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SUMMARY

A study was conducted to determine the effect of eddy distribution on momentum and heat transfer near the wall in turbulent pipe flow. The buffer zone was of particular interest in that it is perhaps the most complicated and least understood region in the turbulent flow field. Six eddy diffusivity relationships are directly compared on their ability to predict mean velocity and temperature distributions in turbulent air flow through a cylindrical, smooth-walled pipe with uniform heat transfer. Turbulent flow theory and the development of the eddy diffusivity relationships are briefly reviewed. Velocity and temperature distributions derived from the eddy diffusivity relationships are compared to experimental data for fully-developed pipe flow of turbulent air at a Prandtl number of 0.73 and Reynold's numbers ranging from 8100 to 25 000.

INTRODUCTION

The transfer of heat and momentum to a fluid flowing in a tube or pipe is a common engineering problem. Although heat and momentum transfer have been well characterized for laminar flow, such is not the case for turbulent flow. Semi-empirical relationships are commonly used to derive the velocity and temperature profiles in turbulent flows. In addition, describing the flow is difficult in the region of the flow near the wall known as the buffer zone where both viscous and inertial effects are important.

The present study was conducted to determine the velocity and temperature distributions in the buffer zone for turbulent pipe flow using various eddy diffusivity relationships. Six diffusivity relationships were used: Deissler's exponential relationship, Von Kármán's linear solution, Von Kármán's logarithmic solution, Lin's linear relationship, Lin's polynomial distribution, and Reichardt's hyperbolic tangent distribution. The profiles were compared to experimental data for air at a Prandtl number of 0.73 and Reynold's numbers from 8100 to 25 000. Through this comparison the accuracy of the various relationships was determined.

BACKGROUND

Basic Equations

Many flows which occur in practical applications are turbulent. The most striking feature of turbulent flows is that velocity, pressure, and density at a fixed point in space do not remain constant, but experience very irregular high frequency fluctuations. A complete theoretical formulation of turbulent motions is nearly impossible, owing to the complexity of turbulent fluctuations. It is therefore convenient to consider time averages of turbulent motion. As such, a turbulent flow can be described in mathematical terms by separating the motion into mean and fluctuating (or eddying) components such that;

$$u_i = \bar{u}_i + u_i'; \; p = \bar{p} + p'; \; \rho = \bar{\rho} + \rho'; \; T = \bar{T} + T' \qquad (1)$$

where $\bar{u}$ denotes the time average of the u-component of velocity and u' denotes the velocity fluctuation. The time averages are formed at a fixed point in space and are given, e.g., by;

$$\bar{u}_i = \frac{1}{t_o} \int_t^{t+t_o} u_i \, dt \qquad (2)$$

where the mean values are taken over a sufficiently long interval of time, t_o, for them to be completely independent of time. The time averaged equations of continuity, motion and energy for a Newtonian fluid with constant k, ρ, $\hat{C}_p$, and μ then become;

Continuity: $$\nabla \bullet \bar{u} = 0 \qquad (3)$$

Motion: $$\rho \frac{D\bar{u}}{Dt} = -\nabla\bar{P} + \mu\nabla^2\bar{u} + \rho g - \nabla \bullet \rho\overline{u'u'} \qquad (4)$$

Energy: $$\rho\hat{C}_p \frac{D\bar{T}}{Dt} = k\nabla^2\bar{T} - \nabla \bullet \rho\hat{C}_p\overline{T'u'} \qquad (5)$$

The viscous dissipation terms of the energy equation have been neglected. As a result of time averaging, additional terms arise from the inertia terms in the equations of motion and energy which account for the fluctuations in the flow. The terms arising from time smoothing the equation of motion are the components of the turbulent momentum flux, $\tau_{ij}^{(t)}$ (commonly referred to as the Reynold's stresses).

$$\bar{\tau}_{ij}^{(t)} = \rho\overline{u_i'u_j'} \qquad (6)$$

Similarly, time smoothing the energy equation results in fluctuation terms which are components of the turbulent energy flux, $\bar{q}_i^{(t)}$.

$$\bar{q}_i^{(t)} = \rho\hat{C}_p\overline{u_i'T'} \qquad (7)$$

As a result of the appearance of these additional terms, the time-smoothed equations of motion and energy cannot be directly solved because there are more unknowns than available equations. Solutions can only be obtained if functional dependencies of $\bar{\tau}_{ij}^{(t)}$ and $\bar{q}_i^{(t)}$ on the system variables can be established. The aim of phenomenological theories is to obtain such expressions directly.

Phenomenological theories assume a simple turbulence mechanism to obtain expressions for $\bar{\tau}_{ij}^{(t)}$ and $\bar{q}_i^{(t)}$ for the purpose of obtaining mean velocity and temperature profiles, respectively. J. Boussinesq was the first to develop such a theory (ref. 1). In it Boussinesq hypothesized that two coefficients of viscosity can be defined in turbulence; one real and the other apparent. The real or molecular coefficient, μ, as defined by Newton's law of viscosity,

$$\tau_{ij}^{(\ell)} = -\mu \frac{du_j}{dx_i} \tag{8}$$

is independant of Reynold's number, boundaries, and position in the fluid whereas the apparent or eddy viscosity, $\rho\varepsilon_\nu$, is dependent on all of these. From this we have Boussinesq's Theory:

$$\bar{\tau}_{ij}^{(t)} = -\rho\varepsilon_\nu \frac{d\bar{u}_j}{dx_i} \tag{9}$$

where ε_ν is the eddy diffusivity for momentum.
The analogy of Boussinesq's theory for heat transfer also follows:

$$q_i^{(\ell)} = -k \frac{dT}{dx_i} \; ; \; \bar{q}_i^{(t)} = -\rho\hat{C}_p\varepsilon_H \frac{d\bar{T}}{dx_i} \tag{10}$$

where ε_H is the eddy diffusivity for heat.

Before Boussinesq's theory can be applied, the dependency of the eddy diffusivities on the variables of the system must be established. This was the subject of further work by Prandtl, Von Kármán and Taylor. Prandtl developed an expression for momentum transfer in a fluid (which can be extended to thermal energy transfer) by assuming that eddies move around in a fluid very much as molecules move around in a gas (ref. 2).

$$\bar{\tau}_{ij}^{(t)} = -\rho\ell^2 \left| \frac{d\bar{u}_j}{dx_i} \right| \frac{d\bar{u}_j}{dx_i} \tag{11}$$

Taylor developed a similar expression by postulating conservation of vorticity rather than conservation of momentum as Prandtl did (ref. 3).

$$\frac{d\tau_{ij}^{(t)}}{dx_i} = -\rho\ell^2 \, 2 \left| \frac{d\bar{u}_j}{dx_i} \right| \frac{d^2\bar{u}_j}{dx_i^2} \tag{12}$$

Finally, Von Kármán, on the basis of dimensional analysis, extended Prandtl's mixing length theory by developing an expression for the mixing length, ℓ (ref. 4).

$$\bar{\tau}_{ij}^{(t)} = -\rho\kappa_2^2 \left| \frac{\left(\frac{d\bar{u}_j}{dx_i}\right)^3}{\left(\frac{d^2\bar{u}_j}{dx_i^2}\right)} \right| \frac{d\bar{u}_j}{dx_i}\,; \qquad \kappa_2 = 0.4 \tag{13}$$

The difficulty with these expressions is that they cannot adequately describe transport processes near solid surfaces where viscous effects are important. In fact, the Prandtl and Taylor theories neglect viscous effects altogether and are only applicable in the large turbulent core of the flow where viscous effects are negligible compared to inertia effects.

A model may be employed which roughly describes the regions of a turbulent flow (ref. 5). The turbulent flow is subdivided into three regions; the turbulent core, buffer zone and the viscous sublayer near the wall. Figure 1 depicts these three regions along with the mean velocity (or temperature) distribution in each region. In the turbulent core, energy is transported very quickly from place to place by virtue of eddy activity. As a result the mean temperature and velocity vary little throughout the turbulent core. Viscous and conduction effects can therefore be ignored in the turbulent core, and the Prandtl Mixing Length Theory can be applied to obtain the well known nondimensional logarithmic velocity and temperature distributions for the turbulent core;

$$u^+ - u_1^+ = \frac{1}{\kappa_1} \ln \frac{y^+}{y_1^+} = T^+ - T_1^+ \qquad y^+ \geq y_1^+ \tag{14}$$

As a consequence of negligible viscosity and application of these well known phenomenological theories, mean velocity and temperature distributions in the core region of a turbulent flow can be predicted.

In the thin viscous sublayer adjacent to the wall, viscous and conduction effects dominate inertia and convection effects (i.e., eddy activity). Since energy transfer by viscous momentum transport and conduction heat transport is a slow process in comparison with eddy transport (inertia and convection), large velocity and temperature gradients occur through the thin viscous zone. By neglecting inertia terms in the equations of motion and energy, one can derive the well known linear velocity and temperature distributions for the viscous sublayer:

$$u^+ = y^+; \qquad T^+ = Pr\,y^+ \qquad 0 \leq y^+ \leq 5 \tag{15}$$

This linear relationship is well accepted.

A situation intermediate between that in the turbulent core and the viscous sublayer exists in the buffer zone. In the buffer zone energy is transported by both viscous (or conduction) and eddy (or convection) effects. As a result of the complexity of this region in the turbulent flow, the buffer zone is not well understood. Numerous functional relationships for the eddy diffusivities in this region of the flow have been proposed in the literature. Each of these relationships which attempt to describe the transport process near solid surfaces is largely empirical and applies only to an experimentally determined region of the flow. Functional relationships for the eddy diffusivities in the buffer zone have ranged from the linear form proposed by Von Kármán (ref. 6),

$$\frac{\varepsilon}{\nu} = \frac{y^+}{5} - 1 \tag{16}$$

and the empirical expression proposed by Reichardt (ref. 7),

$$\frac{\varepsilon}{\nu} = \kappa\left[y^+ - \eta_o \tanh\left(\frac{y^+}{\eta_o}\right)\right]; \qquad \kappa = 0.4,\ \eta_o = 11 \tag{17}$$

to Deissler's well known expression which is valid from the wall to the turbulent core (ref. 8),

$$\frac{\varepsilon}{\nu} = n^2u^+y^+\left[1 - e^{-n^2u^+y^+}\right]; \qquad n = 0.124 \tag{18}$$

The variation in the functional form of these expressions for the eddy diffusivities is indicative of the uncertainty involved in predicting mean velocity and temperature profiles for the buffer region in a turbulent flow. A direct comparison of the various eddy diffusivity relationships in their ability to predict mean velocity and temperature distributions in a turbulent flow is therefore warranted.

THEORY

The intent of this study was to determine the effect of eddy distribution on momentum and heat transfer near the wall in turbulent pipe flow, with particular interest in the buffer zone. For the purpose of this analysis, the region $0 < y^+ < 5$ was assumed to be the viscous sublayer. The region $5 < y^+ < 30$ was taken as the buffer zone, and $y^+ > 30$ was considered the turbulent core, where y^+ is dimensionless distance as measured from the wall. A number of eddy diffusivity relationships from the literature were directly compared on their ability to accurately predict mean velocity and temperature distributions in a cylindrical, smooth-walled pipe with uniform heat transfer. The problem is depicted in figure 2. A fluid is in turbulent flow in a smooth circular pipe of diameter, D , at uniform temperature, T . Beginning at z = 0, there is a cooling device that withdraws heat from the tube at a constant heat flux, q . At some large distance downstream from the start of this constant wall heat flux, the radial temperature profiles will have stabilized and temperature will vary as a linear function of axial distance, z.

$$\bar{T}\,(r,z) = Az + \bar{T}(r) \tag{19}$$

It is at this position in the flow that we compare velocity and temperature distributions calculated from various eddy diffusivity relationships to experimental data. The experimental velocity data used for the comparison are contained in tables I and II. The experimental temperature data are contained in table III. The experimental velocity data are from Laufer (ref. 9) and Deissler (ref. 10) for turbulent flow of air in a smooth, cylindrical pipe at Reynold's numbers ranging from 8 000 to 25 000. The temperature data are from Deissler (ref. 11) for turbulent air flow at Pr = 0.73 and Reynold's numbers ranging from 8100 to 17 000.

An expression relating eddy distribution to velocity distribution in the buffer zone for turbulent pipe flow can be obtained by starting with the Reynold's equation (ref. 12).

$$\frac{r}{R}\, u^{*2} = -\nu \frac{d\bar{u}_z}{dr} + \overline{u_r' u_z'} \tag{20}$$

By applying Boussinesq's theory (eq. (9)), the Reynold's stress term can be written as;

$$\frac{\bar{\tau}_{rz}^{(t)}}{\rho} = -\varepsilon_\nu \frac{d\bar{u}_z}{dr} = \overline{u_r' u_z'} \tag{21}$$

where ε_ν is the coefficient of eddy diffusivity for momentum. Assuming shear stress is constant over the entire flow field, the Reynold's equation becomes;

$$u^{*2} = -\left(1 + \frac{\varepsilon_\nu}{\nu}\right) \nu \frac{d\bar{u}_z}{dr} \tag{22}$$

Introducing dimensionless quantities, transforming coordinates and integrating we arrive at an expression relating velocity distribution to eddy distribution.

$$u^+ = \int_0^{y+} \frac{dy^+}{1 + \frac{\varepsilon_\nu}{\nu}} \tag{23}$$

The problem of calculating the velocity distribution in the buffer zone (or the viscous sublayer or turbulent core) is therefore reduced to finding an adequate eddy distribution, solving equation (23), and comparing the results to the experimental data. The solution may be analytical or numerical depending upon the complexity of the expression for ε_ν/ν.

An expression relating eddy distribution to temperature distribution in turbulent pipe flow is obtained by starting with the energy equation. Several assumptions, which are conventionally made in determining temperature profiles in turbulent flows, are also made here (ref. 5). As previously mentioned, the temperature profiles are assumed to be fully developed. Viscous dissipation is neglected. Finally, heat transfer due to bulk flow is neglected since the buffer zone is in the near wall region. Making these assumptions and applying Boussinesq's theory, the energy equation becomes;

$$q_o = q_r^{(\ell)} + q_r^{(t)} = k \frac{d\bar{T}}{dr} + \rho\hat{C}_p \varepsilon_H \frac{d\bar{T}}{dr} \tag{24}$$

where $q_r^{(\ell)}$ is the laminar contribution to heat transfer (viscous) and $q_r^{(t)}$ is the turbulent contribution (inertial). Introducing dimensionless quantities, transforming coordinate systems, and integrating we arrive at an expression relating temperature distribution to eddy distribution.

$$T^+ = \int_0^{y^+} \frac{dy^+}{\left(\frac{1}{Pr} + \frac{\varepsilon_H}{\nu}\right)} \tag{25}$$

The problem of calculating temperature distributions in turbulent pipe flows is therefore reduced to finding an adequate expression for eddy distribution, solving equation (25) for a given Prandtl number, and comparing the results to experimental data.

If Reynold's analogy applies, there is a direct proportionality between turbulent momentum transfer and turbulent heat transfer. By Reynold's analogy the turbulent Prandtl number ($\varepsilon_\nu/\varepsilon_H$) equals one and:

$$\varepsilon_\nu = \varepsilon_H = \varepsilon \tag{26}$$

The equations of motion and energy then become;

$$\frac{\tau_o}{\rho} = -\,(\nu + \varepsilon)\,\frac{d\bar{u}_z}{dr} \tag{27}$$

$$\frac{q_o}{\rho\hat{C}_p} = \left(\frac{\nu}{Pr} + \varepsilon\right)\frac{d\bar{T}}{dr} \tag{28}$$

A direct proportionality exists between momentum and heat transfer if kinematic viscosity and ν/Pr are negligible compared to eddy diffusivity or if kinematic viscosity and ν/Pr are numerically equal. For turbulent flow in pipes kinematic viscosity and ν/Pr are negligible compared to eddy diffusivity only in the turbulent core. Therefore Reynold's analogy cannot be universally applied in the study of momentum and heat transfer in the buffer zone. However, for gases the kinematic viscosity and ν/Pr are of the same order of magnitude (there is an exact analogy between momentum and heat transfer if $Pr = 1$) and Reynold's analogy will hold in the buffer zone. For liquids, kinematic viscosity is much larger than ν/Pr (the ratio of these two quantities can be as high as 200 for liquids) and Reynold's analogy cannot generally be applied directly to heat transfer in the near wall region (ref. 6). It should also be noted that Reynold's analogy applies if shear stress and heat flux vary with radial direction according to the same law as well as if they are constant with radial direction. In this analysis Reynold's analogy was applied, and identical relationships for eddy diffusivity were used in calculating both velocity and temperature distributions.

Eddy Diffusivity Relationships

Several relationships for eddy distribution in the near wall region (viscous sublayer and buffer zone) were found from a search of the literature.

Each relationship represents a different eddy distribution for the turbulent flow. Other relationships for eddy distribution in the near wall region of turbulent pipe flows which have been suggested in the literature but were not included in the analysis of this paper are reported in references 17 to 20.

Deissler assumed that the analogy between shear stress produced by viscosity and that produced by turbulence is not exact since the mechanism of momentum transfer is different for the two conditions (ref. 8). In the case of viscous shear, momentum transfer takes place suddenly at the instant molecules collide. For the turbulent case the fluid particles can continuously transfer momentum as they travel. He reasoned that this difference can be absorbed in the eddy diffusivity and equation (27) should remain valid. Thus;

$$u^{*2} = \frac{\tau_o}{\rho} = \nu \frac{d\bar{u}}{dy} + \varepsilon \frac{d\bar{u}}{dy} \tag{29}$$

Deissler assumed that near the wall, the effects of the magnitude of fluid velocity and distance from the wall on the turbulent transfer of momentum must be considered, whereas away from the wall only the relative magnitude of velocity at one point as compared to another is important. The experimental data available showed that the turbulent shear stress (turbulent transfer of momentum) becomes very small near the wall. As a result all shear stress is produced by viscous action and the velocity is nearly a linear function of y (distance from the wall). The second and higher velocity derivatives are therefore zero, and the first derivative approaches a constant. By dimensional analysis, Deissler concluded that;

$$\varepsilon = f\left(u, y, \frac{\mu}{\rho}, \frac{du}{dy}, \frac{d^2u}{dy^2}, \ldots \right) \tag{30}$$

Initially Deissler assumed that close to the wall μ/ρ did not influence the turbulent mechanism. The result was;

$$\varepsilon = f(u,y) \tag{31}$$

or, in simplest form;

$$\varepsilon = n^2uy \tag{32}$$

n is a constant determined empirically by Deissler to be 0.124 from tube flow velocity distributions. In later work, Deissler concluded from heat and mass transfer data at high Prandtl and Schmidt numbers that the effect of μ/ρ cannot be neglected in the viscous sublayer. Thus;

$$\varepsilon = \varepsilon\left(u, y, \frac{\mu}{\rho}\right) \tag{33}$$

And by dimensional analysis;

$$\varepsilon = n^2 uyF\left(\frac{n^2 uy}{\frac{\mu}{\rho}}\right) \tag{34}$$

Reasoning that $F(n^2uy/\mu/\rho)$ should approach one as n^2uy increases (the effect of kinematic viscosity becomes negligible at high turbulence levels), Deissler assumed a form for F as:

$$F\left(\frac{n^2 uy}{\frac{\mu}{\rho}}\right) = \left(\frac{n^2 uy}{\frac{\mu}{\rho}}\right) \tag{35}$$

In differential form this becomes;

$$dF = d\left[\frac{n^2 uy}{\frac{\mu}{\rho}}\right] \tag{36}$$

Equation (36) should approach zero as F approaches one, so we multiply by $(1 - F)$ to get;

$$dF = d\left[\frac{n^2 uy}{\frac{\mu}{\rho}}\right](1 - F) \tag{37}$$

Separating variables and integrating yields;

$$F = 1 - e^{[-n^2 uy/\mu/\rho]} \tag{38}$$

and substitution into equation (34) yields;

$$\varepsilon = n^2 uy\left[1 - e^{[-n^2 uy/\nu]}\right] \qquad 0 \leq y^+ \leq 26 \tag{39}$$

When Deissler's expression for eddy distribution is substituted into equations (23) and (25), the expressions for velocity and temperature distribution become;

$$u^+ = \int_0^{y^+} \frac{dy^+}{1 + n^2u^+y^+\left[1 - e^{-n^2u^+y^+}\right]} \tag{40}$$

$$T^+ = \int_0^{y^+} \frac{dy^+}{\frac{1}{Pr} + n^2u^+y^+\left[1 - e^{-n^2u^+y^+}\right]} \tag{41}$$

The solution of these equations is iterative and numerical because of the complexity of Deissler's eddy diffusivity expression. However, the solution proves valid from the wall to the turbulent core region.

Von Kármán assumed the viscous sublayer was purely laminar in turbulent pipe flow.

$$\frac{\varepsilon}{\nu} = 0 \tag{42}$$

and derived the well known linear relationships for velocity and temperature distribution.

$$u^+ = y^+ \tag{43}$$

$$T^+ = Pry^+ \qquad 0 \le y^+ \le 5 \tag{44}$$

Von Kármán extended Nikuradse's (ref. 13) logarithmic velocity distribution for the turbulent core region;

$$u^+ = 5.5 + 2.5\ln y^+ \qquad y^+ \ge 30 \tag{45}$$

to the buffer region. His best estimate of the velocity profile for the buffer region was developed by using a straight line profile that joined the curve $u^+ = y^+$ tangentially at $y^+ = 5$ and that crossed the logarithmic profile at $y^+ = 30$. The result was a logarithmic velocity distribution for the buffer zone (ref. 4).

$$u^+ = 5\ln y^+ - 3.05 \qquad 0 \le y^+ \le 30 \tag{46}$$

The corresponding eddy distribution is;

$$\frac{\varepsilon}{\nu} = \frac{y^+}{5} - 1 \tag{47}$$

When this eddy distribution is used the equation for temperature distribution becomes;

$$T^+ = 5\ln\left[\frac{\frac{1}{Pr} - 1 + \frac{y^+}{5}}{\frac{1}{Pr} - 0.632}\right] \tag{48}$$

Lin proposed a polynomial eddy distribution for the viscous sublayer (ref. 14).

$$\frac{\varepsilon}{\nu} = \left(\frac{y^+}{14.5}\right)^3 \qquad 0 \le y^+ \le 5 \tag{49}$$

Solving for velocity and temperature distribution using this relationship;

$$u^+ = \frac{14.5}{3}\left[\frac{1}{2}\ln\frac{\left(1 + \frac{y^+}{14.5}\right)^2}{1 - \frac{y^+}{14.5} + \left(\frac{y^+}{14.5}\right)^2} + \sqrt{3}\tan^{-1}\frac{\frac{2y^+}{14.5} - 1}{\sqrt{3}} + \frac{\pi\sqrt{3}}{6}\right] \tag{50}$$

$$T^+ = \frac{14.5\mathrm{Pr}^{2/3}}{3}\left[\frac{1}{2}\ln\frac{\left(\frac{1}{\mathrm{Pr}^{1/3}} + \frac{y^+}{14.5}\right)^2}{\left(\frac{y^+}{14.5}\right)^2 - \frac{y^+}{14.5\mathrm{Pr}^{1/3}} + \frac{1}{\mathrm{Pr}^{2/3}}} + \sqrt{3}\tan^{-1}\frac{\frac{2y^+}{14.5} - \frac{1}{\mathrm{Pr}^{1/3}}}{\frac{\sqrt{3}}{\mathrm{Pr}^{1/3}}} + \frac{\pi\sqrt{3}}{6}\right] \qquad (51)$$

The velocity distribution in the sublayer region by using Lin's expression for eddy diffusivity is nearly linear, varying only slightly from $u^+ = y^+$. This suggests that the viscous sublayer is not purely laminar as assumed in Von Kármán's analysis. Figure 3 shows the velocity distribution for turbulent pipe flow in the viscous sublayer and directly compares the Lin velocity distribution and the linear profile to the experimental data of Deissler.

For the buffer zone, Lin modified Von Kármán's logarithmic solution to satisfy the velocity and eddy conditions from equation (49) at $y^+ = 5$. Lin's result for the buffer zone was;

$$\frac{\varepsilon}{\nu} = \frac{y^+}{5} - 0.959 \qquad 5 \le y^+ \le 30 \qquad (52)$$

The corresponding velocity and temperature distributions are,

$$u^+ = 5\ln(y^+ + 0.205) - 3.27 \qquad (53)$$

$$T^+ = 5\ln\left[\frac{\frac{1}{\mathrm{Pr}} - 0.959 + \frac{y^+}{5}}{\frac{1}{\mathrm{Pr}} + 0.041}\right] \qquad (54)$$

Reichardt proposed the following relationship for eddy distribution in turbulent pipe flow (ref. 7).

$$\frac{\varepsilon}{\nu} = \kappa\left[y^+ - n_o \tanh\frac{y^+}{n_o}\right] \qquad 0 \le y^+ \le 50 \qquad (55)$$

where $K = 0.4$ and $n_o = 11$ are experimentally determined constants. Reichardt also suggested a relationship for the turbulent core as;

$$\frac{\varepsilon}{\nu} = \left(\frac{\kappa}{3}\right)y^+ (0.5 - R^2)(1 - R) \qquad (56)$$

For small y, equations (39) and (55) can be simplified by series expansions of the exponential function and hyperbolic tangent, respectively, and by considering only the first two terms in the series.

The result for Deissler's equation (39) is;

$$\varepsilon_\nu \sim (y^+)^4 \tag{57}$$

and for Reichardt's equation (55);

$$\varepsilon_\nu \sim (y^+)^3 \tag{58}$$

Comparisons of the predicted values of heat and mass transfer with experimental data at large Pr and Sc numbers made on the basis of a statistical analysis of the experimental data have shown that the exponent should be in the range from 3.0 to 3.2 for the region close to the wall (ref. 15). Therefore, Reichardt's relationship (eq. (55)) which yields an exponent of 3 is more likely to describe the mechanisms of turbulent transfer near the wall than Deissler's relationship (eq. (39)). In addition, Reichardt's relationship has no discontinuities in the range $y^+ < 50$, which is important for the calculation of heat transfer (ref. 16). The solutions for velocity and temperature distribution using Reichardt's relationship are numerical but prove valid well into the turbulent core region.

$$u^+ = \int_0^{y^+} \frac{dy^+}{1 + 0.4\left[y^+ - 11 \tanh\left(\frac{y^+}{11}\right)\right]} \tag{59}$$

$$T^+ = \int_0^{y^+} \frac{dy^+}{\frac{1}{Pr} + 0.4\left[y^+ - 11 \tanh\left(\frac{y^+}{11}\right)\right]} \tag{60}$$

DISCUSSION

The intent of this study was to compare various relationships for eddy distribution in the near wall region in turbulent pipe flow. The relationships were compared based on their ability to predict velocity and temperature distributions that fit the experimental data. Equations (23) and (25) were derived using classical assumptions to predict velocity and temperature distributions. Few assumptions were made in deriving equation (23) from the equation of motion. However, the temperature effect on fluid properties (turbulent Prandtl number effect), Reynold's number effect (bulk flow), and viscous dissipation were neglected in deriving equation (25) from the energy equation. These classical assumptions are valid for the purpose of comparing relationships for eddy diffusivity, but caution should be exercised in extending the scope of this analysis. For example, using Reichardt's relationship for eddy diffusivity, Petrokhov (ref. 16) calculated several values of Nu and friction factor for the case where q and τ vary along the radius and the case of constant q and τ (i.e., $q = q_o$, $\tau = \tau_o$). The assumption of uniform q and τ produces noticeable errors in Nu and friction factor values, especially for low Re and Pr. Also, figure 4 shows the experimental temperature data used in the analysis of this paper plotted as a function of Reynold's number. As Reynold's number increases, the fluid temperature decreases with wall cooling as a result of enhanced convective heat transfer in the fluid due to bulk flow. The overall temperature difference is ~20 percent at $y^+ = 80$ and decreases to about 5 percent at $y^+ = 10$ for Reynold's numbers ranging from 8100 to 17 000. A rigorous analysis to predict temperature distributions in turbulent pipe flows would include the effects of Reynold's number, variations in fluid properties and viscous dissipation.

The velocity distributions in the near wall region for turbulent pipe flow using the six eddy diffusivity relationships are presented in figures 5 and 6. Figure 5 is a semilogarithmic plot of the velocity distribution while figure 6 is a linear plot showing the actual velocity profiles. From the figures, it can be seen that Von Kármán's linear velocity profile obtained by neglecting eddy effects and Lin's relationship for the viscous sublayer (eq. (49)) fit the experimental data only in the region $0 < y^+ < 5$ (i.e., the viscous sublayer region). This result is expected because these relationships were derived for the viscous sublayer. Lin's cubic relationship also demonstrates that the sublayer is not purely laminar as originally postulated by Von Kármán in deriving the linear velocity relationship for this region. Lin's (eq. (52)) and Von Kármán's (eq. (47)) eddy diffusivity relationships which give logarithmic velocity solutions fit the data only in the buffer zone ($5 < y^+ < 30$) and with reasonable accuracy. However, these solutions diverge quickly from the experimental data in the viscous sublayer and in the turbulent core. The utility of these two relationships is that they permit analytical solutions for velocity (and temperature) distributions in the buffer zone. Both Reichardt's (eq. (55)) and Deissler's (eq. (39)) relationships for eddy distribution fit the data in the buffer zone as well as in the viscous sublayer (i.e., in the entire near wall region). The Reichardt solution also correlates with the data well into the turbulent core ($y^+ > 30$). Although quite accurate and valid over several regions in the turbulent flow field, these relationships require rigorous numerical solutions.

The temperature distributions using the eddy diffusivity relationships are shown in figures 7 and 8. As with the velocity distributions, temperature profiles are given on both semilogarithmic (fig. 7) and linear (fig. 8) plots. Tables IV to IX contain the analytical velocity and temperature data used in figures 5 to 8. Again, Von Kármán's linear temperature distribution and Lin's temperature distribution using the cubic eddy diffusivity relationship fit the data only in the viscous sublayer ($y^+ < 5$). Lin's and Von Kármán's logarithmic temperature distributions roughly fit the data in the region $10 < y^+ < 30$ of the buffer zone. These relationships for eddy diffusivity were experimentally fit to velocity data and do not predict temperature distribution well in the buffer zone (or the other regions). The Reichardt and Deissler relationships for eddy diffusivity fit the data well in both the buffer zone and viscous sublayer. As was the case with the velocity distribution, the Reichardt solution fits the data well into the turbulent core. However, both relationships require rigorous numerical solutions.

CONCLUDING REMARKS

The effect of eddy distribution on velocity and temperature profiles in turbulent pipe flows near the wall was investigated. Six relationships for eddy diffusivity were examined. All relationships examined in the analysis accurately predict velocity distributions in turbulent pipe flows over the regions reported in the literature. The more complex expressions for eddy distribution generally require a more rigorous numerical solution but yield more accurate results over a wider region of the flow field.

By applying Reynold's analogy the six relationships for eddy diffusivity were used to predict temperature distributions. The scatter of the temperature data, the dependence of fluid properties on temperature and the classical assumptions used in the analysis made a comparision of the relationships difficult based on their ability to predict temperature distributions. Future work should be directed toward examining the effects of variations in fluid properties, bulk flow, and viscous dissipation when predicting temperature distributions. However, it appears that the logarithmic solutions are not good choices for predicting temperature distributions in the buffer zone. The Deissler and Reichardt expressions for eddy diffusivity are the only relationships that fit the temperature and velocity data over the entire near wall region.

APPENDIX A

SYMBOLS

A	constant
$\hat{C}_p$	specific heat of fluid at constant pressure, J/kgK
D	pipe diameter, m
F	function of $\varepsilon/(\mu/\rho)$
g	acceleration due to gravity, 9.81 m/s^2
h	heat transfer coefficient, W/m^2K
k	thermal conductivity of fluid, W/mK
ℓ	Prandtl mixing length
Nu	Nusselt number for heat transfer, hD/k
n	Deissler constant (0.124)
P	absolute fluid pressure, N/m^2
Pr	Prandtl number, $\hat{C}_p\mu/k$
q	rate of heat transfer per unit area, W/m^2
$q_r^{(\ell)}$	laminar contribution to heat flux in radial direction, W/m^2
$q_r^{(t)}$	turbulent contribution to heat flux in radial direction, W/m^2
R	pipe radius
Re	Reynolds number, $\rho uD/\mu$
r	radial direction
Sc	Schmidt number, $\mu/(\rho\lambda)$
T	temperature, K
T^+	dimensionless temperature, $\rho\hat{C}_p u^*(\bar{T} - T_o)/q_o$
t	time
t_o	time interval
u	fluid velocity, m/s
u^+	dimensionless fluid velocity, u/u^*

u^* friction velocity, $\sqrt{\tau_o/\rho}$

y distance from wall, m

y^+ dimensionless distance from wall, $yu^*\rho/\mu$

x rectangular coordinate, m

z axial direction

ε coefficient of eddy diffusivity, m^2/s

ε_H coefficient of eddy diffusivity for heat, m^2/s

ε_ν coefficient of eddy diffusivity for momentum, m^2/s

η_o Reichardt constant (11.0)

κ Reichardt constant (0.40)

κ_1 constant

κ_2 Von Kármán constant (0.36 to 0.40)

λ molecular diffusivity, m^2/s

μ fluid viscosity, kg/ms

ν kinematic viscosity, μ/ρ, m^2/s

ρ fluid density, kg/m^3

τ shear stress in fluid, kg/m^2

$\tau_{ij}^{(\ell)}$ laminar contribution to ij component of shear stress, kg/m^2

$\tau_{ij}^{(t)}$ turbulent contribution to ij component of shear stress, kg/m^2

<u>Overlines</u>:

$\hat{}$ per unit mass

$\bar{}$ time smoothed

<u>Superscripts</u>:

' deviation from time-smoothed value

(ℓ) laminar

(t) turbulent

$+$ dimensionless parameter

Subscripts:

i,j	coordinate
r	radial coordinate
z	axial coordinate
o	quantity evaluated at wall
1	quantity evaluated at location 1

APPENDIX B

EDDY DIFFUSIVITY RELATIONSHIPS FOR THE NEAR WALL REGION OF TURBULENT PIPE FLOW

Many relationships for eddy diffusivity in the near wall region of turbulent pipe flow have been suggested in the literature. Six well accepted relationships for eddy diffusivity were considered in the analysis of this paper. Some other relationships which have been suggested in the literature are presented here.

Spalding (ref. 17) has suggested "a single formula for the law of the wall." He suggests the following relationship for the case when the viscosity and density of the fluid are uniform:

$$y^+ = u^+ + 0.1108 \left\{ e^{0.4u^+} - 1 - 0.4u^+ - \frac{(0.4u^+)^2}{2!} - \frac{(0.4u^+)^3}{3!} - \frac{(0.4u^+)^4}{4!} \right\} \qquad \text{(B1)}$$

Spalding used the above equation to fit the experimental data and while the fit was acceptable, it was not clear whether or not to include the term $(0.4u^+)^4/4!$. The author explains that including this term fits the requirement that eddy diffusivity increases with the fourth power of u^+ and y^+ close to the wall.

The viscosity ratio can then be obtained from equation (B1):

$$\frac{\mu_{turbulent}}{\mu_{total}} = \frac{1}{\left[1 + \frac{1}{0.04432}\left\{ e^{0.4u^+} - 1 - 0.4u^+ - \frac{(0.4u^+)^2}{2!} - \frac{(0.4u^+)^3}{3!} \right\}\right]} \qquad \text{(B2)}$$

Mizushina and Ogino (ref. 18) have presented expressions for eddy viscosity based on experimental results and have obtained velocity distributions for the near wall region of turbulent pipe flows. They also analyzed the effect of Reynolds number on the eddy diffusivity. Based on the experimental results, they assumed the following eddy diffusivity relationships for the near wall region:

$$\frac{\varepsilon_\nu}{\nu} = A(y^+)^3 \qquad 0 \leq y^+ \leq y_1^+ \qquad \text{(B3)}$$

$$\frac{\varepsilon_\nu}{\nu} = 0.4y^+\left(1 - \frac{y^+}{R^+}\right) - 1 \qquad y_1^+ \leq y^+ \leq y_2^+ \qquad \text{(B4)}$$

Using the assumptions that eddy diffusivity is continuous and u is continuous, the authors determined the values of A, y_1^+, and y_2^+. By substituting expressions (B3) and (B4) into the velocity distribution equation;

$$u^+ = \int_0^{y^+} \frac{1 - \frac{y^+}{R^+}}{1 + \frac{\varepsilon_\nu}{\nu}} dy^+ \tag{B5}$$

they obtained velocity distributions for the near wall region.

Wasan, et al. (ref. 19) presented a theoretical correlation of velocity and eddy viscosity. They argued that the available equations, while fit the data, do not satisfy the equations of mean motion near the wall. In their work, the velocity distribution for $y^+ \leq 20$ was as follows.

$$u^+ = y^+ - 1.04\text{x}10^{-4}\ (y^+)^4 + 3.03\text{x}10^{-6}\ (y^+)^5 \tag{B6}$$

and the turbulent shear stress distribution:

$$\overline{uv}^+ = 4.16\text{x}10^{-4}\ (y^+)^3 - 15.15\text{x}10^{-6}\ (y^+)^4 \tag{B7}$$

An expression for eddy diffusivity is derived from equations (B6) and (B7):

$$\frac{\varepsilon_\nu}{\nu} = \frac{\overline{uv^+}}{\frac{du^+}{dy^+}} = \frac{1}{1 + \frac{1}{4.16\text{x}10^{-4}(y^+)^3 - 15.15\text{x}10^{-6}\ (y^+)^4}} \tag{B8}$$

Finally, Sherwood, et al. (ref. 20) have investigated the velocity and eddy distribution in the wall region. Their work was experimental and included a flow visualization of minute tracer particles. These researchers measured the instantaneous axial and circumferential components of velocity and turbulent intensity in the wall region, y = 0.2, at different Reynolds numbers (8000 to 50 000). They suggest the following expression for eddy distribution:

$$\frac{\varepsilon_\nu}{\nu} = 7.746(u^+)^3 - 32.51(u^+)^4 + 36.66(u^+)^5 \qquad y^+ < 0.4 \tag{B9}$$

Their values of eddy diffusivity are several times greater than those generally associated with $y^+ < 5$. The authors claim that a derived relation between eddy diffusivity and y^+ in this region is not quantitatively valid. They indicate that their axial intensity data are in agreement with those of Laufer.

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TABLE I. - EXPERIMENTAL VELOCITY DATA FOR AIR (LAUFER, 1953)

Dimensionless distance from wall, y^+	Dimensionless velocity, U^+
3.25	3.30
3.80	3.20
4.30	4.20
5.30	4.90
5.30	5.20
5.80	5.50
6.60	6.40
7.00	6.50
7.70	6.40
8.10	6.60
8.70	8.10
9.40	7.80
10.0	8.80
11.0	9.30
12.5	10.1
13.0	9.60
13.5	10.2
17.0	11.0
17.2	11.5
18.0	11.8
24.0	11.4
25.5	13.2
37.0	13.9
42.5	14.7
54.0	15.2
66.0	15.3

TABLE II. - EXPERIMENTAL VELOCITY DATA FOR AIR
(DEISSLER, 1955)

Re = 8000		Re = 16 000	
Dimensionless distance from wall, y^+	Dimensionless velocity, U^+	Dimensionless distance from wall, y^+	Dimensionless velocity, U^+
5.66	5.83	2.00	2.24
8.50	7.45	3.99	3.17
11.36	9.30	5.99	5.02
14.14	10.68	7.99	6.43
17.00	11.52	9.98	7.71
22.64	12.66	11.98	8.98
28.36	13.49	14.98	10.39
39.64	14.16	20.01	12.09
56.64	14.90	24.92	12.97
85.00	15.90	29.95	13.66
		39.89	14.30
		59.90	15.13
		99.97	16.43
Re = 11 000		Re = 19 000	
y^+	U^+	y+ +	U^+
8.99	7.01	4.80	4.25
12.59	9.37	9.61	7.33
19.78	11.85	12.00	8.75
24.28	12.78	14.41	10.02
31.48	13.90	18.01	11.26
38.65	14.33	24.06	12.66
91.80	16.13	29.97	13.56
		36.02	13.96
		47.99	14.69
		72.05	15.57
Re = 14 000		Re = 25 000	
y^+	U^+	y^+	U^+
4.32	3.98	2.98	3.29
6.49	5.28	5.96	4.74
7.57	7.07	11.94	8.28
9.73	8.03	14.91	9.79
11.90	8.97	17.90	11.07
18.38	11.20	22.38	12.20
23.80	12.63	29.89	13.43
28.10	13.30	37.23	14.15
36.76	14.09	44.75	14.58
45.40	14.60	59.61	15.18
63.80	15.50	89.50	16.10
82.20	16.10		

TABLE III. - EXPERIMENTAL TEMPERATURE DATA FOR AIR

Pr = 0.73

(DEISSLER, 1952)

Re = 8100		Re = 15 000	
Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless distance from wall, y^+	Dimensionless Temperature, T^+
7.59	5.22	8.07	4.81
10.30	6.45	12.56	5.58
13.01	7.68	17.03	7.27
15.10	8.55	21.52	8.65
18.43	9.60	26.01	9.58
21.15	10.13	34.97	10.65
26.57	11.35	43.93	11.73
31.99	11.88	52.91	12.34
42.84	13.11	70.83	13.26
53.68	13.81	88.77	13.73
75.35	14.68		
Re = 10 000		**Re = 17 000**	
y^+	T^+	y^+	T^+
8.97	5.52	19.01	7.78
12.17	6.69	24.02	9.03
15.38	8.05	29.03	9.81
18.58	9.23	39.03	11.05
21.78	9.90	59.05	12.30
24.99	10.58	89.08	13.23
31.39	11.42		
37.81	11.92		
50.62	12.60		
63.43	13.44		
89.94	14.29		
Re = 12 800			
y^+	T^+		
7.08	4.69		
11.02	5.81		
14.95	7.25		
18.88	8.69		
22.82	9.33		
26.75	10.31		
30.69	10.77		
38.55	11.57		
46.43	12.05		
62.16	13.01		
77.90	13.49		

TABLE IV. - ANALYTICAL DATA FOR LINEAR VELOCITY AND TEMPERATURE DISTRIBUTIONS IN VISCOUS SUBLAYER

[VON KÁRMÁN; ε/ν = 0.]

Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+
	Pr = 0.73	Pr = 1.0
0	0	0
1	.73	1
2	1.46	2
3	2.19	3
4	2.92	4
5	3.65	5
6	4.38	6
7	5.11	7
8	5.84	8
9	6.57	9
10	7.30	10
11	8.03	11
12	8.76	12
13	9.49	13
14	10.22	14
15	10.95	15
16	11.68	16
17	12.41	17
18	13.14	18
19	13.87	19
20	14.60	20
21	15.33	21
22	16.06	22
23	16.79	23
24	17.52	24
25	18.25	25
26	18.98	26
27	19.71	27
28	20.44	28
29	21.17	29
30	21.90	30
31	22.63	31
32	23.36	32
33	24.07	33
34	24.82	34
35	25.55	35
36	26.28	36
37	27.01	37
38	27.74	38
39	28.47	39
40	29.20	40

TABLE V. - ANALYTICAL DATA FOR VELOCITY AND TEMPEATURE PROFILES IN VISCOUS SUBLAYER

[LIN, et al.; $\varepsilon/\nu = (y^+/14.5)^3$.]

Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+
	Pr = 0.73	Pr = 1.0
0	0	0
1	.73	1
2	1.46	2.00
3	2.19	2.99
4	2.91	3.98
5	3.62	4.95
6	4.33	5.90
7	5.01	6.82
8	5.67	7.69
9	6.31	8.53
10	6.91	9.31
11	7.49	10.03
12	8.02	10.70
13	8.52	11.31
14	8.98	11.86
15	9.40	12.36
16	9.79	12.81
17	10.14	13.22
18	10.46	13.58
19	10.75	13.90
20	11.01	14.19
21	11.25	14.46
22	11.47	14.69
23	11.66	14.90
24	11.84	15.09
25	12.00	15.26
26	12.15	15.42
27	12.28	15.56
28	12.40	15.69
29	12.51	15.81
30	12.62	15.91
31	12.71	16.01
32	12.80	16.10
33	12.88	16.18
34	12.95	16.25
35	13.02	16.32
36	13.08	16.39
37	13.14	16.45
38	13.19	16.50
39	13.24	16.55
40	13.29	16.60

TABLE VI. - ANALYTICAL DATA FOR VELOCITY AND TEMPEATURE DISTRIBUTIONS IN BUFFER ZONE

[VON KÁRMÁN; $\varepsilon/\nu = y^+/5 - 1.0$.]

Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+
	Pr = 0.73	Pr = 1.0
0	----	-----
1	----	-----
2	0.21	0.42
3	1.37	2.44
4	2.30	3.88
5	3.09	5.00
6	3.77	5.91
7	4.37	6.68
8	4.91	7.35
9	5.39	7.94
10	5.83	8.46
11	6.24	8.94
12	6.61	9.37
13	6.96	9.77
14	7.29	10.15
15	7.59	10.49
16	7.88	10.81
17	8.15	11.12
18	8.41	11.40
19	8.66	11.67
20	8.89	11.93
21	9.12	12.17
22	9.33	12.41
23	9.54	12.63
24	9.73	12.84
25	9.92	13.04
26	10.11	13.24
27	10.28	13.43
28	10.45	13.61
29	10.62	13.79
30	10.78	13.96
31	10.93	14.12
32	11.08	14.28
33	11.23	14.43
34	11.37	14.58
35	11.51	14.73
36	11.64	14.87
37	11.77	15.00
38	11.90	15.14
39	12.02	15.27
40	12.14	15.39

TABLE VII. - ANALYTICAL DATA FOR VELOCITY AND TEMPERATURE DISTRIBUTIONS IN BUFFER ZONE

[LIN, et al.; $\varepsilon/\nu = y^+/5 - 0.959$.]

Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+
	Pr □ 0.73	Pr = 1.0
0	----	-----
1	----	-----
2	0.36	0.68
3	1.46	2.55
4	2.36	3.90
5	3.13	4.97
6	3.79	5.86
7	4.38	6.60
8	4.90	7.25
9	5.37	7.83
10	5.81	8.34
11	6.20	8.81
12	6.57	9.24
13	6.92	9.63
14	7.24	10.00
15	7.54	10.34
16	7.83	10.66
17	8.10	10.96
18	8.35	11.24
19	8.59	11.51
20	8.83	11.76
21	9.05	12.00
22	9.26	12.23
23	9.46	12.45
24	9.66	12.66
25	9.85	12.87
26	10.03	13.06
27	10.20	13.25
28	10.37	13.43
29	10.54	13.60
30	10.70	13.77
31	10.85	13.93
32	11.00	14.09
33	11.14	14.24
34	11.28	14.39
35	11.42	14.54
36	11.55	14.68
37	11.68	14.81
38	11.81	19.94
39	11.93	15.07
40	12.05	15.20

TABLE VIII. - ANALYTICAL DATA FOR VELOCITY AND TEMPERATURE DISTRIBUTIONS NEAR THE WALL

[REICHARDT; $\varepsilon/\nu = \kappa[y^+ - \eta_0 \tanh (y^+/\eta_0)]$ $\kappa = 0.4$, $\eta_0 = 11.$]

Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+	Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+
	Pr = 0.73	Pr = 1.0		Pr = 0.73	Pr = 1.0
0	0	0	46	12.04	14.76
1	.71	1.00	47	12.11	14.82
2	1.43	2.00	48	12.17	14.88
3	2.13	2.98	49	12.23	14.94
4	2.82	3.93	50	12.29	15.00
5	3.49	4.85	51	12.35	15.06
6	4.13	5.70	52	12.40	15.12
7	4.73	6.49	53	12.46	15.17
8	5.28	7.21	54	12.51	15.23
9	5.80	7.86	55	12.57	15.28
10	6.27	8.96	56	12.62	15.33
11	6.70	8.96	57	12.67	15.38
12	7.08	9.42	58	12.72	15.43
13	7.44	9.83	59	12.77	15.48
14	7.77	10.20	60	12.82	15.53
15	8.06	10.54	61	12.86	15.58
16	8.33	10.84	62	12.91	15.62
17	8.58	11.12	63	12.96	15.67
18	8.81	11.37	64	13.00	15.71
19	9.02	11.60	65	13.05	15.76
20	9.22	11.82	66	13.09	15.80
21	9.41	12.02	67	13.13	15.84
22	9.58	12.20	68	13.17	15.89
23	9.74	12.38	69	13.21	15.93
24	9.90	12.54	70	13.25	15.97
25	10.04	12.65	71	13.29	16.01
26	10.18	12.84	72	13.33	16.05
27	10.31	12.97	73	13.37	16.08
28	10.43	13.11	74	13.41	16.12
29	10.55	13.23	75	13.45	16.16
30	10.67	13.35	76	13.48	16.20
31	10.77	13.46	77	13.52	16.23
32	10.88	13.57	78	13.56	16.27
33	10.98	13.67	79	13.59	16.30
34	11.07	13.77	80	13.63	16.34
35	11.17	13.87	81	13.66	16.37
36	11.26	13.97	82	13.69	16.41
37	11.34	14.05	83	13.73	16.44
38	11.43	14.14	84	13.82	16.47
39	11.51	14.22	85	13.86	16.50
40	11.58	14.30	86	13.89	16.54
41	11.69	14.41	87	13.92	16.57
42	11.77	14.48	88	13.95	16.60
43	11.84	14.55	89	13.98	16.63
44	11.91	14.62	90	14.01	16.66
45	11.98	14.69			

TABLE IX. - ANALYTICAL DATA FOR VELOCITY AND TEMPERATURE DISTRIBUTIONS NEAR THE WALL

[DEISSLER; $\varepsilon/\nu = n^2u^+y^+[1 - e^{-n^2u^+y^+}]$; n = 0.124.]

Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+	Dimensionless distance from wall, y^+	Dimensionless temperature, T^+	Dimensionless velocity, U^+
	Pr = 0.73	Pr = 1.0		Pr = 0.73	Pr = 1.0
0	0	0	46	12.58	15.16
1	.69	.95	47	12.66	15.24
2	1.42	1.95	48	12.74	15.33
3	2.15	2.94	49	12.82	15.41
4	2.86	3.91	50	12.90	15.48
5	3.55	4.83	51	12.97	15.56
6	4.20	5.70	52	13.04	15.64
7	4.81	6.48	53	13.11	15.71
8	5.37	7.18	54	13.18	15.78
9	5.87	7.80	55	13.25	15.85
10	6.33	8.35	56	13.32	15.92
11	6.74	8.84	57	13.38	15.99
12	7.12	9.28	58	13.45	16.05
13	7.47	9.67	59	13.51	16.12
14	7.78	10.03	60	13.57	16.18
15	8.08	10.36	61	13.64	16.24
16	8.35	10.67	62	13.69	16.30
17	8.60	10.95	63	13.75	16.36
18	8.84	11.21	64	13.82	16.42
19	9.07	11.45	65	13.87	16.48
20	9.28	11.68	66	13.92	16.54
21	9.48	11.90	67	13.98	16.59
22	9.67	12.10	68	14.03	16.65
23	9.85	12.30	69	14.08	16.70
24	10.02	12.48	70	14.14	16.75
25	10.19	12.66	71	14.19	16.81
26	10.34	12.85	72	14.24	16.86
27	10.50	13.01	73	14.29	16.91
28	10.64	13.16	74	14.34	16.96
29	10.78	13.31	75	14.38	17.01
30	10.92	13.45	76	14.43	17.05
31	11.05	13.56	77	14.48	17.10
32	11.17	13.70	78	14.52	17.15
33	11.29	13.82	79	14.57	17.19
34	11.41	13.95	80	14.61	17.24
35	11.52	14.06	81	14.66	17.29
36	11.63	14.18	82	14.70	17.33
37	11.74	14.29	83	14.74	17.37
38	11.84	14.40	84	14.79	17.42
39	11.95	14.50	85	14.83	17.46
40	12.04	14.60	86	14.87	17.50
41	12.14	14.70	87	14.91	17.54
42	12.23	14.89	88	14.95	17.58
43	12.32	14.89	89	14.99	17.62
44	12.41	14.98	90	15.03	17.66
45	12.50	15.07			

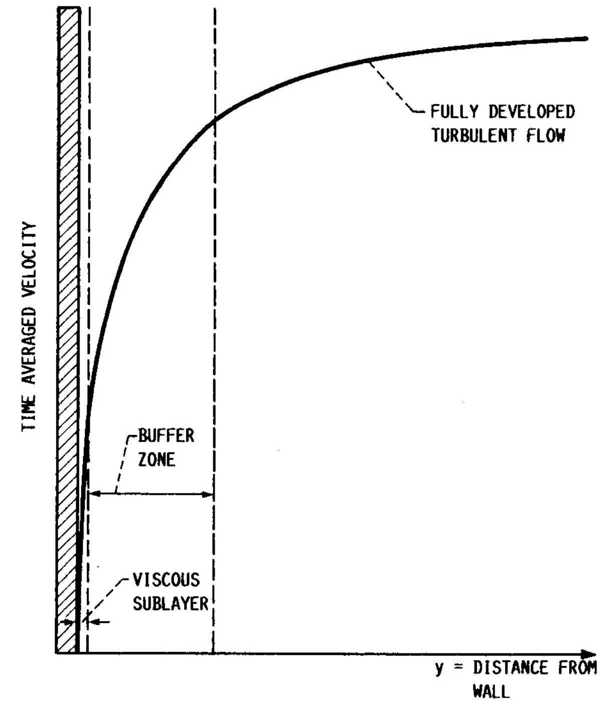

FIGURE 1. - VELOCITY DISTRIBUTION FOR TURBULENT FLOW IN TUBES (REGION NEAR TUBE WALL). (REPRODUCED FROM REF. 5.)

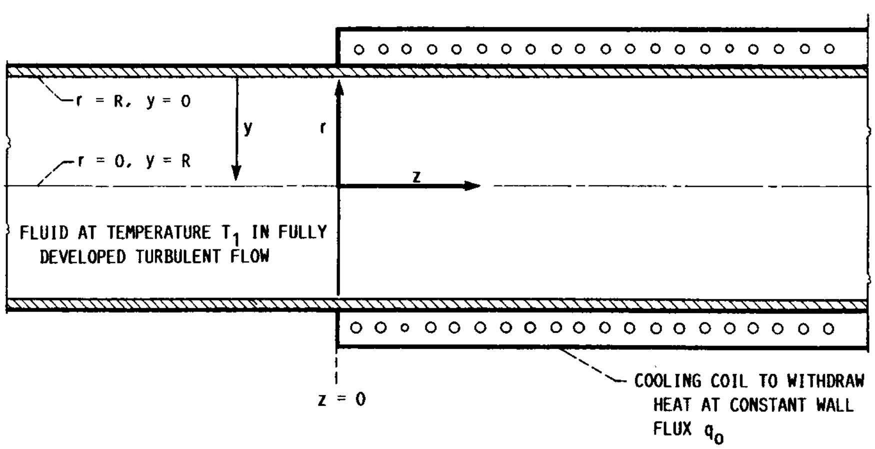

FIGURE 2. - TURBULENT TUBE FLOW WITH CONSTANT HEAT FLUX AT WALL. (REPRODUCED FROM REF. 5)

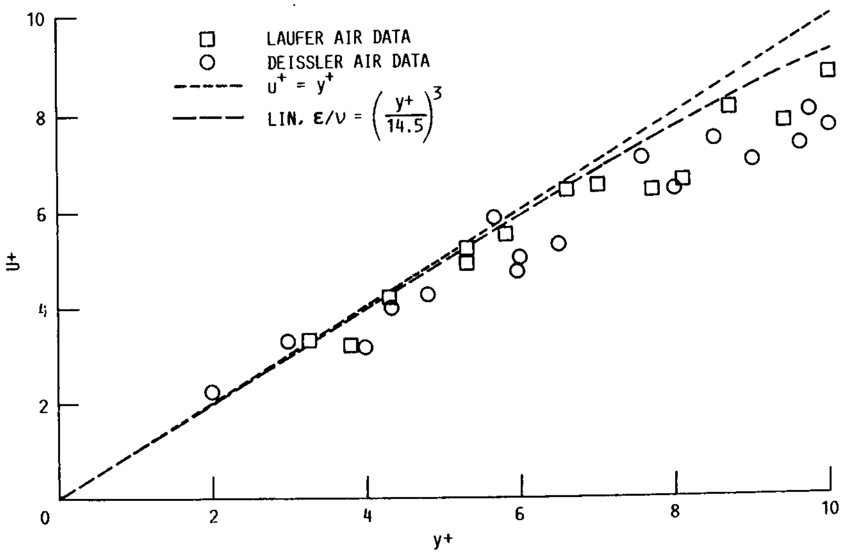

FIGURE 3. - VELOCITY DISTRIBUTION IN THE VISCOUS SUBLAYER FOR TURBULENT PIPE FLOW.

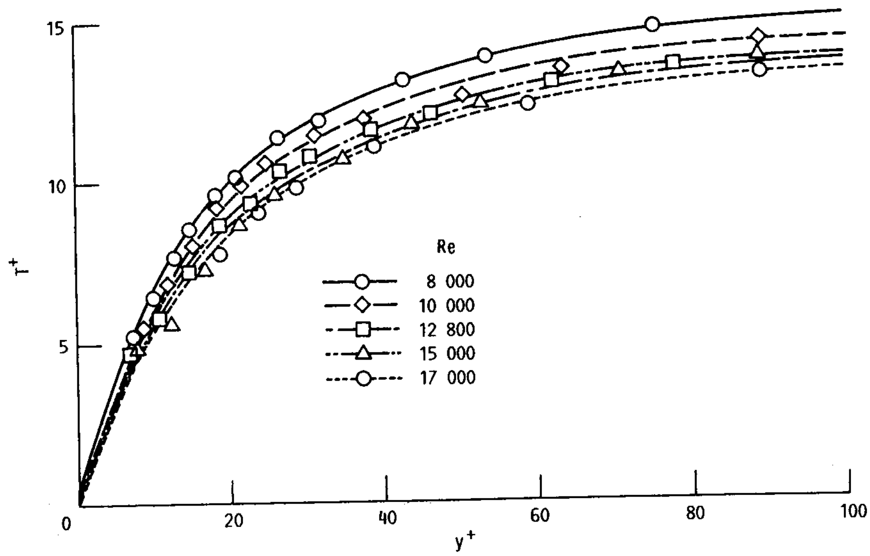

FIGURE 4. - EFFECT OF REYNOLD'S NUMBER ON TEMPERATURE DISTRIBUTION FOR TURBULENT PIPE FLOW.

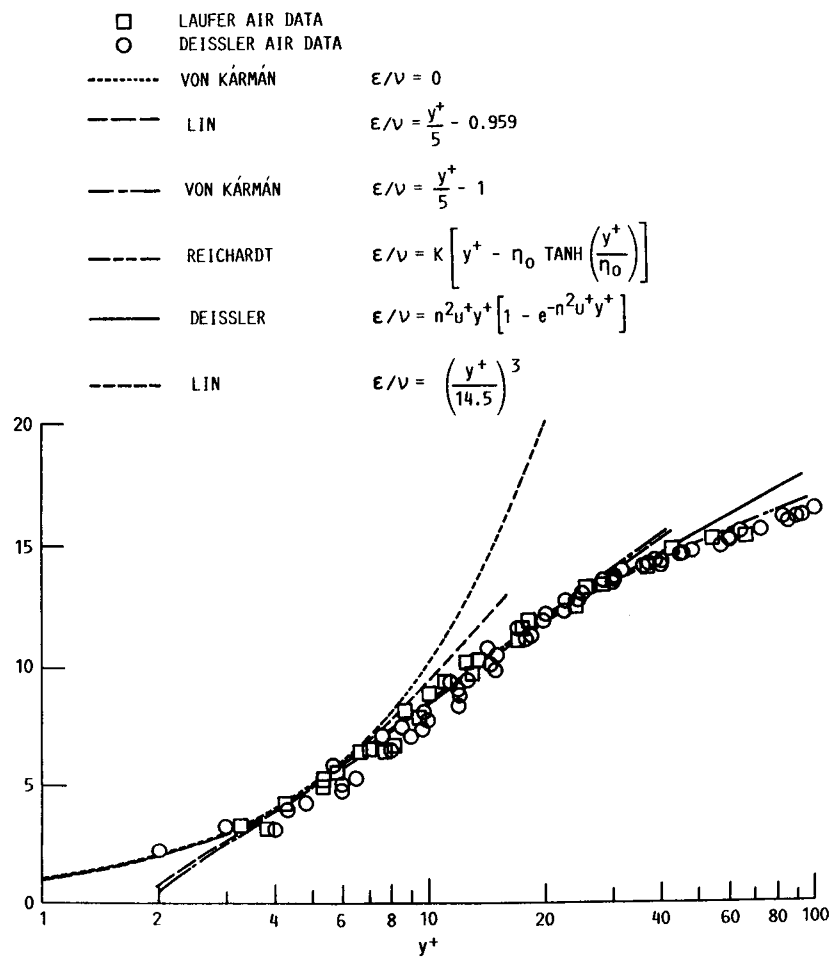

FIGURE 5. - VELOCITY DISTRIBUTIONS FOR TURBULENT PIPE FLOW.

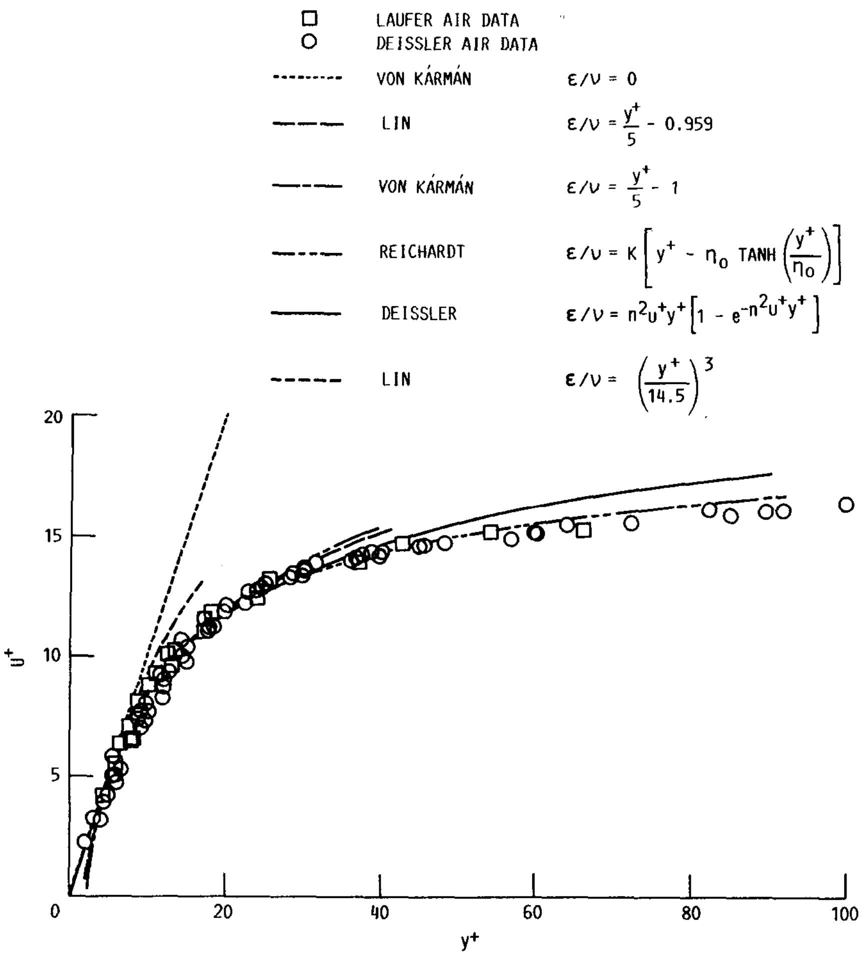

FIGURE 6. - VELOCITY DISTRIBUTIONS FOR TURBULENT PIPE FLOW.

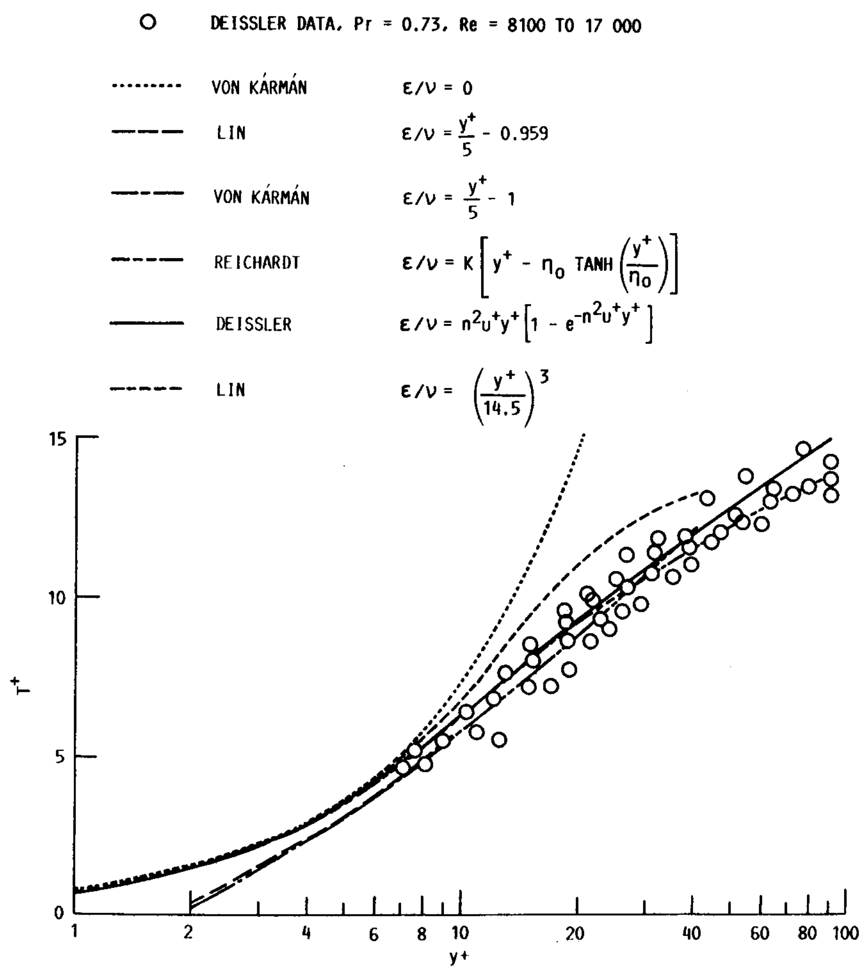

FIGURE 7. - TEMPERATURE DISTRIBUTIONS FOR TURBULENT PIPE FLOW.

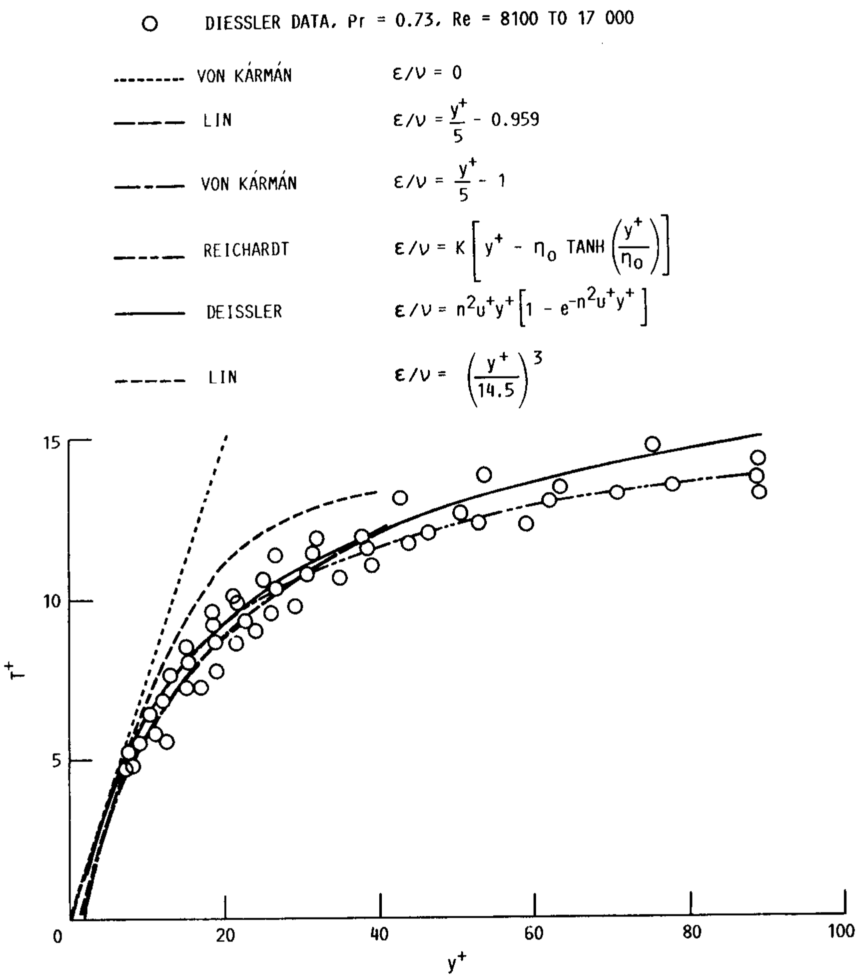

FIGURE 8. - TEMPERATURE DISTRIBUTIONS FOR TURBULENT PIPE FLOW.

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15. Supplementary Notes

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16. Abstract

A study was conducted to determine the effect of eddy distribution on momentum and heat transfer near the wall in turbulent pipe flow. The buffer zone was of particular interest in that it is perhaps the most complicated and least understood region in the turbulent flow field. Six eddy diffusivity relationships are directly compared on their ability to predict mean velocity and temperature distributions in turbulent air flow through a cylindrical, smooth-walled pipe with uniform heat transfer. Turbulent flow theory and the development of the eddy diffusivity relationships are briefly reviewed. Velocity and temperature distributions derived from the eddy diffusivity relationships are compared to experimental data for fully-developed pipe flow of turbulent air at a Prandtl number of 0.73 and Reynold's numbers ranging from 8100 to 25 000.

17. Key Words (Suggested by Author(s))	18. Distribution Statement
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