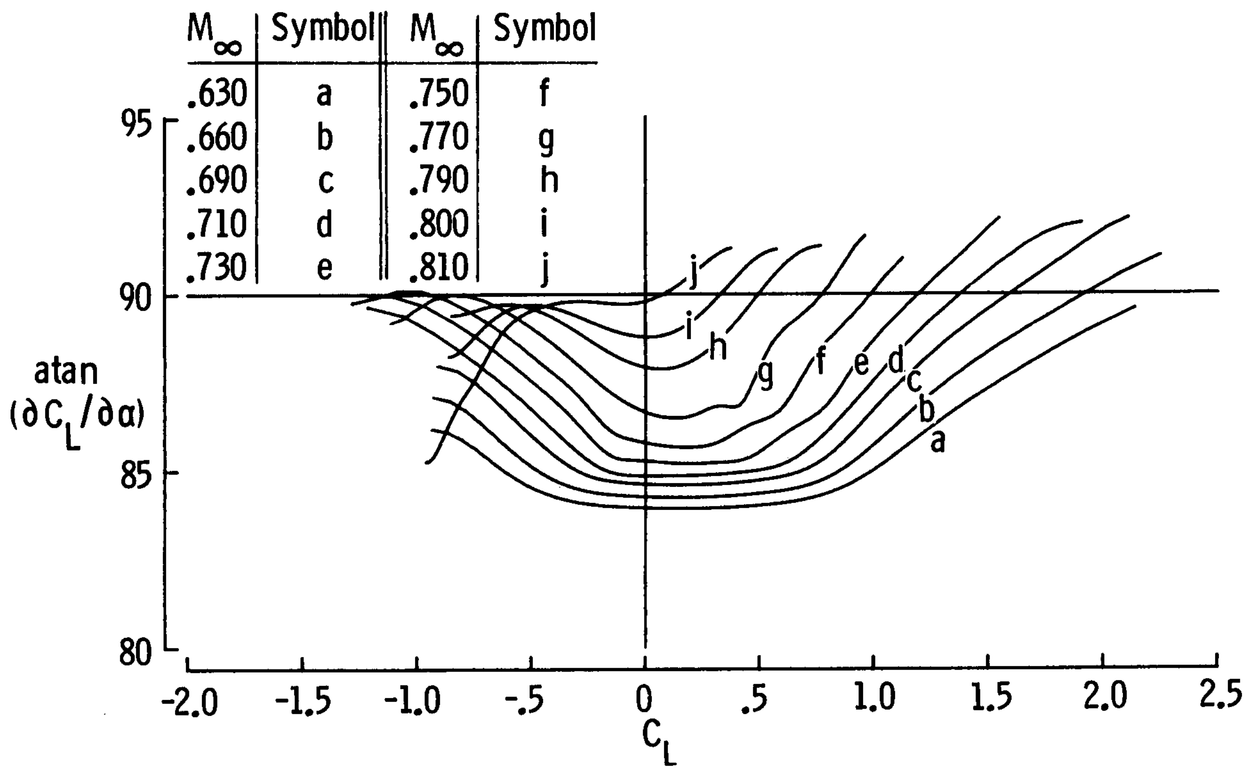
NLR-7301

The angle made by the lift curve plotted as a function of C_L for the NLR-7301 airfoil as predicted by the conservative potential code is shown on this figure.



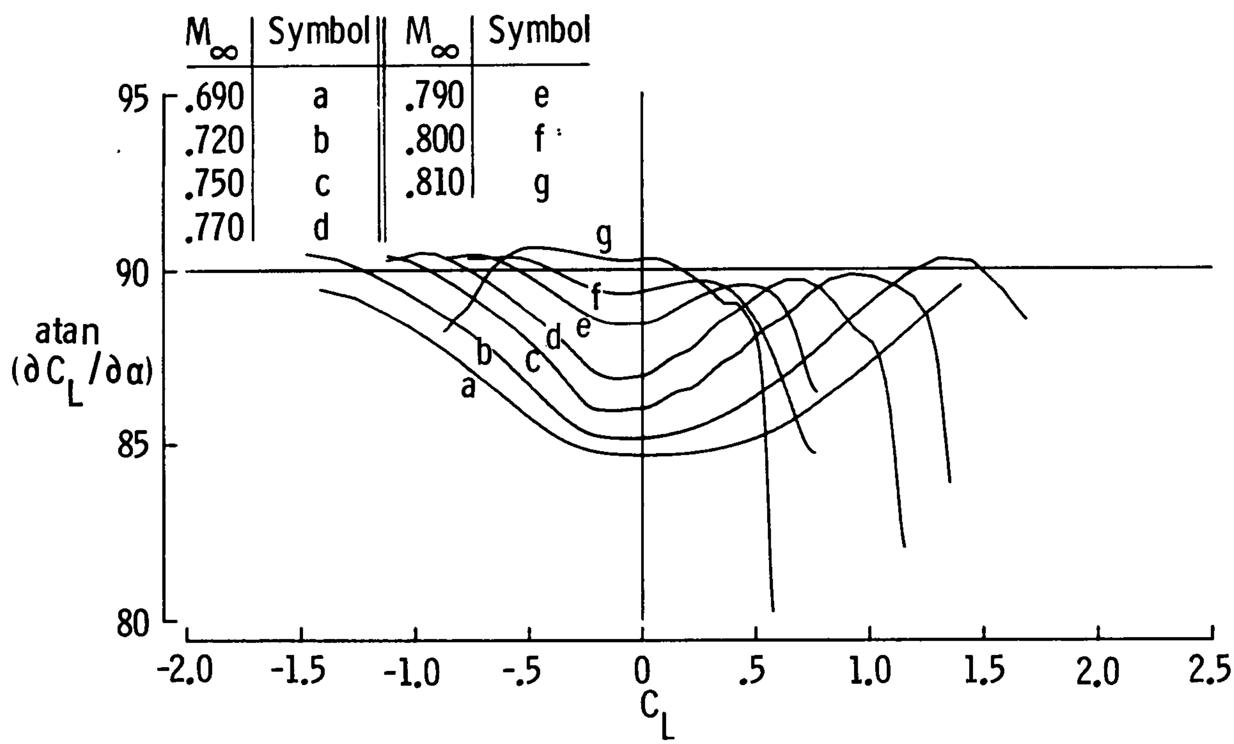
RAE 2822

The angle made by the lift curve plotted as a function of C_L for the RAE 2822 airfoil as predicted by the conservative potential code is shown on this figure.



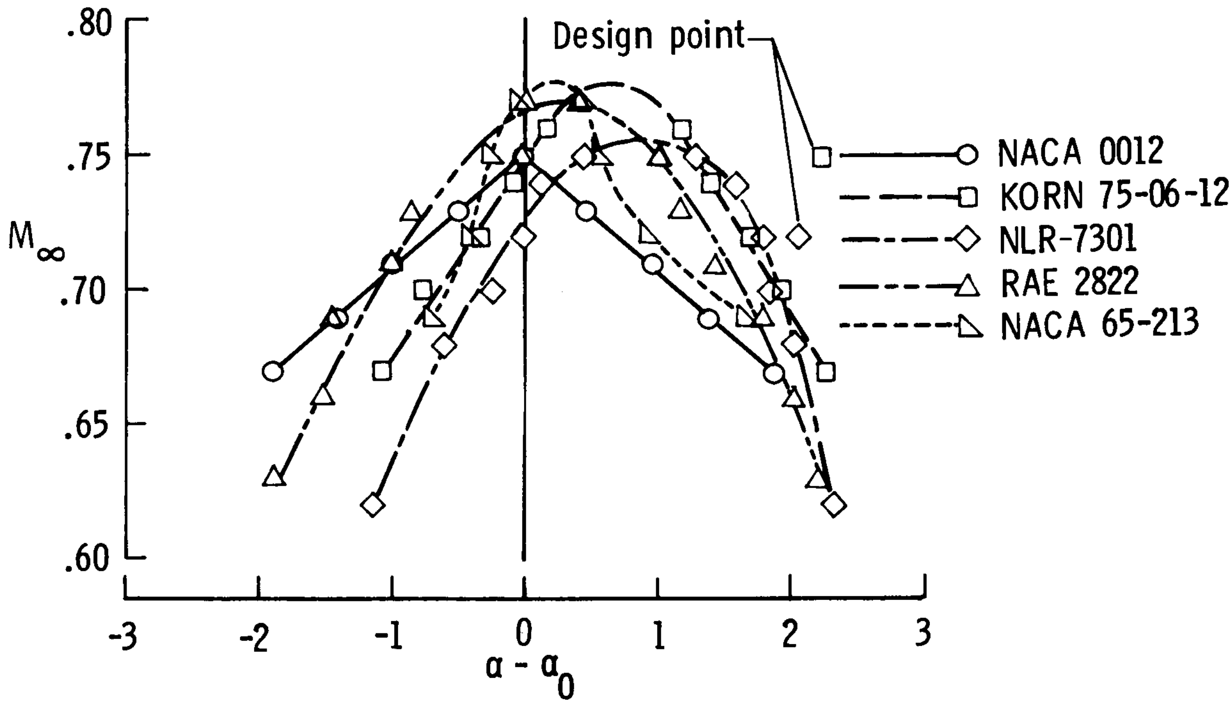
NACA 65-213

The angle made by the lift curve plotted as a function of C_L for the NACA 65-213 airfoil as predicted by the conservative potential code is shown on this figure.



BREAKDOWN REGION IN M_∞, α PLANE

This figure shows where the potential approximation breaks down for each of the five airfoils tested. The shockless design points for the KORN 75-06-12 and NLR-7301 airfoils are also shown in the figure. Note that these design points are isolated shockless flows surrounded by flows with shocks.



CONCLUSIONS

The breakdown of the conservative potential approximation occurs for all airfoils tested. It develops as soon as shock waves appear in the flow field. Since shock waves are not properly represented by the potential approximation, it is conjectured that the breakdown is due to the isentropic shock jump condition of the potential approximation.

- The problem appears universal for conservative potential formulation.
- It occurs as soon as shock waves are developed.
- It is conjectured that the problem is due to the isentropic shock jump condition.

REFERENCES

1. Steinhoff, J.; and Jameson, A.: Multiple Solutions of the Transonic Potential Flow Equation. AIAA Paper 81-1019, June 1981.

2. Salas, M. D.; Jameson, A.; and Melnik, R. E.: A Comparative Study of the Nonuniqueness Problem of the Potential Equation. AIAA Paper 83-1888, July 1983.