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NONLINEAR INPUT-OUTPUT SYSTEMS

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I. INTRODUCTION

Suppose we have a nonlinear control system

(1) $$\dot{x} = f(x) + \sum_{i=1}^{m} u_i g_i(x),$$

when $f, g_1, \ldots, g_m$ are C^∞ vector fields on $\mathbb{R}^n$, $f(0)=0$, and $u_1, u_2, \ldots, u_m$ are controls. Necessary and sufficient conditions that this system be locally (near the origin) feedback equivalent to a controllable linear system having designated Kronecker indices are known [1], [2].

We take a nonlinear system

(2) $$\dot{x} = f(x)$$
$$y = h(x) = (h_1(x), h_2(x), \ldots, h_p(x)),$$

with f a C^∞ vector field on $\mathbb{R}^n$, $f(0)=0$, and $h_1, h_2, \ldots, h_p$ C^∞ functions on $\mathbb{R}^n$, $h(0)=0$. For the single output case ($p=1$) necessary and sufficient conditions that system (2) be locally state space equivalent to an observable linear system with output injection, appear in the literature [3],[4]. A

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recent paper of Krener and Respondek [5] has also solved this equivalence (adding output space coordinate changes) problem for a multi output system. Moreover, through a constructive algorithm, they have given necessary and sufficient conditions for there to exist changes of coordinates taking the nonlinear system

$$(3)\qquad \begin{aligned} \dot{x} &= f(x,u) \\ y &= h(x) \end{aligned}$$

to the system

$$(4)\qquad \begin{aligned} \dot{\xi} &= A\xi + \gamma(\psi,u) \\ \psi &= C\xi, \end{aligned}$$

where (C,A) is an observable pair.

An obvious problem is to find necessary and sufficient conditions for (local) feedback equivalence of

$$(5)\qquad \begin{aligned} \dot{x} &= f(x) + \sum_{i=1}^{m} u_i g_i(x) \\ y &= h(x) = (h_1(x), h_2(x), \ldots, h_p(x)) \end{aligned}$$

and

$$(6)\qquad \begin{aligned} \dot{\xi} &= A\xi + Bv \\ \psi &= C\xi, \end{aligned}$$

where (A,B) is a controllable pair on $\mathbb{R}^n$. Here B is an n x m matrix and $v = (v_1, v_2, \ldots, v_m)$ gives the new controls. In the application of the nonlinear to linear feedback equivalence theory to the automatic flight control of the UH-1H aircraft [6], if position in 3 space and rotation about the runway axis are taken as outputs, then the nonlinear system can be transformed to a controllable linear system with linear outputs. Hence, although our problem seems theoretical by nature, it is also extremely practical.

Taking the viewpoint of kernels of Volterra series, Isidori [7] has developed an algorithm to find feedback and input space coordinate changes so that the input dependent part of the response of the system (5) becomes linear. If system (5) has nonlinear zeroes [8], then this algorithm provides a linear input-output system whose controllable state space realization has dimension less than n. The techniques of this paper often allow us to find a controllable and observable n-dimensional system.

We present necessary and sufficient conditions that system (5) be feedback equivalent to a controllable linear system (6) with linear output. Because of the restricted length of this paper, we consider only the single input (m=1) and single output (p=1) case and omit the proof. However, all results generalize to multi-input, multi-output systems.

It is possible to remove certain nonlinearities of the outputs by feedback as well as nonlinearities in the dynamical state equation. That is, after the dynamical equation has been moved to linear form, it can happen that state space coordinate changes, input space coordinates changes, and feedback produce linear outputs without disturbing the linear form of the dynamical equation. This is totally unsuspected in view of the dual theory of [5] and proves to be a surprise to the authors. An algorithm can be developed to accomplish the equivalence.

Section 2 of this paper contains definitions and examples. In section 3 we present necessary and sufficient conditions for moving from the nonlinear system (5) to the controllable linear system (6) with linear outputs. Again, for the purpose of simplicity we take as

$$(5)\ \begin{cases} \dot{x} = f(x) + ug(x) \\ y = h(x) \end{cases} \quad \text{and as} \quad (6)\ \begin{cases} \dot{\xi} = A\xi + bv \\ y = C\xi \end{cases} .$$

II. DEFINITIONS AND EXAMPLES

Some basic concepts from differential geometry are required for clarity. For C^∞ vector fields f and g on $\mathbb{R}^n$, the Lie bracket is

$$[f,g] = \frac{\partial g}{\partial x} f - \frac{\partial f}{\partial x} g .$$

Successive Lie brackets are $(ad^0 f,g) = g$, $(ad^1 f,g) = [f,g], \ldots, (ad^k f,g) = [f,(ad^{k-1} f,g)]$.

Given a C^∞ function h and a C^∞ vector field f the Lie derivative of h with respect to f is

$$L_f h = \langle dh, f \rangle .$$

Also we have $L_f^0 h = h$, $L_f^1 h = L_f h, \ldots, L_f^k h = L_f L_f^{k-1} h$.

Here $\langle \cdot,\cdot \rangle$ denotes the duality between one form and vector fields. Given another C^∞ vector field g, Lie derivatives such as $L_g L_f h$, $L_g L_f^2 h$, etc. can be taken.

A review of the transformation theory in [2] is quite appropriate. We take system (1) (which is $\dot{x} = f + ug$ in the single input case) and make the following assumptions. Near the origin in $\mathbb{R}^n$

(a) the set $\bar{C} = \left\{ g, [f,g], \ldots, (ad^{n-1} f,g) \right\}$ is linearly independent, and

(b) the set $\hat{C} = \left\{ g, [f,g], \ldots, (ad^{n-2} f,g) \right\}$ is involutive.

Under these assumptions the system $\dot{x} = f + ug$ is feedback equivalent (i.e. state space coordinate changes, state feedback, and input space coordinate changes) to the appropriate linear system.

(7) $$\dot{\xi} = A\xi + bv,$$

where the pair (A,b) is controllable [2]. Moreover, a method for constructing such a transformation is developed. In fact, a close examination of this technique in [2] reveals that it can be separated into two parts:

(i) state space coordinate changes

(ii) renaming the controls and applying feedback.

By just applying (i) we can move system (5) into the following form ($w=(w_1,w_2,\ldots,w_n)$)

(8) $$\begin{bmatrix} \dot{w}_1 \\ \dot{w}_2 \\ \vdots \\ \dot{w}_n \end{bmatrix} = \begin{bmatrix} w_2 \\ w_3 \\ \vdots \\ \alpha(w) \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ \vdots \\ \beta(w) \end{bmatrix}$$

$$y = \gamma(w)$$

with $\beta(w)$ nonvanishing. Setting $v = \alpha(w)+u\beta(w)$ we have system (7) in Brunovsky [9] canonical system.

Definition 2.1. System (8) is called a controllable canonical form of the nonlinear system (5) satisfying conditions (a) and (b).

We emphasize that system (8) was achieved from (5) by coordinate changes on $\mathbb{R}^n$ only. The input space coordinate changes and feedback were later used to move to Brunovsky form. The controllable canonical form is not unique simply because the transformation of (5) to (8) is not unique. As we shall see, it is exactly this nonuniqueness that makes this paper interesting. Moreover, the word canonical is used in the above definition with the understanding that it is modulo all state space coordinate changes taking a given nonlinear system (5) to form (8).

To see if system (5) is feedback equivalent to the controllable linear system (6) with linear output, the first step is to check conditions (a) and (b) for the state equation in (5). If these hold we perform the transformation to move the first equation in (5) to the first equation in (6). One would guess that we have a linear output in (6) if and only if it appears after the transformation (this occurs in the dual problem for observability in [5]). However, as the following example indicates, this is not the case. Remarkably, coordinate changes and feedback can be used to "absorb" certain nonlinearities of the output without disturbing the

linearity in the dynamics of the state equation.

Example 2.2. On $\mathbb{R}^3$ we take the nonlinear system

(9) $$\begin{bmatrix} \dot{x}_1 \\ \dot{x}_2 \\ \dot{x}_3 \end{bmatrix} = \begin{bmatrix} x_2 \\ x_3 \\ x_1^3 \end{bmatrix} + u \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} = f(x) + ug(x)$$

$$y = x_1 + x_1^2 = h(x).$$

The state equation is already in one of the controllable canonical forms. By letting $u = -x_1^3 + v$, we can clear the nonlinear term in the state equation, but we do not have a linear output at this point. Suppose we let $\xi_1 = x_1 + x_1^2 = h$

$\xi_2 = L_f h$

$\xi_3 = L_f^2 h.$

In these coordinates we find (9) becomes

(10) $$\begin{bmatrix} \dot{\xi}_1 \\ \dot{\xi}_2 \\ \dot{\xi}_3 \end{bmatrix} = \begin{bmatrix} \xi_2 \\ \xi_3 \\ q(\xi_1,\xi_2,\xi_3) \end{bmatrix} + r(\xi_1)u \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}$$

$$y = \xi_1,$$

where $q(\xi_1,\xi_2,\xi_3)$ and $r(\xi_1)$ are functions of the variables indicated and $r(\xi_1)$ does not vanish at the origin in $\mathbb{R}^3$. Letting $v = r(\xi_1)u + q(\xi_1,\xi_2,\xi_3)$ (to preserve the linear structure in (10) the choice $v = r(\xi_1)u$ + the nonlinear part of $q(\xi_1,\xi_2,\xi_3)$ is correct) we obtain a linear system with linear output. Hence state coordinate changes, feedback, and input space coordinate changes can be used to eliminate certain nonlinearities of the output equation.

From the point of view of the theory of [2], $x_1 + x_1^2$ is as good as x_1 for moving the state equation to a linear one. Hence the nonuniqueness of the transformation allows us flexibility in trying to linearize the output.

III. MAIN RESULTS

We state necessary and sufficient conditions that the nonlinear system (5) be locally feedback equivalent to the controllable linear system (6) having linear output. All statements hold for a neighborhood of the origin in $\mathbb{R}^n$. Moreover, conditions (*a*) and (b) and set $\hat{C}$ are defined as in section 2.

Theorem 3.1. The nonlinear system (5) is feedback equivalent to the controllable linear system (6) if and only if the following two conditions are satisfied:

(i) Assumptions (a) and (b) hold.

(ii) There exists a C^∞ function T_1 and constants $c_j, 1 \le j \le n$, so that

(11) $\{\langle dT_1, (ad^{n-1} f, g)\rangle\} \neq 0,$

(12) $\{\langle dT_1, \tau\rangle\} = 0$ for all vector fields τ in $\hat{C}$, and the output h of (5) is of the form

$$h = c_1 T_1 + c_2 L_f T_1 + \ldots + c_n L_f^{n-1} T_1.$$

As we mentioned earlier, the authors have derived an algorithm to move from (5) to (6) if these systems are feedback equivalent.

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