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Plasma Contactors for Use with Electodynamic Tethers for Power Generation

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Abstract

Plasma contactors have been proposed as a means of making good electrical contact between biased surfaces such as found at the ends of an electrodynamic tether and the space environment. A plasma contactor is a plasma source which emits a plasma cloud which facilitates the electrical connnection. The physics of this plasma cloud is investigated for contactors used as electron collectors. The central question that is addressed is whether the electrons that are collected by a plasma contactor come from the far field or by ionization of local neutral gas. This question is important to answer because the system implications are quite different for the two mechanisms. It is shown that contactor clouds in space will consist of a spherical core possibly containing a shock wave. Outside of the core the cloud will expand anisotropically across the magnetic field leading to a turbulent cigar shape structure along the field. This outer region is itself divided into two regions by the ion response to the electric field. A two dimensional theory of for the outer regions of the cloud is developed. The current voltage characteristic of an Argon plasma contactor cloud is estimated for several ion currents in the range of 1–100 Amperes. It is suggested that the major source of collected electrons comes by ionization of neutral gas while collection of electrons from the far field is relatively small.

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Nomenclature

$\vec{B}$	Magnetic field
$c_s = \sqrt{2T_e/m_i}$	ion acoustic velocity
D_e	electron diffusion coefficient
D_i	ion diffusion coefficient
$\vec{E}$	Electric field measured in frame moving with source
$\vec{E}_m = \vec{V} \times \vec{B}$	Motional electric field
E_{ion}	ionization energy
f_i	initial ionization fraction
I	total current$=I_e + I_c$
$I_e(r)$	inflowing electron current
$I_c(r)$	outgoing ion current
L_n	density scale length
m_e	electron mass
m_i	ion mass
$n_{ambient}$	ambient ion density
n_e	electron density
n_i	contactor ion density
n_n	neutral density
r	radial distance
r_0	radial dimension of source
r_c	radial dimension of outer boundary of transition region
r_{core}	radial dimension of core
T_e	electron temperature in energy units
T_i	ion temperature in energy units
T_n	neutral temperature in energy units
$\vec{V}$	Velocity of plasma source
v_A	Alfven velocity

v_D	Differential electron ion drift velocity
$\vec{v}_n$	neutral velocity
v_{the}	Electron thermal velocity
v_{thi}	Ion thermal velocity
β	Plasma diamagnetic parameter
κ	electron thermal conductivity
ν_e	Total electron momentum exchange frequency
ν_{ei}	Electron ion momentum exchange frequency
ν_{en}	Electron neutral momentum exchange frequency
$\nu^*_{acoustic}$	Electron momentum exchange frequency due to ion acoustic turbulence
$\nu^*_{Buneman}$	Electron momentum exchange frequency due to Buneman turbulence
ϕ	Potential$(\vec{E} = -\nabla\phi)$
ϕ_0	Potential at center of cloud$(r = r_0)$
ω_{pe}	electron plasma frequency
Ω_e	Electron gyrofrequency
Ω_i	Ion gyrofrequency
ρ_e	Thermal electron gyroradius$(= v_{the}/\Omega_e)$
ρ_i	Thermal ion gyroradius$(= v_{thi}/\Omega_i)$
$\langle\sigma v\rangle_{ionization}$	velocity averaged ionization cross section
$\langle\sigma v\rangle_{recombination}$	velocity averaged recombination cross section
μ_0	Permeability of free space

1 Introduction

One of the most interesting and innovative ideas for the production of power in space is to generate it by using the geomagnetic field. This can be done with an electrodynamic tether where a long cable is swept across the Earth's magnetic field and current is induced to flow through the cable as a result of the motionally generated electric field[1]. In order for the electrodynamic tether to be a practical device for generating power in space it is necessary that the load impedance and the impedance of the current circuit through the tether ends and the ionosphere be not too different. This requires that the tether make efficient electrical contact with the ionosphere. One way of doing this is with a metallized balloon which offers a large collection area to the ionosphere. This suffers from the disadvantage that the neutral drag on the balloon may be large and affect the dynamics of the tether. Another means of effecting electrical contact is with the use of plasma clouds. Devices which emit plasma clouds for the purpose of facilitating electrical contact in space have been named plasma contactors. The expectation is that the plasma cloud will form a large interaction region with the ionosphere and allow the motion of charge through the cloud for a small potential drop without suffering from the problem of neutral drag associated with a metallized balloon.

The expansion of the plasma cloud and the collection of current has been the subject of both experimental and theoretical work. Ground based experiments have proven the basic principle of plasma contactors, that they can act as devices to magnify the collection of current from an ambient plasma[2]. Theoretical work until recently has focused on the plasma flow near the plasma source in the ground based experiments[3] and has obtained qualitative agreement. Recently it has been realized that the use of plasma contactors in the ionosphere where the cloud can freely expand may be different from the ground based experiments and so theoretical analysis has been undertaken for space based contactors[4,5]. These studies show that in order for contactors to work as advertised as electron collection devices it is necessary for the plasma cloud to be highly turbulent in order for the ionospheric electrons to scatter across the restaining influence of the geomagnetic field. This observation is amply confirmed in the ground based experiments. All theoretical studies so far have assumed that the plasma flow is dominantly radial away from the plasma source. This assumption simplifies the analysis considerably and enables simple estimates to be made of the current voltage characteristic of the plasma cloud[5]. In this paper we relax this assumption and construct a model for the anisotropic structure of the plasma cloud across the magnetic field. We show that the cloud physics can be divided into three regimes. The first regime is the core region of the plasma cloud about the plasma source where the plasma cloud is highly diamagnetic and the electrons are effectively unmagnetized either due to exclusion of the geomagnetic field or to turbulent collisions. The ions in this region are unmagnetized because the ion gyroradius is expected to be much larger than the core region. The core region will be isotropic because there is no preferred direction. The second region is where the electrons are magnetized but still experiencing turbulent collisions and the ions are unmagnetized. The ion flow here will still be radial. We shall call this the transition region. Finally the third region will be where the ions and the electrons are magnetized. This is where the plasma cloud will be in contact with the ambient plasma. This region is very important to understand because it is here that some of the electrons will be supplied which will eventually be collected at the center of the cloud. This region we shall call the outer shell. This is because the major effect of the cloud is to facilitate collection of ionospheric electrons and this outer shell provides the collection area akin to the outer surface of a metallized balloon.

In principle, plasma contactors can be used for both electron and ion collection. However in this work we shall concentrate on electron collection. Recent work has dealt with electron emission using plasma contactors[6]. The central question we shall address is the use of plasma contactors for electron collection in the ionosphere. We are concerned with use of plasma contactors for tethers which can draw many amperes of current[1]. There are two possible sources for the electrons which may be collected by such a device. The first is ionospheric electrons. That is the electrons will be pulled in from the far field. The second is by ionization of local neutral gas. In the electric field around the collector the newly born electrons will be collected and the ions repelled. We note that there are two sources of neutral gas, first the ionospheric neutral oxygen and secondly the neutral gas emitted from the contactor itself. It is very important to determine by which mechanism a plasma contactor may work since the system implications are quite different for the two possible mechanisms. We shall show that if plasma contactors collect most of their electrons from the far field then small plasma contactors are more efficient than large ones which suggests use of small multiple tethers for generation of a given current. On the other hand if plasma contactors work by ionization of neutral gas then large collectors are more efficient which drives an electrodynamic tether system towards long single tethers. We shall address this question of how contactors may work in space by constructing a model for the potential in the far field and calculating the electron current which can flow in towards the center. We do this in the presence and absence of ionization of the neutral gas emitted from the contactor.

The outline of the paper is as follows: in section 2 we review and further develop the theory of the inner core region. In section 3 we consider the outer shell and develop a two dimensional model for the potential structure across the magnetic field. In section 4 we solve these models numerically and obtain an estimate of the current voltage characteristic of the contactor cloud. In section 5 we conclude and suggest the impact of this work on engineering use of plasma contactors. We note that there is a large body of work which has been done on electron collection by electron beam-emitting vehicles[7]. This work also suggests the importance of ionization to the creation of the return current necessary for charge conservation. It has been observed that ampere level return currents have been collected for modest vehicle potentials such 100 or 200 Volts. In the case of a plasma contactor used for electron collection there are at least two major differences as compared with the emission of an electron beam and collection of a return current. The first is that there is no perturbing electron beam and hence no possibility of a beam plasma discharge. The second is that we are interested in use of a contactor for steady state electron collection and there is evidence that it may be possible to collect more current in a transient operation than in steady state[8]. These considerations suggest that the physics of electron collection in space by plasma contactor clouds may be different from the experience obtained from electron beam experiments.

2 Review of the core cloud region

In Figure 1 we show a sketch of what the plasma contactor cloud may look like as used on the end of an electrodynamic tether. Near the plasma source the cloud will be very dense and highly diamagnetic. In addition the anodic end of the tether will probably be biased to a voltage which is many times the ambient ion energy. These conditions are such that there will be a core region to the plasma contactor cloud where the expansion is radial. This can be shown by noting that the two directions of anisotropy in the cloud will be the direction of motion of the whole system and the direction of the magnetic field. When the plasma is highly diamagnetic or highly collisional then the magnetic field is shielded out of the plasma so that this direction of anisotropy is destroyed. The other direction of anisotropy is manifested through the motional electric field which is shielded whenever the plasma is dense enough so that it can easily polarize. The manifestation of these two directions of anisotropy divides the plasma cloud into three regions. The first region as we have discussed is a core region where both the magnetic field and the motional electric field are shielded from the plasma. The plasma will expand isotropically in this region and will be determined by the boundary conditions imposed by the plasma source. The boundary of this region will be where the condition $\nu_e/\Omega_e = 1$ is satisfied and where the gyrofrequency is defined in terms of the actual magnetic field as modified by diamagnetic effects. This condition states that the collisional scattering due to either classical or turbulent effects just balances the gyromotion of the electrons. This condition should be regared as the most conservative condition for inclusion of magnetic field effects. A simple model for this core region was examined in Ref [5]. The second or transition region is where the magnetic field is manifested in the plasma but the electric field is still dominated by the imposed electric field from the plasma source. In this region the electrons will be magnetized while the ions will expand radially under the influence of the imposed potential. The boundary of this region will come when the ions have expanded radially a sufficient distance so that they start to turn around under the influence of the magnetic field. Hence the boundary will be at approximately one ion gyroradius with the gyroradius based on the energy that the ions have gained from falling through the self consistent potential. The third region or outer shell will have both the magnetic field and the motional electric field manifested. The ions will be magnetized and drift across the magnetic field with an $\vec{E} \times \vec{B}/B^2$ drift which ultimately is given by the $\vec{E}_m \times \vec{B}/B^2 = -\vec{V}$ while the electrons will flow along the field lines as well as undergo an $\vec{E} \times \vec{B}/B^2$ drift. This ensures that the ions will ultimately come to rest relative to the ambient plasma. This outer region is where the ambient plasma will be contacted.

The core region will satisfy the following equations:

contactor ion density,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 n_i v_i\right) = n_n n_e \langle\sigma v\rangle_{ionization} - n_i n_e \langle\sigma v\rangle_{recombination} \tag{1}$$

neutral density,

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 n_n v_n\right) = -n_n n_e \langle\sigma v\rangle_{ionization} + n_i n_e \langle\sigma v\rangle_{recombination} \tag{2}$$

potential,

$$\frac{\partial}{\partial r}\left(e(\phi_0 - \phi)\right) = \frac{m_e \nu_e}{e n_e}\left(\frac{I}{4\pi r^2}\right) - \frac{T_e}{n_e}\frac{\partial n_e}{\partial r} - \frac{\partial T_e}{\partial r} \tag{3}$$

electron temperature,

$$-\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \kappa \frac{\partial T_e}{\partial r}\right) = \frac{I_e}{4\pi r^2}E - E_{ion} n_n n_e \langle\sigma v\rangle_{ionization} \tag{4}$$

electron density,

$$n_e = n_i + n_{ambient}. \tag{5}$$

where $\langle\sigma v\rangle_{ionization}$ and $\langle\sigma v\rangle_{recombination}$ are given in Ref [5]. In Eq. (4) the electrons are taken to be heated ohmically and to lose their energy mainly as a result of ionization. The small loss due to electron–ion equilibration is ignored. The plasma sources which are used to produce the plasma cloud typically emit cold ions and hot electrons. Hence the ion and neutral temperatures are taken to the same with v_n given by $v_n = \sqrt{2T_i/m_i}$ and the value of the temperature chosen to be the ambient ion temperature. The potential is determined from electron momentum balance(Eq. (3)) and quasineutrality is imposed(Eq. (5)). This means that we are not solving for the potential structure in the sheath close to the anode. Rather we obtain the potential drop associated with the cloud itself. Hence the potential drops calculated should be regarded as the smallest potential drops possible for a given current.

The ion velocity in the core region will be determined from ion momentum balance. For the plasma devices used to produce the plasma clouds the ion are ejected with a velocity which is approximately the ion acoustic velocity. In the appendix we show that if the neutral density is sufficiently low then the ions will accelerate supersonically away from the contactor source. If the neutral density is high than the ion–neutral friction slows the ions down rapidly and an ion shock forms which drops the ion velocity to subsonic values. For parameters of interest the critical neutral density is sufficently high($n_n \gg 10^{20}$ meters^{-3}) that we shall assume that we are below this neutral density everywhere and take the ion to be expanding supersonically. In this case the ion velocity will be given by

$$\frac{1}{2}m_i v_i^2 + e\phi = \frac{1}{2}m_i c_s^2 + e\phi_0. \tag{6}$$

These equations for the core cloud region must be solved subject to the following

$$I = constant \tag{7}$$

while at at $r = r_0$

$$n_i(r_0) = \frac{I_c}{4\pi e c_s r_0^2} \tag{8}$$

$$n_n(r_0) = n_i(r_0)(1 - f_i)/f_i \tag{9}$$

$$T_e = T_e(r_0) \tag{10}$$

and

$$-\kappa \frac{\partial T_e}{\partial r} = \frac{T_e(r_0)}{e}\frac{I_c}{4\pi}. \tag{11}$$

The electron thermal conductivity is taken to the classical expression[9]

$$\kappa = 3.2 n_e T_e/(m_e \nu_e). \tag{12}$$

The differential velocity between electron and ions is

$$v_D = \frac{I}{4\pi r^2 e n_e}. \tag{13}$$

The electron collision frequency is given as

$$\nu_e = \nu_{ei} + \nu_{en} + \nu^*_{acoustic} + \nu^*_{Buneman} \tag{14}$$

where for

$$v_D > c_s/\sqrt{2}(1 + \sqrt{T_e m_i/(m_e T_i)}(T_e/T_i)\exp(-1.5 - T_e/2/T_i))$$

the ion acoustic instability can be triggered and gives[10]

$$\nu^*_{acoustic} = 10^{-2}\frac{T_e}{T_i}\frac{v_D}{v_{the}}\omega_{pe} \tag{15}$$

and for $v_D > v_{the}$ the Buneman instability can be triggered and gives[11]

$$\nu^*_{acoustic} = 0.53\left(\frac{m_e}{m_i}\right)^{0.61}\omega_{pe}. \tag{16}$$

The effect of turbulent scattering is included since it has been experimentally observed and theoretically shown to be necessary for efficient operation of plasma contactors[5]. The boundary of this core region is taken to be the radius at which the collisionality $\nu_e/\Omega_e = 1$ where the magnetic field is chosen to be the diamagnetically modified field(that is,$B = 0$ for $\beta \geq 1$ and $B = B_{ambient}\sqrt{1-\beta}$ for $\beta < 1$). The plasma β is defined as

$$\beta = n_e(T_e + T_i)/(B^2/2\mu_0). \tag{17}$$

The preceeding equations describe the core plasma cloud when the ions are expanding supersonically. The numerical solution of these equations will be discussed in section 4.

3 Two Dimensional Theory of the Outer Region

In the outer shell of the plasma contactor cloud where the ambient environment is contacted the direction of magnetic field and the direction of motion are the two important directions influencing the cloud of plasma which spreads out from a plasma contactor. The magnetic field will influence the cloud by allowing it to expand freely along the field but only with difficulty across the field. Hence we expect that we can write

$$\frac{\partial}{\partial \ell} \ll \frac{\partial}{\partial r_\perp} \tag{18}$$

where ℓ measures along the field $\vec{B}$ and $r_\perp$ measures across $\vec{B}$. Eq. (18) is the statement that perpendicular variations in all plasma quantities will be much more rapid than parallel variations. This statement will only be true where the plasma particles will feel a significant magnetic field as in the outer shell. The plasma cloud projected on the field will be a cigar shape with a possible cross-section as in fig. 1.

The plasma cloud will be described by the state variables n_e, n_i, ϕ, T_e and T_i as in the core region. Since we expect the plasma elongation we can write for any state variable

$$f(r_\perp, \ell) = \bar{f}(r_\perp) + (f - \bar{f}) \tag{19}$$

where

$$\bar{f} = \frac{1}{\int d\ell/B} \int \frac{d\ell}{B} f(r_\perp, \ell) \tag{20}$$

and

$$(f - \bar{f}) \ll \bar{f} \tag{21}$$

Hence we expect that to lowest order the state variables can be written as independent of the position on the field line. The $\int d\ell/B$ average is the correct average that includes the information that $\nabla \cdot \vec{B} = 0$ everywhere. Physically this is the statement that along the magnetic field, the state variables will look the same regardless of location or that variations in physical quantities along the magnetic field will be weak compared to variations in perpendicular direction.

The associated neutral cloud will be expanding spherically and isotropically since the magnetic field will not affect it. For temperature of a few eV and for densities of $10^8 cm^{-3}$, the mean free path is approximately 20 km so that the expansion will be collisionless over the distances of interest to us. In that case we can write the neutral density as

$$n_n = n_n(r_c)(\frac{r_c}{r})^2 \tag{22}$$

where r_c is the outer boundary of the transition region of the cloud.

Since the ion temperature is low on release from most types of contactor so we shall take it as constant (and ambient) as in the core region. Since we do not expect extremely large electric fields, the plasma will be quasineutral, $\bar{n}_e = \bar{n}_i + \bar{n}_{ambient}$. Therefore to complete the state variables we need equations for $\bar{n}_i$, $\bar{\phi}$ and $\bar{T}_e$. These can be obtained by the appropriate averages of the ion continuity equation, the equation for current conservation and the equation for conservation of electron energy.

The $\int d\ell/B$ average can be performed by integrating from the cloud to a distance of the order of $2(v_A/V)r_c$ in each direction. The time $2r_c/V$ is the transit time of the inner part of the cloud across a magnetic field line and v_A (the Alfven velocity) is the speed at which information propagates along the magnetic field. The distance to integrate to is the maximum distance over which electrons can be drawn so that they enter the inner part of the cloud before they sweep past the collector. The ion continuity equation is

$$\frac{\partial n_i}{\partial t} + B\frac{\partial}{\partial \ell}\left(\frac{n_i v_{\|_i}}{B}\right) + \nabla_\perp \cdot (n_i \vec{v}_{i_\perp}) = n_e n_n < \sigma v >_{ionization} - n_e n_i < \sigma v >_{recombination} \tag{23}$$

When the $\int d\ell/B$ average is performed and with the ordering in Eqs. (21) we have

$$\frac{\partial \bar{n}_i}{\partial t} + \nabla_\perp \cdot (\bar{n}_i \vec{v}_{\perp_i}) = \bar{n}_i \bar{n}_n < \sigma v >_{ionization} - \bar{n}_i \bar{n}_e < \sigma v >_{recombination} \tag{24}$$

In Eq. (24) we have assumed that at the radii of interest to us there is no parallel flow of ions along the magnetic field. To complete the ion continuity equation we need the perpendicular ion velocity. If the ions are slowly diffusing across the magnetic field then the solution of the steady state ion momentum balance[12] gives

$$\vec{v}_{i_\perp} = k_1\{e\vec{E}_\perp + \vec{G}_i\} + k_2\{e\vec{E}_\perp + \vec{G}_i\} \times \frac{\vec{B}}{B} \tag{25}$$

where

$$k_1 = \frac{1}{eB}\frac{\kappa_i}{1+\kappa_i^2} \tag{26}$$

and

$$k_2 = \kappa_i k_1 \tag{27}$$

with

$$\kappa_i = \frac{\Omega_i}{\nu_{in}} \tag{28}$$

and

$$\vec{G}_i = -T_i \nabla \ln n_i + m_i \nu_{in} \vec{v}_n \tag{29}$$

The ions are taken to collide mainly with the neutrals (ν_{in}). The first term on the rhs of Eq. (25) is the well known Pederson current while the second term is the Hall current. The pickup current has been neglected since in the far field ionization is expected to be small. Since $k_i \gg 1$ (ion mean free path long compared to a gyroradius) we can write to lowest order in an expansion in $1/k_i$

$$\bar{n}_i \bar{v}_{i_\perp} \simeq -\bar{n}_i(\nabla_\perp \bar{\phi} + \frac{\bar{T}_i}{e} \nabla_\perp \ln \bar{n}_i) \times \frac{\vec{B}}{B^2} \tag{30}$$

We note that this lowest order expression for the ion perpendicular velocity can only be used in the ion continuity equation. For the current balance equation it is well known that the $\vec{E} \times \vec{B}$ drift for the ions cancels out the electron $\vec{E} \times \vec{B}$ drift and the higher order terms from Eq. (25) must be included to get a nontrivial current density.

The lowest order perpendicular ion flux arises due to the $\vec{E} \times \vec{B}$ and diamagnetic drifts. When Eq. (30) is substituted in Eq. (24) we obtain

$$\frac{\partial \bar{n}_i}{\partial t} - \nabla_\perp(\nabla_\perp \bar{\phi} + \frac{\bar{T}_i}{e} \ln \bar{n}_i) \times \vec{B}/B^2) = \bar{n}_i \bar{n}_n < \sigma v >_{ionization} - \bar{n}_i \bar{n}_e < \sigma v >_{recombination} \tag{31}$$

subject to

$$\bar{n}_i(r_c) = n_{i_c}, \tag{32}$$

where the density at $r = r_c$ can be determined from the solution for the ion density in the two inner regions and as $|\vec{r}_\perp| \to \infty$

$$\nabla_\perp \bar{n}_i \to 0 \tag{33}$$

The potential $\bar{\phi}$ can be determined from the $\int d\ell/B$ average of the current conservation relation ($\nabla \cdot \vec{j} = 0$). The average perpendicular electron current is taken to be due to diffusion across the magnetic field. Such diffusion can arise due to classical transport (very small) or due to turbulence induced transport. We write

$$\bar{n}_e \bar{v}_{e_\perp} = -D_e(\nabla_\perp \bar{n}_e - \frac{e\bar{n}_e}{\bar{T}_e} \nabla_\perp \bar{\phi}) + \vec{E} \times \vec{B}\ drifts + diamagnetic\ drifts. \tag{34}$$

The electron diffusion coefficent is given by $D_e = D_{an} + D_{classical}$ where the anomalous diffusion coefficient is D_{an} and the classical one is $D_{classical}$. The electron flux is composed of an $\vec{E} \times \vec{B}$ drift(which cancels the ion drift) as well as a diffusion driven flux from either turbulent or classical collisions. It is this diffusion driven flux which will give rise to the incoming electron current. In this diffusion driven flux we have neglected a temperature gradient term for the purpose of simplicity. The total perpendicular current is

$$\vec{j}_\perp = e(\bar{n}_i \vec{v}_{i_\perp} - \bar{n}_e \vec{v}_{e_\perp}) \tag{35}$$

where for the ion term we use the full expression given in Eq. (25).

The field line average of current conservation is

$$0 = \nabla_\perp \cdot \bar{\mathrm{j}}_\perp + \frac{1}{\int d\ell/B}[\frac{j_\parallel}{B}]_{endpts} \tag{36}$$

This equation contains a contribution from the parallel electron current flowing through the flux tube over which the integration is done. We are looking at the far field and for the magnetic field lines which do not intersect the collector. Hence the parallel current from the two endpoints cancels out since in steady state there can be no accumlation of current and hence all the electron current which is collected from this part of the far field reaches the near field by diffusion across the magnetic field. There is a part of the far field which is on magnetic field lines which intersect the collector. The contribution from this field aligned part of the far field will be included in the next section.

Since the electron $\vec{E} \times \vec{B}$ drifts will cancel out with the ion $\vec{E} \times \vec{B}$ drifts, the Hall terms in the ion current are small and from Eqs. (25), (34), and (36) we obtain for ϕ

$$\begin{aligned} -\nabla_\perp \cdot \left(\frac{e^2}{T_e} \left(\bar{n}_i D_i \frac{\bar{T}_e}{\bar{T}_i} + \bar{n}_e D_e \right) \nabla_\perp \bar{\phi} \right) + \nabla \cdot ((eD_i - eD_e)\nabla_\perp \bar{n}_e) & \\ + \frac{\vec{B}}{B} \times \vec{v}_n \cdot \nabla_\perp \left(e\bar{n}_i \frac{\nu_{in}}{\Omega_i} \right) &= 0 \end{aligned} \tag{37}$$

subject to at $r = r_c$

$$\bar{\phi} = \bar{\phi}_c \tag{38}$$

and in the far field $|\vec{r}_\perp| \to \infty$,

$$-\nabla_\perp \bar{\phi} = \vec{V} \times \vec{B} \tag{39}$$

In Eq. (37) the first set of terms arises from the diffusion driven mobility terms in the fluxes while the second set of terms comes from the density driven diffusion terms in the fluxes.

The boundary condition Eq. (39) is the statement that the field far from the cloud is the unshielded field seen from a moving frame. This boundary condition on the potential is consistent with the condition that in the far field the ions emitted from the contactor will be at rest in a frame fixed to the earth. Any other condition on the potential will require some ion motion in the far field. In Eq. (37) we have

$$D_i = \rho_i^2 \nu_{in} \tag{40}$$

The boundary condition $\bar{\phi}_c$ can be obtained as the potential on the outer boundary of the intermediate zone.

The electron temperature can be obtained from the averaged energy equation

$$\frac{3}{2}\bar{n}_e\left(\frac{\partial}{\partial t}\bar{T}_e - \nabla_\perp\phi \times \frac{\vec{B}}{B^2}\cdot\nabla\bar{T}_e\right) = -e\bar{\Gamma}_{e\perp}E_\perp - E_{ion}\bar{n}_e\bar{n}_n < \sigma v >_{ionization} \tag{41}$$

where

$$\Gamma_{e\perp} = n_e\vec{v}_{e\perp} \tag{42}$$

The electron temperature is determined by a balance between convection, Joule heating and ionization. This equation is to be solved subject to at $r = r_c$

$$\bar{T}_e = \bar{T}_{e_c} \tag{43}$$

and in the far field $|\vec{r}_\perp| \to \infty$,

$$\nabla_\perp \bar{T}_e \to 0 \tag{44}$$

These equations describe the plasma flow in the far field and are the obvious generalizations of the equations for the core cloud. The numerical solution of these equations will be discussed in the next section.

4 Numerical Solutions for the Plasma Cloud

The equations for the core cloud are a coupled set of first order nonlinear equations in the radial variable. These were solved with Gear's method which is well suited to the solution of such coupled equations. The integration was stopped when the collisionality condition was satisfied. In the transition region the ions will still be expanding radially but the electrons will be diffusing across the magnetic field. This region is the most difficult to deal with numerically because the two species obey different sets of equations. We attempted to model the plasma flow in this region by taking the quantities determined by the electrons, namely the potential and the electron temperature to be fixed at their values at the edge of the core region and using the radial equations for the ion and neutral density throughout the intermediate region. The integration of the ion and neutral density equations was terminated when the radial distance was equal to one ion gyroradius as determined by the energy of the ions. By this approximation, boundary values for the outer shell of the plasma cloud were obtained. This approximation for the intermediate region will give an underestimate for the total potential and electron temperature drop and therefore the numerical results reported in this paper should be regarded as being optimistic. The important question of the plasma flow in this transition regime will be the subject of a future publication.

The equations for the cross sectional structure of the outer shell are two dimensional partial differential equations. Successive Point Over Relaxation(SPOR) was used to solve the elliptic current balance equation. The derivatives were approximated by second order finite differences and iterations were carried out until the maximum error was less than 10^{-5} between two sucessive iterations. Neumann boundary conditions were applied in both the x and y directions. We required the potential to match the following boundary conditions in the far field

$$\nabla_{\perp}\bar{\phi} = -\vec{V} \times \vec{B}$$

in the $\vec{V} \times \vec{B}$ direction and

$$\nabla_{\perp}\bar{\phi} = 0$$

in the direction of motion. The potential at the inner boundary of the region was given by the scheme described above for the transition region.

For the convective equations we used the two–dimensional flux correction method(FCT) of Zalesak[13]. The high order scheme was a leapfrog–trapezoidal with fluxes calculated with the flux formulae developed by Zalesak[14] while a donor–cell scheme was used for the lower scheme to complete the FCT algorithm. Symmetric boundary conditions were applied on the density distributions and the flux limiter was applied on every iteration. No assumptions were made about the symmetry of the solutions so that the computations were carried out through the whole mesh.

For a plasma contactor the three quantities of interest are the current, potential drop and gain as a function of the plasma contactor ion current. The gain is defined as $I/I_c(r_0)$ and is a measure of the current multiplication. The gain can be written in a form which illustrates the two different contributions to the total current. From conservation of current we can write

$$I = I_c(r_0) + I_e(r_0) = I_c(r_c) + I_e(r_c) \tag{45}$$

then using Eq. (45) we can write the gain as

$$I/I_c(r_0) = 1 + [(I_c(r_c) - I_c(r_0))/I_c(r_0) + I_e(r_c)/I_c(r_0)]. \tag{46}$$

The first term in brackets on the rhs of Eq. (46) is the gain due to ionization while the second term is the gain due to collection of electrons from the far field.

The electron current flowing inward through the plasma contactor cloud was determined as the surface integral of the current density in the outer shell plus the parallel current flowing into the core region along the magnetic field. This last term is the contribution from the field aligned currents in the far field. Hence we have

$$I_e = \int_{r=r_c} \vec{j}_{\perp_e} \cdot d\vec{S} + I_{\|} = I_{\perp}^{outer} + I_{\|} \tag{47}$$

where $I_{\|}$ is the parallel electron current into the core given by $I_{\|} = j_{\|_e} 2\pi r_{core}^2$ and $j_{\|_e}$ is the electron saturation current density associated with the ambient plasma in the far field. In Eq. (47) the integration surface for computation of the perpendicular outer current was taken to have a length $L_{\|}$ given by the smaller of

$$L_{\|} = 2\sqrt{2T_e/m_e}/\nu_e$$

and

$$L_{\|} = 4r_c(v_A/V).$$

The first length is the twice the mean free path of an electron in the outer region along the magnetic field while the second length is twice the length of the Alfven wing generated by the passage of the contactor cloud. This is the rate at which information about the cloud propagates along the magnetic field[15]. The perpendicular electron current in the outer shell was obtained from the diffusion driven piece of Eq. (34). The electron diffusion coefficent was taken to be the Bohm diffusion coeffient($D_{an} = D_{Bohm} = 6.25 \times 10^{-2} T_e/B$ m^2/sec where T_e is in eV and B is in Telsa). The simulations with the Bohm diffusion coefficient model the effect of turbulence in the outer shell. Such turbulence will probably arise due to electron runaway leading to Buneman turbulence. To model this properly requires a kinetic analysis of the outer shell. We shall defer this for future work. We choose Bohm diffusion since this maximises the perpendicular current that can flow into the device from the outer shell. A choice of only classical diffusion would give much smaller perpendicular currents than we calculate in this section.

In Tables 1 and 2 we give the total current I against ion current I_c for an Argon plasma with $T_e(r_0) = 5$ eV,$f_i = 0.01, 0.99$ and for a range of $I_c = 1, 10, 100$ Amperes. For $I_c \ll 1$ no core region exists because the central density is too low while for $I_c \gg 100$ the gas outflow from the contactor is high enough to render such devices impractical for long term usage in space. In addition for $f_i = 0.01$ and for $I_c > 100$ the central neutral density is so high that ion–neutral friction cannot be neglected in the ion momentum equation. For these and all calculations the ambient density was $n_{ambient} = 2 \times 10^{11}\ m^{-3}$ and the ion temperature was taken as 0.1 eV. In Table 1 we study an Argon plasma contactor cloud which is initially almost totally ionized. The current and potential for the outer shell are those that were calculated as the initial conditions for the state equations in the last section. The core cloud typically reaches a radius of the order of a few meters and therefore allows an parallel electron saturation current of several milliamperes to flow into the current collector at the center. We see that the emitted ion current crossing the boundary of the outer shell drops slightly with increasing ion current. This is because of electron-ion recombination as the electron temperature drops. The electron current from the outer shell drops markedly as the ion current increases. This is a reflection of the fact that as the density of the plasma in the outer shell increases the density gradient driven piece of the current becomes more effective at balancing the inward flowing mobility driven current and therefore the plasma approaches thermodynamic equilibrium. The potential drop for the three ion currents is mainly determined by the high density core and is in the range of several tens of volts. We note that for the all the ion currents the electron current is being supplied mainly by perpendicular flow across the magnetic field. This was observed experimently by Stenzel[8]. From Table 1 we can compute the gain $I/I_c(r_0)$ and see that small ion current contactors have a much larger gain than large ion current contactors. This suggests that there is an advantage in terms of efficiency to using several small contactors in series to replace one large contactor. In Table 2 we consider an Argon plasma which is initially one percent ionized. For small ion currents the system looks like the fully ionized case in Table 1. For large ion currents there is significant ionization of the ejected neutral gas that occurs in the body of the plasma cloud. This is reflected in the potential drop which increases as the initial ionization decreases. The gain for the large ion current contactor case is then much larger than the small ion current case.

From these results we can conclude that if ionization is ignored then the current that can be collected from the far field, even taking it to be driven by turbulence, is small compared to the ion contactor current emitted. This is basically because the ambient ionosphere has too low an electron density to supply a significant current unaided. By contrast if ionization of contactor gas is included then a substantial enhancement of the plasma current is possible. We also note from thses results that most of the current gain due to ionization occurs in the high density core. This suggests that inclusion of the more complex two and three dimensional effects will give only slight improvements on the results from a radial one dimensional model.

5 Discussion and Conclusions

We have developed the theory of the physics of plasma contactor clouds used for electron collection in space. Such clouds have been shown to consist of three regions, a core region, a transistion region and an outer shell. Simple equations were developed for the core region and outer shell. The plasma structure in the transition regime was modelled approximately. The current voltage characteristic of the plasma cloud for a range of ion currents was estimated.

This work indicates that ionization of neutral gas will be the major source of electrons for the current collected to the anode of a plasma contactor. The collection of electrons from the far field for ambient conditions will not give substantial increases in the gain of a contactor.

This work indicates two major conclusions of significance for engineering use of high current contactor clouds. First the biggest current enhancement was obtained with high ion current contactors emitting a weakly ionized gas cloud. This implies that a single tether with a one high ion current contactor will be more efficient than a set of smaller contactors. However most of the gain occurs as a result of ionization of the neutral gas emitted from the contactor itself which indicates a large mass flow rate for the highest ion current cases. This suggests the second conclusion, that if system considerations preclude high mass flow rates and we wish to use a contactor which works by collecting electrons from the far field then the smallest ion contactors are the most efficient.This suggests that an array of electrodyamic tethers spaced far enough apart so that their contactor clouds would not interact would have a much larger gain than one large ion current contactor used on a single tether. This can be illustrated by a simple calculation based on Table 1. A single electrodyanmic tether using a 10 Ampere contactor would allow an 10.2 Ampere current through the tether. An array of ten tethers electrically in series each with a one ampere contactor would enable a 12.1 Ampere current to flow though the tethers.An array of multiple tethers is also more desirable for use in space for reasons of reliability[1].

Future work will concentrate on a more careful treatment of the transition zone. This is important to understand because if the potential drop in this region is too high then operation of electrodynamic tethers will be adversely affected.

6 Acknowledgements

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A Analysis of the Ion Momentum equation

In a spherical geometry the ion momentum equation with spherical expansion for the neutrals is

$$v_i \frac{dv_i}{dr} = -\frac{T_i}{m_i}\frac{d\ln n_i}{dr} - \frac{e}{m_i}\frac{d\phi}{dr} - \nu_{in}(r_0)\left(\frac{r_0}{r}\right)^2 (v_i - v_n) \tag{48}$$

where $\nu_{in}(r_0)$ is the ion–neutral collision frequency evaluated at the center. We shall assume that $n_i = n_e$ which is a very good assumption since we are concerned with the high density core where the ambient ion density can be neglected. If the current driven term in Eq. (3) is small and since the numerical calculations show that $\partial T_e/\partial r \ll T_e/n_e \partial n_e/\partial r$ we write

$$e\frac{\partial \phi}{\partial r} = T_e \frac{\partial \ln n_e}{\partial r}. \tag{49}$$

If we substitute Eqs. (48) into Eq. (49) and take from continuity(ignoring ionization and recombination)

$$\frac{d\ln n_i}{dr} = -\frac{2}{r} - \frac{1}{v_i}\frac{dv_i}{dr} \tag{50}$$

then we obtain

$$(u - \frac{1}{u})\frac{du}{d\eta} = \frac{\eta_0^2}{\eta^2}u + \frac{2}{\eta} + \frac{\eta_0^2}{\eta^2}u_n \tag{51}$$

where $u = v_i/c_s$, $\eta = \nu_{in}(r_0)r/c_s$, $\eta_0 = \nu_{in}(r_0)r_0/c_s$, $u_n = v_n/c_s$ and we have used the more general definition of the ion acoustic velocity as $c_s^2 = (T_e + T_i)/m_i$. Since from Eq. (50)we have

$$n_i = \frac{I}{4\pi r^2 e v_i} \tag{52}$$

we require $u \sim 1/\eta^2$ for $\eta \gg 1$ in order that the density be regular in the far field. It is easy to show that the solution to Eq. (51) which satisfies this is for $\eta \gg 1$,

$$u = \frac{3c}{\eta_0^2 + c\eta^3} \tag{53}$$

where c is an arbitrary constant. Hence we see that in the far field the solution of the full radial momentum equation is deretmines by the neutral collisions. An examination of Eq. (51) indicates that the equation is singular along the line $u = 1$(i.e., the flow is just sonic) except for the point $\eta = \eta_1$, $u = 1$ where

$$\frac{\eta_1}{\eta_0} = \frac{\eta_0}{2}(1 - u_n). \tag{54}$$

The point $\eta = \eta_1$ is the point at which the left and right hand sides of Eq. (51) vanish. If Eq. (51) is wriiten in the form

$$\frac{du}{d\eta} = \frac{\frac{\eta_0^2}{\eta^2}u + \frac{2}{\eta} + \frac{\eta_0^2}{\eta^2}u_n}{u - 1/u} = \frac{f(\eta, u)}{u - 1/u} \tag{55}$$

and if $u_n < 1$(which is reasonable since $T_e/T_i \gg 1$) then $f(\eta, u) = 0$ along the line

$$\eta(u) = \frac{\eta_0^2}{2}(u - u_n). \tag{56}$$

The line $\eta(u)$ divides the (u, η) space into increasing and dcresing trajectories. For $\eta_1 > \eta_0$ the u, η space will look as in Fig 2 where a typical trajectory is sgown. For ions which are emitted with velocity just above the ion acoustic velocity the ion–neutral friction slows them down to sonic speed. Since the sonic line is singular an ion shock forms which connects the supersonic and subsonic trajectories. If the radial distance given by η_1 is inside the core region then this solution is reasonable. For $\eta_1 > \eta_0$ the u, η space will look as in Fig 3. For this case a trajectory which starts slightly supersonically can accelerate because the ion neutral friction never gets high enough to affect it. The condition on whether the ions can accelerate supersonically is $\eta_1 < \eta_0$ which gives

$$\frac{\nu_{in}(r_0)}{2c_s}(1 - \frac{v_n}{c_s}) < 1. \tag{57}$$

For $r_0 = 0.1$ meters, an Argon plasma with $T_e(r_0) = 5$ eV and $T_i = 0.1$ eV and an ion–neutral cross section of 10^{-19} m^2 this gives $n_n(r_0) < 10^{20}$ m^{-3}. hence as long as the initial neutral density is below this the ions can be taken as accelerating supersonically and there is no possibility of an ion shock.

Figure Captions

Figure 1 Possible plasma contactor cloud structure on end of electrodynamic tether

Figure 2 u versus η for $\eta_1 > \eta_0$

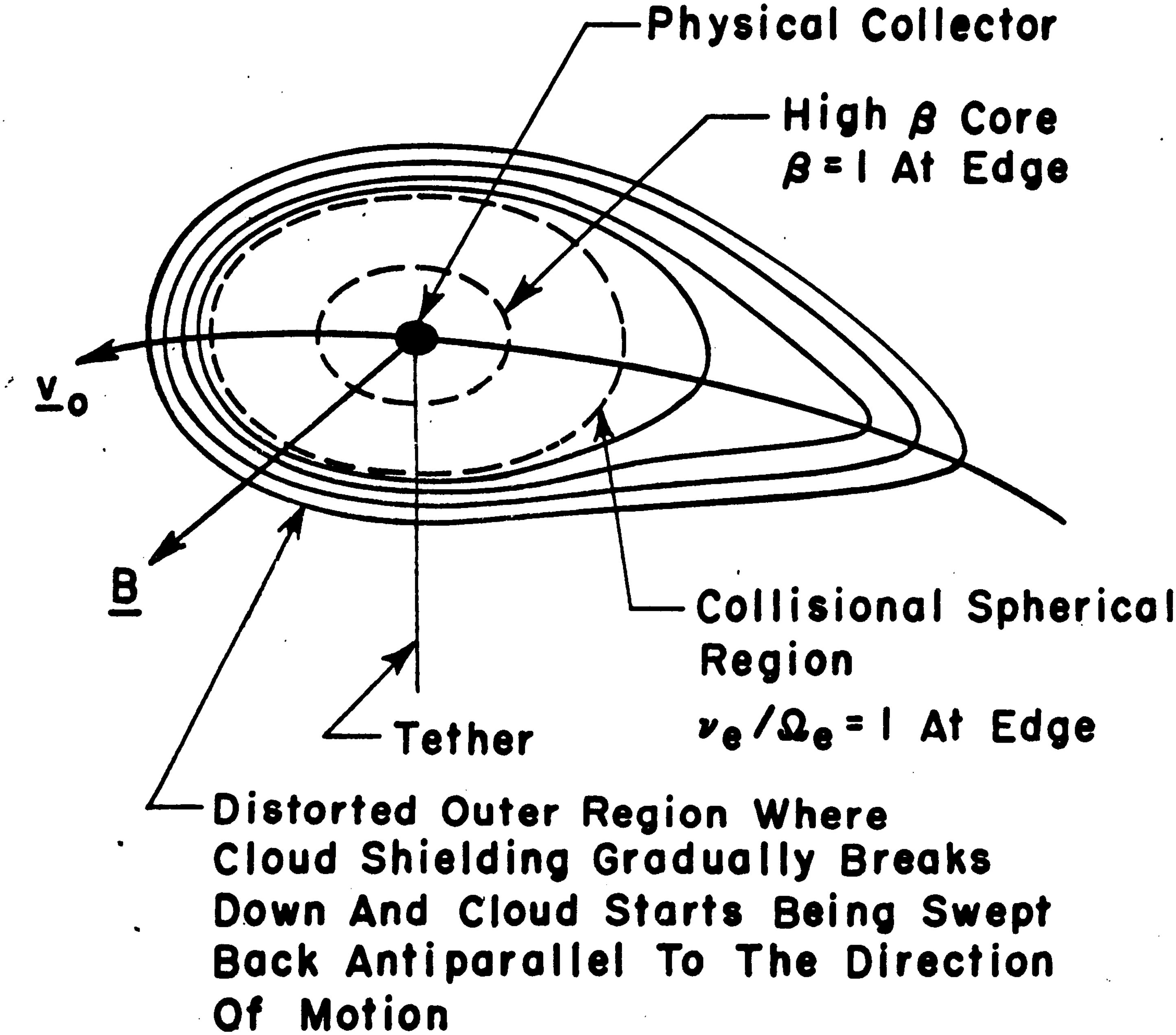
Figure 3 u versus η for $\eta_1 < \eta_0$

$I_c(r_0)$(A)	$I_c(r_{core})$(A)	$I_\parallel$(A)	r_{core}(m)	$I_\perp^{outer}$(A)	I(A)	$\Delta\phi_{core}$(V)	$\Delta\phi_{outer}$(V)	ϕ_0
1	1	1.31×10^{-3}	0.35	0.21	1.21	16.5	1.17	17.67
10	9.99	8.63×10^{-3}	0.95	0.2	10.2	27.13	1.41	28.54
100	99.9	7×10^{-2}	4.8	7.4×10^{-2}	100.04	39.15	1.16	40.31

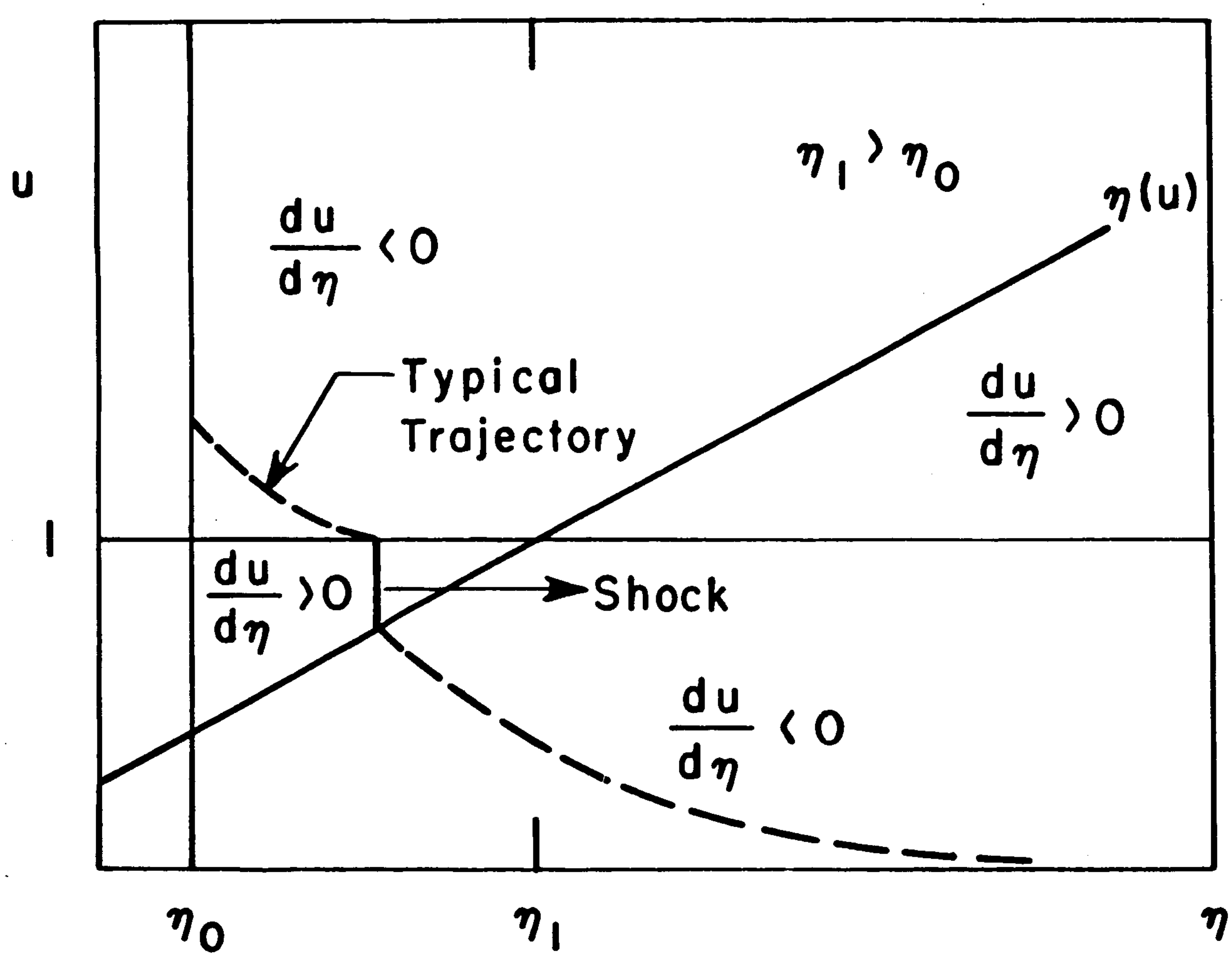
Table 1: Current and voltage against ion currents, r_0=0.1m, $T_e(r_0)=$ 5 eV, f_i=0.99, $n_{ambient} = 2 \times 10^{11}$ m^{-3}, T_i=0.1 eV, Argon

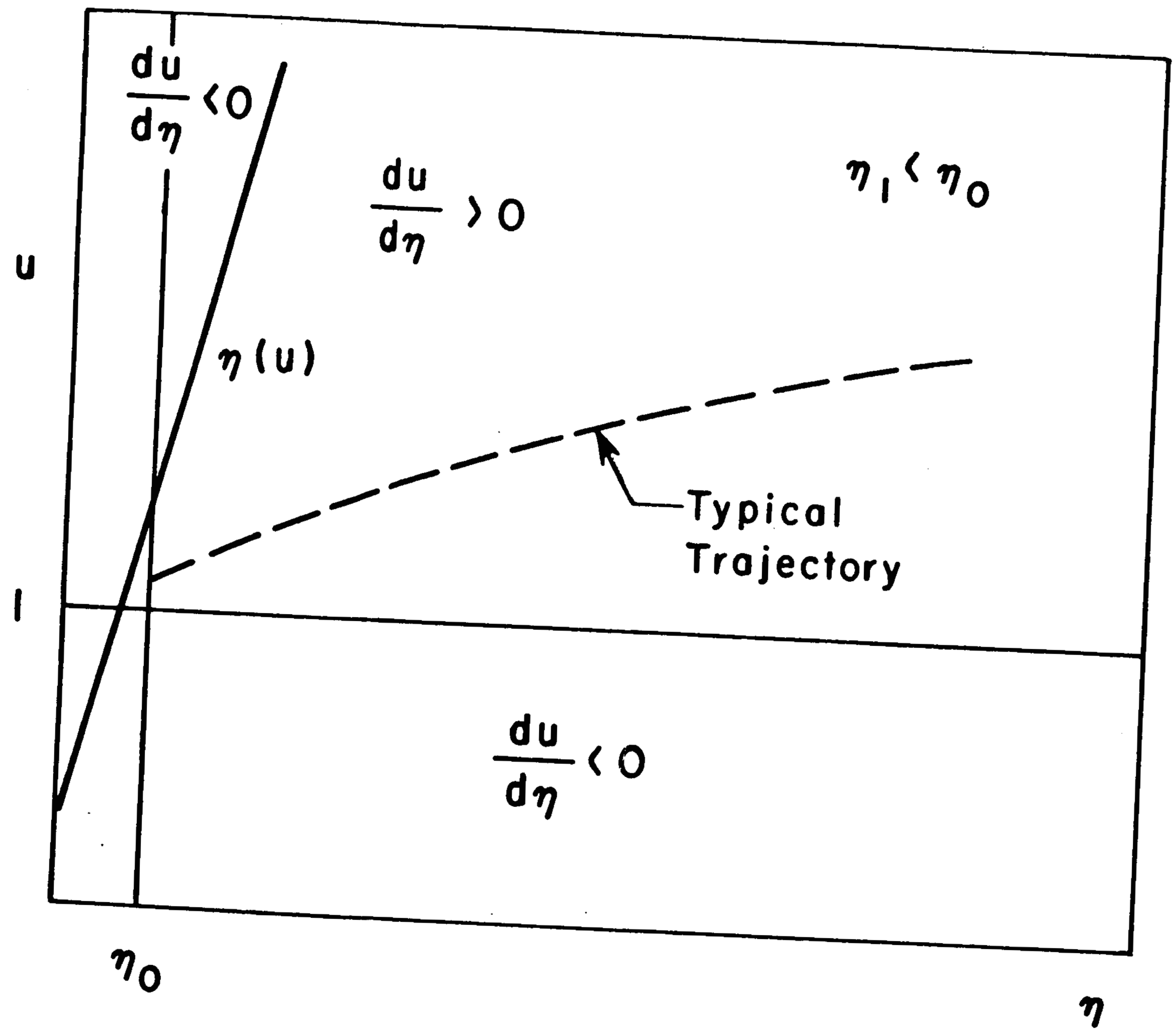
$I_c(r_0)$(A)	$I_c(r_{core})$(A)	$I_\parallel$(A)	r_{core}(m)	$I_\perp^{outer}$(A)	I(A)	$\Delta\phi_{core}$(V)	$\Delta\phi_{outer}$(V)	ϕ_0
1	1.008	1.31×10^{-3}	0.35	0.21	1.22	16.54	1.17	17.71
10	10.81	9.64×10^{-3}	1.0	0.16	10.98	27.45	1.26	28.71
100	219	0.245	8.98	4.8×10^{-2}	219.3	48.1	1.9	50.

Table 2: Current and voltage against ion currents, r_0=0.1m, $T_e(r_0)=$ 5 eV, f=0.01, $n_{ambient} = 2 \times 10^{11}$ m^{-3}, T_i=0.1 eV, Argon



Physical Collector
High β Core
$\beta = 1$ At Edge
$\underline{v}_o$
$\underline{B}$
Collisional Spherical Region
$\nu_e / \Omega_e = 1$ At Edge
Tether
Distorted Outer Region Where Cloud Shielding Gradually Breaks Down And Cloud Starts Being Swept Back Antiparallel To The Direction Of Motion


u
1
$\eta_1 > \eta_0$
$\frac{du}{d\eta} < 0$
Typical Trajectory
$\eta(u)$
$\frac{du}{d\eta} > 0$
$\frac{du}{d\eta} > 0$
Shock
$\frac{du}{d\eta} < 0$
η_0
η_1
η



$\frac{du}{d\eta} < 0$
$\frac{du}{d\eta} > 0$
$\eta_1 < \eta_0$
u
$\eta(u)$
Typical Trajectory
1
$\frac{du}{d\eta} < 0$
η_0
η