RESEARCH ON OUTPUT FEEDBACK CONTROL - Final Report

February, 1988

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SUMMARY

This report presents a summary of the main results obtained during the course of this research effort. The term "output feedback" is used to denote a controller design approach which does not rely on an observer to estimate the states of the system. Thus the order of the controller is fixed, and can even be ze'o order, which amounts to constant gain output feedback.

The emphasis of this research has been on optimal output feedback. That is, a fixed order controller is designed based on minimizing a suitably chosen quadratic performance index. A number of problem areas that arise in this context have been addressed. These include developing suitable methods for selecting an index of performance, both time domain and frequency domain methods for achieving robustness of the closed loop system, developing canonical forms to achieve a minimal parameterization for the controller, two time scale design formulations for ill-conditioned systems, and the development of convergent numerical algorithms for solving the output feedback problem.

Portions of this research were accomplished while the auther was with Drexel University in the Department of Mechancal Engineering and Mechanics.

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SECTION 1
INTRODUCTION

Constrained optimal output feedback, introduced in the early 70's, has received limited attention, despite its intuitive appeal. On the positive side, the designer can specify the complexity of the feedback structure by specifying the order of the compensator. Also, because actuator states are not estimated and fed back, the actuator dynamics are not sped up, which is a common problem in Linear Quadratic Gaussian (LQG) methods. This issue is of special concern in active control systems for vibration suppression. On the negative side is: (1) lack of robustness characterization in the design process, (2) over-parameterization in the compensator representation, and (3) uncertainty in how to penalize the compensator states. The second problem is a direct consequence of the fact that the compensator definition lacks a predefined structure, which invariably results in difficulties with convergence to an optimal solution.

Unlike the algebraic Riccati equation that arises in LQG design, the necessary conditions that result from the optimal output feedback problem are not conducive to analysis in the frequency domain. They also require iterative methods of solution. Thus the development of numerically convergent algorithms is of paramount importance. Moreover, since the solution process is more difficult, it is also more vulnerable to numerical ill-conditioning that may be present due to widely separated modes in the dynamics. Thus we have extended the results for two-time scale design of feedback control systems to the case of output feedback, both for constant gain design and for fixed-order dynamic compensation.

Section 2 of this report presents the formulation and summarizes the results on two-time scale design for constant gain output feedback. In addition, two convergent algorithms are presented for computing the optimal feedback gains subject to a set of constraints on the feedback gain matrix. Section 3 takes up the subject of fixed-order dynamic compensation. Section 4 addresses the issue of robustness, including both time domain and frequency domain results based on loop transfer recovery. The conclusions of this research effort are presented in Section 5. Throughout this report we have intentionally neglected to reference other related research, since adequate referencing is provided by the papers listed in the reference section.

SECTION 2
CONSTANT GAIN OUTPUT FEEDBACK

In this section, the optimal output feedback problem is formulated for a class of problems which includes the standard LQ case. The issues related to two-time scale design are presented and convergent sequential algorithms for solving the necessary conditions are described.

2.1 Problem Formulation and Necessary Conditions

We consider systems of the form

$$\dot{x} = Ax + Bu \qquad x(0) = x_0 \tag{2.1}$$

where $x \varepsilon R^n$ and $u \varepsilon R^m$, with output

$$y = Cx \tag{2.2}$$

where $y \varepsilon R^p$. The control has the form

$$u = -Gy \tag{2.3}$$

The gain, G, is to be chosen to minimize

$$J = \int_0^\infty x^T Q x + u^T R u \, dt + \gamma(G) \tag{2.4}$$

where $Q = \Gamma^T \Gamma$, such that the pair (Γ, A) is detectable, and $R > 0$. Additionally, in order to avoid singularity in the necessary conditions for the optimization problem, we must have

$$\rho(C) = p \tag{2.5}$$

In (2.4), $\gamma(G)$ is any scalar function having a continuous gradient in G, and for which J is bounded below, for all G which render the closed-loop dynamics (2.1-2.3) asymptotically stable.

It is well-known that the integral portion of J satisfies the relation

$$\int_0^\infty x^TQx + u^TRu \, dt = tr\{Kx_ox_o^T\} \tag{2.6}$$

where $K > 0$ is the unique solution of

$$S(G,K) = A_c^TK + KA_c + Q + C^TG^TRGC = 0 \tag{2.7}$$

$$A_c = A - BGC \tag{2.8}$$

and A_c is asymptotically stable. It is customary to relieve (2.6) of its dependence on x_o by assuming that it is a random variable, and modifying the problem statement to that of minimizing $E_{xo}\{J\}$. This amounts to replacing $x_ox_o^T$ in (2.6) by X_o, where $X_o = E\{x_ox_o^T\}$

From (2.6-2.8) we have an equivalent static optimization problem, in which the Lagrangian

$$\mathcal{L}(G,K,L) = tr\{KX_o\} + \gamma(G) + tr\{S(G,K)L^T\} \tag{2.9}$$

is minimized with respect to G, K, and L, where L is a matrix of Lagrange multipliers. If the system (2.1-2.3) can be stabilized by output feedback, the first order necessary conditions for optimality are:

$$\partial\mathcal{L}/\partial G = 0 \qquad \partial\mathcal{L}/\partial K = 0 \qquad \partial\mathcal{L}/\partial L = 0 \tag{2.10}$$

Defining the gradient of $\gamma(G)$

$$\partial\gamma(G)/\partial G = \gamma_G(G) \tag{2.11}$$

the expansion of (2.10) is

$$RGCLC^T - B^TKLC^T + \frac{1}{2}\gamma_G(G) = 0 \tag{2.12}$$

$$A_cL + LA_c^T + X_o = 0 \tag{2.13}$$

$$S(G,K) = 0 \tag{2.14}$$

From (2.12), the optimal value of G will satisfy

$$G^* = R^{-1}[B^T KLC^T - \gamma_G(G)]\ (CLC^T)^{-1} \tag{2.15}$$

where $(CLC^T)^{-1}$ exists because of (2.5) and the fact that $L > 0$ in (2.13), for a suitably chosen X_o.

2.2 Two-Time Scale Design

Consider the system

$$\dot{x}_1 = A_{11}x_1 + A_{12}x_2 + B_1 u, \quad x_1(0) = x_{10}, \quad x_1 \varepsilon R^{n_1} \tag{2.16}$$

$$\varepsilon\dot{x}_2 = A_{21}x_1 + A_{22}x_2 + B_2 u, \quad x_2(0) = x_{20}, \quad x_2 \varepsilon R^{n_2} \tag{2.17}$$

where $0 < \varepsilon \ll 1$, with output

$$y = C_1 x_1 + C_2 x_2 \qquad y \varepsilon R^p \tag{2.18}$$

The feedback law is

$$u = -Gy \qquad u \varepsilon R^m \tag{2.19}$$

If A_{22} is invertible, a reduced order approximation of (2.16-2.18) can be obtained by setting $\varepsilon = 0$ in (2.17):

$$\dot{\xi} = A_o + B_o u \qquad \xi \varepsilon R^{n_1} \tag{2.20}$$

$$\bar{y} = C_o \xi + D_o u \tag{2.21}$$

where

$$A_o = A_{11} - A_{12}A_{22}^{-1}A_{21} \qquad B_o = B_1 - A_{12}A_{22}^{-1}B_2$$
$$C_o = C_1 - C_2A_{22}^{-1}A_{21} \qquad D_o = -C_2A_{22}^{-1}B_2 \tag{2.22}$$

Substituting (2.19) in (2.16,2.17) and setting $\varepsilon = 0$, the reduced feedback control is expressed as

$$\bar{u} = -G^oC_o\xi \tag{2.23}$$

$$G^o = (I + GD_o)^{-1}G \tag{2.24}$$

which necessitates the assumption

$$\rho(I + GD_o) = m \tag{2.25}$$

The inverse of (2.24) is

$$G = G^o(I - D_oG^o)^{-1} \tag{2.26}$$

References [1-5] contain the main results and applications for the above formulation. In particular, it is shown that the output feedback problem does not naturally decompose into separate slow and fast designs. Instead, G^o and G must stabilize the separate systems $\{A_o-B_oG^oC_o\}$ and $\{A_{22}-B_2GC_2\}$ while satisfying the hard constraint in (2.24). Design methods based on "gain spillover suppression" are described in [1-3], where [3] represents the most complete set of results. In these papers, separate performance indices are set up for the slow and fast problems. An alternative design approach based on minimizing a single index of performance, which is more in the spirit of singular perturbation design of optimal linear regulators, is given in [4]. Here we show that the optimal state and control time histories can be approximated to $O(\varepsilon)$ uniformly over time, and the performance index is optimized to $O(\varepsilon^2)$. More complete details concerning these formulations and results are contained in [6].

2.3 Numerical Algorithms

As described above, the necessary conditions for optimality consist of a coupled set of algebraic equations. Two numerical algorithms were developed for the solution of these equations. The algorithms and their associated convergence proofs are given in [7,8]. In the first algorithm, constraints on G are treated indirectly by introducing a penalty term in the performance index as in (2.4). In the second, it is shown that when the constraints are linear, a direct approach may be taken. In this approach, either the constraints are satisfied after a finite number of iterations, or a norm measure of the constraint error can be made arbitrarily small as the number of iterations increases. Linear constraints on G play an important role in eigenvalue/eigenvector assignment and in modal insensitivity design of output feedback controllers to be discussed in Section 4.1.

SECTION 3
FIXED-ORDER DYNAMIC COMPENSATION

The extension of constant gain output feedback to the case of fixed-order dynamic compensation is conceptually straightforward. The compensator dynamics are defined in the form:

$$\dot{z} = -Pz - Ny \qquad z \varepsilon R^{nc} \tag{3.1}$$

$$u = -Hz - Gy \tag{3.2}$$

and adjoined to the plant dynamics. The problem is then reformulated as a constant gain output feedback problem. The structure of the new output feedback gain matrix is

$$\hat{G} = \begin{bmatrix} G & H \\ N & P \end{bmatrix} \tag{3.3}$$

Solution of this new problem yields the matrices needed to define the compensator dynamics. The main difficulty inherent in this approach is that the compensator is overparametized, which invariably leads to convergence problems. In addition, the compensator structure permits direct feedback of the output to the input, which is not desirable from the points of view of sensor noise reduction and robustness. We could invoke the constraint that $G = 0$, but it would be more desirable to avoid this constraint in the beginning by a proper choice of problem formulation. Finally, it is not clear how the compensator states should be penalized in the performance index. If the compensator states are not penalized properly, this normally leads to solutions where the compensator is not coupled to the plant dynamics (either $H = 0$ or $N = 0$).

In [9], it is shown that for a multivariable system described by:

$$\dot{x}_s = A_s x_s + B_s u \qquad x_s \varepsilon R^n \tag{3.4}$$

$$y = C_s x_s \qquad y \varepsilon R^p \qquad (3.5)$$

a fixed-order compensator without direct feedthrough of the output can be formulated in observer canonical form as:

$$u = -\overset{\circ}{H} z \qquad u \varepsilon R^m \qquad (3.6)$$

$$\dot{z} = \overset{\circ}{P} z + u_c \qquad z \varepsilon R^{nc} \qquad (3.7)$$

$$u_c = P_z u - Ny \qquad u_c \varepsilon R^{nc} \qquad (3.8)$$

where

$$\overset{\circ}{H} = \text{block diag}\{[0 \ldots 0 \; 1]_{1 x \nu_i} \; i=1,\ldots,m\} \qquad (3.9)$$

$$\overset{\circ}{P} = \text{block diag } [\overset{\circ}{P}_1,\ldots,\overset{\circ}{P}_m] \qquad (3.10)$$

$$\overset{\circ}{P}_i = \begin{bmatrix} 0 & 0 & \ldots & 0 & 0 \\ 1 & 0 & \ldots & 0 & 0 \\ 0 & 1 & \ldots & 0 & 0 \\ 0 & 0 & \ldots & 1 & 0 \end{bmatrix}_{\nu_i x \; \nu_i} \qquad (3.11)$$

In (3.8), N and P_z are free parameter matrices with dimensions (n_c x p) and (n_c x m), respectively. The dimensions of $\overset{\circ}{H}$ and $\overset{\circ}{P}$ are defined by the observability indices of the compensator, which are chosen to satisfy:

$$\text{i)} \sum_{i=1}^{m} \nu_i = n_c \qquad \text{ii)} \; \nu_i \leq \nu_{i+1}$$

The augmented system matrices:

$$A = \begin{bmatrix} A_s & -B_s\overset{\circ}{H} \\ 0 & \overset{\circ}{P} \end{bmatrix} \qquad B = \begin{bmatrix} 0 \\ I_{nc} \end{bmatrix} \qquad (3.12)$$

$$C = \begin{bmatrix} C_s & 0 \\ 0 & H_\circ \end{bmatrix} \qquad G = [\, N \quad P_z \,] \tag{3.13}$$

define an optimal output feedback problem, with the quadratic performance index:

$$J = E_{x_o} \{ \int_o^\infty [x^t Q x + u_c^t R u_c] \, dt \} \tag{3.14}$$

where the augmented state vector is

$$x^t = [x_s^t \; z^t] \tag{3.15}$$

The control u_c in (3.7) is defined as

$$u_c = -G\, C\, x \tag{3.16}$$

and is used only in designing the compensator parameters, which are packed in the columns of G. The main advantage to this formulation lies in the fact that the problem has been converted to one of constant gain output feedback, and the number of free parameters is the minimum needed to represent a strictly proper (but otherwise arbitrary) transfer function matrix. The necessary conditions for optimality are those given in (2.12-2.14).

This approach can also be easily extended to include frequency shaped cost functionals. In particular, it is shown in [10] that, because of the output feedback formulation, it is not necessary to realize the frequency shaping dynamics as a part of the compensator. The realization is only needed in the problem formulation, and leads to a unique method of selecting the weighting matrix (Q) in (3.14) for penalizing plant and compensator states. The extension of this work to the design of fixed-order dynamic compensators for two-time scale systems is given in [11]. Again, the slow and fast subsystem design problems are coupled. However, the use of canonical forms for defining the compensator dynamics leads to a unique matrix fraction description for the corresponding transfer functions. This leads to a simple procedure for constraining the solution so that the designs decouple, similar to the concept of gain spillover suppression that was used in the constant gain output feedback formulation. The resulting compensators can also be digitally implemented using sample rates appropriate for the dynamics involved in each time scale. These results are shown to be useful in rapid pointing of flexible structures, and in designing tight attitude control systems for aircraft flight control where structural modes (or rotor modes in the case of a helicopter) limit controller bandwidth. More detailed results on the controller and observer canonical compensator formulations can be found in [12].

SECTION 4
ROBUSTNESS IN OUTPUT FEEDBACK DESIGN

Perhaps the greatest criticism of optimal output feedback design methods is their lack of robustness characterization. Two approaches to robust design were examined in this research effort. The first is based on the concept of modal insensitivity design, and can be considered as a time domain approach. The second attempts to recover the loop transmission properties of full state feedback, and can be considered a frequency domain approach. However, the entire formulation is cast in the time domain, similar to the loop transfer recovery approach now popular in Linear Quadratic Gaussian (LQG) design.

4.1 Modal Insensitivity Design

One approach to expressing system sensitivity is in terms of eigenvalue sensitivity to plant parameter variations. This concept is particularly useful in the case of flight control problems where control system requirements are often stated in terms of closed loop eigenvalues. The robust design objective is to synthesize a feedback controller so the specifications are met for the nominal system, and sensitivity of the important closed loop eigenvalues is in some sense minimized. However, it is well-known that the response of a linear system depends on both the eigenvalues and eigenvectors (mode shapes), and this has led to the notion of modal insensitivity. Modal insensitivity implies that the eigenvalues are insensitive, while the associated eigenvectors have variations only in magnitude and not in direction. The design objective then is to assign selected closed loop eigenvalues and achieve modal insensitivity of these selected modes.

In [13,14], it is shown that the requirement for modal insensitivity can be written in the form of a linear constraint on the output feedback gain matrix. Since in most circumstances, the constraint does not completely determine the gain matrix, this allows an optimal output feedback formulation in which selected eigenvalues and eigenvectors are assigned, and in the final design the selected modes are insensitive to a class of variations in the plant parameters. This concept is easily extended to the case of fixed-order dynamic compensation. The use of a dynamic compensator does not increase the dimension of the modal insensitivity subspace; however, it does give greater flexibility in the design (increases the dimension of the free parameter subspace).

In most situations, it is not required that the orientation of the entire eigenvector be insensitive to plant parameter variations. Normally, only certain elements are required to be zero in order to achieve modal decoupling. Thus, a less stringent requirement is that modal decoupling is preserved in the presence of plant parameter variations. Reference [15] extends the concept of modal insensitivity to that of modal decoupling insensitivity. It is shown that the subspace for modal decoupling insensitivity is greater than that for modal insensitivity. Once again, the requirement for modal decoupling insensitivity can be written in terms of a linear constraint on the output feedback gain matrix. More complete details on these problem formulations can be found in [16].

4.2 Approximate Loop Transfer Recovery

Linear Quadratic Regulator (LQR) synthesis methods have guaranteed stability margins. Unfortunately, this requires full-state feedback. It has been shown that the loop transfer properties of an LQR design for nonminimum phase plants can be recovered via an asymptotic design method. This method relies on a cheap control formulation with a subset of the compensator dynamics becoming infinitely fast. It is often stated that the order of the compensator can later be reduced by discarding the fast modes; however, it is not clear how this can be accomplished without introducing direct feedthrough of the measured variables. It is generally good practice to avoid having direct feedthrough of sensor outputs to improve robustness and reduce the effect of sensor noise at high frequency. Aside from robustness issues, the order of the resulting compensator when designed for large order systems may prove unwarranted.

A major objection to optimal output feedback design is that there are no guarantees on stability margins, and there are few guidelines for penalizing plant states and compensator states to improve either performance or robustness. One major contribution in this research is to present a formulation in which the objective of the fixed-order compensator design is to approximate the loop characteristics of a full-state design. Thus, much like the full-order compensator design case, a two-step design is implied -- full-state feedback followed by approximate loop transfer recovery.

Full-state feedback design is often used as a first step in designing an output feedback controller for multivariable systems. A variety of methods exist such as LQR theory, pole placement, eigenvalue/eigenvector assignment, model following control, decoupling control design, etc. The most popular method is LQR design. It is well-known that this approach also yields guaranteed gain and phase margins when measured at the plant input.

The objective in observer-based controller design is to estimate the plant states, and to use the estimated states in place of the actual states. this results in a higher order system where closed-loop eigenvalues and eigenvectors of the full-state design are preserved, and the compensator merely adds its own dynamics to the response. When the compensator is designed, based on loop transfer recovery, it is also possible to recover the robustness properties of the full-state design. This amounts to suitably choosing the weighting matrices in a dual LQR formulation for the observer design. Both full-state and observer designs are decoupled.

In fixed-order compensator design, the notion of state estimation is not present. However, it should be recognized that, so long as the loop transfer properties of a full-state design can be recovered to a sufficient degree of accuracy, then the closed-loop eigenvalues should contain a set of eigenvalues and eigenvectors that approximate those of the full-state design. More importantly, the multivariable gain and phase margin properties should also be approximated. With this in mind, let the return signal in the case of full-state design be

$$u^* = -K^* x_s \tag{4.1}$$

where K^* is the gain corresponding a LQR design. Referring to (3.6), the return signal in the case of fixed-order compensator design is $-H^o z$. Thus, the objective in designing the compensator should be to minimize

$$y_1 = K^* x_s - H^o z \tag{4.2}$$

for a suitably chosen input and for zero initial conditions. Here we select the input waveforms as impulses with magnitudes uniformly distributed on the unit sphere. This naturally leads to selecting the following index of performance:

$$J = E_{xo} \{\int_0^\infty [y_1{}^t y_1 + \rho u_c{}^t u_c]\, dt\} \tag{4.3}$$

Substituting for y_1 from (4.2), and rewriting (4.3) in the form of (3.14), leads to the following expressions for the weighting matrices:

$$Q = \begin{bmatrix} K^{*t}K^* & -K^{*t}H^o \\ -H^{ot}K^* & H^{ot}H^o \end{bmatrix} \qquad R = \rho I_m \tag{4.4}$$

Note that, for zero initial conditions, the effect of the impulses at the system input is to create an initial condition, whose variance matrix is given by

$$X_o = \begin{bmatrix} B_s B_s{}^t & 0 \\ 0 & 0 \end{bmatrix} \tag{4.5}$$

This is used in the necessary condition (2.13) for the distribution on initial conditions. Thus, in this design approach, in addition to approximating loop transfer properties of a full-state design, the state and compensator weighting, and the initial state distribution matrix are all well-defined. Note that, unlike the design of a full-order observer, the design of a fixed-order controller depends on the gain matrix from the full-state design step. Moreover, this gain matrix is not implemented as a part of the final controller. Reference [17] presents the details on this design approach to robustness, and includes several interesting applications.

SECTION 5
CONCLUSIONS

This research has addressed a variety of issues related to the optimal output feedback design problem. Two minimal compensator parameter representations have been derived, and efficient algorithms for solving the optimal output feedback problem were obtained and proven to be convergent. This work has also extended the known results for two-time scale (singular perturbation) analysis and design of full-state and observer based controllers to the case of output feedback. Both the constant gain output feedback problem and the fixed-order compensator design problem have been addressed in this context. Finally, both time domain and frequency domain robustness formulations have been developed.

The main conclusion of this work is that most, if not all, of the objections to design by optimal output feedback have been addressed and resolved. Perhaps the most useful approach is that described for approximate loop transfer recovery. This work combines most of the desirable results of this research: robustness in the design process, canonical compensator representation, unique definition for the state and compensator weightings, and choice for the distribution on initial states.

Several problems still remain that should be addressed in future research. These include further improvements in numerical methods for solving the optimal output feedback problem, analysis of the limitations of the approximate loop transfer recovery process, and extensions of these ideas to the H_∞ problem.

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