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Secondary Subharmonic Instability of Boundary Layers With Pressure Gradient and Suction

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SECONDARY SUBHARMONIC INSTABILITY OF BOUNDARY LAYERS

WITH PRESSURE GRADIENTS AND SUCTION

by

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ABSTRACT

Three-Dimensional linear secondary instability is investigated for boundary layers with pressure gradient and suction in the presence of finite amplitude Tollmien-Schlichting (TS) wave. The focus is on principle parametric resonance responsible for strong growth of subharmonics in low disturbance environment. Calculations are presented for the effect of pressure gradients and suction on controlling the onset and amplification of the secondary instability.

I. INTRODUCTION

In the study of boundary-layer control, it is well known that suction can completely stabilize a laminar boundary layer . Laminarization of the flow by suction, and subsequent viscous drag reduction, is the principle and the most effective mean used for laminar flow control (LFC) [1]. The overall efficiency of an aircraft with LFC is reduced by the power consumed by suction systems. Hence, it is necessary to keep the boundary layer laminar with the least possible suction. To accomplish this one needs to : 1) accurately calculate the stability characteristics of the flow, 2) understand the laminar to turbulent transition process, 3) evaluate the effect of suction control on some of the early stages leading to transition, which is our concern in this paper.

Most of the theoretical work for LFC has relied on linear primary stability theory and the e^n method, where a limited growth of the primary two-dimensional (2D) traveling disturbances (TS waves) is accepted such that the power requierements are minimized and almost all stages of transition are avoided. The effect of suction on the primary TS instability is well established and known to be drastic. This work has been summerized in the report of Saric [2] which lists most of the important papers in the subject.

Three dimensionality is known to be a necessary prerequisite for transition [26] . A key three-dimensional (3D) phenomenon in the early stages of transition is characterized by a strong secondary instability of 3D disturbances in the presence of finite amplitude two-dimensional (2D) primary disturbances. Primary 3D disturbances might be stable or very slowly growing in the absence of the 2D waves. This secondary instability has been recognized experimentally [3-7] in the uncontrolled boundary layer, and observed numerically in the uncontrolled [8,9] and controlled [10] boundary layers. Several routes to transition in boundary layer have been identified. One route is characterized by subharmonic 3D disturbances that are excited in the boundary layer at very low level of the TS amplitude. This mechanism produces either the resonant wave interaction predicted by Craik [11] (C-type), or the secondary instability of Herbert [12] (H-type). The other route, the so called K-type (or peak-valley splitting), occurs as the TS amplitude exceeds a certain threshold value. This represent the path to transition under conditions similar to Klebanoff et al.[3]. At sufficiently large amplitude one of the routes or a mixture of both will appear depending on the disturbance background.

In this paper, we are concerned with the subharmonic secondary instability mechanism as a route to transition that seams to be most dangerous in low-disturbance environments, as they are found in free flight. We investigate the development

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of a subharmonic secondary instability in a boundary layer with pressure gradients controlled by suction. Our objective is to evaluate the effect of suction control on this early stage leading to transition. Several questions need to be answered. Does suction delay the onset of the secondary instability? How sensitive is the growth of the secondary instability to the intensity of suction? What is the effect of the initial amplitude of the primary wave on this sensitivity? What is the effect of slight changes in the pressure gradient? Does the effectiveness of suction as a method for delaying transition depend on where it is applied, or on its intensity? Finally, if the boundary layer would be kept laminar with the least possible suction, then should one allow for a limited growth of the primary wave, or should one increase suction to fully stabilize the primary wave?.

II. PRIMARY INSTABILITY

We consider a 2D boundary-layer flow of an incompressible fluid with inviscid flow field given by $U = U(x)$ and distributed suction given by $v = vw(x)$ at the wall, where x is the streamwise direction and y is the vertical direction. The flow is governed by the nonsimilar boundary-layer equation,

$$(1) \qquad f_{\eta\eta\eta} + ff_{\eta\eta} + \hat{\beta}(1 - f_\eta^2) - \hat{\gamma}f_{\eta\eta} = 2\xi(f_\eta f_{\xi\eta} - f_\xi f_{\eta\eta})$$

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with boundary conditions,

(2) $f_\eta(\xi,0) = 0$, $f(\xi,0) + 2\xi f_\xi(\xi,0) = 0$

    $f_\eta(\xi, \eta \to \infty) = 1$

given in Görtler variables,

(3) $d\xi = U_e dx$ $d\eta = (U_e/\sqrt{2\xi})\, dy$

Then the velocity components u and v in terms of the new variables are,

(4) $u = U_e f_\eta$

(5) $v = vw - (U_e/\sqrt{2\xi})[\, f + 2\xi f_\xi + \eta(\hat\beta - 1)f_\eta\,]$

and

(6) $\hat\gamma(\xi) = (\sqrt{2\xi}/U_e)\, vw(\xi)$

(7) $\hat\beta(\xi) = (2\xi/U_e^2)(dU_e/dx)$

If both the suction and pressure gradient functions are constant (equal to γ_0 and β_0 respectively), then $f(\xi,\eta)$ is a function of η only and we have a similar boundary layer governed by,

(8) $f_{\eta\eta\eta} + f\, f_{\eta\eta} + \beta_0(1 - f_\eta^2) - \gamma_0 f_{\eta\eta} = 0$

(9) $f(0) = f_\eta(0) = 0$, $f_\eta \to 1$ as $\eta \to \infty$

where   the   condition $\hat\gamma(\xi)$ = constant   demands   that   vw   be proportional   to $U_e/\sqrt{2\xi}$.   For   the   case   of   flat   plate   vw is proportional to $1/\sqrt{x}$.

Stability calculations were carried out for similar suction $Y_o =$ constant and similar pressure gradient $\beta_o =$ constant using the similar equations (8)-(9), and for constant continuous suction $vw(\xi) = SL$ (given by equation (6)) using the nonsimilar equations (1)-(2). Negative Y_o and negative SL indicate suction, while negative β_o indicates unfavorable pressure gradient. Equations (8)-(9) were numerically integrated using a shooting technique with a Runge-Kutta integrator, while equations (1)-(2) were solved using a second-order finite difference technique. The mean-flow results were scaled to conform with the way stability equations were nondimensionalized.

Now we consider the primary instability of the calculated mean flow with respect to 2D quasi-parallel spatially growing TS disturbances. Squire's theorem implies that the critical disturbance is 2D. Dimensionless quantities are introduced by using the reference velocity U_e and the referece length $L = (\nu x / U_e)^{1/2}$, so that Reynolds number is given by $R = (U_e x / \nu)^{1/2}$, where x measures the distance from the leading edge of the plate, and ν is the fluid kinematic viscosity.

At sufficiently large distance from the leading edge, primary instability of the laminar flow occurs with respect to TS disturbances. These disturbances take the travelling wave form,

$$(10) \quad (u_p, v_p, p_p) = A_o [u(y), v(y), p(y)] \exp [i(\int \alpha \, dx - \omega t)] + c.c.$$

where for the spatial stability analysis α is a complex wavenumber given by $\alpha = \alpha_r + i\alpha_i$ and ω is a real disturbance frequency, and c.c. denotes complex conjugate terms. The eigensolutions u, v, and p are governed by a fourth-order system of equations that is given in the Appendix. This system is numerically integrated as initial value problem using a combination of shooting [13] and Newton-Raphson iteration technique that employs a Gram-Schmidt orthonormalization procedure.

The linear stability theory of primary instability provides α for a given ω and R. Then the integration of the growth rate $-\alpha_i$ gives the amplification factor (or the amplitude ratio),

$$(11) \quad \ln (A / A_o) = -2 \int_{R_{op}}^{R} \alpha_i \, dR$$

where A_o is an arbitrary initial amplitude of the primary instability at R_{op} (R where the onset of the primary wave). The eigensolutions may be normalized such that A measures directly the maximum r.m.s. value of the streamwise disturbance, that is

$$(12) \quad \max_{0 \le y < \infty} | u(y) |^2 = 1/2$$

Since the primary instability of boundary -layer flows is induced by viscosity, the growth rates and amplification factors here are typically very small compared to the convective length scale.

III. SECONDARY SUBHARMONIC INSTABILITY

The basic state under consideration is the calculated 2D flow with suction and pressure gradient at finite amplitude A of the primary TS wave, that is

$$(13) \qquad (u_b, v_b, p_b) = (U, 0, P) + A \; [\; u(y), v(y), p(y) \;] \; \exp(i\theta)$$

Where

$$A = A_o \; \exp(-\int \alpha_i dx), \qquad \text{assumed constant, and}$$

$$\theta = \int \alpha_r \, dx - \omega t$$

We consider the 3D quasi-parallel spatial subharmonic instability of the basic flow given by (13). The finite amplitude primary wave acts as a parametric exitation on the secondary instability. Following the analysis of Herbert [12,14] and Nayfeh [15], we apply Floquet theory and express the secondary wave using the normal mode concept,

$$(u_s, v_s, p_s) = \exp(\int \gamma \, dx) [\hat{u}(y), \hat{v}(y), \hat{p}(y)] \; \exp(\tfrac{1}{2} i \theta) \; \cos \beta z + c.c.$$

(14)
$$w_s = \exp(\int \gamma \, dx) \hat{w}(y) \exp(\tfrac{1}{2} i \theta) \sin \beta z + c.c.$$

where β is a spanwise real wavenumber, and $\gamma = \gamma_r + i \gamma_i$ is a characteristic exponent. The spatial growth rate of the secondary wave is given by γ_r, while γ_i can be interpreted as a shift in the streamwise wavenumber. In our calculations, we consider only the case of $\gamma_i = 0$, that is the secondary wave is perfectly synchronized with the basic state.

The secondary wave (14) is superposed on the basic state (13) and the result is substituted into the dimensionless Navier-Stokes equations. The meam flow plus the 2D TS quantities are substracted, and the resulting equations are linearized in the secondary disturbances. Then one obtains an eigenvalue problem that can be written as six first-order system of ordinary differential equations in the form,

(15) $DZ_1 = Z_2$

(16) $DZ_2 = Z_1 + R \, DU \, Z_3 + R \left(\gamma_r + \tfrac{1}{2} i \alpha_r \right) Z_6$

$$+ A \, R \left[\left(\gamma_r + \tfrac{1}{2} i \alpha_r \right) u \, \bar{Z}_1 + v \, \bar{Z}_2 + Du \, \bar{Z}_3 \right]$$

(17) $DZ_3 = - \left(\gamma_r + \tfrac{1}{2} i \alpha_r \right) Z_1 - \beta \, Z_4$

(18) $DZ_4 = Z_5$

(19) $DZ_5 = \mathcal{X} Z_4 - R \beta Z_6 + A R [(\gamma_r - \frac{1}{2} i \alpha_r) u \bar{Z}_4 + v \bar{Z}_5]$

(20) $DZ_6 = - R^{-1} (\gamma_r + \frac{1}{2} i \alpha_r) Z_2 - R^{-1} \mathcal{X} Z_3 - R^{-1} \beta Z_5$

$$+ A \{ (\gamma_r - \frac{3}{2} i \alpha_r) v \bar{Z}_1 - [(\gamma_r - \frac{1}{2} i \alpha_r) u + Dv] \bar{Z}_3$$

$$+ \beta v \bar{Z}_4 \}$$

where $\bar{(\ )}$ indicates a complex conjugate quantity, $D = d/dy$,

$$\mathcal{X} = R [(\gamma_r + \frac{1}{2} i \alpha_r) U - \frac{1}{2} i \omega] - (\gamma_r + \frac{1}{2} i \alpha_r)^2 + \beta^2 ,$$

$$Z_1 = \hat{u} \qquad Z_2 = D\hat{u} \qquad Z_3 = \hat{v}$$

$$Z_4 = \hat{p} \qquad Z_5 = \hat{w} \qquad Z_6 = D\hat{w} ,$$

and the boundary conditions are

(21)
$$Z_1 = Z_3 = Z_5 = 0 \qquad \text{at } y = 0$$

$$Z_1 , \quad Z_3 , \quad Z_5 \rightarrow 0 \qquad \text{as } y \rightarrow \infty$$

When A=0, the system of equations (15)-(21) govern a primary subhamonic wave. For A≠0, this system was numerically integrated as initial value problem from $y = y_e$ (edge of the boundary layer) to the wall. The eigenvalue search used a

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Newton-Raphson iteration technique to satisfy the last boundary condition at the wall. A well tested code SUPORT [13] is used which is coupled with an orthonormalization test based on the modefied Gram-schmidt procedure to overcome the stiffness of the integrated system of equations. For more details on the numerical procedure, the reader is refered to El-Hady [16].

The linear stability theory of the secondary instability provides γ for a given β and R. Then the integration of the growth rate γ_r gives the amplification factor,

$$(22) \qquad \ln (B / B_o) = 2 \int_{R_{os}}^{R} \gamma_r \, dR$$

where B_o is an arbitrary initial amplitude of the secondary instability at R_{os} (R where the onset of the secondary wave) . Since the secondary subharmonic instability originates from a strong mechanism of combined tilting and stretching of the vortices [18], the growth rates and amplification factors here are large and occur on a convective length scale.

IV. RESULTS AND DISCUSSION

For the case of no suction and zero pressure gradient, our results are in full agreement with those obtained by Herbert et al.[14] and by Nayfeh and Ragab [17]. The first authers used spectral collecation methods to solve both the primary and

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secondary stability problems,while the others used a numerical technique similar to what is being used in this paper.

Results are presented in this section to show the effect of similar suction parameter γ_{o}, continuous suction SL , and similar pressure gradient parameter β_{o} on the development of the subharmonic secondary instability. All results reported here are for the nondimensional frequency $F = \omega / R = 60 \times 10^{-6}$, that remains fixed as a wave of fixed physical frequency travels downstream. In the present analysis, we limited our calculations to a spesific frequency and to small suction and pressure gradient parameters. This was done to satisfy the assumptions that are inherent in the approximate theory of linear secondary instability, namely the periodicity of the basic state and the weak variation of the TS amplitude. We note that higher frequences and higher suction rates will increase nonparallel effects, violating the periodicity assumption of the basic state. While lower frequences and higher unfavorable pressure gradients will increase the variation of the TS amplitude in the unstable range, violating the second assumption.

At $F=60 \times 10^{-6}$, a primary instability grows between $R_{op} = 554$ and $R_{1p} = 1052$ (first and second neutral points), reaching a maximum amplification factor of $A/A_{o} = 41.679$ for Blasius flow ($\gamma_{o} = 0$, $\beta_{o} = 0$). A broad band of spanwise wavenumbers of

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primary 3D subharmonic waves are subject to amplification in this region, but the time and length scales of this instability is so small to bear any resemblance to experimentally observed transition. A strong growth of subharmonic disturbance can be due to parametrical exitation by the finite amplitude TS wave [25].

4.a Effect of suction

At R=1050, growth rates of the secondary instability is shown in figures 1 and 2 as function of the spanwise wavenumber $b = 10^3 \beta / R$, for various amplitudes A of the primary wave. Figure 1 shows results for Blasius flow, while figure 2 shows results for $\gamma_o = -.1$, $\beta_o = 0$. These figures illustrate first of all the destabilizing effect of A at fixed F and R. Second, at very small amplitudes, considerable growth rates exist in a small band of wavenumbers ,that extends to larger values as the amplitude increases. Third, the maximum growth shifts slightly to occur at higher b as the amplitude A increases. Fourth, small suction rates has strong stabilizing effect on the subharmonic secondary instability. As R increases, a similar increase in the growth rates exist at fixed F and A .

At R = 1050, figure 3 shows the effect of various suction rates on the secondary growth for A =.01. Increasing suction at fixed R and A decreases the growth rate of the subharmonic instability and limit the band of dangerous spanwise wavenumbers . The wavenumber of maximum growth rate is b = .17 and is not affected by suction.

By specifying the initial amplitude of the primary, say A_o=.001, we can combine the effect of increasing amplitude A and increasing R for various suction rates at fixed F. For comparison purposes, the amplification factor of the subharmonic is calculated using equation (22) from R_{os} to any R. All results shown here are for spanwise wavenumber b=.15 which is an avarage value for wavenumbers of maximum growth at various A . Figure 4 shows the variation of growth rates γ_r of the secondary wave with R for various suction rates, while figure 5 shows their amplification factors. The growth rates $-\alpha_i$ and amplification factors of the primary wave (ln A/A_o) are also shown in these figures and indicated by dotted curves. Initially, the primary instability sets in at R_{op} and $-\alpha_i$ grows, then the secondary subharmonic instability sets in at R_{os} and starts to grow strongly due to the increase in A with increasing R. Ultimitly, $-\alpha_i$ decays, while γ_r reaches a maximum at a location where the amplitude of the primary starts to decrease. Small suction rates at fixed A delays the onset of the subharmonic secondary instability , and decreases

significantly its growth rate as well as its amplification factor. Figure 5 shows a reduction of the amplification factor of the secondary wave ($\ln B/B_o$) from nearly 28 to 8 due to an increase in Y_o from 0 to -.05. Increasing suction rate to -.1 dampen completely the subharmonic instability, although the primary shows some growth. This indicates that the onset of the subharmonic instability requires that the primary amplitude exceeds a threshold value. Notice that this threshold value depends on Reynolds number, it decreases as R increases. For example, for $Y_o = 0$ the onset of the subharmonic secondary instability is at R=740 with a threshold amplitude A = .0029, while for $Y_o = -.05$, the onset of the subharmonic secondary instability is at R=850 with a threshold amplitude A = .0024. For $Y_o = -.1$, the primary amplitude reaches a maximum of only .0014 which apparantly is below the value needed to induce secondary subharmonic instability at R > 850.

The onset of the subharmonic secondary instability as well as the maximum amplification factor it reaches are also dependent on the initial amplitude A_o. Figure 6 ,a case of suction rate $Y_o = -.05$, shows a primary instability that sets in at R = 650. The onset of the subharmonic instability occurs at R = 850 for $A_o = .001$, at R = 775 for $A_o = .002$, and at R = 635 for $A_o = .0066$ (well before the onset of the primary) . The amplification factor reaches a value of 8,14.5, and 30 respectively. When the initial amplitude A_o is large enough, the initial instability

can be so strong and secondary instability occurs directly by-passing the usual growth of the linear primary instability. This phenomenon has been documented in cases of roughness and high freestream turbulence. In a situation like this the flow quickly becomes turbulent [23] and transition prediction schemes based on linear primary theory fail completely.

Figure 7 shows the effect of suction on the subharmonic instability at various initial amplitude of the primary . The maximum of ln B/B_o is used as a basis for comparison . The effect of suction on the primary is also shown in the figure and indicated by the dotted curve. The figure suggests that secondary subharmonic instability is very sensetive to and can be controlled by slight suction rates.

The onset of the subharmonic secondary instability is one important feature of the transition process. For LFC purposes, one might try to avoid or delay this instability by using suction. Then, one faces the question whether suction should be applied before or after the onset of the secondary instability. To answer this, we take Blasius case as a basis for comparison where R_{op}= 554, R_{ip}= 1052, R_{os}= 740, R_{is}= 1250, and maximum ln B/B_o = 28.5. Continuous suction SL =-.035(Y_o =-.05 at R = 1000) was applied starting at R = 554 (R_{op} for Blasius flow). As a result, the onset of the subharmonic instability is delayed to R = 800 and continuous to R =1245. reaching maximum ln B/B_o

= 12.5. When continuous suction with the same level SL =-.035 was delayed and applied at R = 740 (R_{os} for Blasius case), the onset of the secondary instability was almost not affected, it occurs at R = 750 but the maximum of ln B/B_o reaches 18.5. Doubling the suction level SL =-.070 (Y_o =-.1 at R = 1000) that begins at R = 740 shows no effect on delaying the onset of secondary instability but bring the maximum of ln B/B_o down to 10.

Two factors can affect the aforementioned behavior of the subharmonic instability ,the initial amplitude A_o of the primary and its evolution. The first factor is held fixed in the previous comparison, its effect is given before in figure 6 . To explain the effect of the evolution of the primary amplitude,we included the amplification factor curves of the primary amplitude (dotted curves) in figure 8 together with those of the secondary instability for the aforementioned cases.The figure indicates that continuous suction resulted in delaying the onset of the primary instability allowing the primary wave to travel further downstream such that its amplitude will reach a threshold value needed to induce a secondary subharmonic instability . While in cases where suction started at R = 740, the primary instability was not delayed but its amplification factor is only enhanced by applying suction, resulting in reduced primary and hence reduced secondary amplification factors. These calculations

show that suction should be applied further upstream near R_{op} to control the development of the primary amplitude and not near the onset of the secondary instability. Similar conclusions were reached by Reed and Nayfeh [19] and by Saric and Reed [20] investigating the effect of suction on primary TS waves, that suction should be concentrated not in the region of maximum growth but near R .

4.b Effect of suction and pressure gradients

At R = 1050, the growth rate γ_r of the secondary subharmonic instability as function of the spanwise wavenumber $b=10^3\beta/R$ for various amplitudes A of the primary exhibits the same features given before in figures 1 and 2. As an example, figure 9 shows a case for $\gamma_o=-.1$ and $\beta_o=-.04$, that illustrates again the destabilizing effect of A at fixed F and R. Notice that $\gamma_o=-.1$ has a stabilizing effect, while $\beta_o=-.04$ has a destabilizing effect on the secondary instability.

Figures 10 and 11 show the effect of pressure gradients alone on the growth rates and amplification factors of the subharmonic instability when combine the effect of increasing A and increasing R at fixed F. Also included in the figure (dotted curves) the primary instability for comparison. These calculations are for $A_o=.001$ and b =.15. Curve b for $\beta_o = -.02$

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reaches a value of maximum ln $B/B_o = 57$, while curve c for $\beta_o = -.04$ is estimated to reach a value of 80. The figure indicates that small unfavorable pressure gradient is strongly destabilizing. Similar results were given by Bertolotti [24] studying the secondary instability of Falkner-Skan flows. Figures 10 and 11 also show that pressure gradients produce small changes in R_{os} but large changes in the maximum of ln (B/B_e).

A similar conclusion was reached by Saric and Nayfeh [21] for the primary 2D instability.

Figures 12 and 13 show the effect of both suction and pressure gradients on γ_r and ln B/B_o at fixed F using $A_o = .001$ and b $= .15$. The figure illustrates the sensitivity of the secondary subharmonic instability to small suction and pressure gradients. Comparing with figure 11, we find that maximum ln (B/B_o) goes from 80 for $\gamma_o = 0$, $\beta_o = -.04$ to 9 for $\gamma_o = -.1$, $\beta_o = -.04$.

Figure 14 shows the effect of suction and pressure gradient on the secondary subharmonic instability using maximum ln B/B_o as a basis for comparison. The primary initial amplitude for these calculations is $A_o = .001$ and the spanwise wavenumber b $= .15$. Increasing A_o will have a destabilizing effect that can be infered from Fig.7.

V. CONCLUDING REMARKS

Previous calculations show that stabilization of the boundary layer   by active means (suction) or by passive means (modifying pressure   gradients) or by a combination is a very sensetive process.   Weak   suction rates produce strong stabilizing effect on the subharmonic   secondary instability (decrease growth rates ,amplification factors and limit the band of dangerous spanwise wavenumbers).   While weak unfavorable pressure gradients produce strong destabilizing effect on the subharmonic secondary instability.

The onset of the subharmonic secondary instability requires that the primary amplitude exeeds a threshold value. This value is Reynolds number dependent, it decreases as Reynolds number increases.

When the initial amplitude of the primary is large enough, the initial instability can be so strong and secondary instability may occur directly by-passing the usual growth of the linear primary instability.

For laminar flow control purposes , suction should be applied near the onset of the primary   instability to control the evolution of the primary amplitude and not near the onset of the secondary instability.

Recent progress in understanding secondary instabilities may prompt modifications to transition prediction schemes to rely on a secondary instability theory instead of the primary instability theory. A modefied e criterion can be reached that envolve the amplitude of the primary disturbance or a measure of the background disturbance. But the means by which freestream disturbances enter the boundary layer (receptivity problem [22]) will remain the key for any transition prediction method.

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APPENDIX

The primary 2D eigenvalue problem can be written as,

$$DX_1 = X_2$$

$$DX_2 = [\ i\ R\ (\ \alpha\ U - \omega\ ) + \alpha^2\ ]\ X_1 + R\ DU\ X_3 + i\ \alpha\ R\ X_4$$

$$DX_3 = -\ i\alpha\ X_1$$

$$DX_4 = -\ R^{-1}\ i\alpha\ X_2 - [\ i\ (\ \alpha\ U - \omega\ ) + R^{-1}\ \alpha^2\ ]\ X_3$$

with boundary conditions

$$X_1 = X_3 = 0 \quad \text{at } y = 0$$

$$X_1\ ,\ X_3 \quad 0 \quad \text{as } y \to \infty$$

where

$$X_1 = u\ ,\quad X_2 = Du\ ,\quad X_3 = v\ ,\quad X_4 = p$$
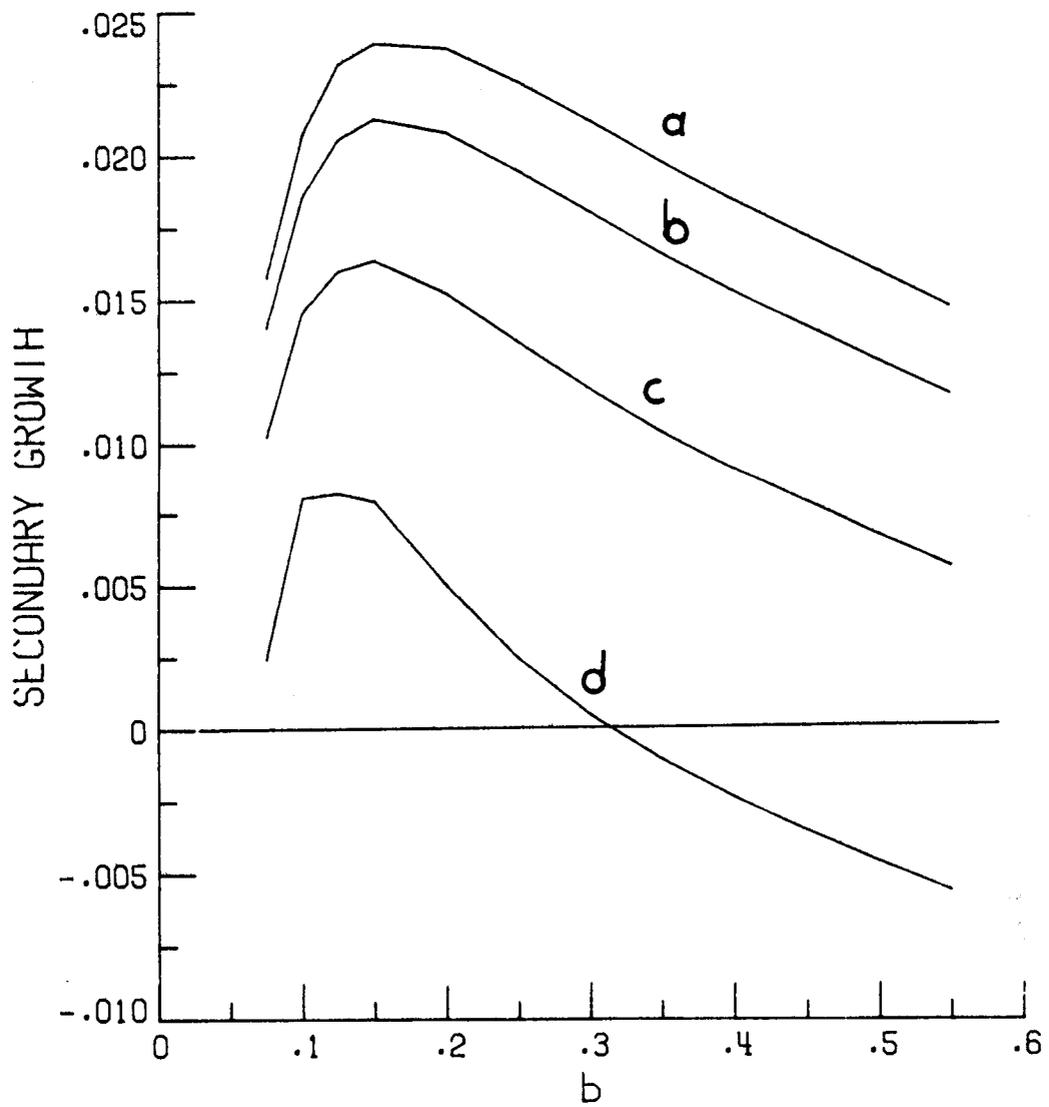
Figure 1 Effect of the amplitude A of the primary on the subharmonic secondary growth for zero pressure gradient and no suction. a) A =.01, b) A =.008, c) A =.005, d) A =.002.
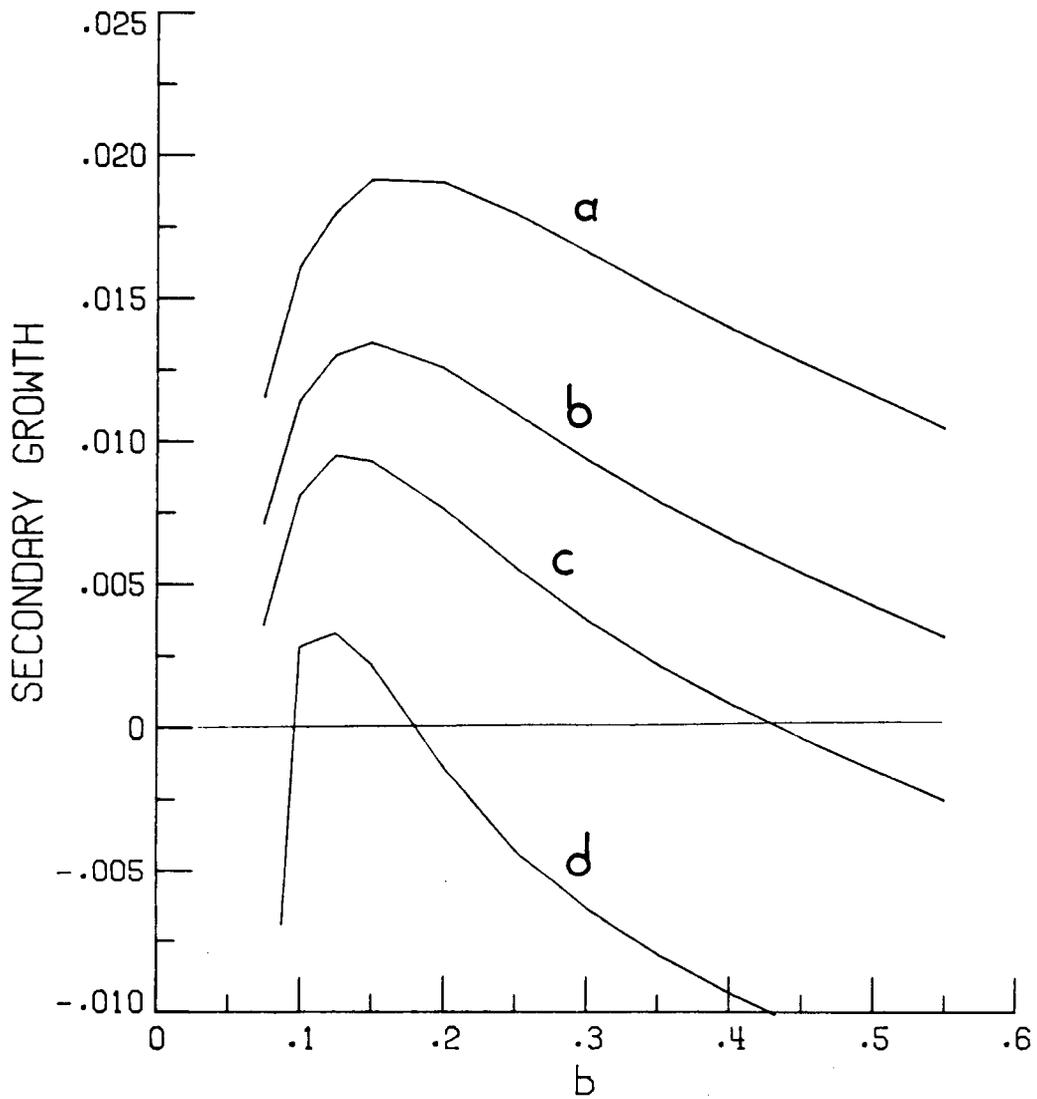
Figure 2 Effect of the amplitude A of the primary on the subharmonic secondary growth for $\gamma_0 = -.1$, and $\beta_0 = 0$. a) A $=.01$, b) A $=.006$, c) A $=.004$, d) A $=.002$.
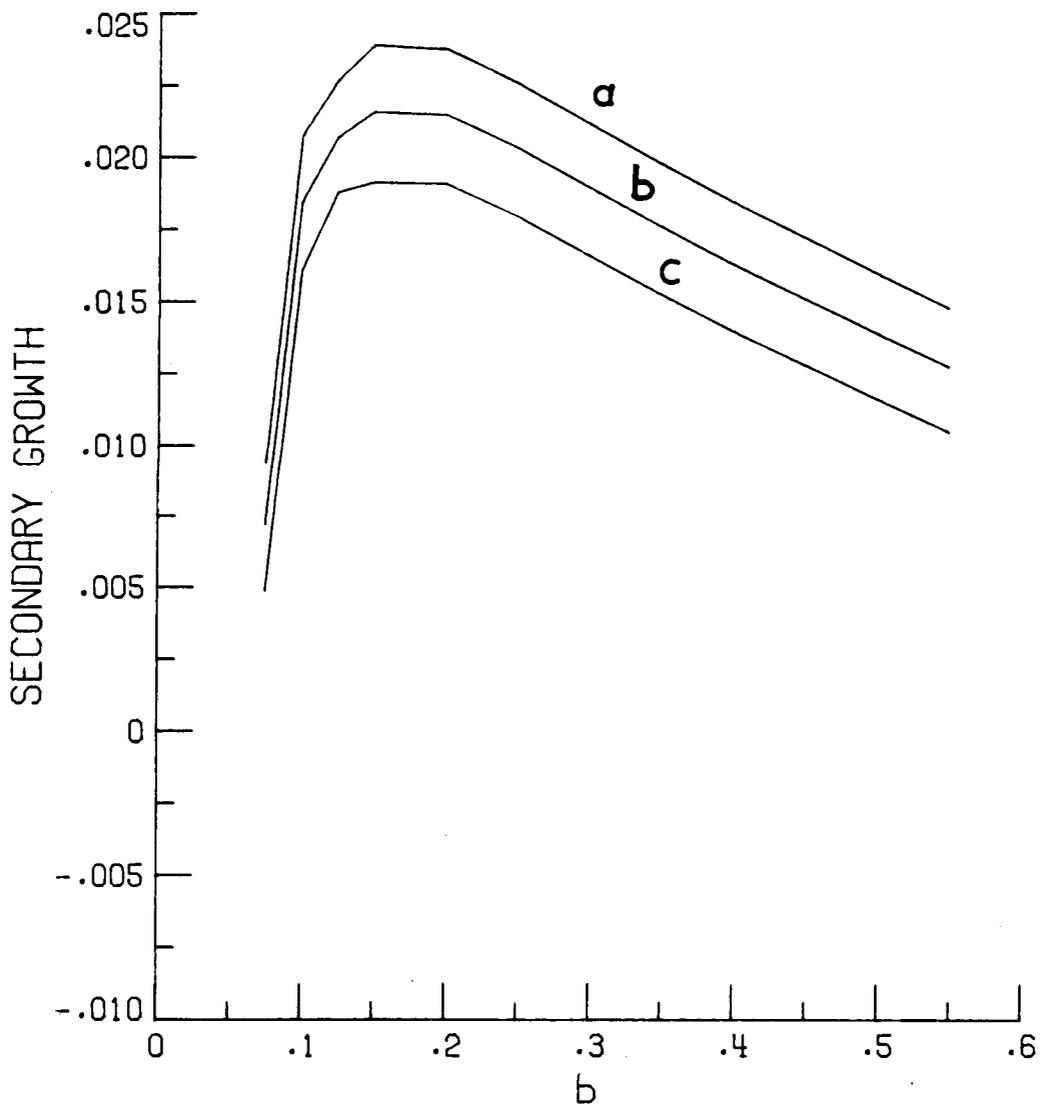
Figure 3 Effect of suction parameter γ_o on the subharmonic secondary growth for a primary amplitude A =.01. a) γ_o =0., b) γ_o =-.05, c) γ_o =-.1.
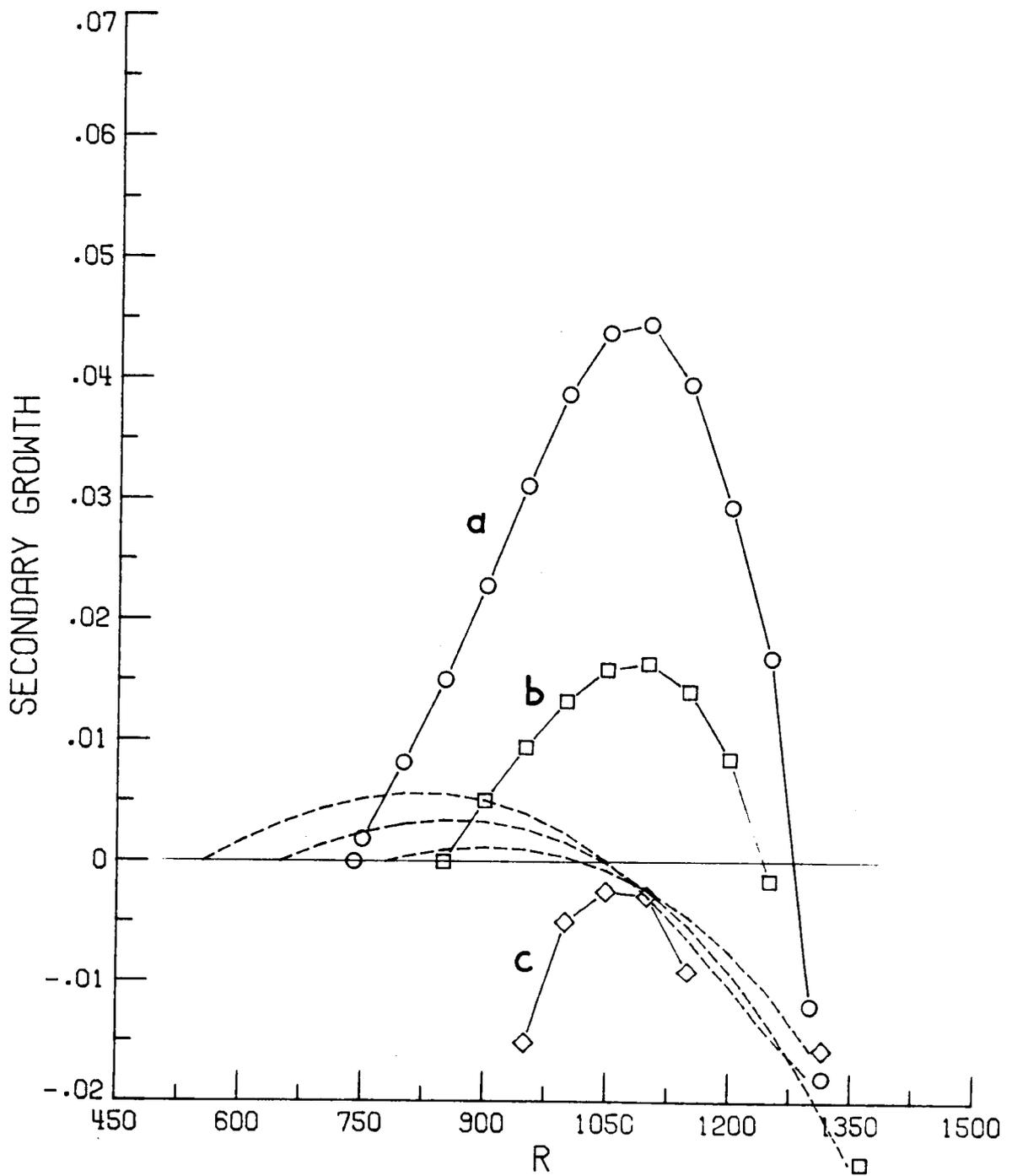
Figure 4 Effect of suction parameter γ_o on the primary and secondary growth rates. The primary initial amplitude $A_o = .001$.
a) $\gamma_o = 0.$, b) $\gamma_o = -.05$, c) $\gamma_o = -.1$.

Figure 5 Effect of suction parameter γ_o on the primary and secondary amplification factors. The primary initial amplitude $A_o = .001$. a) $\gamma_o = 0.$, b) $\gamma_o = -.05$, c) $\gamma_o = -.1$.

Figure 6 Effect of the initial amplitude A_o of the primary on the onset and amplification factors of the subharmonic secondary instability at suction rate $\gamma_o = -.05$. a) $A_o = .001$, b) $A_o = .002$, c) $A_o = .0066$.
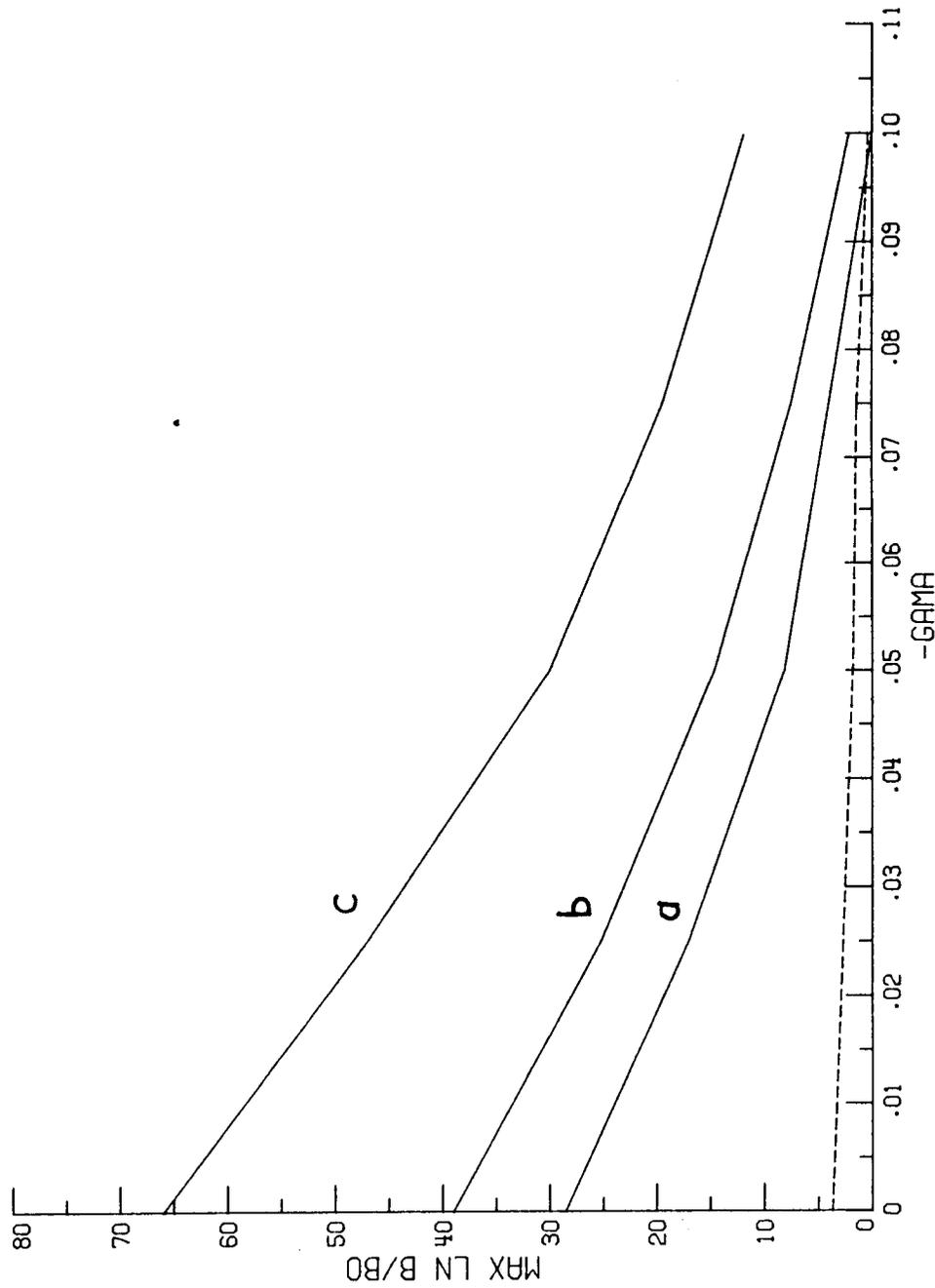
Figure 7   Effect of suction parameter γ_o on maximum amplification factors of the subharmonic at various initial amplitudes of the primary.   a) $A_o = .001$,   b) $A_o = .002$, c) $A_o = .0066$.
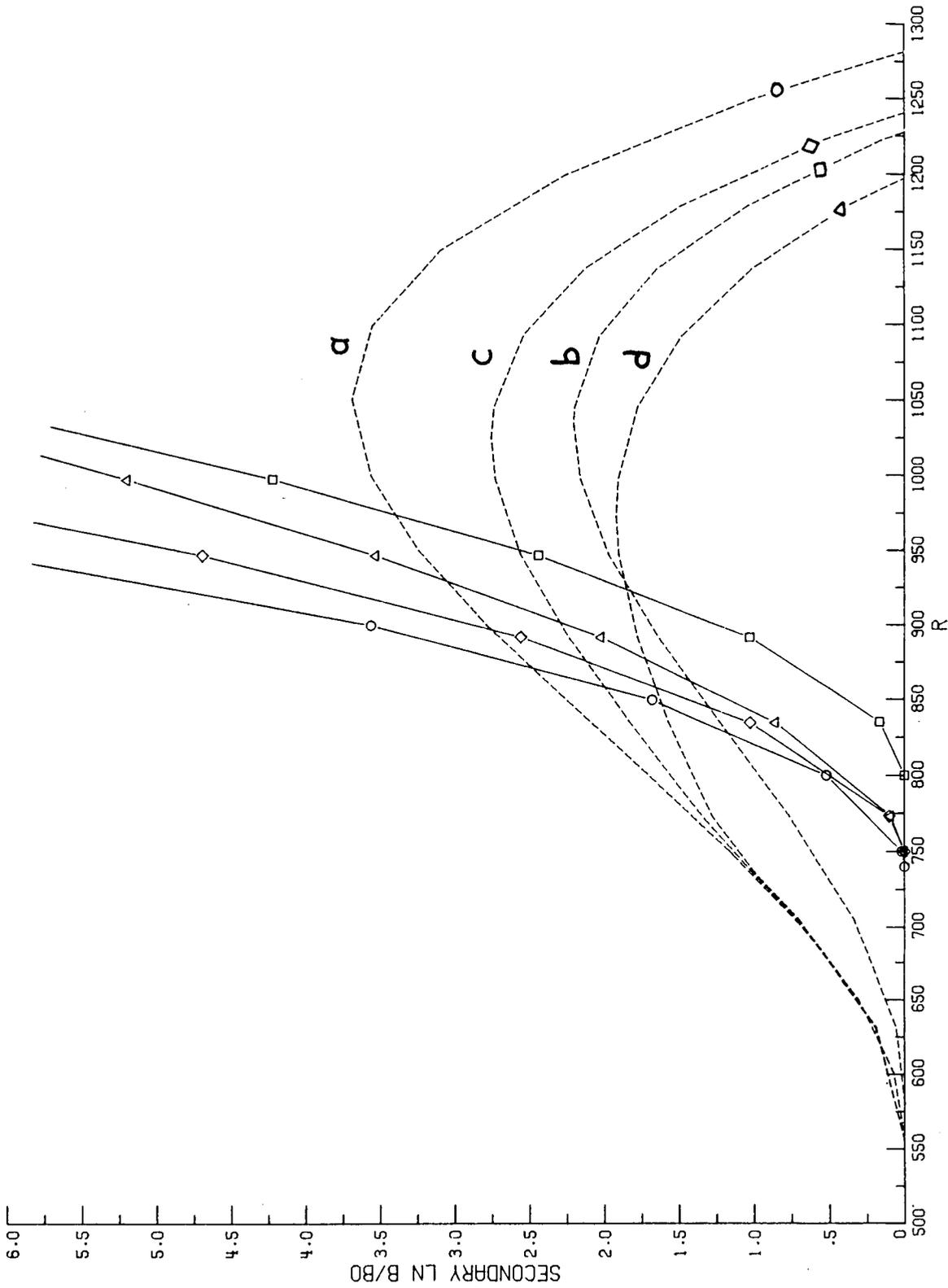
Figure 8 Effect of continuous suction SL on the onset and amplification factors of both the primary and secondary subharmonic. a) no suction, b) SL=-.035 starting at R=554., c) SL=-.035 starting at R=740., d) SL=-.070 starting at R=740.
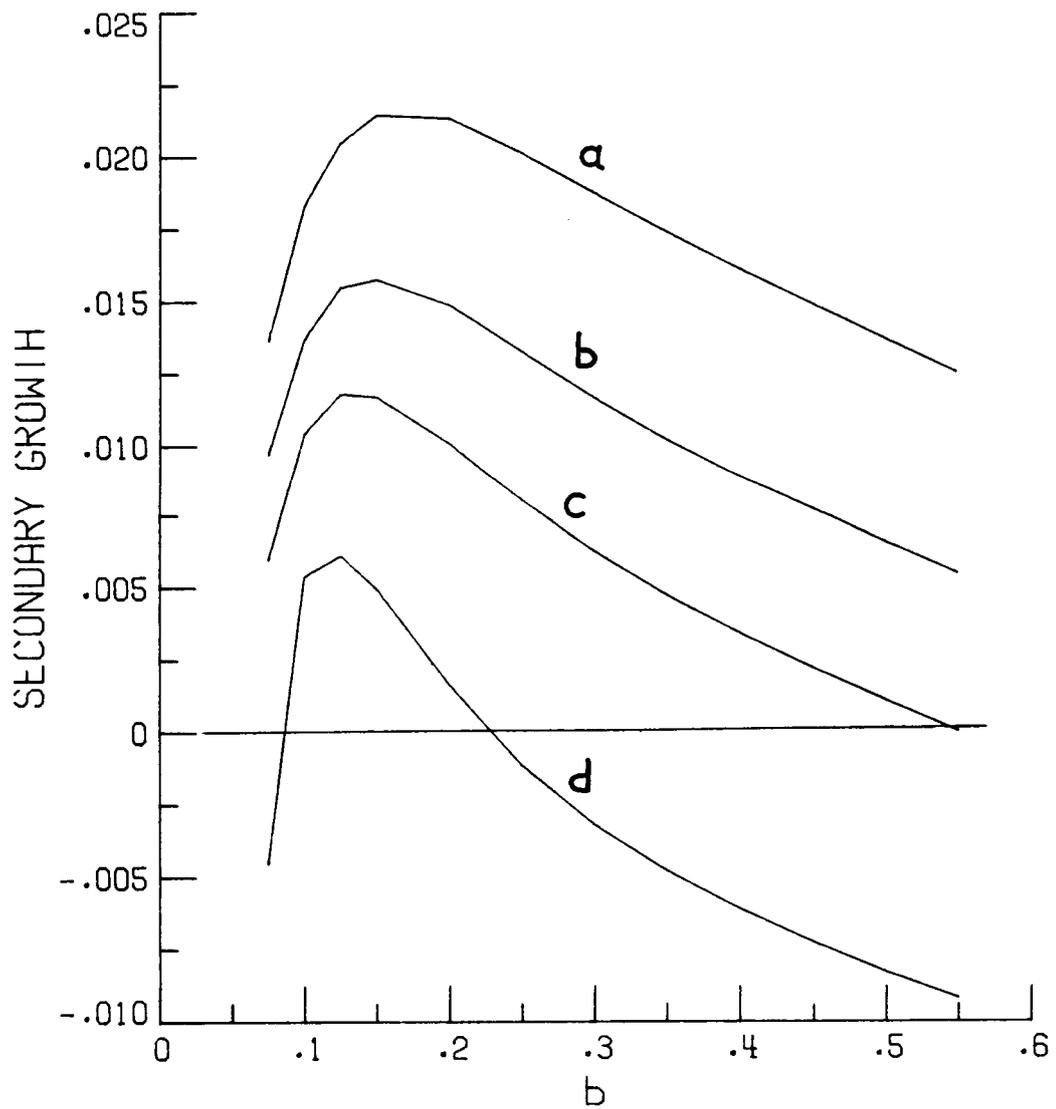
Figure 9 Effect of the amplitude A of the primary on the subharmonic secondary growth for Y_o =-.01, β_o =-.04. a) A =.01, b) A =.008, c) A =.004, d) A =.002.
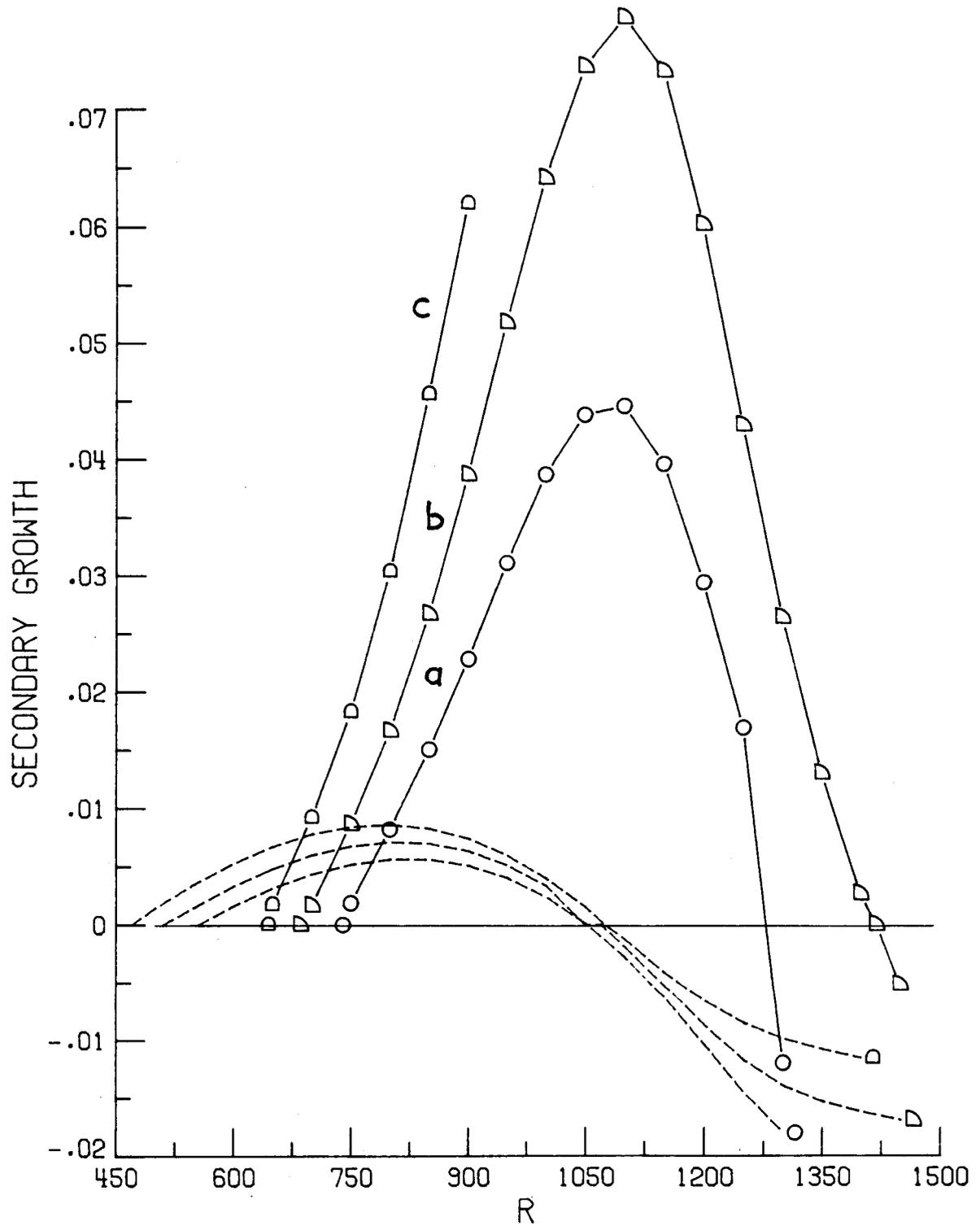
Figure 10 Effect of pressure gradient parameter β_0 on the primary and subharmonic secondary growth rates. The primary initial amplitude $A_0 = .001$. a) $\beta_0 = 0.$, b) $\beta_0 = -.02$, c) $\beta_0 = -.04$.

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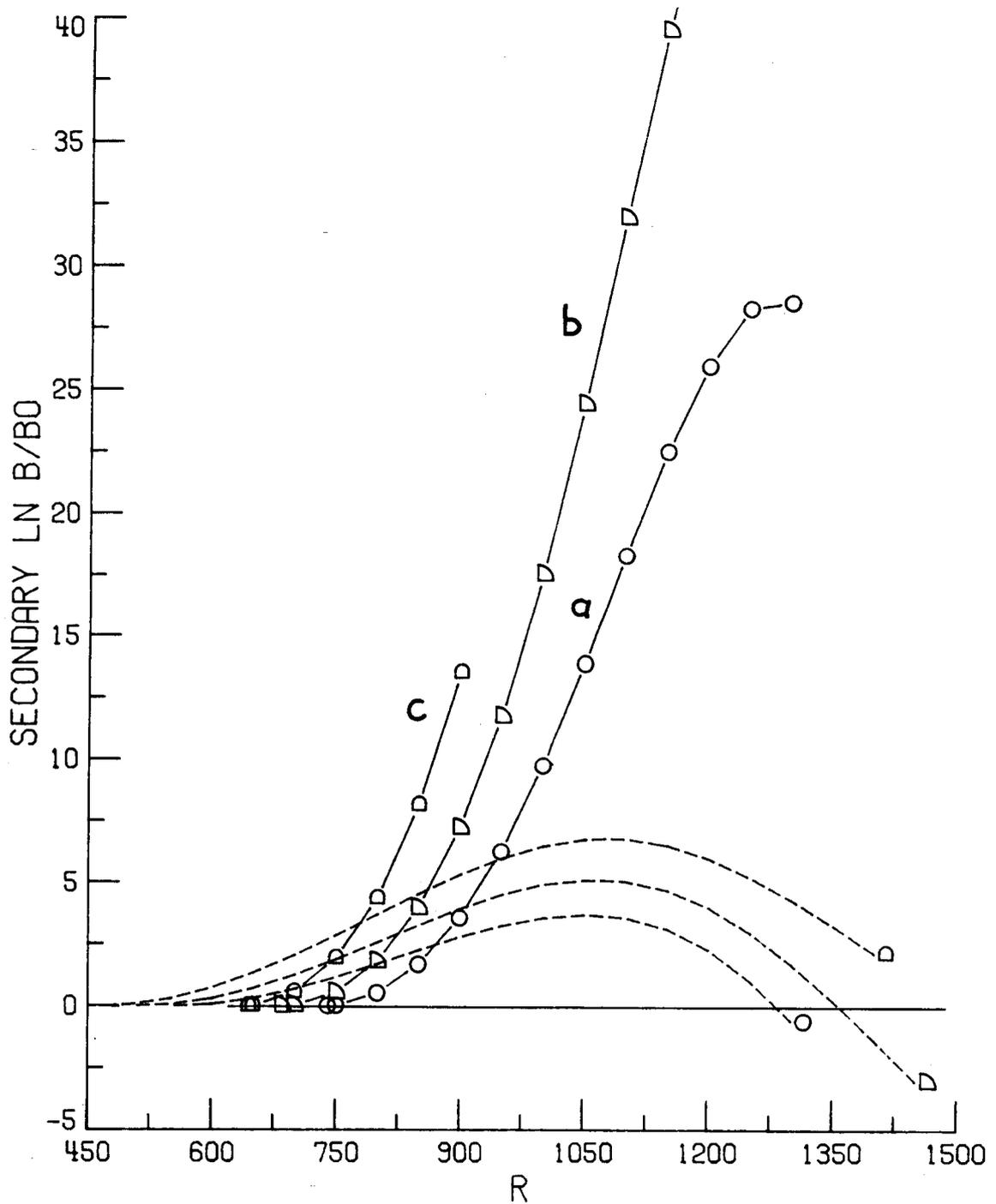
Figure 11 Effect of pressure gradient parameter β_o on the primary and subharmonic secondary amplification factors. The primary initial amplitude $A_o = .001$. a) $\beta_o = 0.$, b) $\beta_o = -.02$, c) $\beta_o = -.04$.
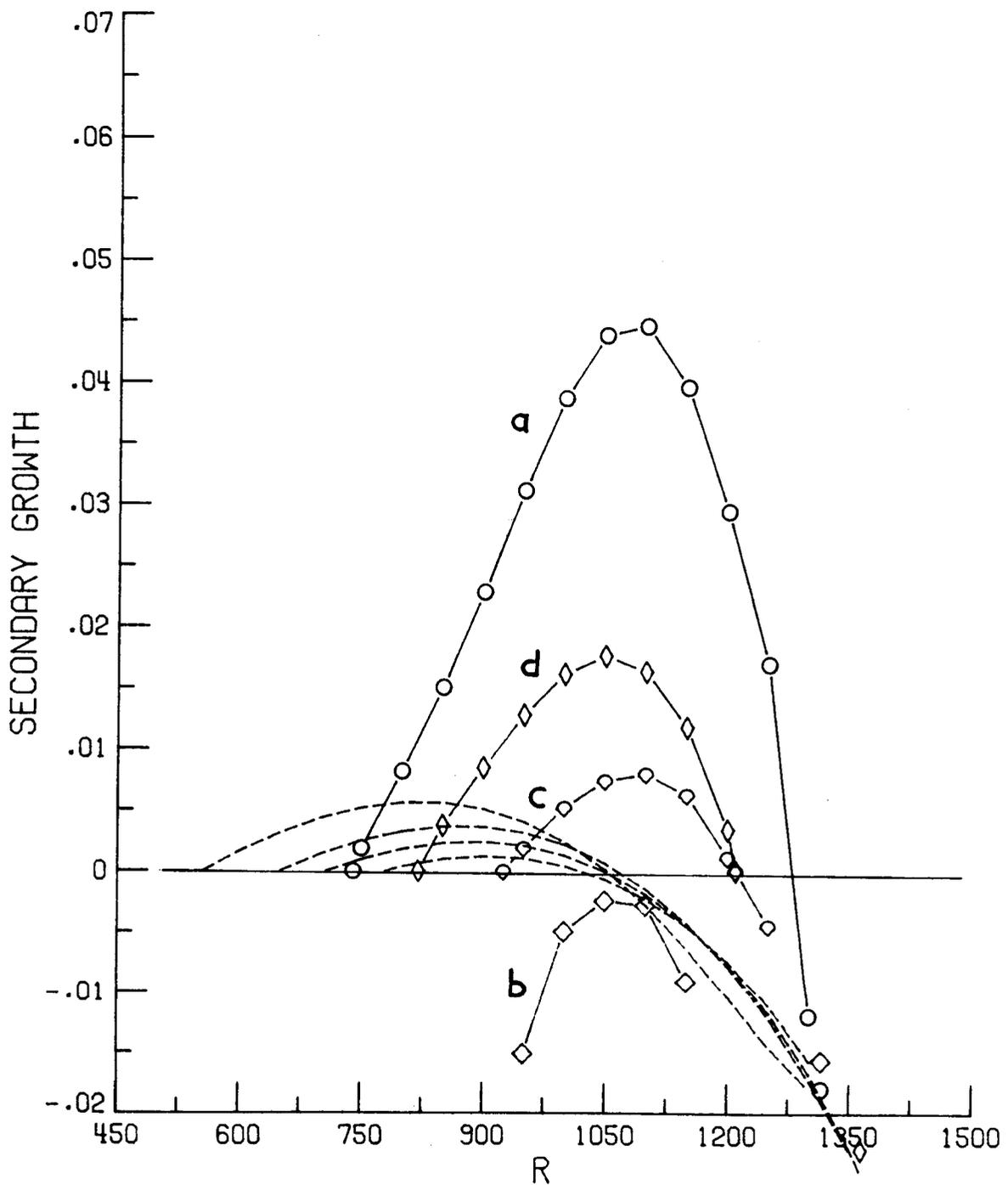
Figure 12 Effect of suction and pressure gradient parameters on the   primary and subharmonic secondary growth rates. The primary initial amplitude $A_o = .001.$ a) $Y_o = 0.$, $\beta_o = 0.$, b) $Y_o = -.1$, $\beta_o = 0.$, c) $Y_o = -.1$, $\beta_o = -.02$, d) $Y_o = -.1$, $\beta_o = -.04.$
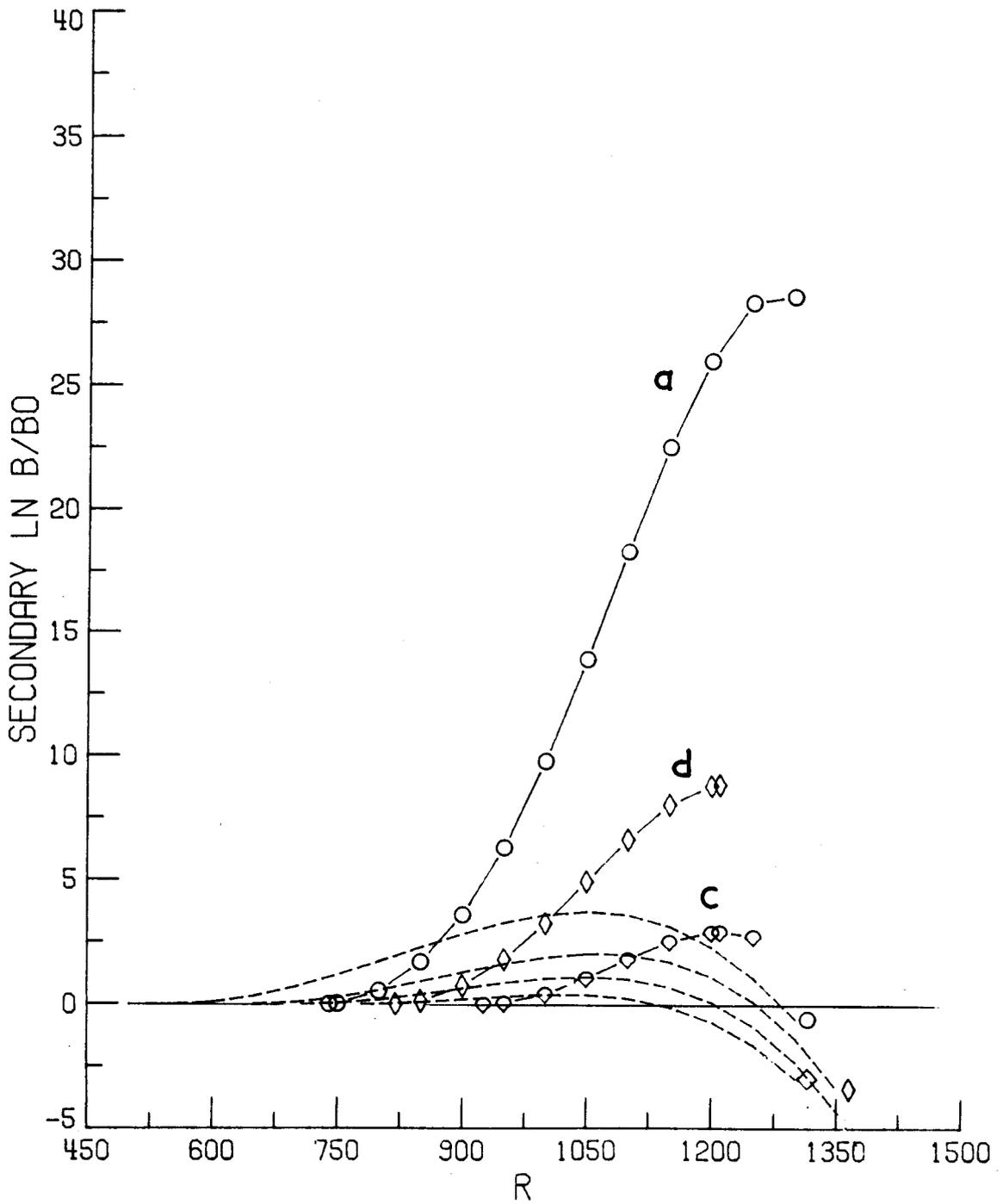
Figure 13 Effect of suction and pressure gradient parameters on
the primary and subharmonic secondary amplification factors.
Same conditions as in Fig. 12
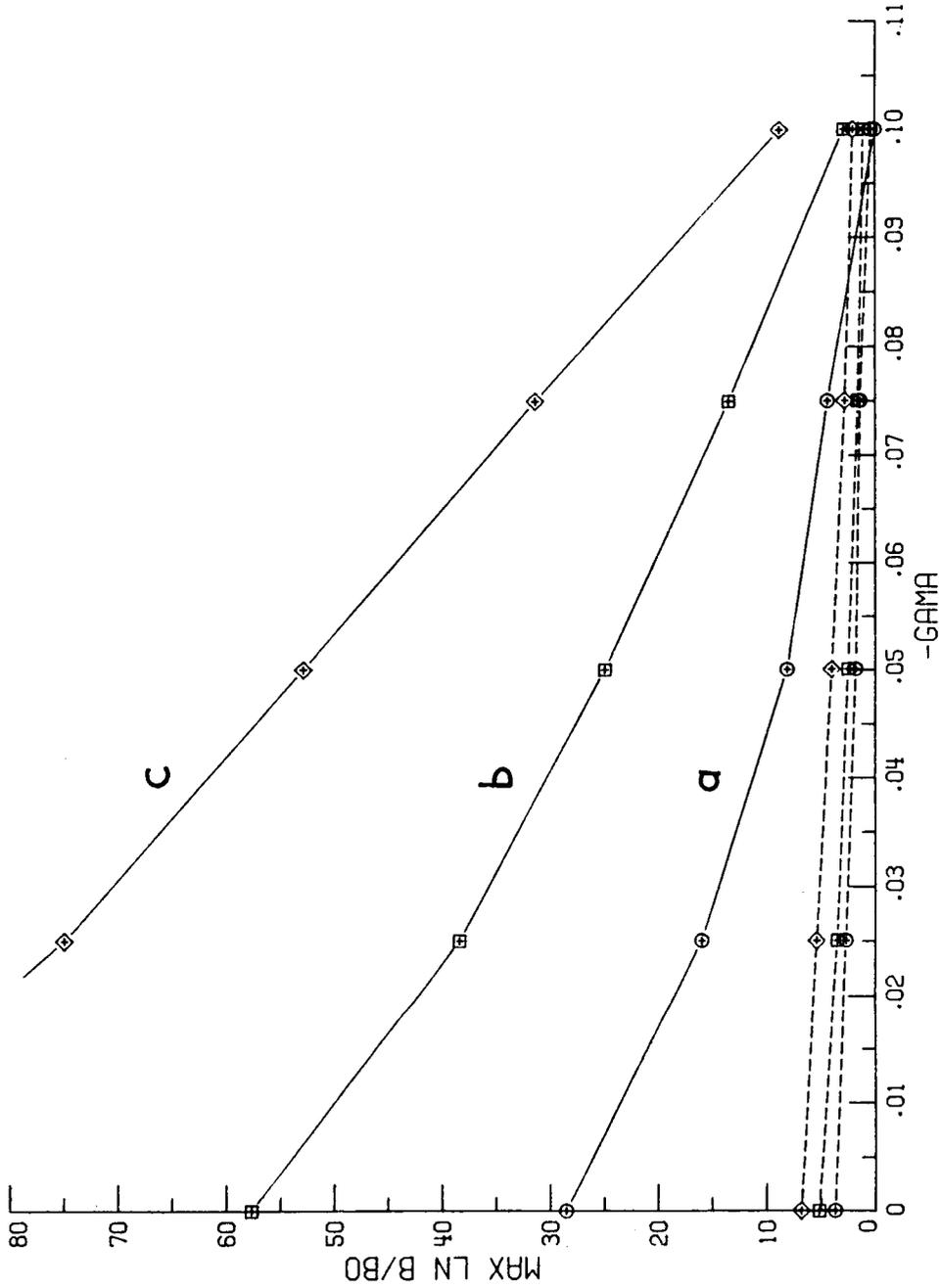
Figure 14   Effect of suction parameter γ_o on the maximum amplification rates of both the primary and secondary subharmonic at various pressure gradient parameters. The primary initial amplitude $A_o = .001$. a) $\beta_o = 0.$, b) $\beta_o = -.02$, c) $\beta_o = -.04$.

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Report Documentation Page

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16. Abstract

Three-dimensional linear secondary instability is investigated for boundary layers with pressure gradient and suction in the presence of finite amplitude TS wave. The focus is on principle parametric resonance responsible for strong growth of subharmonics in low disturbance environment. Calculations are presented for the effect of pressure gradients and suction on controlling the onset and amplification of the secondary instability.

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