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Stress Intensity and Crack Displacement for Small Edge Cracks

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Summary

The weight function method was used to derive stress intensity factors and crack mouth displacement coefficients for small edge cracks in fracture specimen geometries. Of greatest interest were cracks whose lengths were less than 20 percent of the specimen width. The effects of contact stresses due to point application of loads in bend testing were examined. The results are compared with available solutions and equations from the literature and with unpublished boundary collocation results.

Introduction

The purpose of this effort was to support a research study on the fatigue and fracture behavior of small cracks. Wide-range solutions for stress intensity and crack mouth displacement in edge-crack configurations are available. A "wide-range" solution is one that is considered valid for all possible values of the ratio of crack length to specimen width, that is, from zero to unity. These are typically produced by fitting curves from analytic solutions for cracks in semi-infinite bodies through numerical results for cracks in finite bodies. The accuracy depends upon the numerical results used. However, few numerical results for small cracks are published, and those are not supported by results from alternative methods.

Bend specimens have practical advantages in experimental studies of small cracks. First, a fatigue crack may be initiated from a notch at a fairly high load. Next, the cyclic load is reduced, similar to that in a fatigue-crack threshold test, to produce a truly sharp crack. Then, by machining only one edge of the specimen, most evidence of the prior load history may be removed. Although this appears to be a simple way of producing a small, sharp crack, Timoshenko (ref. 1) has noted the presence of additional stresses due to contact forces in bend testing. It is essential to determine just what influence these stresses have on the stress intensity factor and the crack mouth displacement for small cracks.

In this report, Seewald's analysis (ref. 2) of the effect of contact stress was applied to the bend specimen configurations of interest. The stress correction terms were evaluated for three-point and four-point bending. The weight function methods of Bueckner (ref. 3) and Rice (ref. 4) were then used to determine the stress intensity and crack mouth displacement coefficients from the corrected stresses. Finally, results were compared with available numerical solutions and wide-range interpolation equations.

Symbols

A_n	polynomial coefficient
a	crack length
B	beam thickness
E	modulus of elasticity for plane stress
E'	effective modulus
h	half-width of beam
K_I	opening-mode stress intensity factor
M	bending moment (fig. 2)
$M(t)$	weight function
m_1, m_2	coefficients (eq. (4))
P	applied load (figs. 1 and 2)
$p(t)$	crack face pressure
Q	multiplicative term
t	coordinate measured from crack tip toward cracked surface
W	specimen width
v	crack mouth displacement
z	dummy variable
ν	Poisson's ratio
ρ	applied load (figs. 1 and 2)
σ_x	stress normal to load line
$\sigma_{x,0}$	nominal stress normal to load line
σ_x'	stress correction term

Method of Analysis

Contact Stresses in Beams

Timoshenko (ref. 1) notes that when a beam is loaded by a concentrated force the stresses are not exactly as given by elementary beam theory. Specifically, the stresses at and normal to the load line are smaller near the surface opposite the load. He presents curves and a few numerical values from

Seewald (ref. 2) to illustrate this trend. These suggest that contact stresses might affect the stress intensity factor and the crack mouth displacement for small cracks.

Seewald (ref. 2) writes the stress normal to the load line as the sum of two terms,

$$\sigma_x = \sigma_{x,0} + \sigma_x'$$

where $\sigma_{x,0}$ is calculated by elementary beam theory and σ_x' is the stress correction term. His notation and conventions are shown in figure 1. The stress correction term σ_x' is given by

$$\sigma_x' = \frac{2P}{B\pi W}\int_0^{\infty}\left[\frac{(z\cosh z - \sinh z)\cosh\left(\frac{y}{h}\right)z - \left(\frac{y}{h}\right)z\sinh z\sinh\left(\frac{y}{h}\right)z}{\sinh 2z - 2z}\right]\cos\left(\frac{x}{h}\right)z\,dz$$

$$+\frac{2P}{B\pi W}\int_0^{\infty}\left[\frac{(z\sinh z - \cosh z)\sinh\left(\frac{y}{h}\right)z - \left(\frac{y}{h}\right)z\cosh z\cosh\left(\frac{y}{h}\right)z}{\sinh 2z - 2z} + \frac{12\frac{y}{h}}{8z^2}\right]\cos\left(\frac{x}{h}\right)z\,dz \quad (1)$$

where B is the beam thickness and z may be regarded as a dummy variable. The integral was evaluated in piecewise fashion as follows.

For $0 \le z \le 0.1$, the hyperbolic functions were replaced by the first two terms of their series expansions; that is,

$$\sinh z = z + \frac{z^3}{3!} + \frac{z^5}{5!} + \ldots \qquad \cosh z = 1 + \frac{z^2}{2!} + \frac{z^4}{4!} + \ldots$$

(These approximations are accurate to at least five significant figures.) The resulting equation could then be integrated analytically, term by term.

For $0.1 \le z \le 5$, the hyperbolic functions were replaced by their exponential equivalents

$$\sinh z = \frac{e^z - e^{-z}}{2} \qquad \cosh z = \frac{e^z + e^{-z}}{2}$$

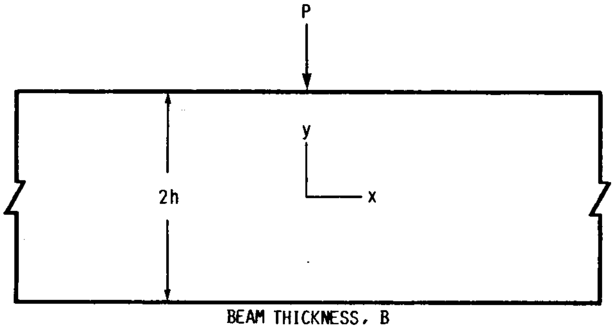


Figure 1.—Notation and conventions of Seewald (ref. 2).

and the integral was evaluated numerically according to Simpson's rule.

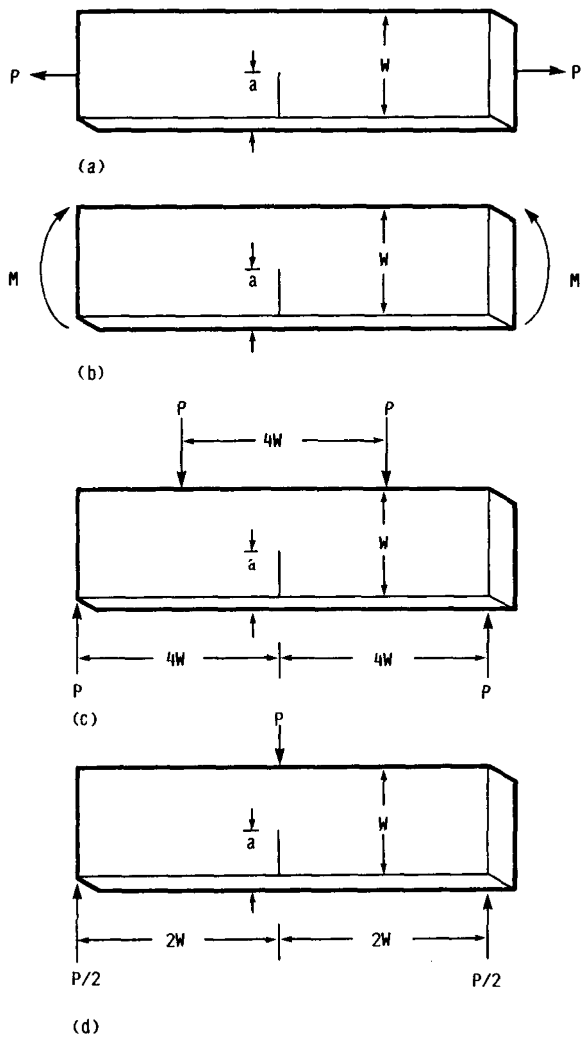
Finally, for $z \ge 5$, the large-argument approximation

$$\sinh z \simeq \cosh z \simeq \frac{e^z}{2}$$

was invoked (the error is less than 0.5 percent for $z \ge 5$). The result could again be integrated analytically, term by term. The piecewise integrals were summed to give the total integral.

Figure 2 shows the configurations of the specimens analyzed. For the three-point-bend specimen, the integral was evaluated in the crack plane (i.e., at $x = 0$). For the four-point-bend specimen, there are two forces located at a distance $x = \pm 2W = \pm 4h$ from the crack plane. By symmetry, the effect of each load is identical at the crack plane. By superposition, the effects may be added. Thus, the stress correction term is twice the value of the integral evaluated at $x/h = 4$. The discrete calculations are given in table I and plotted in figure 3. Note that the dimensionless stress from elementary beam theory is ± 6 at the surfaces ($y/h = \pm 1$). The stress correction term for three-point bending appears significant, but that for four-point bending is quite small and is later shown to be insignificant. The reaction forces at the ends of the specimens (which were at least as far from the crack plane as the four-point-bend loading forces) were disregarded.

To use these discrete values in the weight function analysis to follow, simple equations were fit to them. A bicubic spline was used for three-point bending while a simple parabola was sufficient for four-point bending. The equations for the dimensionless stress correction terms in the crack plane ($x = 0$) are



(a) Uniform tension.
(b) Pure bending.
(c) Three-point bending.
(d) Four-point bending.

Figure 2.—Configurations analyzed.

$$\sigma_x' \frac{BW}{P} = 0.2466 + 1.2566\left(\frac{y}{h}\right) + 0.1338\left(\frac{y}{h}\right)^2 - 0.6092\left(\frac{y}{h}\right)^3 \qquad \text{for } \frac{y}{h} \le 0 \tag{2a}$$

$$\sigma_x' \frac{BW}{P} = 0.2466 + 1.2566\left(\frac{y}{h}\right) + 0.1338\left(\frac{y}{h}\right)^2 - 0.3956\left(\frac{y}{h}\right)^3 \qquad \text{for } \frac{y}{h} \ge 0 \tag{2b}$$

for three-point bending and

TABLE I.—CONTACT STRESS CORRECTION TERM, $\sigma_x' BW/P$ IN THE PLANE $x = 0$

[See fig. 1 for notation.]

Distance, y/h	Three-point bending			Four-point bending	
	Calculated	Fitted spline	Seewald (ref. 2)	Calculated	Fitted parabola
1.00	1.2400	1.2414	1.232	−0.03696	−0.03572
.95	1.2214	1.2220	------	−.03508	−.03478
.90	1.1974	1.1974	------	−.03310	−.03284
.85	1.1684	1.1684	------	−.03108	−.03090
.80	1.1348	1.1350	------	−.02908	−.02898
.70	1.0632	1.0560	------	−.02514	−.02516
.60	.9620	.9632	------	−.02134	−.02136
.50	.8572	.8588	.856	−.01760	−.01760
.40	.7426	.7454	------	−.01394	−.01386
.30	.6234	.6250	------	−.01028	−.01016
.20	.4992	.5002	------	−.00664	−.00648
.10	.3736	.3732	------	−.00298	−.00284
0	.2486	.2466	.242	.00066	.00078
−.10	.1272	.1228	------	.00430	.00436
−.20	.0118	.0056	------	.00794	.00792
−.30	−.0948	−.1018	------	.01154	.01144
−.40	−.1892	−.1956	------	.01510	.01494
−.50	−.2680	−.2722	−.272	.01860	.01840
−.60	−.3272	−.3276	------	.02204	.02184
−.70	−.3544	−.3586	------	.02544	.02524
−.80	−.3678	−.3612	------	.02878	.02862
−.85	−.3574	−.3508	------	.03044	.03030
−.90	−.3730	−.3318	------	.03210	.03196
−.93	−.3196	−.3164	------	.03308	.03296
−.95	−.3056	−.3042	------	.03376	.03362
−.97	−.2896	−.2904	------	.03442	.03428
−.98	−.2810	−.2830	------	.03476	.03462
−.99	−.2716	−.2752	------	.03510	.03494
−1.00	−.2618	−.2670	−.2664	.03544	.03528

$$\sigma_x' \frac{BW}{P} = 0.00078 - 0.036\left(\frac{y}{h}\right) - 0.0015\left(\frac{y}{h}\right)^2 \tag{2c}$$

for four-point bending. All fitted equations are within 0.5 percent of the range of σ_x'.

Weight Function Analysis

Bueckner (ref. 3) gives the stress intensity for a general pressure distribution over the face of a crack as

$$K_I(a) = \sqrt{\frac{2}{\pi}} \int_0^a M(t,a)\, p(t)\, dt \tag{3}$$

where $M(t)$ is the weight function, a is the crack length, $p(t)$ is the crack face pressure, and t is the coordinate measured from the crack tip toward the cracked surface. For the case of an edge crack in a strip of unit width, Bueckner approximates the weight function as

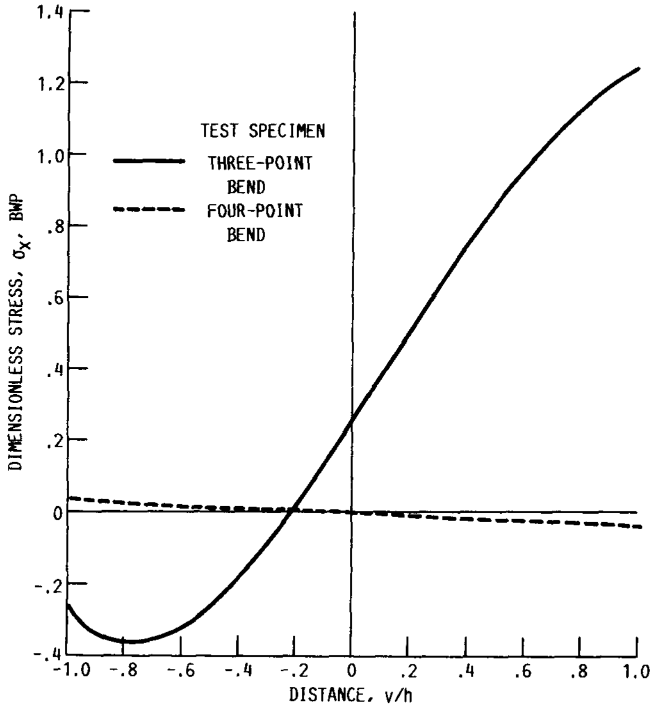


Figure 3.—Stress correction terms for concentrated loading at $x = 0$.

$$M(t,a) \simeq t^{-1/2}\left[1 + m_1 \cdot \left(\frac{t}{a}\right) + m_2 \cdot \left(\frac{t}{a}\right)^2\right] \qquad (4)$$

where

$$m_1 = 0.6147 + 17.1844\left(\frac{a}{W}\right)^2 + 8.7822\left(\frac{a}{W}\right)^6$$

and

$$m_2 = 0.2502 + 3.2889\left(\frac{a}{W}\right)^2 + 70.0444\left(\frac{a}{W}\right)^6$$

and claims an accuracy of 1 percent for $a/W \leq 0.5$.

Rice (ref. 4) extends the weight function method to enable the calculation of crack mouth displacements from the weight function and the stress intensity factor. According to his work, the crack mouth displacement is

$$v = \frac{2}{E'}\sqrt{\frac{1}{2\pi}} \int_0^a K_I(a)M(t,a)\bigg|_{t=a} da \qquad (5)$$

where E' is the effective modulus (modulus of elasticity E for plane stress or $E/(1 - \nu^2)$ for plane strain with ν being Poisson's ratio).

To compute the stress intensity factor, I added the stress from elementary beam theory (evaluated at the crack plane) to the Seewald stress correction term (eq. (1)) and substituted for $p(t)$ in equation (3). Then I computed the crack mouth displacement by substituting the stress intensity factor and the weight function into equation (5). Since both the stress distribution and Bueckner's expression for the weight function are power series, I integrated equations (3) and (4) directly, term by term. The resulting equations, however, were quite tedious.

The stress distributions involved terms up to the third power in a/W (depending on the type of loading). The weight function had terms up to the sixth power. When these were multiplied, the final polynomial for the stress intensity factor had terms to the ninth power in a/W. Multiplying the stress intensity factor by the weight function to calculate the crack mouth displacement gave terms to the 15th power. Because such polynomials are too long for practical use, I reduced them in degree by using Chebyshev economization (ref. 5). To do this, I wrote a program on a NASA mainframe computer by using internally developed subroutines. The exact and economized polynomials agreed within less than 0.33 percent over the range $0 \leq a/W \leq 0.5$. Table II lists the coefficients of the polynomials, which are expressed in the general form

$$P = Q\sum\left[A_n\left(\frac{a}{W}\right)^n\right]$$

Here P is the parameter to be calculated, Q is a multiplicative term, and A_n is the polynomial coefficient.

Results and Discussion

The cases of uniform tension and pure bending were examined first. This was done to verify the application of the weight function method. Also, unpublished boundary collocation results for small cracks obtained by Bernard Gross of NASA Lewis were evaluated. The term $p(t)$ in equation (3) was given a constant value for uniform tension or a linearly varying value for pure bending. Table III lists the values of the stress intensity factor and crack mouth displacement coefficients. Values of wide-range polynomials from the literature and discrete values from numerical analyses are also listed.

Figure 4 shows the case of uniform tension. Over the range $0 \leq a/W \leq 0.5$, the weight function stress intensity and displacement coefficients are within 1.1 and 2.4 percent, respectively, of those from the wide-range equations by Tada (ref. 6) and Koiter (ref. 7). Larger differences in the displacement coefficients were expected, since they depend on the square of the weight function. Numerical results by Keer (ref. 8), using an integral equation method, are within 1 percent for a/W values as low as 0.05. Published (ref. 9) and

TABLE II.—EXACT AND ECONOMIZED POLYNOMIALS FOR STRESS INTENSITY FACTOR AND CRACK MOUTH DISPLACEMENT

[Coefficients of polynomials in form $P = Q \sum [A_n(a/W)^n]$].

(a) Coefficients for stress intensity factor

Polynomial coefficient	Uniform tension		Pure bending		Four-point bending		Three-point bending	
	Parameter to be calculated, P							
	$KBW/P\sqrt{\pi a}$		$KBW^2/6M\sqrt{\pi a}$		$KBW/12P\sqrt{\pi a}$		$KBW/6P\sqrt{\pi a}$	
	Multiplicative term, Q							
	$\sqrt{2}/\pi$		$\sqrt{2}/6\pi$		$\sqrt{2}/6\pi$		$\sqrt{2}/6\pi$	
	Exact	Economized	Exact	Economized	Exact	Economized	Exact	Economized
A_0	2.5099	2.5127	15.059	15.067	15.104	15.112	14.389	14.396
A_1	0	−.27290	−18.310	−19.085	−18.360	−19.138	−20.868	−21.542
A_2	12.772	16.865	76.631	88.778	76.853	89.039	82.423	92.931
A_3	0	−20.376	−59.501	−123.83	−59.664	−124.20	−72.610	−127.82
A_4	0	34.931	0	121.31	−.008568	121.70	21.854	123.77
A_5	0	-------	0	-------	0	-------	−8.9518	-------
A_6	33.873	-------	203.24	-------	203.83	-------	194.19	-------
A_7	-------	-------	−124.16	-------	−124.51	-------	−141.52	-------
A_8	-------	-------	-------	-------	−.014688	-------	38.412	-------
A_9	-------	-------	-------	-------	-------	-------	−13.806	-------

(b) Coefficients for crack mouth displacement

Polynomial coefficient	Uniform tension		Pure bending		Four-point bending		Three-point bending	
	Parameter to be calculated, P							
	$E'BWv/Pa$		$E'BW^2v/6Ma$		$E'BWv/12Pa$		$E'BWv/6Pa$	
	Multiplicative term, Q							
	$2/\pi$		$1/3\pi$		$1/3\pi$		$1/3\pi$	
	Exact	Economized	Exact	Economized	Exact	Economized	Exact	Economized
A_0	4.6807	4.6956	28.084	28.117	28.167	28.200	26.834	26.865
A_1	0	−1.3652	−17.073	−20.024	−17.120	−20.081	−19.459	−22.241
A_2	20.5068	44.378	150.41	190.73	150.85	191.30	149.43	187.10
A_3	0	−88.193	−121.46	−291.20	−121.79	−292.09	−140.66	−295.56
A_4	52.296	183.69	313.78	494.16	314.68	495.69	345.65	498.53
A_5	0	-------	−203.03	-------	−203.59	-------	−250.54	-------
A_6	37.288	-------	223.73	-------	224.36	-------	277.69	-------
A_7	0	-------	−209.35	-------	−209.93	-------	−261.52	-------
A_8	188.92	-------	1133.5	-------	1136.8	-------	1171.6	-------
A_9	0	-------	723.23	-------	−725.22	-------	−864.67	-------
A_{10}	0	-------	0	-------	.087223	-------	228.10	-------
A_{11}	0	-------	0	-------	0	-------	−82.358	-------
A_{12}	205.39	-------	1.2323	-------	1236.0	-------	1177.5	-------
A_{13}	-------	-------	−6.9910	-------	−701.02	-------	−796.80	-------
A_{14}	-------	-------	-------	-------	−.077189	-------	201.86	-------
A_{15}	-------	-------	-------	-------	-------	-------	−68.017	-------

TABLE III.–DIMENSIONLESS STRESS INTENSITY FACTOR AND CRACK MOUTH DISPLACEMENT COEFFICIENTS

(a) Stress intensity factor coefficients

Relative crack length, a/W	Uniform tension, $KBW/P\sqrt{\pi a}$					Pure bending, $KBW^2/6M\sqrt{\pi a}$				Four-point bending, by weight function, $KBW/12P\sqrt{\pi a}$	Three-point bending, $KBW/6P\sqrt{\pi a}$				
	Weight function	Tada (ref. 6)	Koiter (ref. 7)	Gross (ref. 9)	Keer (ref. 8)	Weight function	Tada (ref. 6)	Koiter (ref. 7)	Gross (ref. 9)		Weight function	Tada (ref. 6)	Nisitani (ref. 10)	ASTM (ref. 11)	Gross (ref. 12)
0	1.1311	1.1220	1.1218	-----	-----	1.1304	1.1220	1.1215	-----	1.1338	1.0801	1.0900	1.0758	1.1227	-----
.025	1.1327	1.1218	1.1272	-----	-----	1.0987	1.0937	1.0916	-----	1.1019	1.0439	1.0515	1.0431	1.0827	-----
.050	1.1429	1.1473	1.1418	[a].166	1.140	1.0744	1.0709	1.0691	[a]1.111	1.0776	1.0156	1.0221	1.0169	1.0507	[a]1.060
.075	1.1613	1.1686	1.1643	1.158	-----	1.0569	1.0534	1.0528	[a]1.062	1.0600	.9943	1.0005	.9970	1.0258	[a]1.003
.100	1.1872	1.1957	1.1938	1.190	1.189	1.0455	1.0408	1.0419	1.047	1.0486	.9795	.9858	.9829	1.0074	[a].984
.125	1.2203	1.2289	1.2298	-----	-----	1.0396	1.0329	1.0358	-----	1.0427	.9705	.9771	.9741	.9948	-----
.150	1.2605	1.2682	1.2722	1.268	1.265	1.0388	1.0295	1.0341	1.044	1.0419	.9669	.9738	.9704	.9876	[a].974
.175	1.3077	1.3140	1.3209	-----	-----	1.0426	1.0304	1.0365	-----	1.0457	.9681	.9752	.9715	.9854	-----
.200	1.3620	1.3667	1.3761	1.370	1.367	1.0507	1.0355	1.0429	1.058	1.0539	.9739	.9809	.9769	.9881	.981
.225	1.4237	1.4265	1.4381	-----	-----	1.0630	1.0448	1.0531	-----	1.0661	.9840	.9906	.9866	.9954	-----
.250	1.4930	1.4941	1.5074	1.500	-----	1.0792	1.0582	1.0671	1.084	1.0824	.9982	1.0041	1.0003	1.0073	-----
.275	1.5707	1.5700	1.5845	-----	-----	1.0992	1.0758	1.0850	-----	1.1025	1.0166	1.0214	1.0180	1.0237	-----
.300	1.6573	1.6551	1.6703	1.662	1.660	1.1232	1.0978	1.1070	1.123	1.1266	1.0390	1.0427	1.0396	1.0447	1.043
.325	1.7537	1.7502	1.7656	-----	-----	1.1512	1.1243	1.1333	-----	1.1547	1.0656	1.0680	1.0653	1.0704	-----
.350	1.8608	1.8565	1.8716	1.862	-----	1.1835	1.1557	1.1643	1.184	1.1871	1.0967	1.0979	1.0953	1.1010	-----
.375	1.9799	1.9752	1.9897	-----	-----	1.2202	1.1922	1.2002	-----	1.2239	1.1324	1.1329	1.1300	1.1370	-----
.400	2.1122	2.1080	2.1214	2.110	2.112	1.2618	1.2345	1.2417	1.256	1.2657	1.1731	1.1735	1.1698	1.1786	1.175
.425	2.2591	2.2568	2.2689	-----	-----	1.3087	1.2830	1.2895	-----	1.3127	1.2193	1.2206	1.2155	1.2265	-----
.450	2.4222	2.4241	2.4345	2.420	-----	1.3615	1.3386	1.3442	1.350	1.3657	1.2715	1.2751	1.2680	1.2815	-----
.475	2.6032	2.6128	2.6213	-----	-----	1.4208	1.4023	1.4069	-----	1.4252	1.3304	1.3380	1.3282	1.3443	-----
.500	2.8039	2.8266	2.8328	2.810	2.826	1.4872	1.4752	1.4789	1.497	1.4918	1.3967	1.4106	1.3974	1.4162	1.409

(b) Crack mouth displacement coefficients

Relative crack length, a/W	Uniform tension, $E'BWv/Pa$				Pure bending, $E'BW^2v/6Ma$				Four-point bending, by weight function, $E'BWv/12Pa$	Three-point bending, $E'BWv/6Pa$			
	Weight function	Tada (ref. 6)	Gross (ref. 9)	Keer (ref. 8)	Weight function	Tada (ref. 6)	Gross (ref. 9)	Kapp (ref. 13)		Weight function	Tada (ref. 6)	Gross (ref. 12)	Kapp (ref. 13)
0	2.9893	2.9200	-----	-----	2.9833	2.9200	-----	2.9125	2.9921	2.8505	2.8400	-----	2.9098
.025	2.9844	2.9298	-----	-----	2.9424	2.9066	-----	2.9046	2.9510	2.8034	2.7993	-----	2.8484
.050	3.0102	2.9593	[a]2.820	2.967	2.9241	2.9046	[a]2.840	2.9052	2.9327	2.7785	2.7734	[a]2.733	2.8056
.075	3.0630	3.0090	[a]2.733	-----	2.9264	2.9147	[a]2.707	2.9156	2.9350	2.7736	2.7625	[a]2.589	2.7812
.100	3.1404	3.0796	2.935	3.107	2.9476	2.9376	2.747	2.9370	2.9563	2.7870	2.7669	[a]2.558	2.7753
.125	3.2410	3.1722	-----	-----	2.9864	2.9741	-----	2.9708	2.9952	2.8174	2.7871	-----	2.7880
.150	3.3643	3.2882	[a]3.100	3.336	3.0422	3.0250	[a]2.847	3.0185	3.0512	2.8641	2.8234	[a]2.706	2.8194
.175	3.5112	3.4295	-----	-----	3.1148	3.0914	-----	3.0816	3.1241	2.9270	2.8766	-----	2.8700
.200	3.6835	3.5984	3.625	3.667	3.2046	3.1745	3.156	3.1619	3.2141	3.0064	2.9475	2.898	2.9402
.225	3.8842	3.7977	-----	-----	3.3122	3.2757	-----	3.2612	3.3220	3.1029	3.0371	-----	3.0308
.250	4.1173	4.0310	-----	-----	3.4390	3.3967	-----	3.3816	3.4492	3.2179	3.1467	-----	3.1428
.275	4.3880	4.3024	-----	-----	3.5868	3.5393	-----	3.5254	3.5974	3.3532	3.2778	-----	3.2773
.300	4.7026	4.6171	4.695	4.707	3.7578	3.7059	3.745	3.6952	3.7689	3.5109	3.4323	3.462	3.4361
.325	5.0683	4.9815	-----	-----	3.9547	3.8991	-----	3.8939	3.9665	3.6940	3.6126	-----	3.6212
.350	5.4936	5.4030	-----	-----	4.1808	4.1223	-----	4.1249	4.1933	3.9056	3.8215	-----	3.8351
.375	5.9881	5.8911	-----	-----	4.4400	4.3792	-----	4.3921	4.4532	4.1495	4.0625	-----	4.0811
.400	6.5624	6.4572	6.529	6.549	4.7363	4.6747	4.708	4.7003	4.7504	4.4299	4.3399	4.371	4.3634
.425	7.2283	7.1157	-----	-----	5.0745	5.0144	-----	5.0549	5.0897	4.7517	4.6593	-----	4.6871
.450	7.9984	7.8843	-----	-----	5.4598	5.4056	-----	5.4626	5.4762	5.1200	5.0272	-----	5.0587
.475	8.8868	8.7852	-----	-----	5.8980	5.8571	-----	5.9317	5.9157	5.5405	5.4522	-----	5.4865
.500	9.9086	9.8468	9.880	9.925	6.3952	6.3800	6.366	6.4724	6.4144	6.0195	5.9450	5.943	5.9810

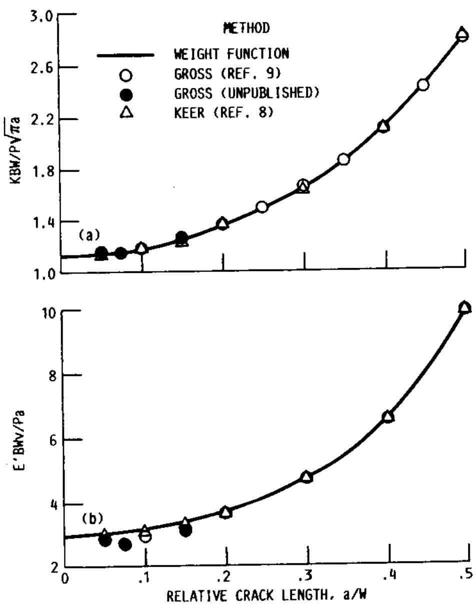
[a]Unpublished data.

unpublished boundary collocation stress intensity factors by Gross are within 2.2 percent at $a/W = 0.05$ and within less than 1 percent for $a/W \geq 0.075$. His displacement coefficients, however, differ by as much as 8.4 percent.

Gross' convergence criterion was based only on the first term of the Williams stress function, which is proportional to the stress intensity factor. Crack displacements, however, are influenced by higher terms as well. Perhaps his displacements would be more accurate if higher-order terms had been included in the convergence criterion.

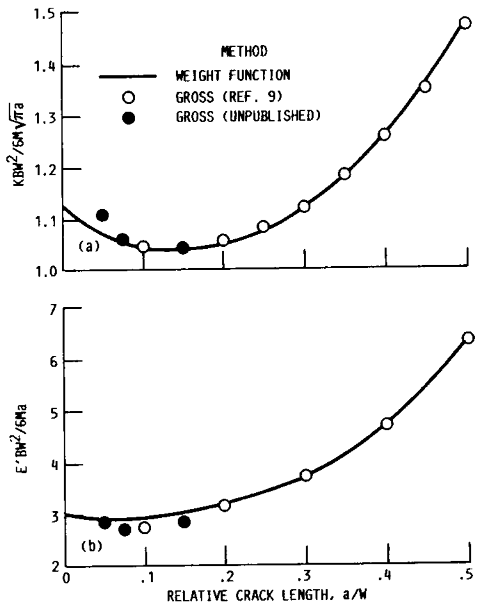
Figure 5 shows similar results for pure bending. Stress intensity factor and crack mouth displacement coefficients derived from the weight function approach are within 2.5 percent of the values from the reference equations. Gross' stress intensity factors are within 4 percent at $a/W = 0.05$ and within 2.6 percent for $a/W \geq 0.5$. Crack mouth displacements show about the same trend as for uniform tension.

The case of four-point bending was analyzed next. The stress correction term of equation (2c) was added to the linear distribution of pure bending. The total stress was substituted for $p(t)$ in equation (3), and the stress intensity factor and crack mouth displacement coefficients were determined as



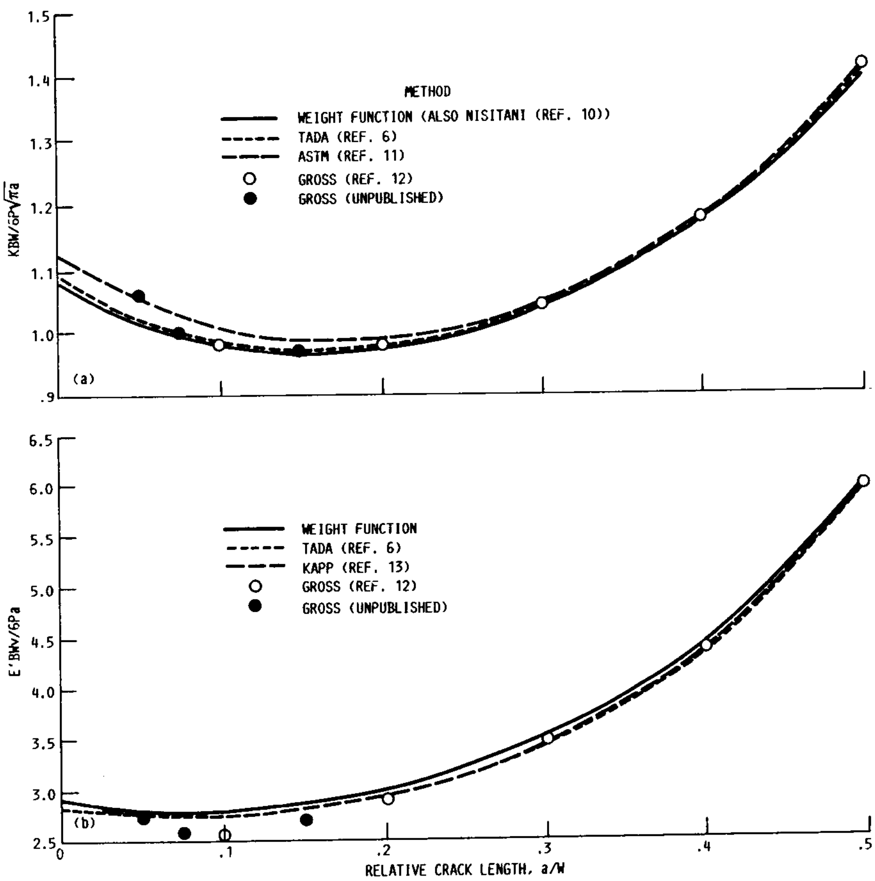
(a) Stress intensity coefficients.
(b) Crack mouth displacement coefficients.

Figure 4.—Coefficients for uniform tension.



(a) Stress intensity coefficients.
(b) Crack mouth displacement coefficients.

Figure 5.—Coefficients for pure bending.



(a) Stress intensity coefficients.
(b) Crack mouth displacement coefficients.

Figure 6.—Coefficients for three-point bending.

before. They are listed in table III but are not plotted. As we might suspect from figure 3, contact stresses in the four-point-bend configuration analyzed had little effect. The stress intensity factor and crack mouth displacement coefficients differed from the case of pure bending by less than 0.33 percent. The reaction loads, which were twice as far from the crack plane as the applied loads, had minimal significance and their effect was disregarded.

The case of three-point bending, however, was more interesting. The stress correction term of equation (2a) was added to the simple bending distribution, and the process was repeated. The results are listed in table III and illustrated in figure 6. There was no analysis for comparison until a paper by Nisitani and Mori (ref. 10) was translated. They used the body force doublet method. Table III also includes their stress intensity results (they did not publish crack mouth displacements). The weight function results agree with theirs within 0.5 percent. Tada's equation (ref. 6) also gives results within 1 percent of the weight function results. Tada's results are also listed in table III and plotted in figure 6. We should note that the 1985 edition of Tada's handbook (ref. 6) repeats the ASTM expression. It would appear, then, that Tada's earlier version might be more accurate. In the limit $a/W = 0$, the results based on the ASTM expression (ref. 11) for stress intensity are nearly 4 percent higher than the weight function results. Gross' collocation results (ref. 12) appear accurate for $a/W \geq 0.075$. For crack mouth displacement, the weight function results and those of Tada (ref. 6) and Kapp (ref. 13) are within 2.5 percent. Gross' collocation results (ref. 8) agree for $a/W \geq 0.2$.

Conclusions

The most important conclusion from this study is that contact stresses influence the stress intensity factor and crack mouth displacement coefficients for small edge cracks in three-point bending. The effect is small but may be significant in small-crack fracture and fatigue-crack propagation studies.

A second conclusion is that collocation values of the stress intensity factor are accurate for $a/W \geq 0.1$ and values of crack mouth displacement are accurate for $a/W \geq 0.2$. A different convergence criterion may be necessary if the collocation method is to be successful for small cracks. It is also evident that the weight function method is useful and effective for crack analysis.

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National Aeronautics and Space Administration
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15. Supplementary Notes

16. Abstract

The weight function method was used to derive stress intensity factors and crack mouth displacement coefficients for small edge cracks (less than 20 percent of the specimen width) in common fracture specimen configurations. Contact stresses due to point application of loads were found to be small but significant for three-point bending and insignificant for four-point bending. The results are compared with available equations and numerical solutions from the literature and with unpublished boundary collocation results.

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