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# On the Global/Local Time Incrementing for Viscoplastic Analysis

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## 1. INTRODUCTION

There has been a great deal of research work in modeling the inelastic deformation behavior of materials at the elevated temperature environment. It appears that a unified approach in the form of viscoplastic relations has been most popular for prediction of material responses. In this context, a number of viscoplastic material models have been published in the literature [e.g. 1-6]. The unified approach differs from the conventional creep and plasticity theory in that both the creep and plastic deformations, or alternately termed inelastic deformations, are treated as time-dependent quantities. Based on the experimental and theoretical studies performed by various investigators [3,4,7-9], it is known that viscoplastic constitutive relations, in principle, are capable of predicting material responses at high temperatures such as cyclic plasticity, rate sensitivity, long-term creep deformations, strain-hardening or softening, etc. The degree of success of a constitutive relationship varies depending on the extent of parameters considered in or mathematical sophistication of a specific model.

Although most of viscoplastic models give improved material response predictions over the classical approach, the associated constitutive differential equations have stiff regimes which present numerical difficulties in time-dependent structural analysis. The numerical difficulty is indeed an important concern when the viscoplastic relations are applied to large scale finite element structural analysis.

In finite element analysis for viscoplastic materials, two issues of primary concern in connection with the associated material nonlinearity: 1) solution convergence in solving the global (incremental) equilibrium equations, 2) integration of the constitutive rate equations at the local material points (or element integration points). Numerically, these two issues are inter-related. On the one hand, global equilibrium can not be achieved if the stresses calculated at local material points are grossly inaccurate. On the other hand, the constitutive relations and stresses are not representative to the material if the strains computed from the nodal displacements are in error.

In view of the above discussion, we have therefore investigated a combined global/local incrementing scheme for the finite element analysis of viscoplastic materials.

## 2. GLOBAL INCREMENTING

Due to the material nonlinearity, a viscoplastic problem is effectively formulated by an incremental approach, in which the finite element equilibrium equations can be linearized. In order to solve these equations successfully, the analyst must be able to specify "appropriate" load steps. If the loading increments are too large, the solution may not converge, or it is far from being accurate. Alternatively, if the loading increments are very small, the computation cost will become prohibitively high. Therefore, it is desirable to implement an automatic incrementing procedure in which the selection of (global) load steps can be made by the program, rather than the analyst.

Use of automatic load stepping for solving nonlinear problems is not new and most of the applications were concentrated at time independent problems [10-11]. Herein, we adopted similar concepts for the solution of time dependent viscoplastic problems. The procedure involves two major steps: 1) initiation of incremental solution, and 2) selection of subsequent load increments. Each of the two steps is briefly outlined below.

(1) Initiation of incremental solution - the solution begins with a specified load vector, i.e.

$$\underset{\sim}{R}_1 = \alpha \cdot \underset{\sim}{R} \tag{1}$$

where $\alpha$ = a load factor, < 1.

$\underset{\sim}{R}$ = a reference load vector.

With the above load vector, solution will proceed with equilibrium iterations. When the number of iterations reaches four and the solution has not yet converged, an estimate is made to project the number of iterations required according to

$$n = i + \ell n(DTOL/d_i)/(\ell n\ d_i - \ell n\ d_{i-1}) \tag{2}$$

where $i$ = number of iterations already performed.

DTOL = iteration tolerance for displacements.

$d_i$ = ratio between the incremental displacement norm of the i-th iteration and total displacement norm.

$= | \Delta \underset{\sim}{U}_i | / | \underset{\sim}{U}_i |$

If n is greater than a maximum number of iteration cycles allowed, then a new load vector is set to be $\underset{\sim}{R}_1^{NEW} = \tau \underset{\sim}{R}_1$, $\tau<1$.

(2) Subsequent load increments - The load increments, subsequent to the first step, are determined on the basis of a constant arc length method [12-14]. In this method, let the current load vector be

$$\underset{\sim}{R}_{i+1} = \lambda_{i+1} \underset{\sim}{R} \tag{3}$$

where $\lambda_{i+1}$ = a load parameter corresponding to the (i+1)-th iteration

$$= \lambda_i + d\,\lambda_{i+1} \tag{4}$$

and d $\lambda_{i+1}$ is calculated from a quadratic algebraic equations [13].

## 3. LOCAL INCREMENTING

Once the global load increment is determined from the method outlined in the above, a sub-incrementing method is incorporated at the material point level to integrate the rate constitutive equation. For the purpose of discussion, the viscoplastic constitutive equations are written in the form

$$\dot{\underset{\sim}{y}} = \underset{\sim}{f}\,(\underset{\sim}{y},\ t) \tag{5}$$

where $\underset{\sim}{y}$ represents the vector of stress, inelastic strain and state variables, and $\underset{\sim}{f}$ is a vector of nonlinear functions. To integrate the preceding equations, we have developed an automatic procedure based on the variable-step Runge-Kutta (R-K) method. In this method, the global time increment $\Delta t$ is divided into a number of sub-increments, i.e. $h = \Delta t/n$. Corresponding to h, the vector $\underset{\sim}{y}$ for iteration (i+1) is evaluated by the 4th order and 5th order R-K formulas, respectively, i.e. $\underset{\sim}{y}_{i+1}^{(4)}$ and $\underset{\sim}{y}_{i+1}^{(5)}$. Then an error can be estimated from

$$E_{st} = ||\ \underset{\sim}{y}_{i+1}^{(5)} - \underset{\sim}{y}_{i+1}^{(4)}\ ||/h \tag{6}$$

The calculated error must be within the following tolerance

$$E_{st} < \varepsilon \, || \underset{\sim}{y_i} || / \Delta t \tag{7}$$

If the above condition is violated, a revised sub-increment h' is then obtained from

$$h' = \tau \, h$$

$$\tau = \left\{ \frac{\varepsilon \, || \underset{\sim}{y_i} ||}{E_{st} \cdot \Delta t} \right\}^{\frac{1}{4}} \tag{8}$$

The foregoing procedure is repeated until the criterion in Eq. (7) is satisfied.

## 4. NUMERICAL EXAMPLE

Several problems have been analyzed using the procedure outlined in the preceding sections. Presented herein is a thick walled cylinder subjected to an internal pressure, varying linearly from 0 to 14.6 psi for $t \in [0, 40 \text{sec.}]$ The cylinder material is assumed to be 2-1/4 Cr-Mo common steel at 811° k and Robinson's viscoplastic model is adopted. For finite element analysis, five 4-noded axisymmetric elements are used.

The analysis was performed by using four different combinations of numerical algorithms:

1) Automatic global and local incrementing (G + L)
2) Automatic global incrementing (G) with constant local steps $h = \Delta t/n_s$, $n_s = 2, 4$, and 8
3) Automatic local sub-incrementing (L) with constant global steps, N = 5, 10, 16, and 20.
4) Constant global and local steps.

Summarized in Table 1 are the algorithm details, CPU time on IBM-3033 computer, and radial displacement at the outer surface of the cylinder. It is seen that with the automatic global/local incrementing algorithm, the lowest CPU time was consumed. Convergence difficulty was experienced if only the automatic global or local incrementing scheme was optioned unless the number of solution steps or the number of sub-increments is significantly increased. Shown in Figure 1 is the load vs. radial displacement at the outer surface of the cylinder calculated from two different algorithms, i.e. automatic global/local incrementing and constant global stepping with automatic local incrementing. Both algorithms gave almost identical results.

## 5. CONCLUSION

Presented in this paper is a global/local time incrementing scheme for viscoplastic analysis of structures. The scheme is very efficient and useful for conducting large scale nonlinear finite element analysis involving viscoplastic materials.

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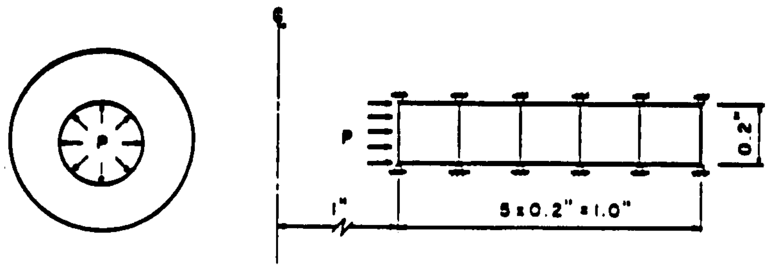
Table 1 A COMPARISON OF DIFFERENT SOLUTION ALGORITHMS FOR A THICK-WALLED CYLINDER

| Case No. | Option | Global Steps | Local Substeps | CPU Unit | $U_o 10^{-2}$ Inch |
|---|---|---|---|---|---|
| 1 | G + L | 16 | -- | 11 | 0.1851 |
| 2A | G | 16 | 8 | 100 | 0.1851 |
| 2B | G | 16 | 4 | 57 | 0.1851 |
| 2C | G | -- | 2 | solution diverged, | (note 5) |
| 3A | L | 20 | -- | 35 | 0.1851 |
| 3B | L | 16 | -- | 36 | 0.1856 |
| 3C | L | 10 | -- | 45 | 0.1879 |
| 3D | L | 5 | -- | solution diverged, | (note 6) |
| 4A | N | 20 | 4 | 37 | 0.1851 |
| 4B | N | 20 | 2 | solution diverged, | (note 5) |

Note:

1. G+L - both global and local automatic incrementing.
2. L - local automatic incrementing only.
3. G - global automatic incrementing only.
4. N - manual incrementing
5. In cases 2C and 4B, solution diverged at steps 6 and 3, respectively, because the values of material state variables are out of bound.
6. In case 3D, solution diverged at step 5 because out-of-balance load was greater than incremental load.
7. $U_o$ is the radial displacement at outer surface of the cylinder.

Geometry and Idealization :

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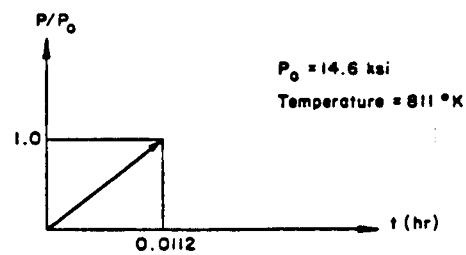


Material Model :

Robinson Viscoplastic Model

CR - MO $2\frac{1}{4}$ common steel

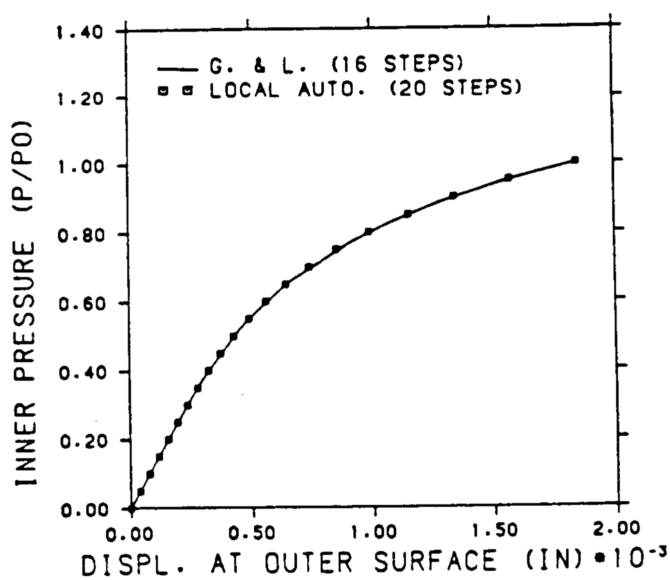


Figure 1. A Thick-Walled Cylinder
Under Internal Pressure