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A Superelement Component Dynamic Synthesis Method

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A method is presented for coupling a broad class of component dynamic models in the manner of direct stiffness assembly and is implemented in a general matrix manipulation program.

INTRODUCTION

A number of methods have been developed in the past to accomplish the basic objectives of the method. These methods are surveyed in detail in [1]. Each of these methods are characterized by the way the component dynamics is input and coupled to adjacent components. Hurty [2,3] proposed that the connect degrees of freedom (DOF) of a component were fixed or had a zero displacement. He then partitioned the modes of the structure into rigid body modes, constraint modes, and normal modes. Craig and Bampton [4] proposed a simplification of Hurty's fixed interface method by dividing component modes into only two groups: constraint modes and normal modes. Bamford [5] added attachment modes to improve the convergence of the method.

Goldman [6] developed the free interface method, employing only rigid body modes and free-free normal modes in component dynamic representation. This technique eliminates the computation of static constraint modes, but their advantage is negated by the poor accuracy of the method. Hou [7] presented a variation of Goldman's free-interface method in which no distinction is made between rigid body modes and free-free normal modes. Gladwell [8] developed "branch mode analysis" by combining free interface and fixed interface methods to reduce the order of the stiffness and mass matrices for individual substructures. The reduction procedure requires the knowledge of topological arrangement and dynamics of all the components in the model. In order to account for adjacent components Benfield and Hruda [9] introduced inertia and stiffness loading of component interfaces. The use of loaded interface modes is shown to have superior convergence characterisics.

MacNeal [10] introduced the use of hybrid modes and inertia relief modes for component mode synthesis. Hybrid modes are substructure normal modes computed with a combination of fixed and free boundary conditions. Inertia relief attachment modes are attachment modes for components with rigid body freedoms. MacNeal also included residual inertia and flexibility to approximate the static contribution of the truncated higher order modes of a component. Rubin [11] extended the residual flexibility approach for free interface method by introducing higher order corrections to account for the truncated modes. Klosterman [12] more fully developed the combined experimental and analytical method introduced by MacNeal. Hintz [13] discussed the implications of truncating various mode sets and developed guidelines for retaining accuracy with a reduced size model.

Recent research has centered on the camparision of the various methods. Baker [14], for example, compares the constrained and free-free approach using experimental techniques and also investigates using mass additive techniques and measured rotational DOF [15]. This investigation was motivated by a need to find the best method for rigidly connected flexible structures. In this connection, the constrained method produced the best result. Klosterman [16] has shown the free-free method to be accurate for relatively stiff structures connected with flexible elements. This supports Rubin's conclusion [11] that the free-free method is at least as accurate when residual effects are accounted for. These conclusions are intuitive because the type of boundary condition imposed in the analysis that best represents the boundary of the assembled structure provides the best accuracy in the modal synthesis.

Meirovitch and Hale [16] have developed a generalized synthesis procedure by broadening the definition of the admissible functions proposed by Hurty [1]. This technique is applicable to both continuous and discrete structural models. The geometric compatibility conditions at connection interfaces are approximately enforced by the method of weighted residuals.

The method due to Klosterman [12] has been implemented in an interactive computer code SYSTAN [17] and that due to Herting [18] is available in NASTRAN. The latter is the most general of the modal synthesis techniques. It allows retention of an arbitrary set of component normal modes, inertia relief modes, and all geometric coordinates at connection boundaries. Both the fixed-interface method of Craig and Bampton, and the MacNeal's residual flexibility method, are special cases of the general method. Other analyses presented in the literature based on modal synthesis techniques are not incorporated into general structural analysis codes.

It is desirable to have a synthesis method to couple different types of component dynamic models in a setting such as that of the finite element method so as to be able to realize the best advantages of both the component synthesis methods and the convenience and generality of the finite element method. The need for a such a capability arises when different types of dynamic models are used to represent the various components of a structure. This variance is dictated by the need to improve accuracy of component dynamic representation, availability of a certain kind of data, etc. Herting's work [18] meets some of these objectives. The procedure presented herein permits a broader class of component models in the manner of direct stiffness finite element assembly and can be implemented in a general matrix manipulation program.

A principal feature of the work developed here is the component dynamic model reduction procedure that leads to an exact and numerically stable synthesis. In order to affect component coupling, neither the specification of external coupling springs nor an user-specified selection of independent coordinate is required. Existing synthesis procedures suffer from these drawbacks. Component dynamic models considered include free-free normal modes with or without interface loading, up to second order stiffness and inertia connections accounting for the effect of modal truncation, fixed interface modes, and also the physical coordinate components. The model reduction procedure involves interior boundary coordinate transformations which explicitly retain connection interface displacement coordinates in the reduced component dynamic representations. Interior coordinates may include physical, modal, or any admissible coordinates. Components in this reduced form are termed "superelement" because they are a generalization of the conventional finite elements of structural mechanics. The problem of component dynamic synthesis is then reduced to the assembly of the superelement. The direct stiffness approach and all subsequent processing operations of the finite element method are then applicable. The formulation, implementation and verification aspects of the superelement component dynamic synthesis method are presented in the following sections.

SUPERELEMENT MODEL REDUCTION

The procedures for reducing component dynamic models to a form involving physical DOFs of the connection interface nodes plus some additional DOFs related to the interior nodes are derived in the following. The component dynamic models in this reduced form possess a structure similar to that of the displacement finite element method and are therefore termed superelements. Four types of component dynamic models are considered: (1) finite element model; (2) fixed interface modal model with static constraint modes; (3) free interface modal model with residual flexibility attachment modes; and (4) free interface model with normal modes. The last type may also include any general admissible shape vectors and corresponding dynamic matrices as long as certain requirements for matrix partitioning and invertibility are satisfied. A system may involve any combination of the above types of component dynamics model since the models are reduced to a common form before assembly.

The finite element and the constrained interface modal models naturally contains the connection interface DOFS and are already in the required superelement form. Constrained interface modal model based superelement synthesis is treated in [19]. The free interface modal model with residual flexibility attachment modes is reduced to the superelement as follows. Expressing component displacements $\underline{X}$ in partitioned form can be witten as

$$\begin{Bmatrix} \underline{X}_O \\ \underline{X}_B \end{Bmatrix} = \begin{bmatrix} \underline{\underline{\phi}}_{KO} & \underline{\underline{G}}_{OB} \\ \underline{\underline{\phi}}_{B} & \underline{\underline{G}}_{BB} \end{bmatrix} \begin{Bmatrix} \underline{q}_K \\ \underline{q}_B \end{Bmatrix} \tag{1}$$

where $\underline{\underline{\phi}}$ and $\underline{\underline{G}}$ are respectively the normal mode and residual flexibility attachment mode matrices, $\underline{q}_K$ and $\underline{q}_B$ are the associated generalized coordinates, and the subscripts B and O refer to the boundary and other DOFs. To obtain the superelement reduction transformation, following Martinez [20] solve the lower partition of Eq. (1) for $\underline{q}_B$. Thus,

$$\underline{q}_B = -\underline{\underline{G}}_{BB}^{-1} \underline{\underline{\phi}}_{KB} \underline{q}_K + \underline{\underline{G}}_{BB}^{-1} \underline{X}_B \tag{2}$$

substituting the above in Eq. (1) leads to the superelement form associated with the residual flexibility attachment model:

$$\begin{Bmatrix} \underline{X}_O \\ \underline{X}_B \end{Bmatrix} = \underline{\underline{T}} \begin{Bmatrix} \underline{q}_K \\ \underline{X}_B \end{Bmatrix} \quad \text{where,} \quad T = \left[\begin{array}{c|c} \underline{\underline{\phi}}_{KO} - \underline{\underline{G}}_{OB} \underline{\underline{G}}_{BB}^{-1} \underline{\underline{\phi}}_{KB} & \underline{\underline{G}}_{OB} \underline{\underline{G}}_{BB}^{-1} \\ \hline \underline{\underline{0}} & \underline{\underline{I}} \end{array}\right] \tag{3}$$

The generalized coordinates $\underline{q}_K$ are now the participation factors of the modified modes $(\underline{\underline{\phi}}_{KO} - \underline{\underline{G}}_{OB} \underline{\underline{G}}_{BB}^{-1} \underline{\underline{\phi}}_{KB})$. The internal partitions of the modified normal modes are in effect free interface normal mode partitions minus a set of constraint modes internal partitions and therefore represent modes constrained at boundary degrees of freedom. The superelement equations of motion are obtained by transforming the component modal equation of motion as

$$\begin{bmatrix} \underline{\underline{M}}_{KK}+\underline{\underline{\phi}}_{KB}^{T}\,\underline{\underline{J}}_{BB}\,\underline{\underline{\phi}}_{KB} & -\underline{\underline{\phi}}_{KB}^{T}\,\underline{\underline{J}}_{BB} \\ \text{sym.} & J_{BB} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{q}}_{K} \\ \ddot{\underline{X}}_{B} \end{Bmatrix} + \begin{bmatrix} \underline{\underline{\Lambda}}_{KK}+\underline{\underline{\phi}}_{KB}^{T}\,\underline{\underline{G}}_{BB}^{-1}\,\underline{\underline{\phi}}_{KB} & -\underline{\underline{\phi}}_{KB}^{T}\,\underline{\underline{G}}_{BB}^{-1} \\ \text{sym.} & \underline{\underline{G}}_{BB}^{-1} \end{bmatrix} \begin{Bmatrix} \underline{q}_{K} \\ \underline{X}_{B} \end{Bmatrix} = \begin{Bmatrix} \underline{0} \\ \underline{f}_{B} \end{Bmatrix} \tag{4}$$

where $J_{BB} = \underline{\underline{G}}_{BB}^{-1}\,\underline{\underline{H}}_{BB}\,\underline{\underline{G}}_{BB}^{-1}$,and $\underline{\underline{m}}_{KK}$ and $\underline{\underline{\Lambda}}_{KK}$ are generalized mass and stiffness matrices respectively, $\underline{\underline{G}}_{BB}$ and $\underline{\underline{H}}_{BB}$ are the generalized stiffness and mass associated with the residual flexibility attachment modes. The modification of the component generalized mass and stiffness matrices by the contributions from the residual flexibility of the deleted modes is clearly seen in the above equation.

In the event only a truncated set of free interface normal modes is used to represent component displacements, the superelement reduction is obtained from a partial inversion of the modal matrix as described in the following. Consider the free interface normal mode transformation

$$\begin{Bmatrix} \underline{X}_{O} \\ \underline{X}_{B} \end{Bmatrix} = \begin{bmatrix} \underline{\underline{\phi}}_{KO} \\ \underline{\underline{\phi}}_{KB} \end{bmatrix} \underline{q}_{K} \tag{5}$$

where $\underline{\underline{\phi}}_{KO}$ and $\underline{\underline{\phi}}_{KB}$ are $(n_O x m_K)$ and $(n_B x m_K)$size partitions of the component modal matrix corresponding to the interior and boundary degrees of freedom $\underline{X}_O$ and $\underline{X}_B$, respectively. Consider the lower portion of Eq. (5)

$$\underline{X}_{B} = \underline{\underline{\phi}}_{KB}\,\underline{q}_{K} \tag{6}$$

and partition $\underline{\underline{\phi}}_{KB}$ into a nonsingular invertible square matrix $\underline{\underline{\phi}}_{KB1}$ and remainder $\underline{\underline{\phi}}_{KB2}$. Thus,

$$\underline{X}_{B} = \begin{bmatrix} \underline{\underline{\phi}}_{KB1} & : & \underline{\underline{\phi}}_{KB2} \end{bmatrix} \begin{Bmatrix} \underline{q}_{K1} \\ \underline{q}_{K2} \end{Bmatrix} \tag{7}$$

which requires that the number of kept modes be greater than the number of boundary degrees of freedom. Solving the above equation for $\underline{q}_{K1}$ gives

$$\underline{q}_{K1} = -\underline{\underline{\phi}}_{KB1}^{-1}\,\underline{\underline{\phi}}_{KB2}\,\underline{q}_{K2} + \underline{\underline{\phi}}_{KB1}^{-1}\,\underline{X}_{B} \tag{8}$$

and

$$\underline{q}_K = \begin{Bmatrix} \underline{q}_{K1} \\ \underline{q}_{K2} \end{Bmatrix} = \begin{bmatrix} -\underline{\underline{\phi}}_{KB1}^{-1} \underline{\underline{\phi}}_{KB2} & \underline{\underline{\phi}}_{KB1}^{-1} \\ \underline{\underline{I}} & \underline{\underline{0}} \end{bmatrix} \begin{Bmatrix} \underline{q}_{K2} \\ \underline{X}_B \end{Bmatrix}$$

or

$$\underline{q} = \underline{\underline{T}}_2 \begin{Bmatrix} \underline{q}_{K2} \\ \underline{X}_B \end{Bmatrix} \tag{9}$$

the generalized coordinates $\underline{q}_{K2}$ are called reduced modal coordinates since they are associated with modified modes as seen in the above equation. Combining transform Eqs. (6) and (9), a net transformation from physical coordinates $\underline{X}$ to free interface normal mode superelement coordinates can be written as

$$\underline{\underline{T}} = \underline{\underline{\phi}}_K \cdot \underline{\underline{T}}_2 = \begin{bmatrix} (-\underline{\underline{\phi}}_{K01} \underline{\underline{\phi}}_{KB1}^{-1} \underline{\underline{\phi}}_{KB2} + \underline{\underline{\phi}}_{K02}) & \underline{\underline{\phi}}_{K01} \underline{\underline{\phi}}_{KB1}^{-1} \\ \underline{\underline{0}} & \underline{\underline{I}} \end{bmatrix} \tag{10}$$

where $\underline{\underline{\phi}}_{K01}$ and $\underline{\underline{\phi}}_{K02}$ are partitions of $\underline{\underline{\phi}}_{K0}$ created using the partitioning information generated in Eq. (8). Analogous to the Ritz transformation for constrained interface modal models, and the transformation of Eq. (4) for residual flexibility model modal, the left and right partitions of the transformation of Eq. (10) may be interpreted as expressing modal matrices of modified normal modes and boundary constraint modes, respectively. The component equations of motion in superelement coordinates ($\underline{q}_2$, $\underline{X}_B$) can be obtained using the transform of Eq. (11) as

$$\underline{\underline{T}}^T \begin{bmatrix} \underline{\underline{M}}_{00} & \underline{\underline{M}}_{0B} \\ \underline{\underline{M}}_{B0} & \underline{\underline{M}}_{BB} \end{bmatrix} \underline{\underline{T}} \begin{Bmatrix} \ddot{\underline{q}}_{K2} \\ \ddot{\underline{X}}_B \end{Bmatrix} + \underline{\underline{T}}^T \begin{bmatrix} \underline{\underline{K}}_{00} & \underline{\underline{K}}_{0B} \\ \underline{\underline{K}}_{B0} & \underline{\underline{K}}_{BB} \end{bmatrix} \underline{\underline{T}} \begin{Bmatrix} \underline{q}_{K2} \\ \underline{X}_B \end{Bmatrix} = \underline{\underline{T}}^T \underline{f} \tag{12}$$

using physical coordinate component model, or

$$T_2^{\,T} \begin{bmatrix} \underline{\underline{M}}_{q11} & \underline{\underline{M}}_{q12} \\ \underline{\underline{M}}_{q21} & \underline{\underline{M}}_{q22} \end{bmatrix} \underline{\underline{T}}_2 \begin{Bmatrix} \ddot{\underline{q}}_{K2} \\ \ddot{\underline{X}}_B \end{Bmatrix} + \underline{\underline{T}}_2^T \begin{bmatrix} \underline{\underline{K}}_{q11} & \underline{\underline{K}}_{q12} \\ \underline{\underline{K}}_{q21} & \underline{\underline{K}}_{q22} \end{bmatrix} \underline{\underline{T}}_2 \begin{Bmatrix} \underline{q}_{K2} \\ \underline{X}_B \end{Bmatrix} = \underline{\underline{T}}_2^T \underline{\underline{\phi}}_K^T \underline{f} \tag{12}$$

from modal coordinate model. In the above the generalized mass and stiffness matrices are used to include the case where the columns of the modal matrix in Eq. (12) are not orthogonal. (Mass and stiffness loaded component modes are nonorthogonal with respect to the original unloaded mass and stiffness matrices, for example.). Further, it is to be emphasized again that the reduced modal representation given in Eqs. (9) through (12) is obtainable from any given free interface "modal" representation provided the partitioning conditions leading to Eq. (8) can be met. The columns of modal matrix in Eq. (5) need not be normal modes. The only necessary condition is that they be linearly independent, adequate in number, and be from a complete set, i.e., a linear combination of them should be capable of representing the deformation shapes of the component undergoing motion within the compound of the built-up system.

SUPERELEMENT SYNTHESIS

The reduction procedures described in the preceding section yield component models with a common characteristic: the boundary degrees of freedom are explicitly retained and the internal degrees of freedom are transformed to reduced generalized coordinates. The equations of motion of a superelement component can be expressed as

$$\begin{bmatrix} \underline{\underline{m}}_{qq}^{(\alpha)} & \underline{\underline{m}}_{qB}^{(\alpha)} \\ \text{sym} & \underline{\underline{m}}_{BB}^{(\alpha)} \end{bmatrix} \begin{Bmatrix} \ddot{\underline{q}}^{(\alpha)} \\ \ddot{\underline{X}}_B^{(\alpha)} \end{Bmatrix} + \begin{bmatrix} \underline{\underline{C}}_{qq}^{(\alpha)} & \underline{\underline{C}}_{qB}^{(\alpha)} \\ \text{sym} & \underline{\underline{C}}_{BB}^{(\alpha)} \end{bmatrix} \begin{Bmatrix} \dot{\underline{q}}^{(\alpha)} \\ \dot{\underline{X}}_B^{(\alpha)} \end{Bmatrix} + \begin{bmatrix} \underline{\underline{K}}_{qq}^{(\alpha)} & \underline{\underline{K}}_{qB}^{(\alpha)} \\ \text{sym} & \underline{\underline{K}}_{BB}^{(\alpha)} \end{bmatrix} \begin{Bmatrix} \underline{q}^{(\alpha)} \\ \underline{X}_B^{(\alpha)} \end{Bmatrix} = \begin{Bmatrix} 0 \\ \underline{f}_B^{(\alpha)} \end{Bmatrix} \tag{13}$$

or

$$\underline{\underline{M}}^{(\alpha)} \ddot{\underline{p}}^{(\alpha)} + \underline{\underline{C}}^{(\alpha)} \dot{\underline{p}}^{(\alpha)} + \underline{\underline{K}}^{(\alpha)} \underline{p} = \underline{F}^{(\alpha)}$$

Each component substructure of a given built-up system can be expressed in the above format. Analogous to the matrix equation of a displacement based finite element model, the generalized coordinates may be interpreted as internal DOFs. Assuming that the global structure as partitioned into N components, the assembled global matrices $\underline{\underline{M}}$, $\underline{\underline{C}}$, $\underline{\underline{K}}$ relating the global displacement vector $\underline{X}$ and the global force vector $\underline{F}$ can now be synthesized by the direct stiffness approach from the component matrices of Eq. (13). The direct stiffness approach postulates equal nodal displacement and nodal equilibrium of forces at the connection interfaces of adjacent components, thus

$$\underline{X}_B^{(\alpha)} = \beta_B^{(\alpha)} \underline{X}_B \text{ , and } \underline{f}_B = \sum_{N=1}^{N} \beta_B^{(\alpha)} \underline{f}_B^{(\alpha)} \tag{14}$$

No compatibility conditions exist among the reduced generalized coordinates $\underline{q}^{(\alpha)}$ and remain intact. The coupled system coordinate vector thus becomes

$$\underline{X}^T = \{\underline{q}^{(1)^T}, \underline{q}^{(2)^T}, \ldots, \underline{q}^{(N)^T} \quad \underline{X}_B^T\} \tag{15}$$

so that the global mass, damping, and stiffness matrices are given by

$$\underline{\underline{M}} = \sum_{\alpha=1}^{N} \underline{\underline{\beta}}^{(\alpha)^T} \underline{\underline{M}}^{(\alpha)} \beta^{(\alpha)} \quad , \underline{\underline{C}} = \sum_{\alpha=1}^{N} \underline{\underline{\beta}}^{(\alpha)^T} \underline{\underline{C}}^{(\alpha)} \beta^{(\alpha)} \text{ , } \underline{\underline{K}} = \sum_{\alpha=1}^{N} \underline{\underline{\beta}}^{(\alpha)^T} \underline{\underline{K}}^{(\alpha)} \beta^{(\alpha)} \tag{16}$$

where $\underline{\underline{\beta}}^{(\alpha)}$ is the portion of the component to global transformation matrix $\underline{\underline{\beta}}$ corresponding to the α^{th} component.

$$\underline{Y} = \underline{\underline{\beta}}\, \underline{X} \text{ , } \quad \beta^{(\alpha)} = \left[\begin{array}{c:c:c:c:c:c:c:c:c} \cdot & \cdot & \underline{\underline{I}}^{(\alpha)} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hdashline \cdot & \cdot & \underline{\underline{0}} & \cdot & \cdot & \cdot & \cdot & \cdot & \underline{\underline{\beta}}^{(\alpha)} \end{array}\right] \tag{17}$$

$\underline{Y}$ being the vector of uncoupled superelement coordinates.

$$\underline{Y}^T = \{\underline{q}^{(1)^T} \; \underline{X}_B^{(1)^T} \quad \underline{q}^{(2)^T} \; \underline{X}_B^{(2)^T} \; \ldots \; \underline{q}^{(N)^T} \underline{X}_B^{(N)^T}\} \tag{18}$$

and $\underline{\underline{\beta}}_B^{(\alpha)}$ representing the compatibility conditions for the degrees of freedom at the boundaries of the component α.

The built-up system mass, damping, and stiffness matrices are of the following form.

$$\underline{\underline{A}} = \begin{bmatrix} \underline{\underline{a}}_{qq}^{(1)} & \underline{\underline{0}} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{a}}_{qB}^{(1)} \\ & \underline{\underline{a}}_{qq}^{(2)} & \underline{\underline{0}} & \cdots & \underline{\underline{0}} & \underline{\underline{a}}_{qB}^{(2)} \\ & & & & \vdots & \vdots \\ & & & \ddots & \underline{\underline{a}}_{qq}^{(N)} & \underline{\underline{a}}_{qB}^{(N)} \\ & & \text{sym.} & & & \underline{\underline{a}}_{BB} \end{bmatrix} \tag{19}$$

where the coefficients $a_{ij}^{(\alpha)}$ are the mass, damping, or stiffness coefficients matrices given in Eq. (14), and $\underline{\underline{A}}$ is the corresponding global matrix. The diagonal submatrices $\underline{\underline{a}}_{qq}^{(\alpha)}$ in Eq. (19) correspond to the reduced generalized coordinates of the components and are uncoupled from other components.

$$\underline{\underline{a}}_{BB} = \sum_{\alpha=1}^{N} \underline{\underline{\beta}}_B^{(\alpha)^T} \; \underline{\underline{a}}_{BB}^{(\alpha)} \; \underline{\underline{\beta}}_B^{(\alpha)}$$

The submatrices are symmetric and in general fully populated. The system governing equations can be expressed as

$$\underline{\underline{M}} \, \ddot{\underline{X}} + \underline{\underline{C}} \, \dot{\underline{X}} + \underline{\underline{K}} \, \underline{X} = \underline{f} \tag{20}$$

which itself is in a superelement form since the system basis coordinate $\underline{X}$ contains the boundary degrees of freedom explicitly. This is particularly useful from the standpoint of coupling the system of Eq. (20) to a higher level superelement. The recovery of component displacements from the system displacement vector $\underline{X}$ is simply a back substitution and transformation process.

NUMERICAL EXAMPLE AND CONCLUSIONS

The above formulation is implmented as a pre- and post-processor to a general matrix manipulation computer program. The following presents a sample synthesis problem involving free interface and residual flexibility modal models solved using the developed software to demonstrate its working. In Figure 1 the component A represents a lumped parameter model of an aircraft, while that labeled B represents a model of a store to be attached at the tip of the wing. The types of dynamic models employed for each component are as follows. For Aircraft: (1) free interface normal modes followed by a transformation to superelement coordinates, and (2) Free interface normal modes plus a residual flexibility attachment mode followed by a transformation to superelement coordinates. And for the Store: (1) free interface normal modes followed by superelement coordinate transformation, and (2) physical coordinate model. For comparison purposes the problem is also solved using existing MacNeal Method and Rubin Method of synthesis. The results of the synthesis are shown in Table 1. The exact results are obtained by solving the eigenproblem of the built-up system without partitioning. The superelement synthesis using complete mode sets of the components leads to exact system synthesis as expected. Superelement method using severely truncated component mode set along with residual

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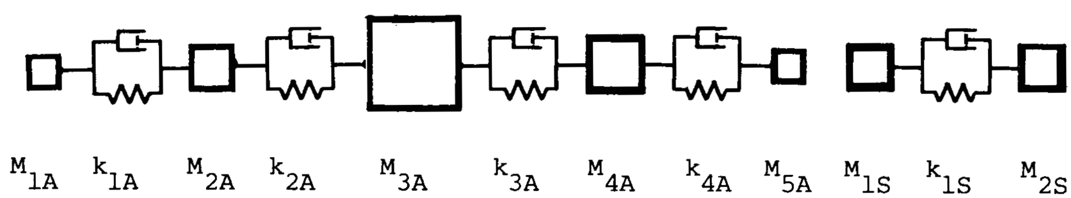
flexibility attachment modes predicts system frequency with an accuracy better than that of the MacNeal method and equal to that of the Rubin's second order method.

It must be noted that in this study different types of component dynamics models are synthesized by virtue of the superelement formulation, and that the accuracy of system synthesis is entirely governed by the type of component dynamic models and not by the coupling procedure.

TABLE 1 DIRECT AND SYNTHESIZED SYSTEM NATURAL FREQUENCIES (RADS./SEC)

MODE NO.	EXACT FREQ.	SUPERELEMENT SYNTHESIS A	SUPERELEMENT SYNTHESIS B	MACNEAL METHOD (B)	RUBIN METHOD (B)
1	0.	0.	0.	0.	0.
2	52.417	52.417	52.418	52.419	52.418
3	69.348	69.348	69.829	69.886	69.829
4	72.818	72.818	291.697	504.67	291.697
5	418.029	418.03			
6	775.678	775.678			

A: (1 Rigid body + 4 elastic) modes of the aircraft
B: (1 Rigid body + 1 elastic + 1 residual flexibility) mode of the aircraft



$m_{1A} = m_{5A} = 0.1$, $m_{2A} = 36$, $m_{3A} = 100$, $m_{4A} = 400$
$k_{1A} = 60{,}000$, $k_{2A} = 120{,}000$, $k_{3A} = 100{,}000$, $k_{4A} = 50{,}000$
$m_{1S} = 0.2$, $m_{2S} = 0.4$, $k_{1S} = 2{,}000$

Fig. 1 Lumped Parameter Aircraft-Store System

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