Satellite Interference Analysis and Simulation Using Personal Computers

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ABSTRACT

This report presents the complete analysis and formulas necessary to quantify the interference experienced by a generic satellite communications receiving station due to an interfering satellite. Both satellites, the desired as well as the interfering satellite, are considered to be in elliptical orbits. Formulas are developed for the satellite look angles and the satellite transmit angles generally related to the land mask of the receiving station site for both satellites. Formulas for considering Doppler effect due to the satellite motion as well as the Earth's rotation are developed. The effect of the interfering-satellite signal modulation and the Doppler effect on the power received are considered. The statistical formulation of the interference effect is presented in the form of a histogram of the interference to the desired signal power ratio. Finally, a computer program suitable for microcomputers such as IBM AT is provided with the flowchart, a sample run, results of the run, and the program code.
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SECTION 1
INTRODUCTION

In the late seventies, the number of communications satellites in service increased at a rapid pace. Most of these satellites were geosynchronous. While interference is always experienced by the communications systems where they share frequencies of transmission or use channels adjacent to those of powerful transmitters, interference from geosynchronous satellites is becoming an increasingly important factor in establishing and operating a ground station. In the eighties the interference problem at the ground station has become worse because low Earth orbiter satellites have been added. These satellites are even more disruptive than those in geosynchronous orbits because of their low altitude above the Earth. The interference produced by all these satellites is generally intolerable to such a sensitive communications link as the Deep Space Network System at Goldstone, California.

The interference experienced by the victim receiving station may be quantified in two different ways: one characterizes the interference in terms of statistics of the problem; the other simulates the interference phenomena with a computer. The statistical characterization of the interference problem suffers from the fact that it is quite difficult to statistically model the interfering signals and situations. This attempt introduces approximations and hand-waving in the modeling and, consequently, results in inaccurate as well as limited quantification of the interference effects. Simulation of the interference effects on a computer also is not trouble-free, because the computer must be run for considerable amounts of CPU time to generate the desired histogram.
There have been many efforts to quantify interference phenomenon, some quite good, but in general they are either too restrictive in their scope of applications (statistical-result programs) or too cumbersome to use (main-frame computer simulations). Computer simulations become more acceptable if they are transferred to a microcomputer where the user generally has exclusive use of the computer, and the CPU time does not enter the decision to run the simulation.

Recently, microcomputers have become reasonably fast and their memory capacity has increased considerably; hence it has become feasible to write a simulation on a microcomputer that would normally be written on a mini or a main-frame computer. The following is an attempt to provide formulas and simulation for the histogram of interference experienced at a ground station. The histogram generates the mean and variance of interference along with such parameters as the cumulative distribution function (CDF), the probability density function (PDF), and the probability of the interference exceeding a given level.

This is a complete simulation containing computation of orbital positions of the desired and interfering satellites using the classical orbital elements, calculation of the satellite antenna look angles for both satellites and look angles (elevation angles) at the desired-satellite ground-station antenna, and computation of Doppler effect due to the motions of the satellites and the Earth's rotation; it also computes the interference-to-signal-power ratio, taking into account losses suffered by the links. After computing the interference-to-signal-power ratios, the program then computes the statistical quantities. The program is expected to be of general use to system designers and frequency managers in selecting the proper frequency under an interference scenario.
SECTION 2
INTERFERENCE PARAMETERS

2.1 Satellite Interference Problem Definition

A satellite communications system generally has a satellite in an orbit and a ground station in communication with it. The downlinking from the satellite to the ground station is done at an assigned frequency and at a particular time slot in the orbit of the satellite. The ground-station antenna tracks the satellite and receives the telemetry or ranging signal. Tracking is necessary to maximize the gain of the antenna and consequently minimize the power required from the satellite for transmission.

During the time period when the ground-station antenna, presumably a good-sized antenna dish with reasonable gain, is tracking the desired satellite, interference may result if an interfering satellite enters the antenna beamwidth. The magnitude of interfering power received depends upon the following factors: (1) The frequency separation between the desired-satellite frequency and the interfering-satellite frequency, (2) the RF power radiated by the interfering-satellite antenna, (3) gain of the interfering-satellite antenna in the direction of the desired-satellite ground station, (4) gain of the desired-satellite ground station in the direction of the interfering satellite, and finally, (5) the losses of the desired-satellite ground system. It is well known that the magnitude of the interfering power alone does not determine the interference to the desired ground station, but it is the ratio of the interfering power to the desired signal power that quantifies the interference. Accordingly, we define the interference to signal power ratio (ISP) as:

\[
\text{ISP} = \frac{\text{interference power at the desired satellite ground station antenna terminal}}{\text{desired satellite signal power at the ground station antenna terminals}}
\]
In the following computer simulation, this ratio is computed in decibels and used as the system degradation measurement parameter. It is true that there are many computer programs for large machines and minicomputers that do something similar to what is proposed here, but there are only a handful, if any, programs available for microcomputers.

### 2.2 Orbital Determination of Satellite Position

Elliptical orbits are assumed for both the interfering satellite and desired satellite. Using the classical theory of satellite orbits, one can compute the position of the satellites in a straightforward manner. No attempts will be made here to explain the theory behind orbital mechanics of a satellite; there are many excellent books and publications in the literature toward that end. We will use the already developed results and formulas for our purpose.

The first step toward orbital position determination is to compute the position of the satellite in the orbital plane of the satellite. The x axis passes through the orbital perigee and the z axis is perpendicular to the orbital plane. The coordinates of the satellite in the orbital plane \((x_0, y_0, z_0)\) are obtained by using the classical theory of orbital mechanics of the satellites. The following steps are needed to generate the position of the satellite.

Solve for the eccentric anomaly \(E\) of the satellite. Eccentric anomaly is obtained as follows. Locate the point where a line perpendicular to the semimajor axis of the orbit and passing through the position of the satellite intersects the circle that circumscribes the satellite orbit. A line from the center of the ellipse to this point makes an angle \(E\) (the eccentric anomaly).
with the semimajor axis. The eccentric anomaly of the satellite is obtained from the following equation:

$$E - e \sin (E) = \frac{1}{a} \left[ \frac{398613.52}{a} \right]^{1/2} (t - t_p)$$  \hspace{1cm} (1)$$

where \(a\) is the semimajor axis of the orbit, \(e\) is the eccentricity of the elliptical orbit (\(e = 0\) is a circle), and \(t - t_p\) is the time of perigee passage. The radial distance of the satellite from the origin (the Earth's center), \(r_0\), can be calculated from

$$r_0 = a \left(1 - e \cos (E)\right)$$  \hspace{1cm} (2)$$

This distance will become useful in computing the power received by the receiving station from the radiating satellite. The orbital plane coordinates can be computed by using the following equations:

$$\begin{align*} x_0 &= \frac{a(1 - e^2) - r_0}{e} \\ y_0 &= \left[ (1 - e^2) \left( (a - r_0)^2 + a^2 e^2 \right) / e^2 \right]^{1/2} \\ z_0 &= 0 \end{align*}$$  \hspace{1cm} (3)$$

It should be noted that the \(z\) coordinate is 0 because the satellite is in the orbital plane.

The next step involves conversion of the orbital coordinates into the geocentric coordinate system. The plane of the orbit will also be defined within this system. In this system, the \(x\) axis always points towards the first point of Aries, and the \(z\) axis passes through the Earth's geographic north pole. This coordinate system translates as the Earth revolves around the
Sun, but it does not rotate. Figure 1 shows the orbit of the satellite around the Earth. Angular distance measured eastward in the equatorial plane is called the right ascension. The two points at which the orbit penetrates the equatorial plane are called nodes; the satellite moves upward through the equatorial plane at the ascending node and downward through the equatorial plane at the descending node. The right ascension of the ascending node and the angle that the orbital plane makes with the equatorial plane together locate the orbital plane with respect to the equatorial plane.

To convert the orbital plane coordinates \((x_0, y_0, z_0)\) of the satellite into the geocentric coordinates \((x_i, y_i, z_i)\), we use the following equations:

\[
\begin{align*}
x_i &= \left[ \cos (\Omega) \cos (\omega) - \sin (\Omega) \cos (i) \sin (\omega) \right] x_0 \\
&\quad - \left[ \cos (\Omega) \sin (\omega) + \cos (\omega) \cos (i) \sin (\Omega) \right] y_0 \\
y_i &= \left[ \sin (\Omega) \cos (\omega) + \cos (\Omega) \cos (i) \sin (\omega) \right] x_0 \\
&\quad + \left[ -\sin (\Omega) \sin (\omega) + \cos (\Omega) \cos (i) \cos (\omega) \right] y_0 \\
z_i &= \left[ \sin (i) \sin (\omega) \right] x_0 + \left[ \sin (i) \cos (\omega) \right] y_0
\end{align*}
\]

The next step is to compute the rotational coordinate system coordinates of the satellite. In this system, the \(x\) axis passes through the point at which the prime geographic meridian passes through the equator. The \(z\) axis coincides with the rotational Earth's axis. The transformation from the geocentric coordinates \((x_i, y_i, z_i)\) to the rotational coordinates \((x_r, y_r, z_r)\) can be made in the following way:

\[
\begin{align*}
x_r &= x_i \cos (\Omega_e T_e) + y_i \sin (\Omega_e T_e) \\
y_r &= -x_i \sin (\Omega_e T_e) + y_i \sin (\Omega_e T_e) \\
z_r &= z_i
\end{align*}
\]
Figure 1. Satellite orbital geometry.
where

\[ \Omega_e \] is the angular velocity of the rotating system, and

\[ T_e \] is the time elapsed since the x axis of the rotational system coincided with the x axis of the geocentric system.

\[
T_e = 99.6909833 + 36000.7689 T^2_c \\
+ 0.00038708 T_c + 0.25068447 t \quad \text{(degrees)}
\] (6)

where

\[ T_c = (\text{Julian Day} - 24150207)/36525 \]

and t is measured in minutes after midnight universal time (UT).

2.3 The Subsatellite Point

This is the point at which a line joining the Earth's center to the satellite intersects the Earth's surface. This subsatellite point will be very useful in computing various angles and distances in the sections to follow. The subsatellite point will be characterized in terms of the north latitude, \( L_s \), and west longitude, \( l_s \). Using the rotational coordinates \( (x_r, y_r, z_r) \), we may compute the latitude and longitude mentioned above by using the following formulas (Ref. 1):

\[
\text{subsatellite point} = \begin{align*}
L_s &= 90 - \cos^{-1} \left[ \frac{z_r}{r_0} \right] \\
\end{align*}
\] (7)
1, subsatellite point longitude (degrees) = \( l_s \)

\[
\begin{align*}
-\tan^{-1}\left[\frac{y_r}{x_r}\right] & \quad x_r \geq 0 \text{ and } y_r \geq 0 \\
180 + \tan^{-1}\left[\frac{y_r}{|x_r|}\right] & \quad x_r \leq 0 \text{ and } y_r \geq 0 \\
90 + \tan^{-1}\left[\frac{|x_r|}{y_r}\right] & \quad x_r \leq 0 \text{ and } y_r \leq 0 \\
\tan^{-1}\left[\frac{|y_r|}{x_r}\right] & \quad x_r \geq 0 \text{ and } y_r \leq 0 \\
\end{align*}
\]

It should be realized that there will be one subsatellite point for the desired satellite and another one for the interfering satellite.

2.4 Visibility of the Satellites

Satellite visibility determines when communications to and from the satellite occur. For interference to occur, the interfering satellite and the desired satellite must transmit downlink data (e.g., telemetry and ranging) at the same time. This implies that for interference to occur at the desired-satellite ground station, the ground station should be able to "see" the desired satellite plus the interfering satellite and at the same time the interfering-satellite ground station should "see" the interfering satellite. This is a necessary condition for interference, but obviously not sufficient in itself. More often than not, satellite transmissions are not scheduled to begin as soon as the satellite is above the horizon, but the transmission begins when the satellite is above a predetermined elevation angle. Primarily, this is because of the land mask around the
receiving station or some particular multipath condition around the receiving station. This elevation angle sets the limits of visibility of the satellite. The visibility limits of a given satellite may be different for the two ground stations.

Figure 2 shows the interference geometry for the desired-satellite ground station. Let the desired-satellite ground-station latitude be \( L_d \) and the corresponding west longitude be \( \lambda_d \). Let \( L_i \) and \( \lambda_i \) be the latitude and west longitude of the interfering-satellite ground station. In the previous section, we computed the subsatellite point; using those formulas we may compute latitude and longitude pairs for the subsatellite points of the desired and interfering satellites. Let \( L_{sd} \) and \( \lambda_{sd} \) be the latitude and longitude of the subsatellite point of the desired satellite and \( L_{si} \) and \( \lambda_{si} \) the latitude and longitude of the subsatellite point of the interfering satellite. Using the geometry and notations for distances and angles used in Figure 2 we get:

\[
\begin{align*}
\eta_1 &= \cos^{-1} \left[ \cos (L_d) \cos (L_{si}) \cos (L_{si}) \cos (\lambda_d - \lambda_{si}) + \sin (L_d) \sin (L_{si}) \right] \\
\eta_2 &= \cos^{-1} \left[ \cos (L_i) \cos (L_{si}) \cos (\lambda_i - \lambda_{si}) + \sin (L_i) \sin (L_{si}) \right] \\
\eta_3 &= \cos^{-1} \left[ \cos (L_d) \cos (L_{sd}) \cos (L_{sd}) \cos (\lambda_d - \lambda_{sd}) + \sin (L_d) \sin (L_{sd}) \right] \\
\eta_4 &= \cos^{-1} \left[ \cos (L_i) \cos (L_{si}) \cos (L_{si}) \cos (\lambda_i - \lambda_{si}) + \sin (L_i) \sin (L_{si}) \right] \\
\end{align*}
\]

\[(9)\]

\[
\begin{align*}
r_{01} &= \left[ x_{rd}^2 + y_{rd}^2 + z_{rd}^2 \right]^{1/2} \\
r_{02} &= \left[ x_{ri}^2 + y_{ri}^2 + z_{ri}^2 \right]^{1/2} \\
\end{align*}
\]

\[(10)\]
Figure 2. Interference geometry.
\[
\begin{align*}
    r_1 &= \left[ \sqrt{r_{01}^2 + r_e^2 - 2 r_e r_{01} \cos (\eta_3)} \right]^{1/2} \\
    r_2 &= \left[ \sqrt{r_{02}^2 + r_e^2 - 2 r_e r_{02} \cos (\eta_1)} \right]^{1/2} \\
    r_3 &= \left[ \sqrt{r_{02}^2 + r_e^2 - 2 r_e r_{02} \cos (\eta_2)} \right]^{1/2}
\end{align*}
\]

(11)

It should be noted that the \(d\) subscripts in the above formulas denote the desired satellite quantity while the \(i\) subscripts denote the interfering satellite quantity. The pertinent elevation angles can now be computed with ease.

\[
\begin{align*}
    El_1 &= \cos^{-1} \left[ \left( \frac{r_{01} \sin (\eta_3)}{r_1} \right) \right] \\
    El_2 &= \cos^{-1} \left[ \left( \frac{r_{02} \sin (\eta_1)}{r_2} \right) \right] \\
    El_3 &= \cos^{-1} \left[ \left( \frac{r_{02} \sin (\eta_2)}{r_3} \right) \right]
\end{align*}
\]

(12)

Once \(El_1\), \(El_2\), and \(El_3\) are known, they may be used in deciding when the downlink transmission from the satellite starts, i.e., if

\[
El_1 > a_1, \text{ then desired-satellite transmission begins}
\]

\[
El_3 > a_3, \text{ then interfering-satellite transmission begins}
\]

where \(a_1\) and \(a_3\) are the predetermined angles at which the desired-satellite and interfering-satellite transmissions begin. These angles basically depend upon the land mask of the respective ground station. Thus the necessary condition for interference to occur at the desired-satellite ground station is that the following set of theoretical conditions results in "true":

\[
(El_1 > a_1) \cap (El_3 > a_3) \cap (El_2 > 0)
\]

(13)
The last term is necessary because the desired-satellite ground station should be seen by the interfering satellite to produce interference. If the answer to the above condition is "false," there is no interference possibility at the desired-satellite ground station.

2.5 Antenna Angles

To determine the power received at the desired-satellite ground-station antenna terminals from the desired satellite as well as the interfering power from the interfering satellite, we need the following off-the-boresite angles of various antennas: angle $\theta_3$ off the boresite of the desired-satellite ground-station antenna to the desired satellite, angle $\theta_2$ off the boresite of the desired-satellite ground station antenna to the interfering satellite, angle $\theta_4$ off the boresite of the desired-satellite antenna to the desired satellite ground station, and finally, angle $\theta_1$ off the boresite of the interfering-satellite antenna to the desired satellite ground station. We will assume that $\theta_3 = \theta_4 = 0$. This amounts to assuming that the antenna pointing systems on the desired satellite and the desired-satellite ground station are working perfectly and hence both these antennas are aligned properly. It will be assumed that the interfering-satellite antenna pointing system is working perfectly and, as a consequence, the interfering satellite antenna will be pointing to the interfering satellite ground station perfectly. Figure 2 shows the pertinent angles. The computation of the angles is done as follows:

The distance $x$ in Figure 2 can be computed by

$$x^2 = r_0 [2(1 - \cos (L_1) \cos (L_d) \cos (l_i - l_d) - \sin (L_1) \sin (L_d))]$$

(14)
Using the law of cosines we get,

\[ \theta_1 = \cos^{-1} \left( \frac{r_2^2 + r_3^2 - x^2}{2 r_2 r_3} \right) \]  

To compute \( \theta_2 \) we will need the distance \( r_4 \) in Figure 2. Since we know the coordinates of both the satellites, we can compute this distance quite easily.

\[ r_4 = [(x_{rd} - x_{ri})^2 + (y_{rd} - y_{ri})^2 + (z_{rd} - z_{ri})^2] \]

Using the law of cosines again we get,

\[ \theta_2 = \cos^{-1} \left( \frac{r_1^2 + r_2^2 - r_4^2}{2 r_1 r_2} \right) \]  

It should be noted that these antenna angles can be computed in many different ways, and some of these ways lend themselves to computer applications with ease and accuracy.

2.6 Computation of Doppler Effect

It will be assumed that the desired-satellite station will be tracking the Doppler effect introduced by its own (desired) satellite motion and hence the Doppler effect will not be considered for the desired satellite while computing the desired signal power. Doppler effect introduced by the interfering satellite motion will cause some variation in the interfering power received at the desired-satellite ground station and will be computed. The velocity component of the interfering satellite along the line \( r_2 \) in Figure 2 causes the Doppler effect at the desired-satellite ground station. It should be noted that this is not the only velocity causing the Doppler effect. The Earth's rotation also adds to the Doppler effect. Both these components are computed below.
The magnitude of the velocity vector of the interfering satellite $V$ can be computed using the following formula:

$$V = 398613.52 \left[ \left( \frac{2}{r_0} \right) - \left( \frac{1}{a_I} \right) \right]$$

(17)

where $a_I$ is the semimajor axis of the interfering-satellite orbit. We need the component of $V$ along $r_2$ to compute the Doppler effect. Toward this end, we will need to compute the angle $\phi$ and the flight path angle $\delta$ shown in Figure 2. Since we have the subsatellite points for both satellites, the computation of these angles is not difficult. The distance $y$ shown in Figure 2 can be computed from:

$$y^2 = r_e \left( 1 - 2 \cos(\eta_1) \right)$$

Using the law of cosines (see Figure 2) we obtain

$$\phi = \cos^{-1}\left[ \left( \frac{(r_0 - r_e)^2 + r_2^2 - y^2}{2 \, r_2 \, (r_0 - r_e)^2} \right) \right]$$

(18)

The flight path angle for the interfering satellite would also be needed for computing the Doppler-effect-producing velocity component. The flight path angle $\delta$ is shown in Figure 2.

$$\delta = \tan^{-1}\left[ \frac{e \sin (\phi_0)}{\sqrt{1 + 2 \, e \cos (\phi_0) + e^2}} \right]$$

(19)

where $\phi_0$ is the angle between the semimajor axis and the line joining the focus of the orbit to the satellite.

Thus the velocity component of the interfering satellite toward the desired-satellite ground station is given by

$$V_c = V \cos \left( \left( \frac{\pi}{2} \right) + \delta - \phi \right)$$

(20)

$V_c$ is one Doppler-effect-producing velocity component; the other component is due to the Earth's rotation. These two
components add vectorially to produce the velocity that is responsible for the total Doppler effect. It should be noted that in this report, the Doppler due to the Earth's rotational component will be computed only approximately. Assuming a spherical Earth with an average radius of 6378 km, one may compute the Earth's rotational speed at the equator to be about 0.4638 km/s.

Figure 3 shows the geometry of Doppler effect due to the two velocity components. The vector along $r_2$ is the component $V_c$ and the component along the line TP is the Earth's rotational component. These two components are at an angle $\sigma$. The line TP is tangential to point T (the desired-satellite ground station) and the point P is directly east of T. The latitude of point P is the same as that of point T and the longitude of point P is slightly different than the longitude of point T ($\Delta$ is assumed to be small). The distance between S and P, $d_{SP}$, may be computed from

$$d_{SP}^2 = [x_{ri} - r_e \cos (L_d) \cos ((1 - \Delta)1_d)]^2$$
$$+ [y_{ri} - r_e \cos (L_d) \sin ((1 - \Delta)1_d)]^2$$
$$+ [z_{ri} - r_e \sin (L_d)]^2$$  \hspace{1cm} (21)

The distance between T and P, $d_{TP}$, may be computed from

$$d_{TP}^2 = [r_e \cos (L_d) \sin (1_d)]^2$$
$$- r_e \cos (L_d) \sin ((1 - \Delta)1_d)]^2$$
$$+ [r_e \sin (L_d) - r_e \sin ((1 - \Delta)1_d)]^2$$  \hspace{1cm} (22)

Using the law of triangles we compute the angle $\sigma$.

$$\sigma = \cos^{-1} \left[ \frac{(r_{02}^2 + d_{TP}^2 - d_{SP}^2)}{2 \, r_{02} \, d_{TP}} \right]$$  \hspace{1cm} (23)
Figure 3. Geometry of Doppler components.
The velocity component, $V_r$, which is due to the Earth's rotation and contributes to the Doppler effect, can be computed as

$$V_r = 0.4638 \cos (\sigma)$$  \hspace{1cm} (24)

It should be noted that the velocity components $V_c$ and $V_r$ are along the line joining the desired-satellite ground-station antenna center to the interfering satellite and hence can be added algebraically. Thus the Doppler causing velocity, $V_d$, is found from

$$V_d = V_c + V_r$$  \hspace{1cm} (25)

If the interfering satellite transmits at $f_c$ carrier frequency, then the frequency received at the desired-satellite ground station, $f_r$ (which is modified by the interfering-satellite motion), is calculated from:

$$f_r = \frac{(1 + (V_d/c))}{[1 - (V_d/c)^2)]^{1/2}}$$  \hspace{1cm} (26)

where $c$ is the velocity of light.

2.7 The Effect of Interfering Modulation

The interfering power received by the desired-satellite ground station from the interfering satellite depends upon the modulation of the interfering signal and the front-end bandwidth of the receiver among many other things. Let the desired-satellite ground-station receiver front end have a bandwidth of $BW$ Hz and the desired-satellite frequency be $f_d$ Hz. It is assumed that the receiver bandwidth, $BW$, is large enough for successful carrier tracking of the received doppler-corrupted desired signal. The receiver does not employ adaptive bandwidth optimization, i.e., once the bandwidth is selected, it remains
constant. Figure 4 shows the baseband equivalent spectrum, and the shaded area shows the interfering power collected by the desired-satellite ground-station receiver.

\[
\text{Interfering power collected by the receiver} = \frac{2 E_s}{P_s} \int_{\text{delf-}\frac{BW}{2}}^{\text{delf+}\frac{BW}{2}} (\text{PSD}) \, df = \text{FOF} \tag{27}
\]

where PSD is the baseband power spectral density of the modulation present on the interfering signal, delf is the frequency offset given by \(|f_d - f_r|\), and \(E_s\) is the energy in the pulse and \(P_s\) is the power in the pulse. For the NRZ pulse modulation of the signal, \(E_s = P_s/\text{data rate}\).

It should be noted that the received frequency, \(f_r\), used in the above calculation includes the Doppler effect. This implies that the shaded area will move according to the Doppler shift experienced by the incoming interfering signal. With all these parameters computed, we are ready to compute the interfering power received by the desired-satellite ground station and compare it with the desired signal power received.

2.8 Power Received Calculations

Suppose the desired satellite radiates \(P_d\) watts of RF power (this includes all the desired-satellite antenna inefficiencies). Using the previously defined parameters, the power input to the receiver, \(P_{dr}\), can be computed as

\[
P_{dr} = P_d L_d g_1(0) g_2(0) \left[\frac{\lambda_1}{(4\pi r_1)}\right]^2 \tag{28}
\]

where \(L_d\) signifies all the losses of the receiving ground station system (this may be frequency dependent). \(g_1(0)\) is the gain of
Figure 4. Interference modulation power spectrum.
the ground-station antenna and $g_2(0)$ is the gain of the desired-satellite antenna. These antenna patterns are assumed to be symmetric in $\phi$ direction. A similar equation for the interfering power received by the desired-satellite ground station may be written:

$$P_{di} = P_i \text{FOF} L_i \left[ \frac{\lambda_2}{(4\pi r_1)} \frac{\lambda_1}{(4\pi r_2)} \right]^2 g_3(\theta_1) g_1(\theta_2)$$  \hspace{1cm} (29)$$

where $P_i$ is the interfering radiated RF power, $L_i$ signifies all the losses caused by the receiving ground station system to the interfering signal, $g_3(\theta_1)$ is the gain of the interfering-satellite antenna toward the desired-satellite ground-station antenna, $g_1(\theta_2)$ is the gain of the desired-satellite ground-station antenna toward the interfering satellite, and FOF is the factor computed in equation (27). Dividing equation (29) by the equation (28) we get a ratio ISR:

$$\text{ISR} = \frac{P_i L_i}{P_d L_d} \left[ \frac{\lambda_2}{\lambda_1} \frac{r_1}{r_2} \right]^2 \frac{g_3(\theta_1) g_1(\theta_2)}{g_1(0) g_2(0)} \text{FOF}$$  \hspace{1cm} (30)$$

Normalizing the ISR by the factor $(P_i L_i / P_d L_d)$ we obtain the previously defined interference measurement factor, ISP:

$$\text{ISP} = \left[ \frac{\lambda_2}{\lambda_1} \frac{r_1}{r_2} \right]^2 \frac{g_3(\theta_1) g_1(\theta_2)}{g_1(0) g_2(0)} \text{FOF}$$  \hspace{1cm} (30)$$
In the theory described above we have assumed that the antenna pointing systems on the satellites as well as those on the ground stations function perfectly. This implies that the main lobe of the desired- or interfering-satellite antenna is always properly focused on the pertinent ground station as soon as the satellite becomes "visible" to the ground-station antenna. The interfering power received by the desired-satellite ground-station antenna depends upon the angles $\theta_1$ and $\theta_2$. These two angles are functions of time and are constantly changing because of the change in the satellite orbital position. The interfering power also depends upon the Doppler effect on the interfering carrier, and this effect is also time dependent. Thus the received interfering power will change from orbit to orbit and from point to point in any particular orbit of these two satellites. Hence, even though the parameters of the interference scenario are constants, a histogram of the ratio ISP may be created giving rise to many statistical parameters of interest. There are many excellent books on the subject of histograms (Ref. 2); we will follow the procedure described below for constructing a histogram for ISP.

Suppose we are interested in the range of ISP with the lower limit of ISPLL (dB) and the upper limit of ISPUL (dB). Let $N$ be the total number of sample ISP values gathered out of which $N_0$ be the number of ISP values such that $\text{ISP} < \text{ISPLL}$

$N_1$ be the number of ISP values such that $d_0 < \text{ISP} < d_1$
$N_2$ be the number of ISP values such that $d_1 < ISP < d_2$

$N_3$ be the number of ISP values such that $d_2 < ISP < d_3$

\[ \ldots \]

$N_k$ be the number of ISP values such that $d_{k-1} < ISP < d_k$

$N_{k+1}$ be the number of ISP values such that $ISPUL < ISP$

where

\[
N = \sum_{j=0}^{k+1} N_j \quad \text{and} \quad d_j = ISPLL + j \text{DELISP}, \quad j = 1, \ldots, k
\]  

(31)

\[
\text{DELISP} = \frac{ISPUL - ISPLL}{k}, \quad d_0 = ISPLL, \quad \text{and} \quad d_k = ISPUL
\]

(32)

The following figure depicts the bins produced by this process:

\[ \begin{array}{ccc}
\text{----------} & \text{----------} & \text{----------} \\
N_0 : N_1 : N_2 : N_3 : \ldots : N_{k-1} : N_k : N_{k+1} \\
\text{----------} & \text{----------} & \text{----------} \\
\end{array} \]

Let $d_{j\text{mid}}$ be the middle point of the interval $(d_{j-1}, d_j)$; hence,
\[ d_{jmid} = \text{ISPLL} + \text{DELISP} \left( j - 0.5 \right) j = 1, \ldots, k+1 \] (33)

\[ \text{sample mean} = m = \frac{1}{N} \sum_{j=1}^{k} N_j d_{jmid} \]

\[ = \text{ISPLL} + \text{DELISP} \left[ 0.5 + \left( \sum_{j=1}^{k} \frac{j N_j}{N} \right) \right] \]

\[ \text{sample variance} = \frac{1}{N-1} \sum_{j=1}^{k} \left[ N_j (d_{jmid} - m)^2 \right] \] (35)

\[ \text{sample probability} = p_j = \text{Prob} \left[ d_{j-1} < \text{ISP} < d_j \right] \]

\[ = \frac{N_j}{N} \quad j = 1, \ldots, k+1 \] (36)

\[ \text{cumulative distribution} = F_j = \frac{1}{N} \sum_{i=1}^{k+1} N_i \quad j = 1, \ldots, k+1 \] (37)

\[ \text{sample probability density} = p_j = \frac{N_j}{N \ \text{DELISP}} \quad j = 1, \ldots, k+1 \] (38)

Using similar techniques we can also obtain the mean time between interference and average time the interference lasts. All these values are invaluable to the system designers and system analysts.

It should be noted that the accuracy of the histogram depends upon the selection of DELISP and ISPLL as well as ISPUL. The information obtained above is sufficient to obtain some statistical parameters of interest such as the probability of interference ISP exceeding a given ISP level.
The computer program is written specifically for a microcomputer such as an IBM AT. To make use of the program easier, it is written in IBM BASICA basic language. The program generally uses the theory and equations presented above; however, the theory does not discuss the complexity for computer implementation. The computer program harbors several changes to the equations described above to make them suitable for computer implementation, always producing the same result as predicted by the theory.

The basic flow of the computer simulation is to pass through a question and answer session for the user and gather all the parameters needed for the equations; this gives the user a fair amount of control over the execution of the program. All the computations are made and stored in double-precision arithmetic, but the printing is done in single precision. The inputs necessary for this program are listed below:

Necessary inputs for the program:

(1) Satellite orbital elements for the desired satellite.
   (a) Orbital eccentricity.
   (b) Semimajor axis.
   (c) Elapsed time since perigee.
   (d) Right ascension of the ascending node.
   (e) Argument of perigee.
   (f) Inclination of orbital plane to the equatorial plane.
   (g) Julian day of the first observation of the satellite.
(2) Satellite orbital elements for the interfering satellite.
   (a) Orbital eccentricity.
   (b) Semimajor axis.
   (c) Elapsed time since perigee.
   (d) Right ascension of the ascending node.
   (e) Argument of perigee.
   (f) Inclination of orbital plane to the equatorial plane.
   (g) Julian day of the first observation of the satellite.

(3) Inputs for sampling the orbits.
   (a) Number of sample points per orbit.
   (b) Number of orbits to be sampled.

(4) Desired-satellite ground-station parameters.
   (a) Gain of the ground-station antenna.
   (b) Ground-station receiver front-end bandwidth.
   (c) North latitude of the ground station.
   (d) West longitude of the ground station.

(5) Interfering-satellite ground-station parameters.
   (a) North latitude of the ground station.
   (b) West longitude of the ground station.

(6) Desired-satellite system parameters.
   (a) Frequency in use (MHz).
   (b) Data rate of the link (bits/s).
   (c) Gain of the antenna (dB).
   (d) Elevation angle at the ground station at which transmission begins (deg).
(7) Interfering-satellite system parameters.
(a) Frequency in use (MHz).
(b) Data rate of the link (bits/s).
(c) Gain of the antenna (dB).
(d) Elevation angle at the ground station at which transmission begins (deg).

(8) Miscellaneous parameters.
(a) Lower limit of range of interest of ISP (dB).
(b) Upper limit of range of interest of ISP (dB).
(c) Name of the file to store the histogram results.

After these inputs are fed to the computer program, all the parameters necessary to evaluate IS/P at every sample point are computed and the following outputs are generated:

Outputs of the program:

(1) Average value of ISP (dB).
(2) Standard deviation of ISP (dB).
(3) Average duration of interference (minutes).
(4) Average duration free of interference (minutes).
(5) Print the following outputs:

\[
\begin{array}{cccccc}
\text{d}_{j} & \text{N}_j & \text{P}_j & \text{P}_j & \text{F}_j \\
\text{.} & \text{.} & \text{.} & \text{.} & \text{.} \\
\text{.} & \text{.} & \text{.} & \text{.} & \text{.} \\
\text{.} & \text{.} & \text{.} & \text{.} & \text{.} \\
\text{.} & \text{.} & \text{.} & \text{.} & \text{.} \\
\text{.} & \text{.} & \text{.} & \text{.} & \text{.} \\
\end{array}
\]

(6) Store \( \text{d}_{j} \) \( \text{N}_j \) \( \text{P}_j \) \( \text{P}_j \) \( \text{F}_j \) in the user-specified file, in the order specified above.
The program allows the user to enter any interfering-modulation spectrum. The user has to locate in the program the reminder statement, "Power spectral density of the interfering modulation" and starting from the next line the user may enter any interfering modulation. If nothing is entered then default is the spectrum of the NRZ pulse. Similarly, the user may enter any antenna pattern (symmetrical in $\theta$ direction) for the desired-satellite ground-station antenna and the interfering-satellite antenna after the reminder statement, "Computing gain of the desired ground station" or "Computing gain of the interfering satellite" is found.

It should be noted that there are 1000 bins made from the user-specified range of ISP, and the user does not have any control of this number. Hence the user should make sure that his storage device has enough space to store about 5000 double-precision numbers.

The flow chart for the program is Figure 5, which shows the program's steps to compute the ISP value at each sample point of the orbit.
QUESTION AND ANSWER SESSION FOR GATHERING DESIRED SATELLITE PARAMETERS

YES

TIME ≥ STOP

NO

ORBITAL POSITION DETERMINATION FOR DESIRED AS WELL AS INTERFERING SATELLITE

INCREMENT TIME BY ΔT

COMPUTATION OF SUBSATELLITE POINT FOR DESIRED AS WELL AS INTERFERING SATELLITE

COMPUTATION OF ELEVATION ANGLES AT THE GROUND STATION ANTENNA FOR DESIRED AS WELL AS INTERFERING SATELLITE

IS INTERFERENCE POSSIBLE

NO

YES

Figure 5. Flowchart for the program.
COMPUTATION OF ANTENNA ANGLES $\theta_1$, $\theta_2$, $\theta_3$, AND $\theta_4$

COMPUTATION OF DOPPLER EFFECT
- DOPPLER DUE TO SATELLITE MOTION
- DOPPLER DUE TO EARTH'S ROTATION

COMPUTATION OF THE EFFECT OF INTERFERER MODULATION (FOF)

COMPUTATION OF ISP

INCREMENT TIME BY $\Delta t/10$

COMPUTATION OF STATISTICAL QUANTITIES

PRINT, STORE DATA

Figure 5 (contd)
Let us suppose that we need to scope the interference environment of a geosynchronous satellite communications system. The interference will be assumed to be generated by a low Earth orbiter satellite with a semimajor axis of orbit of 10000 km. We would like to compute the average value of the ISP (dB), the standard deviation of ISP (dB), the maximum value of ISP (dB), the minimum value of ISP (dB), the average duration of interference experienced by the desired-satellite ground station in minutes, and the average duration in minutes that is free of interference.

We would also like to compute the histogram of the interference (essentially the probability density of the interference) and the cumulative distribution function of the interference. The last curve would be helpful in finding such values as the probability of the interference exceeding a certain level.

Figure 6 shows the inputs and Figures 7, 8, and 9 the outputs of the program for the example at hand.
<table>
<thead>
<tr>
<th><strong>SATELLITE ORBITAL ELEMENTS INPUTS FOR THE DESIRED SATELLITE</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ORBITAL ECCENTRICITY</strong> = 0.0001</td>
<td></td>
</tr>
<tr>
<td><strong>SEMI-MAJOR AXIS (KM)</strong> = 42000.0</td>
<td></td>
</tr>
<tr>
<td><strong>ELAPSED TIME SINCE PERIGEE (SECS)</strong> = 27916</td>
<td></td>
</tr>
<tr>
<td><strong>RIGHT ASCENSION OF ASCENDING NODE (DEG)</strong> = 70</td>
<td></td>
</tr>
<tr>
<td><strong>ARGUMENT OF PERIGEE (DEG)</strong> = 130</td>
<td></td>
</tr>
<tr>
<td><strong>ORBITAL PLANE INCLINATION (DEG)</strong> = 0</td>
<td></td>
</tr>
<tr>
<td><strong>THE JULIAN DAY OF FIRST OBSERVATION OF SATELLITE (DAYS)</strong> = 2443869.5</td>
<td></td>
</tr>
</tbody>
</table>

<table>
<thead>
<tr>
<th><strong>SATELLITE ORBITAL ELEMENTS INPUTS FOR THE INTERFERING SATELLITE</strong></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>ORBITAL ECCENTRICITY</strong> = 0.0001</td>
<td></td>
</tr>
<tr>
<td><strong>SEMI-MAJOR AXIS (KM)</strong> = 10000.0</td>
<td></td>
</tr>
<tr>
<td><strong>ELAPSED TIME SINCE PERIGEE (SECS)</strong> = 27916</td>
<td></td>
</tr>
<tr>
<td><strong>RIGHT ASCENSION OF ASCENDING NODE (DEG)</strong> = 50</td>
<td></td>
</tr>
<tr>
<td><strong>ARGUMENT OF PERIGEE (DEG)</strong> = 80</td>
<td></td>
</tr>
<tr>
<td><strong>ORBITAL PLANE INCLINATION (DEG)</strong> = 80</td>
<td></td>
</tr>
<tr>
<td><strong>THE JULIAN DAY OF FIRST OBSERVATION OF SATELLITE (DAYS)</strong> = 2443869.5</td>
<td></td>
</tr>
</tbody>
</table>

Enter the number of desired satellite orbits to be considered and the number of points per orbit = 5,500

**Receiving Station Parameters for the Desired Satellite**

Enter gain of the antenna (dB) = 40
Enter the front end bandwidth = 2000.0
Enter the north latitude (deg) = 37
Enter the west longitude (deg) = 80

**Receiving Station Parameters for the Interfering Satellite**

Enter the north latitude (deg) = 45
Enter the west longitude (deg) = 80

**Desired Satellite System Inputs**

Frequency in use (MHz) = 1
Bit rate in use = 1000
Gain of the antenna (dB) = 15
Elevation angle at which transmission begins (deg) = 5

**Interfering Satellite System Inputs**

Frequency in use (MHz) = 1
Bit rate in use = 1000
Gain of the antenna (dB) = 15
Elevation angle at which transmission begins (deg) = 5

Enter the lower limit of range of interest of ISP (dB) = 0
Enter the upper limit of range of interest of ISP (dB) = 20

Enter the file name to store the histogram results = ABCD

Figure 6. Input--output of the program.
ORBIT NUMBER 1 IS IN PROGRESS

ORBIT NUMBER 2 IS IN PROGRESS

ORBIT NUMBER 3 IS IN PROGRESS

ORBIT NUMBER 4 IS IN PROGRESS

ORBIT NUMBER 5 IS IN PROGRESS

% POINTS ABOVE THE SPECIFIED ISP UPPER LIMIT OF 20 (dB) = 0
% POINTS BELOW THE SPECIFIED ISP LOWER LIMIT OF 0 (dB) = 1.075269
MINIMUM VALUE OF ISP = -0.630883 (dB)
MAXIMUM VALUE OF ISP = 18.90298 (dB)
MEAN VALUE OF ISP = 11.15138 (dB)
STANDARD DEVIATION OF ISP = 4.5661 (dB)
AVERAGE DURATION OF INTERFERENCE = 16.5712 MINUTES
AVERAGE DURATION FREE OF INTERFERENCE = 152.6125 MINUTES

Figure 6  (contd)
Figure 7. Plot of the histogram envelope.
Figure 8. Plot of sample probability density function.
Figure 9. Plot of cumulative distribution function.
REFERENCES


APPENDIX

SYMBOLIC CODE OF THE PROGRAM
10 DEFDBL A-H,O-Z
20 DEFINT I
30 DIM IPDF(1001), POFY(1001), PDF(1001), DJMID(1001), CDF(1001)
40 REM DATA 0.001181,42164.765,27916.3413,84.178,138.167,0.802,2443869.5
50 REM DATA 0.1,10000.0,27916.3413,84.211,50,80.0,2443869.5
60 REM DATA 5,10
70 REM DATA 40,2000,37.229,80.438
80 REM DATA 45,80
90 REM DATA 1,1000,15,5
100 REM DATA 1,1000,15,5
110 REM DATA 5,20,ABCD
120 FOR I=1 TO 1001: IPDF(I)=0: NEXT
130 NDIV=100: RE=6378: ICONT=0: ICOMT=0: IORB=1: IACCOUNT=0: MSPECIAL=0:
NSPECIAL=0: PI=3.1415926542#: CONVERT=PI/180: TOTTIME56=0:
140 TIME3=0: TIME4=0: TIME5=0: TIME6=0: IDB=0: SUMDB=0: PRINT"SATELLITE ORBITAL ELEMENTS INPUTS FOR THE DESIRED SATELLITE : : :: ": PRINT
150 GOSUB 210
160 EDSA=EEE: ADSA=AAA: TMTPDSA=TTT: CAPOMDSA=CCC: OMDSA=OOO:
AINCLDSA=AI11: AJDDSA=AJJJ
170 PRINT"SATELLITE ORBITAL ELEMENTS INPUTS FOR THE INTERFERING SATELLITE :: :: ": PRINT
180 GOSUB 210
190 EISA=EEE: AISA=AAA: TMTPISA=TTT: CAPOMISA=CCC: OMISA=OOO:
AINCLISA=AI11: AJDISA=AJJJ
200 FOR I=1 TO 1001: IPDF(I)=0: NEXT
210 GOTO 280
220 INPUT"ORBITAL ECCENTRICITY = ", EEE: INPUT"SEMI-MAJOR AXIS (KM) = ", AAA
230 IF AAA<RE THEN INPUT"SEMI-MAJOR AXIS SHOULD BE GREATER THAN RADIUS OF THE EARTH, I.E., 6738 KM. PLEASE RE-ENTER ": ", AAA
240 INPUT"ELAPSED TIME SINCE PERIGEE (SECS) = ", TTT
250 INPUT"RIGHT ASCENTION OF ASCENDING NODE (DEG) = ", CCC:
CCC=CCC*CONVERT: INPUT"ARGUMENT OF PERIGEE (DEG) = ", OOO:
OOO=OOO*CONVERT
260 INPUT"ORBITAL PLANE INCLINATION (DEG) = ", AI11:
AI11=AI11*CONVERT
270 INPUT"THE JULIAN DAY OF FIRST OBSERVATION OF SATELLITE (DAYS) = ", AJJJ: PRINT
280 REM READ EEE, AAA, TTT, CCC, OOO, AI11, AJJJ: REM PRINT EEE;AAA;TTT;
CCC; OOO; AI11; AJJJ: CCC=CCC*CONVERT; OOO=OOO*CONVERT; AI11=AI11*CONVERT
290 RETURN
300 REM GOTO 320
310 PRIN
PRINT "ENTER THE NUMBER OF DESIRED SATELLITE ORBITS TO BE CONSIDERED AND THE NUMBER OF POINTS PER ORBIT = ", NUMORB, NPOINTS: PRINT
320 REM READ NUMORB,NPOINTS: REM PRINT NUMORB; NPOINTS
330 TMTPO=TMTPISA: C1=COS(CAPOMISA): S1=SIN(CAPOMISA): C11=
COS(CAPOMDSA): S11=SIN(CAPOMDSA)
350 C3=COS(AINCLISA): S3=SIN(AINCLISA): C33=COS(AINCLDSA):
S33=SIN(AINCLDSA)
360 FAJDISA=AJDISA-INT(AJDISA): FAJDDSA=AJDDSA-INT(AJDDSA):
ORBPD=2*PI*ADSA*SQR(ADSA/398613.5):TIME=0:TIME6=0:
DELT=ORBPD/NPOINTS:SSTOP=NUMORB*ORBPD
370 PRINT"RECEIVING STATION PARAMETERS FOR THE DESIRED
SATELLITE :::::":PRINT
380 REM GOTO 400
390 INPUT"ENTER GAIN OF THE ANTENNA (dB) = ",GNDST:
INPUT"ENTER THE FRONT END BANDWIDTH = ",BW
400 REM READ GNDST,BW:REM PRINT GNDST,BW
410 GOSUB 470
420 DSTLAT=DDD:DSTLON=EEE
430 PRINT"RECEIVING STATION PARAMETERS FOR THE INTERFERING
SATELLITE :::::":PRINT
440 GOSUB 470
450 AISTLAT=DDD:AISTLON=EEE
460 GOTO 510
470 REM GOTO 490
480 INPUT"ENTER THE NORTH LATITUDE (DEG) = ",DDD=DDD*CONVERT:
INPUT"ENTER THE WEST LONGITUDE (DEG) = ",EEE=EEE*EEE*CONVERT:
PRINT
490 REM READ DDD,EEE:PRINT DDD;EEE:DDD=DDD*CONVERT:EEE=
EEE*CONVERT
500 RETURN
510 GNDST=10^(GNDST/10)
520 PRINT:PRINT"DESIR ED SATELLITE SYSTEM INPUTS ::::: :::::":PRINT
530 GOSUB 590
540 FRDST=FFF*1000000!:GNDSA=GGG:ANGDSA=AAA:DSTBR=BBB
550 PRINT"INTERFERING SATELLITE SYSTEM INPUTS ::::: :::::":PRINT
560 GOSUB 590
570 FRIST=FFF*1000000!:GNISA=GGG:ANGISA=AAA:AISTBR=BBB
580 GOTO 640
590 REM GOTO 620
600 INPUT"FREQUENCY IN USE (MHZ) = ",FFF:INPUT"BIT RATE IN
USE = ",BBB
610 INPUT"GAIN OF THE ANTENNA (dB) :",GGG:INPUT"ELEVATION ANGLE
AT WHICH TRANSMISION BEGINS (DEG) = ",AAA=AAA*AAA*CONVERT:PRINT
620 REM READ FFF,BBB,GGG,AAA:PRINT FFF;BBB;GGG;AAA=AAA*AAA*CONVERT
630 REM GOTO 620
640 GNDSA=10^(GNDSA/10):GNISA=10^(GNISA/10):GNIST=10^(GGG/10)
650 REM GOTO 690
660 PRINT:INPUT"ENTER THE LOWER LIMIT OF RANGE OF INTEREST OF
ISR (dB) = ",AISPLL
670 PRINT"ENTER THE UPPER LIMIT OF RANGE OF INTEREST OF ISR
(dB) = ",AISPUL:DELAISP=(AISPUL-AISPLL)/1000
680 PRINT:INPUT"ENTER THE FILE NAME TO STORE THE HISTOGRAM
RESULTS = ",FILEA$
690 REM READ AISPLL,AISPUL,FILEA$:PRINT AISPLL,AISPUL,FILEA$:
DELAISP=(AISPUL-AISPLL)/1000
700 PRINT:XC=C1*C2-S1*S2*C3:XS=-C1*S2-S1*C2*C3:YC=S1*C2+C1*S2*C3:
YS=-S1*S2+C1*C2*C3:ZC=S2*S3:ZS=C2*S3
710 Xc11=C11*C22-S11*S22*C33:XS11=-C11*S22-S11*C22*C33:YC11=S11*
C22+C11*S22*C33:YS11=-S11*S22+C11*C22*C33:ZC11=S22*S33:
ZS11=C22*S33
720 PRINT:PRINT:PRINT:PRINT
730 REM CONTINUE
740 ISEDSADST=0:ISEISAIST=0:ISEISADST=0
750 DISDSTIST=RE*SQR(2*(1-COS(AISTLAT)*COS(DSTLAT)*
COS(AISTLON-DSTLON)-SIN(AISTLAT)*SIN(DSTLAT)))
760 IF (TMTPISA-TMTP) > (SSTOP-DELT/600) THEN GOTO 1870
770 IF (TIME>(IORB-1)*ORBPD/60) AND (TIME<(IORB*ORBPD/60))
THEN IORB=IORB+1:PRINT:PRINT"ORBIT NUMBER "IORB-1;
"IS IN PROGRESS :::::"
780 REM PREPARATION FOR SUBROUTINE COM1+COM2 FOR DESIRED
SYSTEM :::::
790 AAS=ADS:TMTP=TMTPD:TIMES=TIME2:EPSIS=EDSA:XCS=XCS11:
XSS=XS11:YCS=YCS11:YSS=YS11:ZCS=ZCS11:ZSS=ZS11:FAJDS=FAJDDSA
800 AJDS=AJDDSA:DLATS=AISTLAT:DLONS=AISTLON:VLATS=DSTLAT:
VLONS=DSTLON
810 GOSUB 940
820 DELTADSA=DELTA:ROD=ROS:DISDSAIST=DISDS:DISDSADST=DISVS:
ELISTDSA=ELDS
830 ELDSTDSA=ELEVSA:GMDSTSDSA=GAMAD:GMDSTSDSA=GAMAV:XR2=XR:YR2=XR:
ZR2=ZR:DDDI=DDD2
840 IF ELDSTDSA >= ANGDSA THEN ISEDSADST=1
850 REM PREPARATION FOR SUBROUTINE COM1+COM2 FOR
INTERFERING SYSTEM ::::
860 AAS=AISA:TMTP=TMTPISA:TIMES=TIME:EPSIS=EISA:XCS=XCS:
YCS=YCS:ZCS=ZCS:ZSS=ZS:FAJDS=FAJDDSA
870 GOSUB 940
880 DELTAISA=DELTA:ROI=ROS:DISISAIST=DISDS:DISISADST=DISVS:
ELISTISA=EELDS:ELDSTISA=ELEVSA:DDE=DDD2
890 IF ELISTISA >= ANGISA THEN ISEISAIST=1
900 IF ELDSTISA >= 0 THEN ISEISADST=1
910 XRI=XR:YRI=YR:ZRI=ZR
920 ELDSTISA=ELEVSA:GMDSTISA=GAMAD:GMDSTISA=GAMAV
930 GOTO 1280
940 REM SUBROUTINE COM1+COM2 ::::
950 REM SUBROUTINE COM1 ::::
960 ETA=SQR(398613.52#/AAS)/AAS:AM=ETA*TMTPS:E=PI
970 FUN=E-EPSIS*SIN(E)
980 IF ABS(FUN-AM)<1E-10 THEN GOTO 1030
990 IF FUN>AM THEN E=E-DEL
1000 IF FUN<AM THEN E=E+DEL
1010 DEL=ABS(FUN-AM)/2
1020 GOTO 970
1030 ROS=AAS*(1-EPSIS*COS(E))
1040 SINPHIO=AAS*SIN(E)*SQR(1-EPSIS^2)/ROS:
COSPHIO=AAS*(COS(E)-EPSIS)/ROS
1050 XO=ROS*COSPHIO:YO=ROS*SINPHIO:ZO=0
1060 XI=XCS*XO+XSS*YO:YI=YCS*XO+YSS*YO:ZI=ZCS*XO+ZSS*YO:
ZZZ=(EPSIS*SINPHIO)/(SQR(1+2*EPSIS*COSPHIO+EPSIS^2))
1070 DELTAS=ATN(ZZZ/SQR(1-ZZZ^2)):ISIGN=1:IF FAJDS<.5
THEN ISIGN=-1
1080 TC=(INT(AJDS)+.5*ISIGN-2415020!)/36525!:ALFA=99.6909833#+
36000.7689#:TC=3.8708E-04*TC*TC
1090 OMETE=ALFA+.25068447#*(FAJDS-.5*ISIGN)*1440!+TIME):
OMETE=OMETE/360:OMETE=(OMETE-INT(OMETE))*360
1100 OMETE=OMETE*CONVERT:XR=XI*COS(OMETE)+YI*SIN(OMETE):
YR=XI*SIN(OMETE)+YI*COS(OMETE):ZR=ZI
1110 REM SUBROUTINE COM2:::
1120 ZZZ=ZR/SQR(XR^2+YR^2+ZR^2): SLAT=ATN(ZZZ/SQR(1-ZZZ^2))
1130 IF XR=0 THEN GOTO 1190
1140 IF (XR>=0) AND (YR>=0) THEN SLON=-ATN(YR/XR)
1150 IF (XR<=0) AND (YR>=0) THEN SLON=PI+ATN(YR/ABS(XR))
1160 IF (XR<=0) AND (YR<=0) THEN SLON=PI/2+ATN(ABS(XR/YR))
1170 IF (XR>=0) AND (YR<=0) THEN SLON=ATN(ABS(YR)/XR)
1180 GOTO 1210
1190 IF YR>O THEN SLON=-PI/2
1200 IF YR<O THEN SLON=PI/2
1210 ZZZ=COS(DLATS)*COS(SLAT)*COS(SLON-DLONS)+SIN(DLATS)*
       SIN(SLAT): COSGAMAD=ZZZ:
1220 DISDS=ROS*SQR(1+(RE/ROS)^2-2*(RE/ROS)*COSGAMAD): ZZZ=(DISDS^2
       +RE^2-ROSA^2)/2/RE/DISDS: PSID=PI/2-ATN(ZZZ/SQR(1-ZZZ^2))
1230 ELEDS=PSID-PI/2
1240 ZZZ=COS(VLATS)*COS(SLAT)*COS(SLON-VLONS)+SIN(VLATS)*
       SIN(SLAT): COSGAMAV=ZZZ:
1250 DISVS=ROS*SQR(1+(RE/ROS)^2-2*(RE/ROS)*COSGAMAV): ZZZ1=
       (DISVS^2+RE^2-ROSA^2)/2/RE/DISVS: PSIV=PI/2-ATN(ZZZ1/
       SQR(1-ZZZ1^2))
1260 ELEV5=PSIV-PI/2:DDD2=SQR(2*RE^2*(1-ZZZ))
1270 RETURN
1280 IF (ISEISAIST=1) AND (ELISTISA<ANGISA) THEN ISEISAIST=0
1290 ISEE=ISEISAIST*ISEISADST*ISEDADST
1300 REM INTERFERENCE TO SIGNAL POWER CALCULATIONS: ::::::::::
1310 IF ISEE=1 AND ICONT=0 THEN TOTTIME56=TOTTIME56+TIME5:
1320 IF ISEE=1 THEN IMM=IMM+1
1330 ICONT=1: TIME6=TIME-TOTTIME56:
1340 ICONT=1: TIME6=TIME-TOTTIME56:
1350 REM PRINT"TIME3 = "CSNG(TIME3);"TIME4 = "CSNG(TIME4);
       "TOTTIME56 = "CSNG(TOTTIME56)
1360 IF ISEE=0 THEN GOTO 1850
1370 IF ICONT=0 THEN TIME5=TIME
1380 IF ICONT=0 THEN INN=INN+1
1390 ICONT=1: TIME6=TIME-TOTTIME34:
1400 REM PRINT"TIME5 = "CSNG(TIME5);"TIME6 = "CSNG(TIME6);
       "TOTTIME34 = "CSNG(TOTTIME34)
1410 ZZZ=(DISISAIST^2+DISISADST^2-DIST3^2)/2/DISISAIST/
       DISISADST: THDSTISA=PI/2-ATN(ZZZ/SQR(1-ZZZ^2))
1420 DIST3=SQR((XR2-XR)^2+(YR2-YR)^2+(ZR2-ZR)^2)
1430 ZZZ=(DISISADST^2+DISISADST^2-DIST3^2)/2/DISISADST:
       THDSTISA=PI/2-ATN(ZZZ/SQR(1-ZZZ^2))
1440 REM COMPUTING GAIN OF THE DESIRED GROUND STATION: :::::
1450 IF THDSTISA=0 THEN GV=1 ELSE GV=(SIN(THDSTISA)/THDSTISA)^2
1460 REM COMPUTING GAIN OF THE INTERFERING SATELLITE: ::::::
1470 IF THDSTISA=0 THEN GD=1 ELSE GD=(SIN(THDSTISA)/THDSTISA)^2
1480 GV=GNDST*GV: GD=GNIAS*GD
1490 REM COMPUTING DOPPLER EFFECT INCLUDING THE EARTH ROTATION ::
1500 ZZZ=(DISISAIST^2+(ROI-RE)^2-DADDI^2)/2/DISISAIST/(ROI-RE):
       TH=PI/2-ATN(ZZZ/SQR(1-ZZZ^2))
1510 DSTLONC=DSTLON-CONVERT*:1:ZZZ=COS(DSTLAT)*COS(SLAT)*
       COS(DSTLONC-SLON)+SIN(DSTLAT)*SIN(SLAT):
       DOP=ROI*SQR(1+(RE/ROI)^2-2*ZZZ*(RE/ROI))
1520 DNP=RE*SQR(2*(1-COS(DSTLAT)*COS(DSTLAT)*COS(CONVERT)-
1530 SIN(DSTLAT) * SIN(DSTLAT))
1540 VELO=SQR(398613.52*(2/ROI-1/AISA)) * COS(DELTISA+PI/2-THT) + .463603*COSANG
1550 FR=FRIST*(1+VELO/300000!)/(SQR(1-(VELO/300000!)^2)): DOPPLER=FRIST-FR
1560 REM COMPUTATION OF FREQUENCY OFFSET FACTOR FOF (INCLUDING DOPPLER EFFECT): :::::
1580 SU=0: FOR I=1 TO (NDIV-1) STEP 2: F=I*DF+LRLMT
1590 GOSUB 1670
1600 SU=SU+4*RESULT:NEXT: FOR I=2 TO (NDIV-2) STEP 2: F=I*DF+LRLMT
1610 GOSUB 1670
1620 SU=SU+2*RESULT:NEXT: F=LRLMT
1630 GOSUB 1670
1640 SU=SU+RESULT:NEXT: F=UPLMT
1650 GOSUB 1670
1660 SU=SU+RESULT: FOF=DF*SU/3/PI: GOTO 1710
1670 REM POWER SPECTRAL DENSITY OF THE INTERFERING MODULATION: :::::
1680 IF F=0 THEN GOTO 1700
1690 RESULT=(SIN(F)/F)^2: RETURN
1700 RESULT=1: RETURN
1710 AISPB=(FRDST*DISDSADST/FR/DISISADST)^2*GV*GD*FOF/GNDSA/GNDST:
1720 SUMDB=SUMDB+AISPB: IDB=IDB+1: IF IDB>1 THEN GOTO 1740
1730 AIMAX=AISPB: AIMIN=AISPB
1740 IF AISPB<AIMIN THEN GOTO 1770
1750 IF AISPB>AIMAX THEN GOTO 1780
1760 GOTO 1790
1770 IPDF(0)=IPDF(0)+1: IJK=0: MSPECIAL=MSPECIAL+1: GOTO 1800
1780 IPDF(1001)=IPDF(100)+1: IJK=1001: NSPECIAL=NSPECIAL+1: GOTO 1800
1790 IJK=INT((AISPB-AISPLL)/DELAISP): IPDF(IJK+1)=IPDF(IJK+1)+1
1800 MCOUNT=MCOUNT+1
1810 IF AISPB>AIMAX THEN AIMAX=AISPB
1820 IF AISPB<AIMIN THEN AIMIN=AISPB
1830 TMTPISA=TMTPISA+DELT/10: TMTPDSA=TMTPISA+DELT/10: TIME=TIME+DELT/10: TIME2=TIME2+DELT/10/60:
1840 GOTO 730
1850 TMTPISA=TMTPISA+DELT: TMTPDSA=TMTPISA+DELT: TIME=TIME+DELT/60: TIME2=TIME2+DELT/60: IPDF(0)=IPDF(0)+10
1860 GOTO 730
1870 REM COMPUTING STATISTICAL QUANTITIES: :::::
1880 PRINT: PRINT: PRINT"% POINTS ABOVE THE SPECIFIED ISP UPPER LIMIT OF "AISPUL;": (dB) = "CSNG(100*NSPECIAL/MCOUNT)
1890 PRINT"% POINTS BELOW THE SPECIFIED ISP LOWER LIMIT OF "AISPLL;": (dB) = "CSNG(100*MSPECIAL/MCOUNT)
1900 PRINT"MAXIMUM VALUE OF ISP = "CSNG(AIMIN);": (dB)": PRINT
1910 SUM=0: SQ=0: ICOUNT=0
1920 FOR I=1 TO 1000: ICOUNT=ICOUNT+IPDF(I): DJMID(I)=AISPLP+ DELAISP*(1+2*I)/2: SUM=SUM+IPDF(I)*DJMID(I): NEXT
1930 AMEAN=SUM/IDB: FOR I=1 TO 1000: SQ=SQ+IPDF(I)*(DJMID(I)-AMEAN)^2: NEXT: VARIANCE=SQ/(ICOUNT-1)

43
DEVIAT = SQR(VARIANCE)
PRINT "MEAN VALUE OF ISP = "CSNG(AMEAN);"(dB)"
PRINT "STANDARD DEVIATION OF ISP = "CSNG(DEVIAT);"(dB)"
REM COMPUTING PROBABILITY DISTRIBUTION OF ISP:::
SUM = 0: IF ICOUNT = 0 THEN GOTO 2000
FOR I = 1 TO 1000: POFY(I) = IPDF(I) / ICOUNT:
    PDF(I) = POFY(I) / DELAISP: NEXT
CDF(1) = POFY(1): FOR I = 2 TO 1000: CDF(I) = CDF(I - 1) + POFY(I):
    NEXT: GOTO 2010
PRINT "THERE WAS NO INTERFERENCE ::::": GOTO 2070
TOTTIME56 = TOTTIME56 + TIME6 - TIME5:
TOTTIME34 = TOTTIME34 + TIME4 - TIME3
PRINT: PRINT "AVERAGE DURATION OF INTERFERENCE =
"CSNG(TOTTIME56/INN);"MINUTES"
PRINT "AVERAGE DURATION FREE OF INTERFERENCE =
"CSNG(TOTTIME34/IMM);"MINUTES": PRINT
INPUT ABC
FOR I = 0 TO 1001: PRINT CSNG(DJMID(I)); CSNG(IPDF(I));
    CSNG(POFY(I)); CSNG(PDF(I)); CSNG(CDF(I)): NEXT
OPEN "O", #1, FILEA$: FOR I = 0 TO 1001: PRINT #1, DJMID(I),
    IPDF(I), POFY(I), PDF(I), CDF(I): NEXT
END
This report presents the complete analysis and formulas necessary to quantify the interference experienced by a generic satellite communications receiving station due to an interfering satellite. Both satellites, the desired as well as the interfering satellite, are considered to be in elliptical orbits. Formulas are developed for the satellite look angles and the satellite transmit angles generally related to the land mask of the receiving station site for both satellites. Formulas for considering Doppler effect due to the satellite motion as well as the Earth's rotation are developed. The effect of the interfering-satellite signal modulation and the Doppler effect on the power received are considered. The statistical formulation of the interference effect is presented in the form of a histogram of the interference to the desired signal power ratio. Finally, a computer program suitable for microcomputers such as IBM AT is provided with the flowchart, a sample run, results of the run, and the program code.