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PROBLEMS AND POSSIBLE SOLUTIONS INVOLVED IN HARD TARGET CALIBRATION OF COHERENT DOPPLER LIDAR

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ABSTRACT

In this paper the whole field of radiometry is analyzed in light of coherence for our surfaces are irradiated with coherent, polarized light. Definitions of some concepts had to be modified. In light of these modifications the problems in calibration suggested by Kavaya were analyzed and solutions suggested. The most important task is to develop hard targets that exhibit minimal specular reflection (mirror-like and retroreflection) and follows closely a "Lambertian" scattering curve. Bistable reflectometer experiments and integrating sphere measurements should be used to optically characterize the target. Optical and electron microscopy will be used to physically characterize the targets. Since no one is measUring the BRDF matrix, this capability must be developed for perferrably both bistatic and monostatic reflectometer measurements. The equipment is expensive and must be developed. If one can prove for a diffuse scatterer that the BRDF matrix is diagonal, the calibration is simplified.

ACKNOWLEDGEMENTS

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LIST OF FIGURES

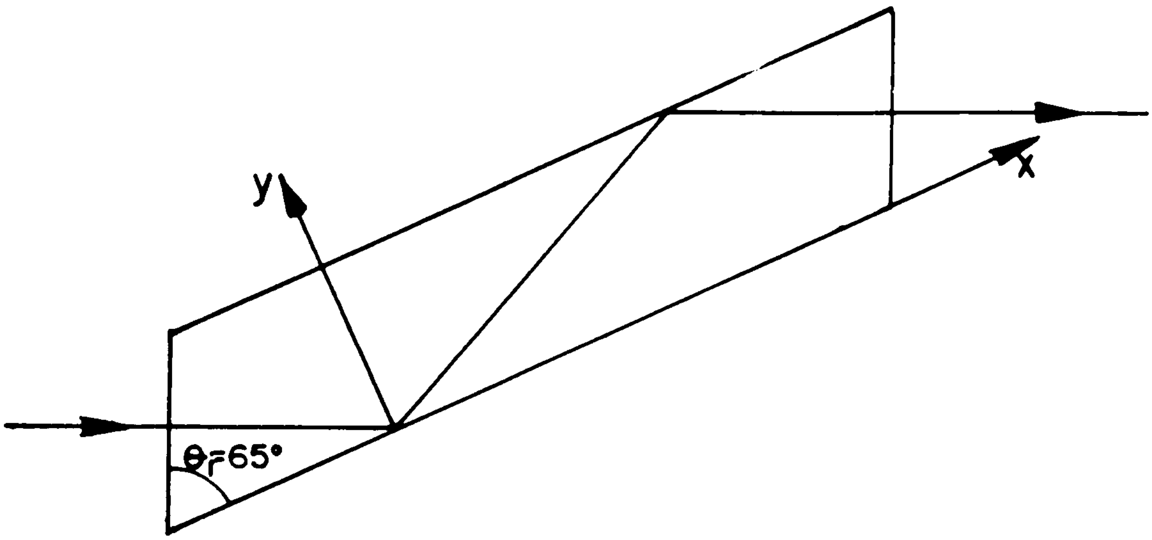
LIST OF TABLES

INTRODUCTION

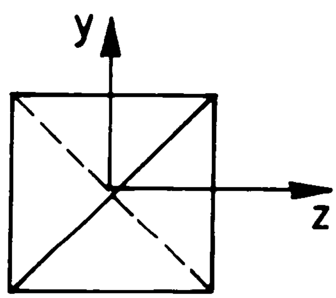
The usual quarter waveplate for the 90-11 μm CO_2 laser emission region is made of CdS and since its ordinary and extraordinary indices of refraction vary over this spectral region, it is necessary to construct a quarter waveplate for each laser line. These devices are custom made and they are very expensive. As a result, in order to produce and analyze circular polarized light it has been suggested that a Fresnel rhomb[1] be used in the place of the usual quarter waveplate. In this paper the phase variation of a 10.6 μm quarter waveplate will be evaluated when it is used at other wavelengths than the designed 10.6 μm. Also the effects of having a beam strike the quarter waveplate at angles between 0-5^{o} at 10.6 μm will be evaluated.

By using a linear polarizer and a ZnSe Fresnel rhomb it is possible to produce right and left hand circular polarized light and analyze this light. The Fresnel rhomb is sketched in Figure 1 with the appropriate angles indicated.

ZnSe has a nearly constant index of refraction over the 9-11 μm region of the CO_2 laser, so the rhomb exhibits no chromatic effects in this spectral region. The index of refraction of ZnSe is given in Table 1. These values are crude but they will indicate accurately enough the phase angle variation of the rhomb. If the rhomb is cut at the proper angle θ_i and light is polarized along a face diagonal and is incident normally on the entrance face, it is possible to have the light internally reflected twice with a phase change of ± 45° between the electric field components parallel to and perpendicular to the plane of incidence. As a result, the rhomb introduces a phase change of ± 90° between the field components and since the two field components have equal amplitude, circular polarized light results.



Side View
(a)



End View
(b)

FIGURE 1 Fresnel rhomb

Table 1. Index of refraction of ZnSe versus wavelength[2]

Wavelength μm	Index of refraction n
2.75	2.44
5.00	2.43
7.50	2.42
9.50	2.41
11.0	2.40
12.5	2.39
13.5	2.38
15.0	2.37
16.0	2.36
16.9	2.35
17.8	2.34
18.5	2.33
19.3	2.32
20.0	2.31

OBJECTIVES

The objectives of this research will be to develop and determine the operational characteristics of the Fresnel rhomb.

1. The theory of operation of the rhomb is developed.
2. The phase angle variation with input angle will be examined.
3. The change in phase angle of a 10.6 μm quarter waveplate at various other CO_2 laser lines will be calculated.

THEORY

The incident beam can be divided into two field components as indicated in the above discussion above when the incident beam is incident along the diagonal to the crystal ($\pm 45^{o}$). The light enters the end face of the crystal normal to the surface and it undergoes two internal reflections before it re-emerges parallel to its incident direction, but the beam is laterally displaced. At each internal reflection the reflection and transmission Fresnel coefficients are

$$r_{\parallel} = \frac{\left(n_t \text{ Cos } \theta_i - n_i \text{ Cos } \theta_t\right)}{\left(n_i \text{ Cos } \theta_t + n_t \text{ Cos } \theta_i\right)}$$

$$t_{\parallel} = \frac{2 n_i \text{ Cos } \theta_i}{\left(n_i \text{ Cos } \theta_t + n_t \text{ Cos } \theta_i\right)} \qquad (1)$$

$$r_{\perp} = \frac{\left(n_i \text{ Cos } \theta_i - n_t \text{ Cos } \theta_t\right)}{\left(n_i \text{ Cos } \theta_i + n_t \text{ Cos } \theta_t\right)}$$

and

$$t_{\perp} = \frac{\left(2 n_i \text{ Cos } \theta_i\right)}{\left(n_i \text{ Cos } \theta_i + n_t \text{ Cos } \theta_t\right)}$$

where in our case $n_t = 1$ and $n_i = n_{ZnSe} = n = 2.40$.

Now for ZnSe the critical angle becomes Sin θ_c = 1/2. 40 and $\theta_c = 24.6^{\circ}$. This means that internal reflection will occur for a crystal cut with the small corner angle at an angle greater than $\theta_i > 24.6^{\circ}$. This does not tell us the phase shift that occurs on each internal reflection. It will be assumed that $\theta_i > \theta_c$ and Sin $\theta_i > 1/n$. Now Cos $\theta_t = [1 - (\text{Sin } \theta_t)^2]^{1/2}$ and from Snell's law Cos $\theta_t = [1 - (n\text{Sin } \theta_i)^2]^{1/2}$. Now since Sin $\theta_i > 1/n$, the Cos θ_t is imaginary or

$$\text{Cos } \theta_t = j\ n \left[\text{Sin}^2\ \theta_i - 1/n^2\right]^{1/2} \tag{2}$$

The Fresnel coefficient becomes

$$r_{\parallel} = \frac{\left[\text{Cos } \theta_i - j\ n^2 \left\{\text{Sin}^2\ \theta_i - 1/n^2\right\}^{1/2}\right]}{\left[\text{Cos } \theta_i + j\ n^2 \left\{\text{Sin}^2\ \theta_i - 1/n^2\right\}^{1/2}\right]}$$

$$= \exp - \left\{2j\ \delta_{\parallel}\right\}, \tag{3}$$

where the phase shift on reflection is

$$\text{Tan } \delta_{\parallel} = \frac{n^2 \left\{\text{Sin}^2\ \theta_i - 1/n^2\right\}^{1/2}}{\text{Cos } \theta_i} \tag{4}$$

and

$$r_{\perp} = \frac{\left[n\ \text{Cos } \theta_i - j\ n \left\{\text{Sin}^2\ \theta_i - 1/n^2\right\}^{1/2}\right]}{\left[n\ \text{Cos } \theta_i + j\ n \left\{\text{Sin}^2\ \theta_i - 1/n^2\right\}^{1/2}\right]}$$

$$= \exp - \left\{2j\ \delta_{\perp}\right\}, \tag{5}$$

where

$$\tan \delta_{\perp} = \frac{\left[\sin^2 \theta_i - 1/n^2\right]^{2/2}}{\cos \theta_i} \tag{6}$$

The amplitude of the incident waves are equal and each is represented by an equation of the form

$$E_i = A_i \exp j \left[\omega t - k \left\{y \cos \theta_i + z \sin \theta_i\right\}\right] \tag{7}$$

The two reflected waves have amplitudes

$$E_{r\parallel} = r_{\parallel} \; E_i = A_i \exp j\left[\omega t - k\left\{y \cos \theta_i + z \sin \theta_i\right\} - 2 \delta_{\parallel}\right]$$

and (8)

$$E_{r\perp} = r_{\perp} E_i = A_i \exp j\left[\omega t - k\left\{y \cos \theta_i + z \sin \theta_i\right\} - 2 \delta_{\perp}\right]$$

The phase difference between the field components is

$$\delta = 2 \left\{\delta_{\parallel} - \delta_{\perp}\right\} \tag{9}$$

so

$$\tan(1/2 \; \delta) = \tan \left\{\delta_{\parallel} - \delta_{\perp}\right\}$$

$$= \frac{\left\{\tan \delta_{\parallel} - \tan \delta_{\perp}\right\}}{\left\{1 + \tan \delta_{\parallel} \; \tan \delta_{\perp}\right\}} \tag{10}$$

This last equation can be reduced to a form which allows the calculation of the rhomb angle θ_i if the desired phase shift at each reflection is known. From equation (4) and (6) the relation between the field component phase angles is

$$\operatorname{Tan} \delta_{\parallel} = n^2 \operatorname{Tan} \delta_{\perp}$$

so

$$\operatorname{Tan} (1/2\ \delta) = \frac{\operatorname{Tan} \delta_{\perp} \left\{n^2 - 1\right\}}{\left\{1 + n^2 \operatorname{Tan}^2 \delta_{\perp}\right\}}$$

$$= \frac{\left[\left\{\operatorname{Sin}^2 \theta_i - 1/n^2\right\}^{1/2} \left\{n^2 - 1\right\}\right] \operatorname{Cos}^2 \theta_i}{\operatorname{Cos} \theta_i \left[\operatorname{Cos}^2 \theta_i + n^2 \left\{\operatorname{Sin}^2 \theta_i - 1/n^2\right\}\right]}$$

or

$$\operatorname{Tan} (1/2\ \delta) = \frac{\operatorname{Cos} \theta_i \left\{\operatorname{Sin}^2 \theta_i - 1/n^2\right\}^{1/2}}{\operatorname{Sin}^2 \theta_i}$$

Thus if the phase shift on each reflection is known, the rhomb angle θ_i can be calculated. This theory assumes the crystal is isotropic and this is the case for ZnSe is cubic.

DISCUSSION

A. Fresnel Rhomb

In the design the phase angle is chosen and the two internal reflection give a phase angle of 2 δ. Then it is possible to calculate the rhomb angle θ_i to produce this phase change when light is incident normally on the face and is linearly polarized along an end face diagonal. From equation (11), θ_i is

$$\theta_i = \mathrm{Sin}^{-1}\left[\frac{\left(n^2+1\right) \pm \left[\left(n^2+1\right)^2 - 4n^2\left(\mathrm{Tan}^2(1/2\,\delta)+1\right)\right]^{1/2}}{2n^2\left(\mathrm{Tan}^2(1/2\delta) + 1\right)}\right]$$

In order to make a quarter waveplate which produces right or left hand circular polarized light, then 2 $\delta = \pi/2$ or 1/2 $\delta = \pi/8$. In this case $\theta_i = 25.1^\circ$ which is smaller than the critical angle so no internal reflection occurs or $\theta_i = 65^\circ$ which is the desired rhomb angle.

An interesting device can be produced by placing two rhombs end to end as shown in Figure 2. This device eliminates the lateral displacement of the beam as occurs for a single rhomb but there will be increased beam attenuation. In constructing a quarter waveplate the incident beam is linearly polarized as described previously but on each reflection a phase shift of $\pi/8$ occurs and from equation (12) the rhomb angle is 77.6°.

In Figure 3 is shown a polarized beam which is not incident normally on the face of the rhomb but has makes a small angle above or below the normal. Then from Snell's law

$$\mathrm{Sin}\ \theta_i' = n\ \mathrm{Sin}\ \theta_t' \tag{13}$$

and

$$\mathrm{Tan}\left(1/2\ \delta\right) = \frac{\mathrm{Cos}\ \theta_i'' \left[\mathrm{Sin}^2\ \theta_i'' - 1/n^2\right]^{1/2}}{\mathrm{Sin}^2\ \theta_i''}$$

$$= \mathrm{Cos}\ \frac{\left(\theta_i \pm \theta_t'\right) \left[n^2\ \mathrm{Sin}^2\left(\theta_i \pm \theta_t'\right) - 1\right]^{1/2}}{n\ \mathrm{Sin}^2\left(\theta_i \pm \theta_t'\right)} \tag{14}$$

The results for light not incident normally on the face of the rhomb are given in Table 1, where the rhomb corresponds to a quarter waveplate for $\theta_i = 65^\circ$. Angles of incidence above the normal are positive and below are negative in the data presented in Table 2. These numbers give an indication of the importance of having a collimated light beam incident normally on the rhomb face.

A topic which needs to be considered is whether a Fresnel rhomb behaves the same as a quarter waveplate. The Jones' matrix[3] of a quarter waveplate oriented with the fast axis $+45^\circ$ to the incident polarized beam is

$$T = \begin{bmatrix} \exp - (j\ \delta_{T\perp}) & 0 \\ 0 & \exp - (j\ \delta_{T\parallel}) \end{bmatrix}, \tag{15}$$

where the $\perp$ and field components are $E_\perp$ and E or E_x and E_y and each equals A (x,y,z,t) then

$$E=T\ E_i=\begin{bmatrix}\exp-(j\ \delta_{T\perp}) & 0\\ 0 & \exp-(j\ \delta_{T\parallel})\end{bmatrix}\begin{bmatrix}E_\perp\\ E_\parallel\end{bmatrix}=\begin{bmatrix}E_\perp\ \exp-(j\ \delta_{T\perp})\\ E\ \ \exp-(j\ \delta_{T\parallel})\end{bmatrix}, \quad (16)$$

where δ_T is the total phase change of each field component. In the case of the Fresnel rhomb the Jones' matrix is

$$T=\begin{bmatrix}\exp-(2j\ \delta_\perp) & 0\\ 0 & \exp-(2j\ \delta_\parallel)\end{bmatrix}=\begin{bmatrix}\exp-(j\ \delta_{T\perp}) & 0\\ 0 & \exp-(j\ \delta_{T\parallel})\end{bmatrix}, \quad (17)$$

where $2\ \delta_\perp = \delta_{T\perp}$ and $2\ \delta_\parallel = \delta_{T\parallel}$. The matrix is identical to that of the quarter waveplate. The Mueller matrix for the Fresnel rhomb with a general phase shift δ is

$$M(\text{Fresnel rhomb}) = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & \cos\delta & -\sin\delta\\ 0 & 0 & \sin\delta & \cos\delta\end{bmatrix} \quad (18)$$

and for a phase shift of 90° it is

$$M(\text{Fresnel rhomb}) = \begin{bmatrix}1 & 0 & 0 & 0\\ 0 & 1 & 0 & 0\\ 0 & 0 & 0 & -1\\ 0 & 0 & 1 & 0\end{bmatrix} \quad (19)$$

With light incident at $+45^\circ$ to the horizontal the Stoke's vector is

$$S = 1/2 \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} \tag{20}$$

where the 1/2 factor is the attenuation of the polarizer so the product of the Mueller matrix times the Stoke's vector is

$$S\ (\text{R Circular}) = 1/2 \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 1 \\ 0 \\ 1 \\ 0 \end{bmatrix} = 1/2 \begin{bmatrix} 1 \\ 0 \\ 0 \\ 1 \end{bmatrix}, \tag{21}$$

which is right hand circular polarized light.

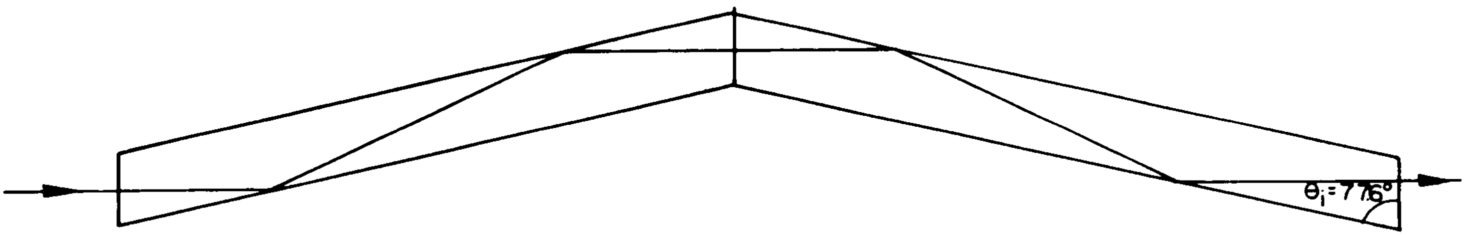
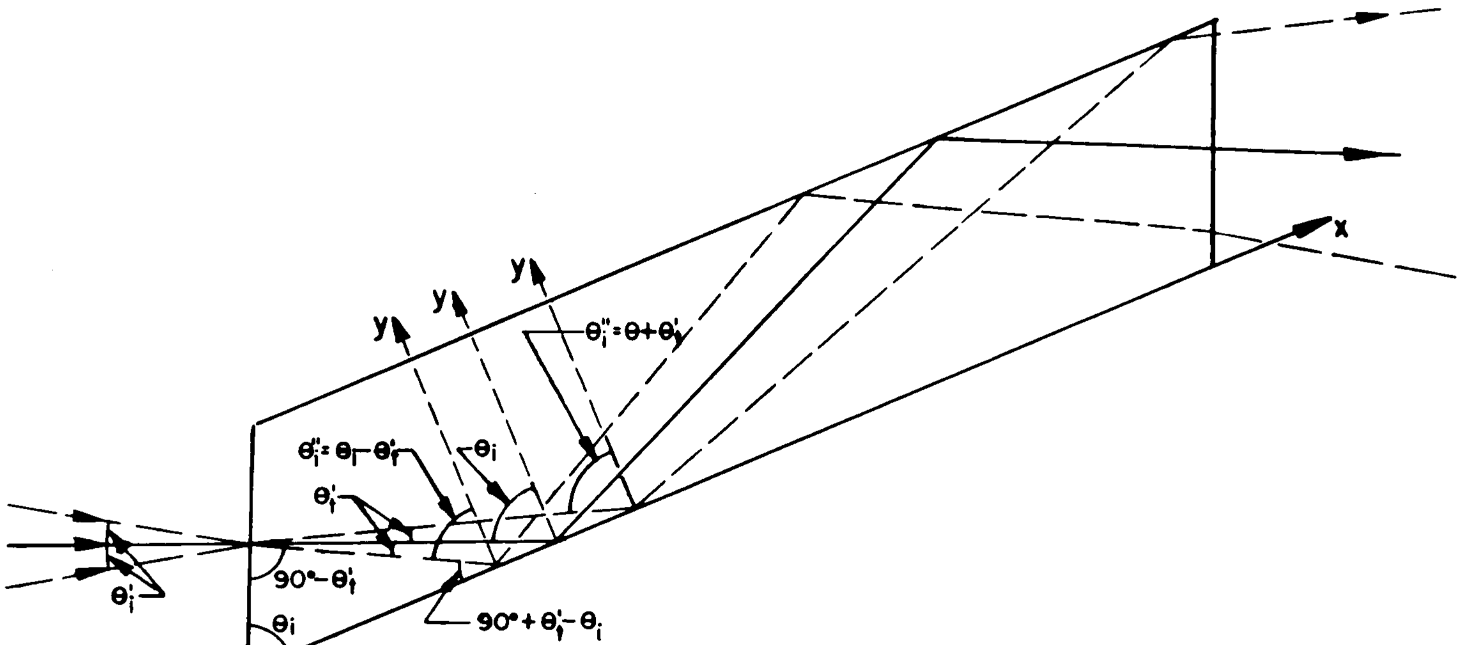


FIGURE 2 Dual Fresnel rhomb



θ_i' Above normal $\quad \theta_i'' = \theta_i - \theta_t'$

θ_i Below normal $\quad \theta_i'' = \theta_i + \theta_t'$

FIGURE 3 Input angle deviation

Table 2. Deviations of the phase with angles above and below the normal to the Fresnel rhomb.

Angle θ_i' Degrees	θ_t' Degrees	$\theta_i+\theta_t'$ Degrees	δ Degrees	2δ Degrees	Deviation 2δ from 90° Degrees
+0.5	-0.21	64.79	45.36	90.72	+0.72
-0.5	+0.21	65.21	44.62	89.24	-0.76
+1.0	-0.42	64.58	45.72	91.44	+1.44
-1.0	+0.42	65.42	44.24	88.48	-1.52
+1.5	-0.63	64.38	46.08	92.16	+2.16
-1.5	+0.63	65.62	43.90	87.80	-2.20
+2.0	-0.83	64.17	46.44	92.88	+2.88
-2.0	+0.83	65.83	43.52	87.04	-2.96
+2.5	-1.04	63.96	46.82	93.64	+3.64
-2.5	+1.04	66.04	43.16	86.32	-3.68
+3.0	-1.25	63.75	47.18	94.36	+4.36
-3.0	+1.25	66.25	42.78	85.56	-4.44
+3.5	-1.46	63.54	47.54	95.08	+5.08
-3.5	+1.46	66.46	42.42	84.84	-5.16
+4.0	-1.67	63.33	47.92	95.84	+5.84
-4.0	+1.67	66.67	42.04	84.08	-5.92
+4.5	-1.87	63.13	48.26	96.52	+6.52
-4.5	+1.87	66.87	41.70	83.40	-6.60
+5.0	-2.08	62.92	48.64	97.28	+7.28
-5.0	+2.08	67.08	41.32	82.64	-7.36

It should be mentioned that the Mueller matrix[4-5] for a quarter waveplate with the fast axis at some angle θ to the horizontal is

$$M\ \lambda/4\ (\theta) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & C^2 & CS & -S \\ 0 & CS & S^2 & C \\ 0 & S & -C & 0 \end{bmatrix}, \qquad (22)$$

where $C = \mathrm{Cos}\ 2\theta$ and $S = \mathrm{Sin}\ 2\theta$. If the fast axis is assumed vertical, then

$$M\ \lambda/4\ (90^{\circ}) = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & -1 \\ 0 & 0 & 1 & 0 \end{bmatrix},$$

which is identical to the Fresnel rhomb. Thus the Fresnel rhomb has all of the polarization characteristics of the quarter waveplate.

B. Quarter Waveplate

The quarter waveplate in the CO_2 wavelength region is constructed of CdS. The index of refraction for the ordinary ordinary and extraordinary rays of CdS were supplied by II-VI with the original data coming from Cleveland Crystal. These data are listed in Table 3. A true quarter waveplate is too thin to be constructed so plates with a thickness which are multiples of a quarter wavelength are made. The plates supplied by II-VI are 5 $\lambda/4$ plates of thickness 0.1018 cm or 0.0401 in.

With a plate of this thickness the deviation of the phase angle with the deviation of the beam from normal incidence on the plate is negligible. As an example, for a beam incident at ± 3.5° from the normal the phase angle deviation is ± 0.44° and for an incident angle deviation of ± 5° the phase angle deviation is ± 0.88°.

Table 4 shows the deviation of the phase angle between the field components E and $E^{\perp}$ of other CO_2 wavelengths for a quarter waveplate designed for the CO_2 10.591 μm line which at this wavelength gives a phase shift of 450°. From these data it is evident that a quarter waveplate designed for a specific wavelength may not be used at another CO_2 wavelength. This means that for each wavelength a new quarter waveplate must be purchased and these are special runs and are expensive. This means that a Fresnel rhomb must be used and the rhomb has input angle limitations and these limitations must be considered for they can lead to large phase shifts that would limit its usefulness.

CONCLUSIONS

The Fresnel rhomb in the 9-10 μm CO_2 laser emission region is a nearly wavelength independent phase retarder as is seen in Table 1. A single quarter waveplate of ZnSe may be made with a small rhomb corner angle of 65^o. With light normally incident upon the face and polarized along either diagonal right and left hand polarized light is produced. By using two rhombs with corner angles of 77.6^o it is possible to produce a quarter waveplate and this design avoids the lateral displacement of the beam. The greatest problem with the rhomb is that the beam must collimate and incident normally. It is easy to calculate the Mueller matrix for the rhomb and show that it is identical to the quarter waveplate at the same orientation.

It is evident from this study that a quarter waveplate designed for the 10.591 μm CO_2 line cannot be used over the entire emission region of the laser. As a result, it is necessary to use a nearly wavelength insensitive devise like the Fresnel rhomb, even though, it is subject to stringent irradiation condition.

Table 3. Extraordinary and ordinary indices of refraction of CdS[6]

Wavelength μm	Index of refraction n_o	n_e	Difference Δn
9.20	2.2395	2.253	0.0135
9.50	2.237	2.250	0.013
10.0	2.232	2.245	0.013
10.5	2.226	2.239	0.013
10.591	2.226	2.239	0.013
11.0	2.220	2.234	0.014

Table 4. Deviation of phase angle for a 10.591 μm CO_2 laser laser line quarter waveplate at other CO_2 wavelengths.

CO_2 wavelength μm		Degrees	Degrees
R(26)	9.239	535.5	85.5
R(24)	9.250	534.9	84.9
P(16)	9.519	500.5	50.5
P(22)	9.569	497.9	47.9
P(28)	9.621	495.2	45.2
R(36)	10.148	469.5	19.5
R(14)	10.289	463.0	13.0
P(14)	10.532	452.4	2.4
P(20)	10.591	450.0	0.0
P(34)	10.741	460.6	10.6

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